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Exchange Degeneracy and Oscillating Total Cross Sections*

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The hypothesis that the Pomeranchukon consists, in part, of a pair of complex conjugate Regge poles, as suggested by Chew and Snider, is shown to allow exact exchange-degenerate fits to the high-energy total-cross-section data. A unique prediction of this hypothesis is that the ratio of real to imaginary parts for the amplitude for $\pi^- p \to \pi^- p$ scattering should not rise uniformly but rather rise to plateau, remain approximately constant from $p_L = 20$ to 60 GeV/c, then resume its rise.

In recent years much theoretical and experimental work has led to the attractively simple theory of exchange-degenerate,[1] straight-line Regge trajectories. The elements of this theory, $\alpha_\rho(0) = \alpha_f(0) = \alpha_{A_2}(0) = \alpha_\omega(0) \cong 0.5$ and the complete cancellation of the contributions of the four secondary trajectories to σ_{K^+p} and σ_{pp}, have been successful in accounting for the experimental data below $p_L = 30$ GeV/c.

Recent results at higher energies,[2] however, show that the fall to a constant of σ_{π^-p} and σ_{K^-p} (proportional to $s^{-1/2}$ at lower energies) has slowed drastically and is no longer in agreement with the simple Regge theory described above. For example, if we continue to assume that the leading odd charge-conjugation Regge singularity has $\alpha(0) \cong 0.5$, then we conclude that σ_{K^+p} must rise by at least 1 mb from $p_L = 25$ to 60 GeV/c in contrast to the constant behavior predicted by exchange degeneracy and the observed behavior from 6 to 25 GeV/c.

In an attempt to explain these data Barger and Phillips[3] have invoked absorptive-like Regge cuts. The constant plateau in σ_{K^+p} from 6 to 25 GeV/c is then a consequence of a cancellation between the cut and the secondary Regge poles, no longer exchange degenerate. The needed breaking of exchange degeneracy is quite large: The ratio of the net contribution of the secondary trajectories to the forbidden reaction over their contribution to the allowed reaction is 0.41 for KN and 0.45 for NN. In addition their cuts are much stronger than those usual in an absorptive model.

We show here that the suggestion of Chew and Snider[4] that, in a multiperipheral model, pairs of complex conjugate Regge poles may appear along with the usual Pomeranchuk pole at $\alpha(0) = 1$, allows successful few-parameter fits to the data within the context of exact exchange degeneracy. Thus we retain simplicity for the secondary Regge poles, but sacrifice it for the Pomeranchukon.

A pair of complex conjugate Regge poles contributes to the cross section the form

$$\gamma[(s/s_0)^{\alpha_r+i\alpha_i} + (s/s_0)^{\alpha_r-i\alpha_i}] = \gamma(s/s_0)^{\alpha_r}\cos(\alpha_i \ln s - \alpha_i \ln s_0)\,.$$

This implies oscillations in $\ln s$ of period $2\pi/\alpha_i$. For $\alpha_r \neq 1$, the oscillations will be damped. We fix $\alpha_\rho(0) = \alpha_f(0) = \alpha_{A_2}(0) = \alpha_\omega(0) = 0.5$, and the resultant expressions for the various cross sections are

$$\sigma_{\pi^-p} = \gamma_\pi^P + \gamma_\pi^O s^{\alpha_r}\cos(\alpha_i \ln s + \varphi_\pi) + (\gamma_\pi^f + \gamma_\pi^\rho)s^{-1/2},$$
$$\sigma_{\pi^+p} = \sigma_{\pi^-p} - 2\gamma_\pi^\rho s^{-1/2},$$
$$\sigma_{K^+p} = \gamma_K^P + \gamma_K^O s^{\alpha_r}\cos(\alpha_i \ln s + \varphi_K),$$
$$\sigma_{K^-p} = \sigma_{K^+p} + \gamma_K^S s^{-1/2},$$
$$\sigma_{pp} = \gamma_p^P + \gamma_p^O s^{\alpha_r}\cos(\alpha_i \ln s + \varphi_p),$$
$$\sigma_{\bar{p}p} = \sigma_{pp} + \gamma_p^S s^{-1/2},$$

where

$$\gamma_{K,p}^S = \gamma_{K,p}^f + \gamma_{K,p}^\rho + \gamma_{K,p}^\omega + \gamma_{K,p}^{A_2} = 2(\gamma_{K,p}^f + \gamma_{K,p}^\omega)\,.$$

Here the superscripts P, O, and S refer to the Pomeranchukon, oscillatory, and secondary trajectories; the subscripts refer to the projectile.

