a point outside the integration region. However, a closer examination reveals that $(t_l + t_r)$ reaches its maximum at a point in which neither t_l nor t_r is stationary. It is therefore possible that $t_l - t_r$ undergoes sizable variations in the integration region in which $(t_l + t_r)$ stays close to its maximum value.

[7]These contributions were lumped in the central distribution in Ref. 2. We have considered them separately in this paper for the following reasons: (a) In Ref. 2 the distribution 2, e.g., was considered as the limit of 3, where $s_l' \to m_1^2$, and the correspondent extrapolation was performed in $\sigma_{M\pi}(s_l')$; this is certainly a dangerous procedure (the extreme case of the distribution 4B was computed in the old scheme extrapolating the $p\bar{p}$ cross section at a value of the c.m. energy equal to m_π). (b) In the reasonable assumption that the baryon exchange does not extend more than three steps from the end, we can suppress the need of using the $p\bar{p}$ annihilation cross section as an input. (c) In the particular problem under examination, the charge distribution plays a very important role. In the present case of an incoming π^-, assuming $I=1$ exchanges to be dominant, the leading π (position 1) will be a π^- an average of 50% of the time and a π^0 50% of the time; the pions emitted in the position 2 are also rather charge asymmetric (50% π^-, 25% π^0, 25% π^+). However, the particles emitted from the chain more than two steps away from the ends have to a large extent "lost memory" of the incoming one, and can be reasonably considered charge symmetric.

[8]We assume nucleon exchange to be dominant over Δ exchange on the basis of the experimentally large π^+p (as compared to the π^-p) backward peak and of the charge asymmetry in $p\bar{p}$ annihilation into $\pi^+\pi^-$ at low energy. For the meson trajectory we remark that an elementary π exchange would do just as well; what is really needed to get the asymmetry is a rapid decrease in the momentum transfers.

[9]The existence of the forward-backward asymmetry and its size do not depend critically on any of these assumptions.

[10]See also A. Ajduk, L. Michejda, and W. Wojcik, Acta Phys. Polon. A37, 285 (1970).

[11]Our distributions have a maximum close to $p_L=0$, and therefore a definition in terms of slopes at $p_L=0$ is meaningless.

[12]J. W. Elbert, A. R. Erwin, W. D. Walker, and J. W. Waters, Nucl. Phys. B19, 85 (1970).

[13]These features are more easily understood in terms of the rapidity distribution. The central particles 3 contribute to $d\sigma/dw$ an approximately constant central plateau, the length of which increases with ln$s$ whereas the height remains unchanged. The (asymmetric) end effects get more and more displaced in opposite directions with increasing s, and keep their shape unchanged. Therefore, the displacement required to go from the center-of-mass frame (which is the symmetry frame of the central plateau) to a frame in which the asymmetry of the central part compensates the asymmetry of the end effects is energy independent. On the contrary, to increase the number of particles at fixed s corresponds to increasing the height of the central contribution. Therefore, the larger the number of particles at a given s, the smaller the displacement in w (and correspondingly the β of the Lorentz transformation) required to compensate for the asymmetry of the end effects. The author is very grateful to James Bjorken for a conversation on this point.

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Some Quark-Parton-Model Inequalities For the Neutron and Proton Inelastic Structure Functions *

D. P. Majumdar

Physics Department, Syracuse University, Syracuse, New York 13210

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We summarize several inequalities for the neutron and proton inelastic structure functions $W_2(\nu, Q^2)$ that must be satisfied if the partons are identified with quarks. Some of these inequalities provide us with very useful constraint equations for the $P(N)$ function of the parton model. Due to lack of sufficient data, it is not certain at present if all the inequalities are satisfied. The importance of these inequalities lies in the fact that if any of them are confirmed not to satisfy the experimental data, the concept of individual quark-parton association has to be abandoned.

I. INTRODUCTION AND DATA

We discuss here a few inequalities for the inelastic neutron and proton structure functions that follow from the quark-parton concepts[1,2] of the inelastic lepton-nucleon scattering and the isodoublet character of the nucleons.

First let us summarize the experimental observations on the neutron data[3] that we shall require.

