Forward-Backward Asymmetry in πp Inelastic Reactions*

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It is shown that the recently observed asymmetry between forward π^+ and backward π^- in inelastic $\pi^- p$ interactions can be simply explained by kinematical considerations. The effect is a characteristic of the multiperipheral model, and a quantitative estimate is in excellent agreement with the data.

A recent bubble-chamber experiment[1] on inelastic $\pi^- p$ interactions at 25 GeV/c has provided interesting information on the spectra of the produced pions. The possibility of separating the different multiplicities (or at least the different topologies) has important consequences.

The measured longitudinal distribution of the negative pions is shown in Fig. 1. The sharp peak at $p_L = 0$ is partly due to the factor $(E_\pi)^{-1}$ in the invariant phase space [i.e., the more significant distribution in the rapidity variable $w = \tanh^{-1}(p_\pi / E_\pi)$ should look much smoother].

As expected, the distribution is very asymmetric, showing a clear trend of the negative particles to follow the direction of the incoming negative one. In order to eliminate the effect of the leading negative pions so as to compare the spectra of the particles produced by really inelastic mechanisms, the authors of Ref. 1 compare the spectrum of the backward π^- with the spectrum of the forward π^+. In Fig. 2(a) we see that the asymmetry persists (actually the decrease of the π^+ spectrum is comparable with the one of the π^- in the region $0 < p_L < 1$).

As a Lorentz transformation along the z direction tilts the distributions for positive and negative p_L in opposite directions, it is possible to find a frame in which the backward π^- and the forward π^+ look symmetric. The fact that in this frame the ratio of the momenta of the initial p and π^- happens to be close to $\frac{3}{2}$ is proposed in Ref. 1 as an argument in favor of a composite (quark) structure of the hadrons.

Here we would like to point out that the asymmetry in question is very easily understood as a kinematical effect within the general framework of the multiperipheral model and corresponds to the physically intuitive expectation that very few particles should be emitted backward *in the rest frame of either initial particle*.[2] After a Lorentz transformation to the c.m. system, the above statement corresponds to a limitation of the order of $(m_\pi / 2m_p) \times \sqrt{s}$ for negative values of p_L, whereas no corresponding limitation is expected for $p_L > 0$.

To be more quantitative let us consider the multiperipheral graph of Fig. 3(a). The general spirit of the discussion is the same as the multiperipheral model for inclusive distributions proposed some time ago,[3] to which we refer for details. The double differential cross section for particle 1 of Fig. 3(a) can be written

$$\frac{d\sigma}{dp_T^2 dp_L} = c_1 e^{2at_1} \left(\frac{s}{s'}\right)^{2\alpha_M(t_1)} \frac{\sigma_{Mp}(s')}{pE_\pi \sqrt{s}}, \tag{1}$$

where p is the c.m. momentum of the initial particles, s' is the missing mass squared, and $\sigma_{Ms}(s')$ is the total cross section for the "scattering" of the (off-mass-shell) meson M on protons.

The same form [Eq. (1)] holds for the distribution of particle 5; however, as we are interested in produced π only, the relevant exchange in this case is a baryon one.

Following the procedure of Ref. 2, the distribution of the central pions[4] (3 in Fig. 3) can be cast in the form

$$d\sigma = \frac{c}{p^2 s} \left(\frac{s_l}{s_l'}\right)^{2\alpha_M(t_l)} \left(\frac{s_r}{s_r'}\right)^{2\alpha_M(t_r)} e^{2\alpha(t_l + t_r)} \times \sigma_{M\pi}(s_r') \sigma_{Mp}(s_l') ds_l ds_r dt_l dt_r ds_l' ds_r'. \tag{2}$$

