Is There a Vector-Dominance Frame Problem?

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We argue that it is not meaningful to discuss the vector-dominance model using the language of longitudinal projections. We find that a more suitable language for vector dominance is in terms of electric and magnetic couplings rather than in terms of longitudinal and transverse ones. We advocate the use only of the off-shell invariant amplitudes ("Ball amplitudes"). The experimental failure of the model for polarized photoproduction of pions then implies either that some of the Ball amplitudes require subtractions in k^2, the mass of the current, or that states other than ρ, ω, and φ also couple to the electromagnetic current.

I. INTRODUCTION

The vector-dominance model (VDM) has served as a good qualitative guide over the last ten years in its applications to processes involving photons and vector mesons. There seems to be a large body of truth in the model, far too much for it to be coincidental. Recently however, a disturbing number of "failures" have been recorded.[1] Apart from doubts (which have now almost disappeared) as to the numerical value of the direct coupling, $g_{\gamma V}$, the two major difficulties may broadly be described as the frame problem and the behavior of form factors as $k^2 \to -\infty$, where k^2 is the mass of the electromagnetic current. While we make no attempt here to treat the nucleon form factor, we note that applying vector dominance in this region requires a large extrapolation in k^2. In this work we shall advocate that the other major difficulty, namely the failure of VDM in polarized photoproduction of pions, is in a sense related to its failure in the form-factor region; i.e., we may say that even though we do not know the reason for the observed behavior of the form factors, whatever that reason is, it also influences the off-shell Ball invariants of photoproduction. This may then manifest itself in some of the Ball amplitudes needing subtractions in k^2, or through contributions of states other than ρ, ω, and φ to the k^2 dispersion relations. The aim of this paper is actually very limited. We are unable to predict when subtractions may be required, and have no hope of calculating the subtraction constants. Like the form-factor problem this requires knowledge of hadron dynamics. We can only use the experimental data to infer that they may be required. Thus we do not set out to fit the polarized photoproduction data to those of $\pi^- p \to \rho^0 n$. We endeavor to show, however, that this failure is not due to a nonuniqueness in the choice of frame, and that though the data still represent a challenge to VDM, we cannot meet this challenge by playing around with frames.

By using the off-shell Ball invariants we shall (like many previous authors) automatically recover the helicity-frame predictions, but only in the limit as $s \to \infty$. For small s there are still predictions, but they cannot be expressed in terms of a frame at all. As pointed out by Cho and Sakurai[2] and also (though in a slightly different context) by Mannheim and Nussinov,[3] we even get more information this way than by using a longitudinal projection procedure, i.e., we are also able to relate the helicity-zero vector-meson amplitudes to those of helicity one. Thus we reject the impression that has become somewhat prevalent in the literature, namely that the choice of frame is itself a dynamical problem, and suggest that the two major difficulties mentioned above are in fact related and together provide a single dynamical problem for VDM. We are well aware that most of the arguments we shall present are known or have been derived as "back of envelope" calculations. The only point of this paper is where we put the emphasis.

We aim to show that it is kinematically incorrect to apply VDM in any frame at all. We then show that none of the kinematical difficulties obtain if we apply VDM to the off-shell invariant amplitudes (Ball amplitudes[4]). These should not be confused with the amplitudes for electroproduction (FNW amplitudes[5]) or the on-shell photoproduction amplitudes (CGLN amplitudes[6]). Though we find that this procedure is kinematically consistent, we leave open completely the question of whether it is dynamically correct. The Ball amplitudes are known to be free of kinematic singularities in k^2(which is not the case for the FNW amplitudes) and hence we can write dispersion relations for them. Whether or not they are smooth in k^2, and whether or not they are dominated by the vector-meson poles, we cannot *a priori* say. This is why vector dominance is still only a model.

In Sec. II, we discuss the nature of the frame problem from the viewpoint of Lorentz invariance, little groups, parity, and crossing. In Sec. III, we examine a "yes-frame" philosophy for the processes $\gamma N \to \pi N$, $\gamma N \to VN$, and $\gamma N \to \pi\Delta$, and show that the only genuine breakdown is in polarized $\gamma N \to \pi N$. Finally in Sec. IV, we adopt a "no-frame" philosophy and discuss the nature of the predictions we can make using the invariant amplitudes. We find a description in terms of electric and magnetic couplings to be the most convenient one.

II. STATEMENT OF THE PROBLEM

A physical photon has two degrees of freedom, whereas a vector meson has three. Comparisons in which both the photon and the vector meson are on-shell in their respective processes thus require a commitment as to which degrees of freedom we compare. This problem never arises in "form-factor physics," since here the current is off-shell, and hence has three degrees of freedom also. There is a gauge-invariance problem and this has been resolved by the current-field identity of Kroll, Lee, and Zumino.[7] Further, we showed in Ref. 3 how current conservation and smoothness of the invariants in k^2 lead to successful predictions for 3-point functions when both the V and γ are respectively on-shell without the need to specify a frame. For the 3-point function, k^2 is the only dynamical variable, so the only possibility is to disperse in k^2. It is instructive for the subsequent study of the 4-point function that in the above case VDM for invariants did not yield helicity-frame VDM. We return to this point again in Sec. IV.