(a) $D/H - 1 \simeq W_{2n}/W_{2p}$ data are consistent (within errors) with a single function of ω. Further, $\nu W_{2n}/\nu W_{2p}$ starts from about 0.5 at $\omega \simeq 1.5$ and increases gradually towards $\sim$0.95 at $\omega \simeq 12$.

(b) $(2 - D/H)$ $(\nu W_{2p}) \simeq \nu W_{2p} - \nu W_{2n}$ is roughly consistent with a function of ω.

(c) Experimental data indicate that

$$\int_1^{20} F^p(\omega)\frac{d\omega}{\omega} = 0.72 \pm 0.05, \quad (1.1)$$

$$\int_1^{20} F^p(\omega)\frac{d\omega}{\omega^2} = 0.17 \pm 0.01, \quad (1.2)$$

$$\int_1^{12} F^{n,p}(\omega)\frac{d\omega}{\omega} = 0.45,\ 0.58 \pm \text{errors}, \quad (1.3)$$

$$\int_1^{12} F^{n,p}(\omega)\frac{d\omega}{\omega^2} = 0.10,\ 0.14 \pm \text{errors}. \quad (1.4)$$

Here $F(\omega) = \nu W_2(\nu, Q^2)$ in the scaling limit.

There are possibilities of large errors in the data, especially for the neutron data. It has been suggested from an analysis of the data that

$$\nu W_{2p} - \nu W_{2n} = 0.03(12/\omega)^\alpha \quad \text{for } \omega \geq 12, \quad \text{where } \alpha > 0. \quad (1.5)$$

Since both the neutron and proton data roughly obey scale invariance, the parton concept seems to be quite useful for the inelastic lepton-nucleon scattering. It is therefore worthwhile to investigate what can be learned from a comparison of the inelastic e-p and e-n data from such models. It is known that an understanding of the discrepancy between the e-p and e-n data will give us some insight into the nucleon structure relevant to the deep-inelastic phenomena. With this object in mind, we shall discuss here several inequalities which can test whether partons can be identified with quarks.

II. PARTON-MODEL FORMULAS

From the parton-model considerations, it can be derived[2] that the inelastic structure function $W_2(\nu, Q^2)$ of the lepton-nucleon scattering is given by

$$\nu W_2(\nu, Q^2) = \sum_N P(N)\left\langle \sum_1^N Q_i^2 \right\rangle x f_N(x) = F(x), \quad (2.1)$$

where $x = Q^2/2M\nu = 1/\omega$. Here $P(N)$ is the probability of finding N partons in the nucleon; $\langle \sum_1^N Q_i^2 \rangle$ is the average value of the sum of the squared charges of the partons in a configuration of N partons, and $f_N(x)$ is the probability density function for finding a parton with longitudinal fraction x of the nucleon's four-momentum. Note that

$$\sum_N P(N) = 1 \quad (2.2)$$

and

$$\int_0^1 f_N(x)dx = 1. \quad (2.3)$$

Further, if the momentum distribution of the N partons is symmetric, which we shall assume here, then

$$\int_0^1 x f_N(x)dx = \frac{1}{N}. \quad (2.4)$$

$\left\langle \sum_1^N Q_i^2 \right\rangle$ can be written as

$$\left\langle \sum_1^N Q_i^2 \right\rangle = \left\langle \sum_1^N [(I_i^3)^2 + \tfrac{1}{4}(Y_i)^2 + I_i^3 Y_i] \right\rangle, \quad (2.5)$$

where I_i^3 is the third component of isotopic spin and Y_i is the hypercharge of the ith parton. Now the fact that neutron and proton form an isodoublet implies that

$$\left\langle \sum_1^N (I_i^3)^2 \right\rangle_p = \left\langle \sum_1^N (I_i^3)^2 \right\rangle_n,$$
$$\left\langle \tfrac{1}{4}\sum_1^N Y_i^2 \right\rangle_p = \left\langle \tfrac{1}{4}\sum_1^N Y_i^2 \right\rangle_n, \quad (2.6)$$
$$\left\langle \sum_1^N I_i^3 Y_i \right\rangle_p = -\left\langle \sum_1^N I_i^3 Y_i \right\rangle_n.$$