The integration over the two "masses s_l' and s_r' has to be done numerically, but the ingenious procedure of Chan, Kajantie, and Ranft[5] allows us to perform the t integrations analytically at fixed s_l and s_r. Unfortunately, we are interested in $d^2\sigma / dp_L dp_T^2$ which reads

$$\frac{d^2\sigma}{dp_L dp_T^2} = \frac{c}{E_\pi p \sqrt{s}} \left[\frac{s_l(p_L, p_T, t_l, t_r)}{s_l'}\right]^{2\alpha_M(t_l)} \left[\frac{s_r(p_L, p_t, t_l, t_r)}{s_r'}\right]^{2\alpha_M(t_r)} e^{2\alpha_M(t_l + t_r)} \sigma_{M\pi}(s_r') \sigma_{Mp}(s_l') dt_l dt_r ds_l' ds_r' \tag{3}$$

with

$$s_{l,r} = s'_{l,r} + 2(p_{a,b} \cdot p_\pi) + t_{r,l} - t_{l,r} .$$

In order to be able to use the procedure of Ref. 5, we approximate the value of s_l and s_r in the integration over t_l and t_r with their values at the point in which $t_l + t_r$ is maximum. The exponential behavior in $(t_l + t_r)$ makes us confident of the validity of this approximation.[6] The distribution of the particles emitted in the positions 2 and 4 in Fig. 3(a) is given by the same expression (3) in which one of the masses s'_l or s'_r is fixed at the value of the mass of the particle 1 or 5, and the correspondent subenergy s_l or s_r is scaled by the usual s_0.[7] For the distribution of particle 4 we must distinguish the case in which particle 5 is a baryon ($4M$) or a π ($4B$).

We are now ready to compute. Our general attitude was *not to attempt to fit* the experimental distribution, but only to insert reasonable values of the parameters and to see whether a reasonable answer (in particular a forward-backward asymmetry comparable with the experimental one) was predicted by the model. We have therefore *a priori* chosen[8] $a=2$, $a_M(t)=0.5+0.8t$, $a_B(t)=-0.38+8t$. The total meson-proton cross section $\sigma_{Mp}(s')$ has been parametrized with a Δ resonance at $s'<2$ GeV^2, and a smooth form $c(1+0.5/s')$ for $s'>2$ GeV^2, and correspondingly the total meson-pion cross section $\sigma_{M\pi}(s')$ has been described[9] by a vector-meson (ρ) resonance for $s'<1$ GeV^2, and the same form $\sigma_{M\pi}(s')=c(1+0.5/s')$ for $s'>1$ GeV^2.

The couplings have been fixed by normalizing the 1, 2, $4M$, $4B$, and 5 distributions to 18 mb, the total inelastic $\pi^- p$ cross section at 25 GeV/c. The distribution 3 has been normalized to $(\bar{n}-4) \times \sigma_{\pi^- p}^{\text{inelastic}} \simeq 54$ mb, where $\bar{n}$ is the average final multiplicity at 25 GeV. We have therefore no free parameter left. The distributions obtained are shown in Fig. 3(b). The expected rapid decrease of the π spectrum with negative p_L is clear. The way in which this effect is built in multiperipheralism is also clear; it is obvious from Fig. 3 that in order to minimize the momentum transfers all the final particles must be emitted in the direction of the incoming one in the rest frame of either initial particle.[10]

In order to compute the π^+ and π^- spectra from the distributions of Fig. 3(b), some assumption on the isospin structure of the dominant exchanges must be made. In Ref. 1 it is observed that the leading pions (i.e., the π emitted in position 1 in our model) appear to be on the average 50% negative and 50% neutral. This result seems to indicate a dominance of $I=1$ exchange, and we will make that assumption. The charge distribution for the particles emitted at position 2 can therefore be computed.[7] For the "central" pions 3, we assume uniform sharing between the possible charges. At the proton end, we have already committed ourselves to $I=\frac{1}{2}$ dominance. From the proton distributions[2] it seems likely that, in the framework of this model, baryon-exchange graphs account for about 50% of the total cross section. We assume