Owing to uncertainties in the Glauber correction

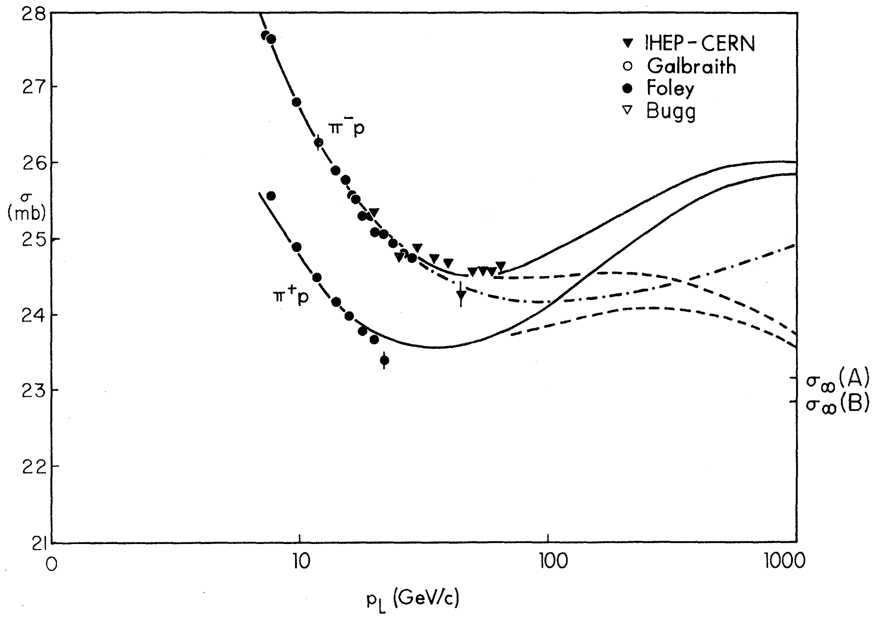

FIG. 1. The two fits to the $\pi^{\pm}p$ total-cross-section data. Error bars are shown only for one point for each group of data. In all figures the solid line is fit *A*, the dashed line is fit *B*. Where the dashed line is omitted the two are indistinguishable. The dot-dash line is a fit of the strong absorptive type to $\pi^{-}p$ only.

we ignore in our fits the data for π^-, K, and p scattering on neutrons, though our results can easily be extended to these data. The usual data[2,5] for scattering on protons have been included, along with those on the ratio of real to imaginary parts of the forward scattering amplitude.[5]

Successful fits have been found for wide ranges of α_r and α_i:

$$0.7 \leq \alpha_r \leq 1, \quad 0.7 \leq \alpha_i \leq 1.$$

Two of these are illustrated in Figs. 1–5, along with their extrapolations to 1000 GeV/c. For comparison, we have also graphed the results of a fit of the strong-absorptive-cut type to the $\pi^- p$ total-cross-section data alone.

The parameters of the fit *A* (*B*) are $\alpha_r = 1.0$ (0.8), $\alpha_i = 0.7$ (1.0), $\gamma_\pi^P = 23.2$ mb (22.9 mb), $\gamma_\pi^O = 5.0$ mb (2.6 mb), $\gamma_\pi^f = 25.8$ mb (22.7 mb), $\gamma_\pi^\rho = 4.4$ mb (4.4 mb), $\varphi_\pi = 0.92$ (-0.45), $\chi^2(\pi p) = 55$ (66) for 60 degrees of freedom; $\gamma_K^P = 18.6$ mb (17.9 mb), $\gamma_K^O = 3.2$ mb (1.4 mb), $\gamma_K^S = 23.9$ mb (24.0 mb), $\varphi_K = 1.25$ (0.3), $\chi^2(Kp) = 51$ (55) for 20 degrees for freedom; $\gamma_p^P = 38.8$ (38.8), $\gamma_p^O = -4.3$ mb (-2.7 mb), $\gamma_p^S = 61.1$ mb (60.9 mb), $\varphi_p = 2.0$ (0.85), $\chi^2(pp) = 52$ (48) for 27 degrees of freedom. Only the statistical error has been used in calculating χ^2.