We shall discuss here inequalities based on the assumption that partons are quarks. For this case we note that $(I^3)^2 + \frac{1}{4}Y^2$ is $\frac{5}{18}$ for each of $\mathcal{P}$, $\mathfrak{N}$, $\bar{\mathcal{P}}$, and $\bar{\mathfrak{N}}$ quarks, and it is $\frac{1}{9}$ for λ and $\bar{\lambda}$ quarks. Similarly, $I_3 Y$ is $\frac{1}{6}$ for $\mathcal{P}$ and $\bar{\mathcal{P}}$ quarks, $-\frac{1}{6}$ for $\mathfrak{N}$ and $\bar{\mathfrak{N}}$ quarks, and zero for λ and $\bar{\lambda}$ quarks.

III. INEQUALITIES

A. Inequalities for $\nu W_2(\nu, Q^2) = F(\omega)$

Two sum rules that follow immediately[2,4] from the parton-model formula are

$$\int_1^\infty \nu W_2 \frac{d\omega}{\omega^2} = \sum_N P(N)\frac{\left\langle \sum_1^N Q_i^2 \right\rangle}{N}$$
$$= \text{mean square charge per parton} \quad (3.1)$$

and

$$\int_1^\infty \nu W_2 \frac{d\omega}{\omega} = \sum_N P(N)\left\langle \sum_1^N Q_i^2 \right\rangle. \quad (3.2)$$

Several useful inequalities can be derived from these sum rules by exploiting the maximum and minimum values of Q_i^2 for the quarks. We observe that $\langle \sum_1^N Q_i^2 \rangle$ satisfies the following bounds:

$$Q_0^2 + \tfrac{1}{9}(N-3) \leq \left\langle \sum_1^N Q_i^2 \right\rangle \leq Q_0^2 + \tfrac{4}{9}(N-3), \tag{3.3}$$

where Q_0^2 is equal to 1 for proton and $\frac{2}{3}$ for neutron. Thus if partons are quarks, we must have

$$\tfrac{1}{9} + \tfrac{2}{3}\langle 1/N \rangle \leq \int_1^\infty F^p(\omega)\frac{d\omega}{\omega^2} \leq \tfrac{4}{9} - \tfrac{1}{3}\langle 1/N \rangle \tag{3.4}$$

for the proton, and

$$\tfrac{1}{9} + \tfrac{1}{3}\langle 1/N \rangle \leq \int_1^\infty F^n(\omega)\frac{d\omega}{\omega^2} \leq \tfrac{4}{9} - \tfrac{2}{3}\langle 1/N \rangle \tag{3.5}$$

for the neutron. The upper bounds correspond to assuming all the partons beyond the first three as $\mathcal{P}$ or $\overline{\mathcal{P}}$ quarks while the lower bounds correspond to assuming that there are no $\mathcal{P}$ and $\overline{\mathcal{P}}$ quarks in the cloud. We have made this oversimplification to obtain the bounds. However, they furnish us with some useful information. For example, these inequalities can be turned around to produce an inequality for $\langle 1/N \rangle$ and hence a constraint equation for $P(N)$. Taking the experimental data for the proton, we find that $\langle 1/N \rangle \leq 0.1$. Thus Eqs. (3.4) and (3.5) will put a severe constraint on the parameters of the $P(N)$ function. We shall illustrate this by the following example. Suppose $P(N) = \text{const}/N^2$; then, for the Bjorken-Paschos[2] model,

$$\begin{aligned} \langle 1/N \rangle &= 0.22 \quad \text{for } N_0 = 3 \\ &= 0.12 \quad \text{for } N_0 = 5 \\ &= 0.08 \quad \text{for } N_0 = 7\,, \end{aligned} \tag{3.6}$$

where N_0 is the minimum number of partons allowed in the model. Therefore only $N_0 = 7$ models should be allowed. However, we shall show later that large-N_0 ($N_0 > 5$) models may not satisfy an inequality Eq. (3.16) and thus these kinds of models should be rejected. In Fig. 1 we plot $\langle 1/N \rangle$ values for several $P(N)$ functions which display clearly the effect of these inequalities.