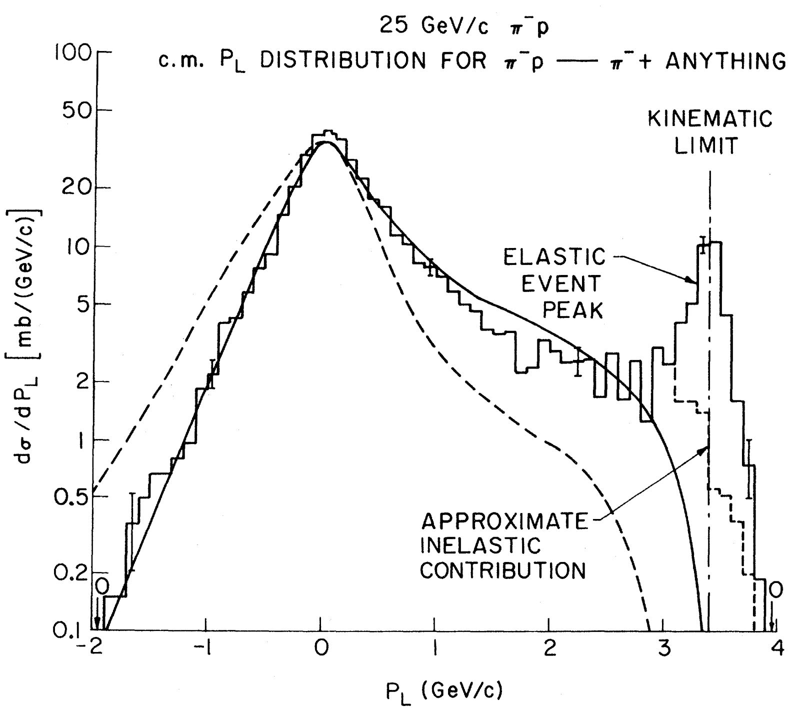

FIG. 1. The longitudinal distribution of the π^-. The solid and dashed lines represent the π^- and π^+ spectra calculated in our model.

the probability of the baryon's traveling n steps along the chain to be given by $(2)^{-n-1}$, and neglect this probability for $n \geq 3$. With these assumptions the distributions shown in Figs. 1 and 2(a) are obtained.

We would like to stress that all the assumptions made to obtain these distributions (the parametrization of $\sigma_{M\pi}$ or σ_{Mp}, and the quantum number structure of the relevant exchange) have been made *a priori*, on a plausibility basis, not *a posteriori* to fit the data. Actually it turns out that the size of the predicted asymmetry is quite independent of these details (e.g., flat σ_{Mp} and $\sigma_{M\pi}$ would do as well). The only parameter on which the asymmetry depends is a, which in turn cannot be changed from its assumed value by more than 10% without forcing an unrealistic p_T dependence (with $a=2$, $\langle p_T \rangle$ is 250 MeV at $p_L=0$ and rises to about 350 MeV at $|p_L|=1$, owing to the effect of the E_π^{-1} factor). Therefore, the multiperipheral scheme not only provides a dynamical mechanism that realizes the physically expected asymmetry, but also establishes a quantitatively successful relation between its size and the limitation in p_T.

Bubble-chamber experiments have the advantage of being able to separate the different charged multiplicities. It has been observed in Ref. 1 that the forward-backward asymmetry shows a clear trend to disappear at large multiplicities (an embarrassing feature in the quark framework).

We would like to show now that this trend is quite natural in the multiperipheral scheme. Noting that the experimented longitudinal distributions are well fitted by exponentials, at least for $-1 < p_L < 1$