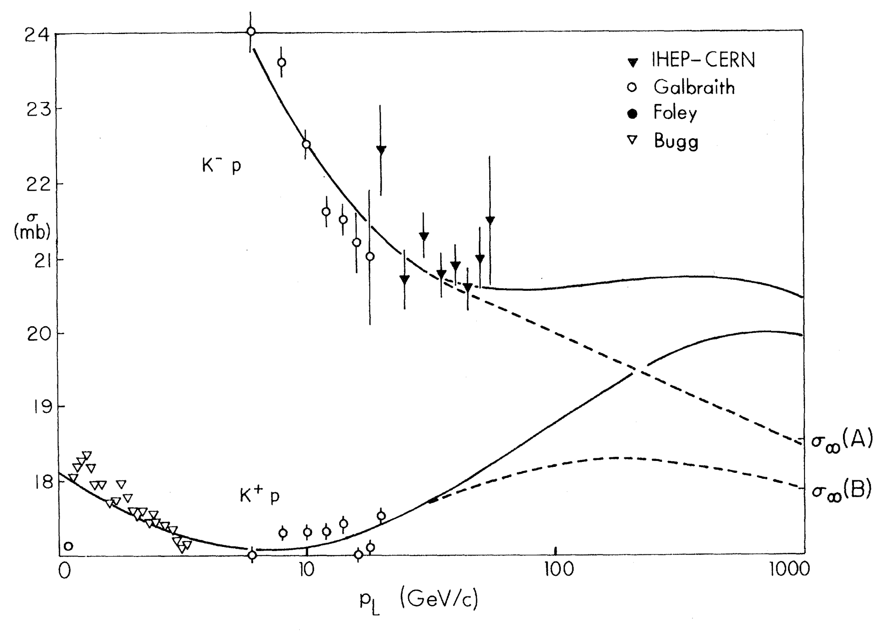

FIG. 2. The two fits to the $K^{\pm}p$ total-cross-section data. The quoted errors on the low-energy $K^{\pm}p$ data are of the order of the size of the triangles.

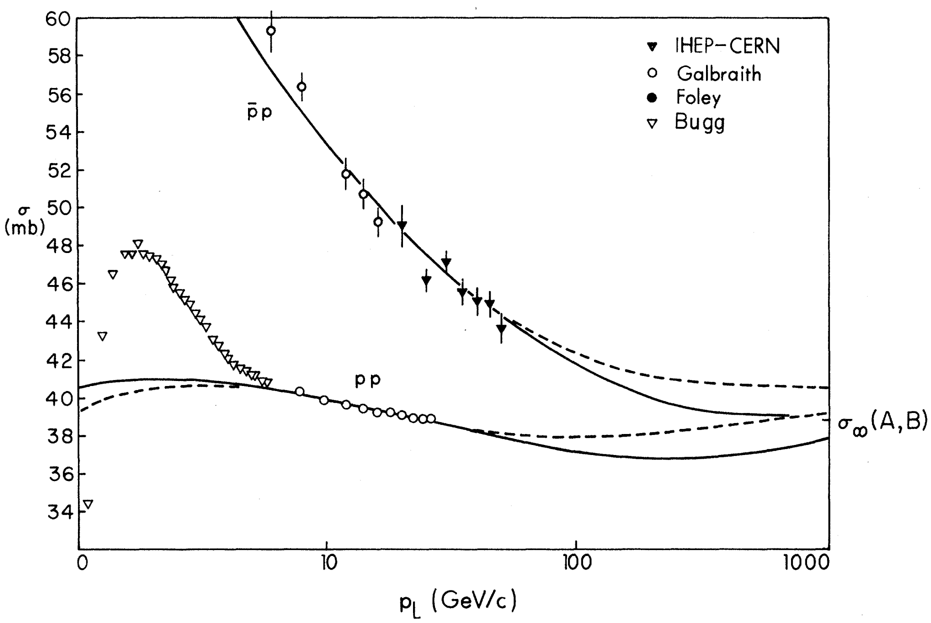

FIG. 3. The two fits to the pp and $\bar{p}p$ total-cross-section data.

We make the following observations about these fits:

(1) Both are very good fits (low χ^2) to the $\pi^{\pm}p$ data.

(2) Both extrapolate well through the low-energy data for K^+p,[6] but badly for pp.[7] As these two channels are empty of resonances, we should expect a successful extrapolation.

(3) The values for σ_∞ are intermediate between pre-Serpukhov predictions and those of Barger and Phillips.

(4) The momentum at which the next oscillation occurs and the rising (or flat) total cross sections start to drop again varies from $p_L = 200$ to 800 GeV/c.