From the second sum rule we obtain an inequality,

$$\tfrac{2}{3} + \tfrac{1}{9}\langle N \rangle \leq \int_1^\infty F^p(\omega)\frac{d\omega}{\omega} \leq \tfrac{4}{9}\langle N \rangle - \tfrac{1}{3}\,. \tag{3.7}$$

A similar relation can be derived for the neutron. It says that if the integral diverges, then $\langle N \rangle$, the expectation value of N, must also diverge and vice versa. Also, since for quark-parton models there must have at least three quarks, we obtain

$$\begin{aligned} \int_1^\infty F(\omega)\frac{d\omega}{\omega} &\geq 1 \quad \text{for the proton} \\ &\geq \tfrac{2}{3} \quad \text{for the neutron.} \end{aligned} \tag{3.8}$$

The left-hand side evaluated for the proton up to $\omega = 20$ is only 0.72, but the trend of the data is not conclusive enough to make any definite statement

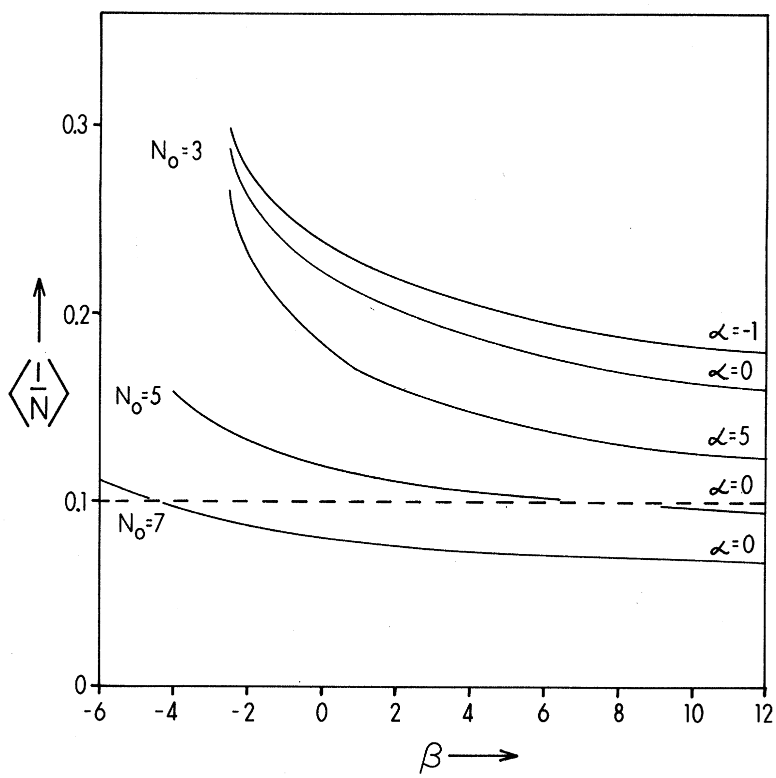

FIG. 1. $\langle 1/N \rangle$ values are plotted for some $P(N)$ functions given by $P(N) = \text{const}/[(N+\alpha)(N+\beta)]$, with α and β as parameters. N is given by N_0, N_0+2, $N_0+4, \ldots, \infty$. Note that only certain sets of values of N_0, α, and β are allowed to satisfy the requirement $\langle 1/N \rangle \leq 0.1$.

regarding these inequalities.

$\langle N\rangle$ should be directly related to the average multiplicity in hadron collisions. It is usually believed that the multiplicities will become infinite (~ln$s$) at infinite incident energies. However, Eq. (3.7) gives us an upper bound for $\langle N\rangle$. Since the parton concept of the hadrons is independent of lepton-hadron and hadron-hadron scattering, this bound for $\langle N\rangle$ will be very useful in settling the question whether the average multiplicity can be finite[5] even when the incident energy approaches infinity.