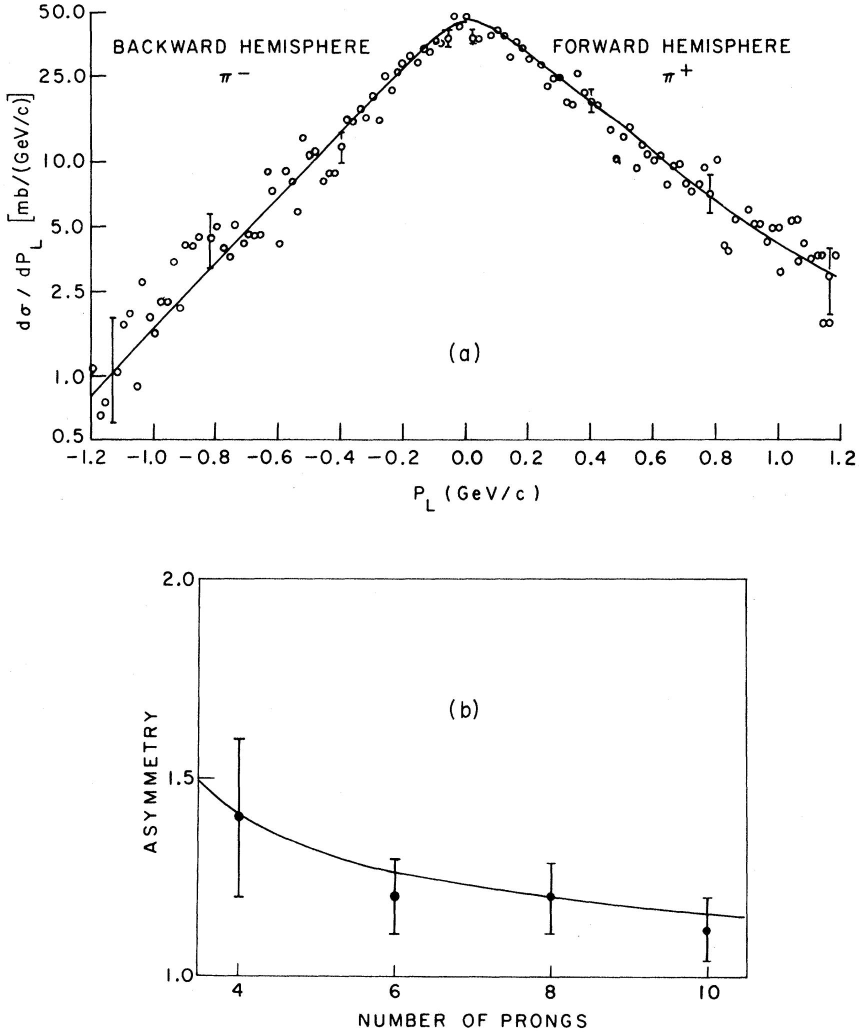

FIG. 2. (a) Forward π^+ and backward π^- distributions with our predictions. (b) Dependence of the asymmetry parameter on the number of prongs. The solid line is the multiperipheral prediction; the values of the asymmetry (and the errors) have been estimated by the author on the basis of the data kindly provided to him by J. W. Elbert.

[Fig. 2(a)], we define an asymmetry parameter[11]

$$A = \frac{\ln[\sigma^-(0)/\sigma^-(-1)]}{\ln[\sigma^+(0)/\sigma^+(+1)]},$$

where $\sigma^-(p_L)$ and $\sigma^+(p_L)$ are the differential cross sections for π^- and π^+.

In the framework of our model the distributions for the different topologies can be obtained by inserting in Eqs. (1) and (3) the values of the total cross section *with a definite number of prongs in the final state*, for which good data are available. To obtain the distribution of particle 3, all the possible ways of sharing the number of prongs between the left and the right blobs must be added. Preliminary calculation has shown that the shape of the distributions plotted in Fig. 3(b) does not depend very strongly on the number of prongs in the final state (the general trend, for instance for distribution 3, is to get sharper when the number of prongs is increased, but the effect is rather small). As a first approximation we can therefore assume that only the normalization of the "central" distribution 3 changes with the number of prongs. From the same experiment we learn[12] that in the region $4 < n_p < 10$ (where n_p is the number of prongs) the total multiplicity is well represented by $\bar{n} = 1.25 n_p + 1$. We also find that the value of $\sigma^{\pm}(0)$ is well approximated by the contribution of distribution 3 only, which, on the contrary, contributes very little to $\sigma^+(1)$ and $\sigma^-(-1)$. With these further approximations we obtain in our model the asymmetry as a function of the number of prongs

$$A(n_p) \simeq 1 + \frac{0.7}{1.4 + \ln(n_p - 2.4)},$$

which compares reasonably well with the estimated experimental one [Fig. 2(b)].