(5) At low momenta, unique structures for α are predicted. In both fits $\alpha_{\pi^- p}$ rises to a plateau, remains constant from 20 to 60 GeV, and then falls again, though the value and width of this plateau vary from fit to fit. In all other fits to these data $\alpha_{\pi^- p}$ rises uniformly. We also predict $\alpha_{\bar{p}p}$ to be small and *positive*; other fits predict it to be small and *negative*. Both these predictions are verifiable at presently accessible energies.

(6) The oscillating component of the Pomeranchukon is a 5–10% effect, and thus the experi-

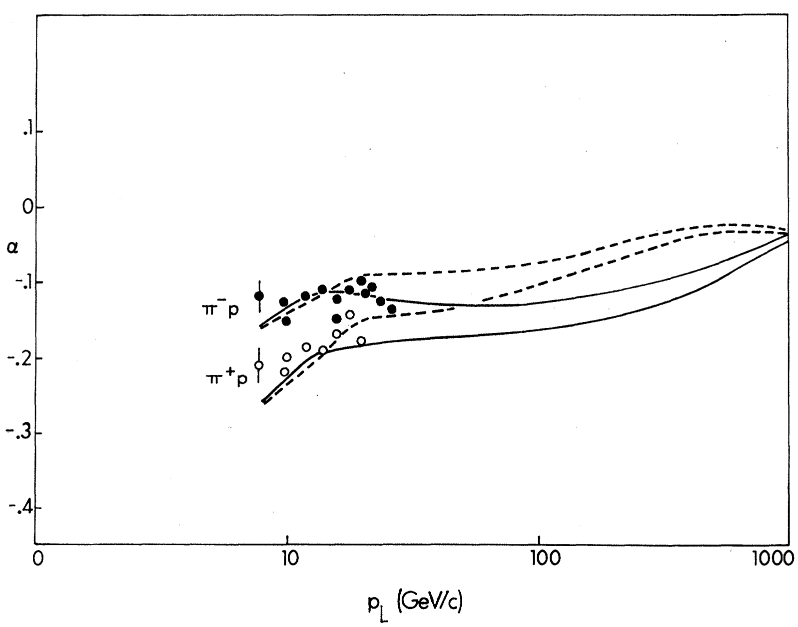

FIG. 4. The two fits to $\alpha = \mathrm{Re}F/\mathrm{Im}F$ for $\pi^{\pm}p$.

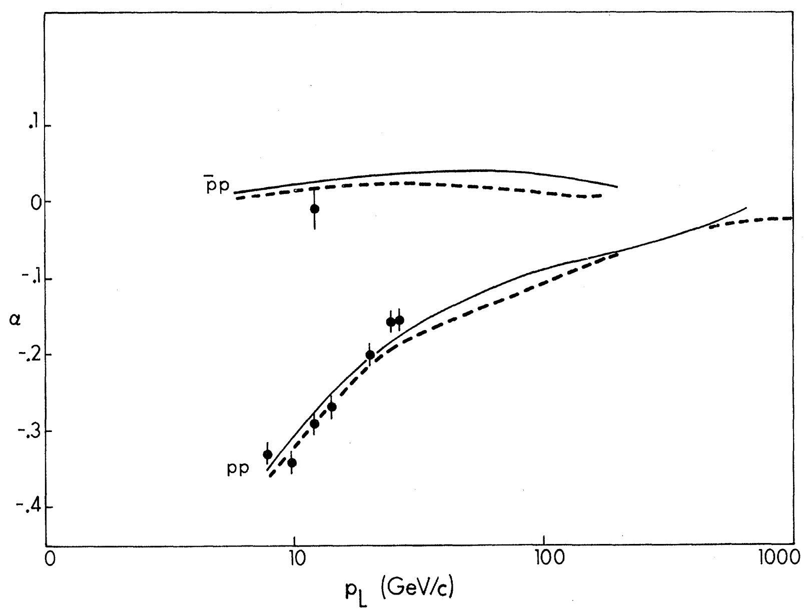

FIG 5. The two fits to $\alpha = \mathrm{Re}F/\mathrm{Im}F$ for pp and $\bar{p}p$.

mentally observed factorization of the Pomeranchukon[8] should still hold approximately for the combination of three poles proposed here.

We have not included the various sum-rule constraints[9] in the above fits, but have checked that both give acceptable results for the Igi sum rule.[10] A complete analysis might be useful in determining α_r and α_i.

To describe the experimental data,[11] the pair of complex conjugate poles must have their real part close to 1. Estimates of the real part of the complex conjugate poles found in the multiperipheral model[4,12] are much less than 1. Thus the oscillatory effect introduced here is much less damped than expected.

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