B. Inequalities for $F^p(\omega)+F^n(\omega)$

For the sum of the neutron and proton structure functions the $I_i^3 Y_i$ term cancels out, and we obtain

$$\nu W_{2p}+\nu W_{2n}=2\sum_N P(N)\left\langle\sum_1^N[(I_i^3)^2+\tfrac{1}{4}Y_i{}^2]\right\rangle x f_N(x). \quad (3.9)$$

Now utilizing the values of $(I_i^3)^2+\frac{1}{4}(Y_i)^2$ for the quarks and using a similar procedure to that of Eq. (3.3), we find that

$$\tfrac{2}{9}+\langle 1/N\rangle \leq \int_1^\infty(\nu W_{2p}+\nu W_{2n})\frac{d\omega}{\omega^2}\leq\tfrac{5}{9}. \quad (3.10)$$

If there are no strange quarks (λ's), this integral would be exactly $\frac{5}{9}$, and so this integral gives us a measure of the number of strange quarks present in the nucleon. Experimentally, the integral for ω up to 12 is ~0.24, satisfying the inequality quite well, and indicating only that there must be a large number of λ and $\bar{\lambda}$ quarks in a strict quark-parton picture of the nucleon.

Bjorken[6] derived a constant-q^2 inequality which can be presented as

$$\int_1^\infty(\nu W_{2p}+\nu W_{2n})\frac{d\omega}{\omega}\geq\tfrac{1}{2}\ \text{for constant } q^2. \quad (3.11)$$

For the quark-parton models, since one knows exactly what the constituents are, one can improve on this kind of inequality. In the parton picture,

$$\int_1^\infty(\nu W_{2p}+\nu W_{2n})\frac{d\omega}{\omega} =2\sum_N P(N)\left\langle\sum_1^N[(I_i^3)^2+\tfrac{1}{4}Y_i{}^2]\right\rangle. \quad (3.12)$$

Therefore,

$$1+\tfrac{2}{9}\langle N\rangle\leq\int_1^\infty(\nu W_{2p}+\nu W_{2n})\frac{d\omega}{\omega}\leq\tfrac{5}{9}\langle N\rangle. \quad (3.13)$$

This inequality as well as the inequality (3.7) will have important consequences for those $P(N)$ functions where $\langle N\rangle$ is finite. For a Poisson distribution given by $P(N)=\text{const}\,a^N/N!$, for example, $\langle N\rangle=a+N_0P(N_0)$, which is finite. Now, since there must be at least three quarks in a nucleon, we obtain the minimum requirement

$$\int_1^\infty(\nu W_{2p}+\nu W_{2n})\frac{d\omega}{\omega}\geq\tfrac{5}{3}. \quad (3.14)$$

This is a much stronger result than Bjorken's inequality, but, of course, this is less general. Experimentally, the integral is not known exactly, but the present indications are that it is about $1.25+\int_{20}^\infty\pm$ large errors.

If the three-quark configuration of the partons is rejected due to its main contributions to elastic and quasielastic scattering, then quark-parton models must start with five quarks. In that case,

$$\int_1^\infty(\nu W_{2p}+\nu W_{2n})\frac{d\omega}{\omega}\geq\tfrac{19}{9}. \quad (3.15)$$

From the trend of the data it seems that this inequality may not be satisfied. In that case strict quark-parton models should always start with three quarks. Equation (3.14), then, becomes very crucial and must be satisfied in order to keep the quark-parton identification. In other words, Eq. (3.13) enables us to write an inequality for the minimum number of partons N_0, namely,

$$3\leq N_0\leq\tfrac{9}{2}\left[\int_1^\infty(\nu W_{2p}+\nu W_{2n})\frac{d\omega}{\omega}-1\right]. \quad (3.16)$$

We stressed before[4] that the minimum number of partons plays an important role in the parton picture of the inelastic scattering. In our previous work we fixed N_0 to be 4 by looking at the data near $\omega\sim1$. This inequality will now give us more ideas in fixing N_0 for a quark-parton model.