The predictions of the multiperipheral model for future higher-energy π-p inelastic experiments are the following:

(a) An exponential parametrization for $d\sigma/dp_L$ should become less and less acceptable, and a clear $1/E_\pi$ dependence should show up for small values of $x = 2p_L/\sqrt{s}$.

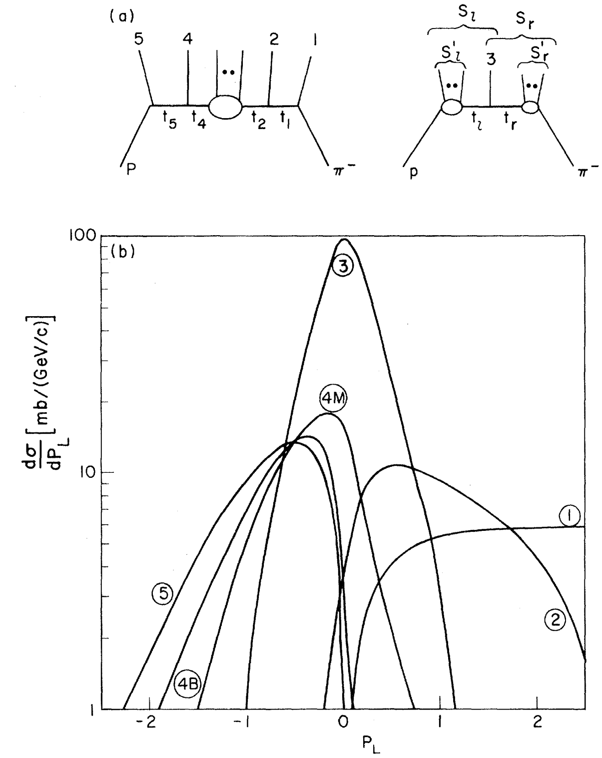

FIG. 3. (a) Multiperipheral diagram considered and (b) resulting longitudinal distributions.

(b) Correspondingly, thanks to the Lorentz invariance of d^3p/E, the shape of the central peak should depend less and less on the reference frame.

(c) The asymmetry as defined here in terms of ratios of $d\sigma/dp_L$ at $p_L = 0$ and at a given p_L (or also at a given x) should decrease to 1 as $1 + c/\ln s$.

(d) Nevertheless, the total number of forward π^+ will differ from the total number of backward π^- by an energy-independent constant δ in the center-of-mass frame, and a Lorentz transformation of s-independent parameter will be needed to transform to a frame in which $\delta = 0$.[13]

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[1]J. W. Elbert, A. R. Erwin, and W. D. Walker, Phys. Rev. D 3, 2042 (1971).

[2]The same argument has been previously proposed by C. N. Yang in *Proceedings of the Third International Conference on High-Energy Collisions, Stony Brook, 1969*, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1969).

[3]L. Caneschi and A. Pignotti, Phys. Rev. Letters 22, 1219 (1969).

[4]We define a particle as "central" in the present work if it is neither an extreme one nor adjacent to an extreme one. This definition is somehow different from the one of Ref. 2. See Ref. 7 for a justification of this modification.

[5]Chan Hong-Mo, K. Kajantie, and G. Ranft, Nuovo Cimento 49A, 157 (1967).

[6]This is certainly an improvement over the procedure of Ref. 2, in which s_l and s_r were computed at $t_l = t_r = 0$,

a point outside the integration region. However, a closer examination reveals that $(t_l + t_r)$ reaches its maximum at a point in which neither t_l nor t_r is stationary. It is therefore possible that $t_l - t_r$ undergoes sizable variations in the integration region in which $(t_l + t_r)$ stays close to its maximum value.