C. Some Other Inequalities

The ratio of the neutron and proton structure functions[7] can be written as

$$\frac{\nu W_{2n}}{\nu W_{2p}}=1+\frac{2\sum P(N)\langle\sum_1^N I_i^3 Y_i\rangle_n x f_N(x)}{\sum P(N)\langle\sum_I^N Q_i{}^2\rangle_p x f_N(x)} \quad (3.17)$$

or

$$\tfrac{1}{4}+\tfrac{15}{16}\frac{\sum P(N)xf_N(x)}{\sum P(N)(N-\frac{3}{4})xf_N(x)}\leq\frac{\nu W_{2n}}{\nu W_{2p}} \leq 4-30\frac{\sum P(N)xf_N(x)}{\sum P(N)(N+6)xf_N(x)}. \quad (3.18)$$

Experimentally, $\nu W_{2n}/\nu W_{2p}$ is greater than 0.25, satisfying the lower bound, but unless we know

$p(N)$ and $f_N(x)$ explicitly this inequality cannot be verified. However, we can derive an integrated inequality from here. We note that

$$\frac{\nu W_{2p}}{\sum P(N)(N+6)xf_N(x)} \geq \frac{1}{9}. \qquad (3.19)$$

Therefore,

$$\tfrac{1}{4}\nu W_{2p} + \tfrac{5}{48}\sum_N P(N)xf_N(x) \leq \nu W_{2n} \leq 4\nu W_{2p} - \tfrac{10}{3}\sum_N P(N)xf_N(x). \qquad (3.20)$$

Integrating Eq. (3.20), we obtain our final inequality,[8]

$$\int_1^\infty (\nu W_{2p} - \tfrac{1}{4}\nu W_{2n})\frac{d\omega}{\omega} \geq \tfrac{5}{6}. \qquad (3.21)$$

In deriving[9] this inequality, we have used extensively the quark character of the parton, while for the sum and difference of the neutron and proton structure functions, the $I_i Y_i$ and $(I_i^3)^2 + \frac{1}{4}Y_i^2$ terms were, respectively, cancelled out.[10] From the experimental point of view, we can make an estimate of this integral as follows:

$$\int_1^\infty (\nu W_{2p} - \tfrac{1}{4}\nu W_{2n})\frac{d\omega}{\omega} = \int_1^{12} (\nu W_{2p} - \tfrac{1}{4}\nu W_{2n})\frac{d\omega}{\omega} + \tfrac{1}{4}\int_{12}^\infty (\nu W_{2p} - \nu W_{2n})\frac{d\omega}{\omega} + \tfrac{3}{4}\int_{12}^\infty \nu W_{2p}\frac{d\omega}{\omega} \simeq 0.6. \qquad (3.22)$$

So this sum rule is apparently not satisfied by the data, although we cannot be certain about it at present.

IV. CONCLUSION

In this paper we have discussed several inequalities for the inelastic neutron and proton structure functions that must be satisfied if the partons are identified as quarks. These inequalities should be valid for all quark-parton models. The basic ingredients in deriving these inequalities had been the I^3 and Y properties of the quarks and the isodoublet nature of the nucleons. The naive derivation of these inequalities leads us to feel that these are weak bounds and perhaps much stronger bounds can be derived. However, some of these inequalities are of considerable interest because they provide us information about the $P(N)$ function of the parton model and also because it is not certain at present whether these are all satisfied by the data. The beauty of these inequalities lies in the fact that if any of these inequalities is not satisfied, the individual quark-parton association picture must be rejected. At least these inequalities will serve, at present, as guidelines for numerical estimates of the structure functions.

Added note. Llewellyn Smith has kindly pointed out to the author some similar work by him[11] and by Gourdin.[12] The bound given in Eq. (3.8) can be found in Llewellyn Smith's paper, Eq. (16). He communicated to the author that he has recently been able to derive the upper bound in Eq. (3.10) in the gluon model and the lower bound in Eq. (3.10) in a field-theory model. Gourdin has independently derived the upper bound in Eq. (3.10). He mainly discusses the neutrino cross sections, and from the neutrino data (neglecting errors) he concludes that $\langle 1/N\rangle \geq 0.09$, whereas our conclusion from e-p data is that $\langle 1/N\rangle \leq 0.1$. Thus improved data on νp and ep scattering can critically test the quark-parton concept of the inelastic scattering.