[7]These contributions were lumped in the central distribution in Ref. 2. We have considered them separately in this paper for the following reasons: (a) In Ref. 2 the distribution 2, e. g., was considered as the limit of 3, where $s_l' \to m_1^2$, and the correspondent extrapolation was performed in $\sigma_{M\pi}(s_l')$; this is certainly a dangerous procedure (the extreme case of the distribution $4B$ was computed in the old scheme extrapolating the $p\bar{p}$ cross section at a value of the c.m. energy equal to m_π). (b) In the reasonable assumption that the baryon exchange does not extend more than three steps from the end, we can suppress the need of using the $p\bar{p}$ annihilation cross section as an input. (c) In the particular problem under examination, the charge distribution plays a very important role. In the present case of an incoming π^-, assuming $I = 1$ exchanges to be dominant, the leading π (position 1) will be a π^- an average of 50% of the time and a π^0 50% of the time; the pions emitted in the position 2 are also rather charge asymmetric (50% π^-, 25% π^0, 25% π^+). However, the particles emitted from the chain more than two steps away from the ends have to a large extent "lost memory" of the incoming one, and can be reasonably considered charge symmetric.

[8]We assume nucleon exchange to be dominant over Δ exchange on the basis of the experimentally large $\pi^+ p$ (as compared to the $\pi^- p$) backward peak and of the charge asymmetry in $p\bar{p}$ annihilation into $\pi^+\pi^-$ at low energy. For the meson trajectory we remark that an elementary π exchange would do just as well; what is really needed to get the asymmetry is a rapid decrease in the momentum transfers.

[9]The existence of the forward-backward asymmetry and its size do not depend critically on any of these assumptions.

[10]See also A. Ajduk, L. Michejda, and W. Wojcik, Acta Phys. Polon. A37, 285 (1970).

[11]Our distributions have a maximum close to $p_L = 0$, and therefore a definition in terms of slopes at $p_L = 0$ is meaningless.

[12]J. W. Elbert, A. R. Erwin, W. D. Walker, and J. W. Waters, Nucl. Phys. B19, 85 (1970).

[13]These features are more easily understood in terms of the rapidity distribution. The central particles 3 contribute to $d\sigma/dw$ an approximately constant central plateau, the length of which increases with $\ln s$ whereas the height remains unchanged. The (asymmetric) end effects get more and more displaced in opposite directions with increasing s, and keep their shape unchanged. Therefore, the displacement required to go from the center-of-mass frame (which is the symmetry frame of the central plateau) to a frame in which the asymmetry of the central part compensates the asymmetry of the end effects is energy independent. On the contrary, to increase the number of particles at fixed s corresponds to increasing the height of the central contribution. Therefore, the larger the number of particles at a given s, the smaller the displacement in w (and correspondingly the β of the Lorentz transformation) required to compensate for the asymmetry of the end effects. The author is very grateful to James Bjorken for a conversation on this point.

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Some Quark-Parton-Model Inequalities For the Neutron and Proton Inelastic Structure Functions *

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We summarize several inequalities for the neutron and proton inelastic structure functions $W_2(\nu, Q^2)$ that must be satisfied if the partons are identified with quarks. Some of these inequalities provide us with very useful constraint equations for the $P(N)$ function of the parton model. Due to lack of sufficient data, it is not certain at present if all the inequalities are satisfied. The importance of these inequalities lies in the fact that if any of them are confirmed not to satisfy the experimental data, the concept of individual quark-parton association has to be abandoned.

I. INTRODUCTION AND DATA

We discuss here a few inequalities for the inelastic neutron and proton structure functions that follow from the quark-parton concepts[1,2] of the inelastic lepton-nucleon scattering and the isodoublet character of the nucleons.

First let us summarize the experimental observations on the neutron data[3] that we shall require.

(a) $D/H - 1 \simeq W_{2n}/W_{2p}$ data are consistent (within errors) with a single function of ω. Further, $\nu W_{2n}/\nu W_{2p}$ starts from about 0.5 at $\omega \simeq 1.5$ and increases gradually towards ~ 0.95 at $\omega \simeq 12$.

(b) $(2 - D/H)\,(\nu W_{2p}) \simeq \nu W_{2p} - \nu W_{2n}$ is roughly consis-