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[7]Two inequalities can be derived for the difference of neutron and proton structure functions. We first note that

$$\nu W_2^p - \nu W_2^n = 2\sum_N P(N)\left\langle \sum_1^N I_i^3 Y_i \right\rangle_p xf_N(x). \qquad (a)$$

Then, using the same procedure as that of Eq. (3.3), we obtain

$$-\tfrac{1}{3}+\tfrac{4}{3}\langle 1/N\rangle \leq \int_1^\infty [F^p(\omega)-F^n(\omega)]\frac{d\omega}{\omega^2} \leq \tfrac{1}{3}-\tfrac{2}{3}\langle 1/N\rangle \qquad \text{(b)}$$

and

$$\tfrac{4}{3}-\tfrac{1}{3}\langle N\rangle \leq \int_1^\infty [F^p(\omega)-F^n(\omega)]\frac{d\omega}{\omega} \leq -\tfrac{2}{3}+\tfrac{1}{3}\langle N\rangle\,. \qquad \text{(c)}$$

The λ and $\bar{\lambda}$ quarks have I^3Y value zero and so these expressions give us an overestimation, thus making the bounds too weak.

[8]There are several other inequalities that can be derived from Eq. (3.20) but they do not give us new information.

[9]Note that Eq. (2.4) has not been used in Sec. III C. Only Eqs. (3.1), (3.4), (3.5), and (3.10) make use of the assumption that the momentum distribution of the N partons are symmetric.

[10]We have found that this relation can also be derived with the help of Eq. (3.7) and the following inequality:

$$\tfrac{1}{9}\langle N\rangle+\tfrac{1}{3} \leq \int_1^\infty F^n(\omega)\frac{d\omega}{\omega} \leq \tfrac{4}{9}\langle N\rangle-\tfrac{2}{3}\,.$$

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General Treatment of the Breaking of Chiral Symmetry and Scale Invariance in the $SU(3)$ σ Model*

J. Schechter and Y. Ueda
Physics Department, Syracuse University, Syracuse, New York 13210
(Received 22 January 1971)

We study the model of nine scalar and nine pseudoscalar fields interacting by means of the most general nonderivative chiral $SU(3)\times SU(3)$ invariant and any particular symmetry-breaking term. Two basic "generating" equations which express the complete content of chiral symmetry are derived. The masses and coupling constants of arbitrary order for this model are simply found by differentiating the "generating" equations an arbitrary number of times and using an equation which expresses the stability of the "ground" state. In this way, previous results on this model can be easily recaptured and a systematic framework for the investigation of different symmetry-breaking terms is provided. Numerical estimates are made for scalar-meson widths, $\pi\pi$ and πK scattering lengths, and $\eta'\to\eta 2\pi$ decays. The consequences of imposing scale invariance on the invariant part of the interaction are also investigated by writing down a scale invariance "generating equation." Finally, we discuss the relation between our approach and the method of using the divergences of currents and trace of the energy-momentum tensor.

I. INTRODUCTION

The subject of chiral $SU(3)\times SU(3)$ symmetry breaking[1] has recently been one of the most actively pursued branches of strong-interaction theory. There is great interest in this field not only because it searches for a way to estimate corrections to the interesting "current-algebra" results but also because it is hoped that the answer to the symmetry-breaking problem will elucidate some deep mysteries of elementary-particle structure.

Now, once we depart from the exact symmetry limit of any theory, a large number of alternatives usually present themselves. Therefore, in order not to get lost in a maze of complications, it is normally desirable to study a relatively simple model which contains (it is hoped) the key features of the problem. For the case of $SU(3)\times SU(3)$ breaking, the model which is generally taken as a prototype is the so-called "$SU(3)$ σ model"[2] which contains nine pseudoscalar and nine scalar fields transforming *linearly* under the chiral $SU(3)\times SU(3)$ group of transformations.

The advantage of the $SU(3)$ σ model over the quark-model approach to symmetry breaking (as exemplified by the recent work of Gell-Mann, Oakes, and Renner[3] and its descendants) is that everything is explicit in the case of the σ model so that results can be obtained relatively easily. Otherwise the structure of the two models, as we shall illustrate, is very similar. One example of the practical advantage of the σ model lies in the specification of the "ground state" or "vacuum state" of the system. In the σ-model approach the symmetry breaking of the "vacuum" is correlated to the choice of symmetry-breaking interaction by means of a "stability" or "extremum" equation. On the other hand, in the quark-model approach this physical condition is more difficult to enforce and often it is just assumed that the "vacuum" has a certain symmetry