The physical photon has no rest frame, and so for it the natural quantization axis is the direction of motion. In fact, gauge invariance demands the use of the helicity basis for a massless particle, with its helicity being a Lorentz invariant.[8] The helicity of a massive particle however is not a Lorentz invariant, as it can always be suitably Lorentz-transformed to its rest frame, where we are free to pick any axis we like to quantize the states. These states then mix under rotations. The choice of quantization axis is arbitrary and requires some additional direction (such as the line of flight of some second particle), and, unlike the photon case, this direction bears no reference to the massive particle itself. All directions are equivalent and *a priori* we can choose any one we like. This is in fact the frame problem, and various authors have exploited this ambiguity like an adjustable free parameter so as to fit the data. We shall discuss such a purely phenomenological approach in more detail in Sec. III. The current theoretical understanding of the frame ambiguity is that since all frames are in principle equally good, we need more information (possibly dynamical) in order to select the best one. We prefer to take the view that all frames are equally bad, and that since there is no preferred frame, there is no frame at all; i.e. there is no way of formulating VDM for any set of transversely polarized states in a Lorentz-invariant manner.

The second kinematic difficulty is provided when we try to continue from $k^2 = m_V^2$ to $k^2 = 0$. The little group for massive particles is R_3, whereas that of a photon is noncompact E_2. So even if we pick a frame and choose two out of a set of three polarization states, we are still faced with the task of continuing a compact group into a noncompact one. This is of course a far more serious problem than the ambiguity in picking the frame.

Our third kinematic objection is provided by the demands of parity. Though we have stated that a physical photon has two degrees of freedom, strictly speaking the little group only provides us with one. For the other helicity state we need the parity transformation (cf. the two-component neutrino theory) as it is not possible to connect the ± 1 helicity states of a photon by a restricted Lorentz transformation. However, for massive particles the eigenvalue of the helicity operator (or, for that matter, of any other basis operator) may be reversed by an ordinary $180°$ rotation in the rest frame. Thus the comparison between ± 1 states of massive and massless particles is not immediate.

We have also noted in a previous publication[9] that massive and massless particles have different crossing properties. These arise because a transverse vector meson acquires a longitudinal component in the s-t crossing. Consequently the kinematic singularities of both the s- and t-channel helicity amplitudes at the respective s- and t-channel thresholds and pseudothresholds are not continuous in the limit $m_V^2 \to 0$. We took this into account in Ref. 9 by proposing VDM for the kinematic singularity free (KSF) amplitudes, and found that we could then maintain consistency with crossing. Thus for calculations in which VDM is coupled to the Regge-pole model it is necessary to use the t-channel KSF amplitudes. At large s the s-channel thresholds and pseudothresholds are far away, and our construction of Ref. 9 actually only showed that it is necessary to use the t-channel KSF amplitudes, if we start with the assumption that it is legitimate to apply VDM to the full helicity amplitudes in the s channel. Thus the KSF prescription is also a helicity-frame prescription, and not a Jackson-frame prescription as claimed in Ref. 10.

As we noted above, the frame problem is the ambiguity in picking the quantization axis, and arises only because the vector meson has a rest frame. As such, this ambiguity is independent of the momentum of the vector meson in the process in which it is compared to a photon, and is independent of how many other particles are involved in the scattering. It is unfortunate that the major attention to VDM in the literature has been to 4-point functions in which the meson is relativistic. Then claims for the helicity frame in dynamical models will always be enhanced (as we shall see in Sec. IV). We already find[3] for the 3-point function that there is no preferred frame, and here the vector meson cannot be moving relativistically. Further there is as yet no prescription for the n-point function in which a slow vector meson is emitted. From the point of view of VDM, it is hard to understand why the momenta and dynamical properties of all the other particles in the process should be relevant to an ambiguity which exists independently of them, especially if the vector meson couples universally to matter.

III. YES-FRAME PHILOSOPHY

To make an experimental comparison we may compare the reaction $\pi^- p \to \rho^0 n$ to the reactions $\gamma p \to \pi^+ n$ and $\gamma n \to \pi^- p$. We define $\sigma_\perp$ ($\sigma_\parallel$) as the sum of the differential cross sections for the last two above reactions for incoming photons polarized perpendicular (parallel) to the production plane. Using time-reversal invariance and ρ dominance, we then have

$$\begin{aligned} \sigma_\perp &= g_{\gamma\rho}{}^2[\rho_{11} + \rho_{1-1}]\sigma(\pi^- p \to \rho^0 n), \\ \sigma_\parallel &= g_{\gamma\rho}{}^2[\rho_{11} - \rho_{1-1}]\sigma(\pi^- p \to \rho^0 n), \qquad (1) \\ \sigma(\text{unpolarized}) &\equiv \sigma_{\text{un}} = \tfrac{1}{2}(\sigma_\perp + \sigma_\parallel), \end{aligned}$$

where the frame of the density matrix of the outgoing meson has to be prescribed. Apart from the helicity frame, there is also in the literature another popular choice, the Donohue-Högaasen frame (DH),[11] advocated by Białas and Zalewski.[12] This frame is such that in the new basis, $\text{Re}\rho_{10} = 0$. When the predictions were first tested a few years ago a remarkable fact was that the prediction for σ_{un} worked in the helicity frame over a wide range of s and t using the value of $g_{\gamma\rho}{}^2$ given by the colliding-beam experiments.[1,13] We now use the word remarkable because of historical hindsight, since the predictions for $\sigma_\perp$ and $\sigma_\parallel$ were later found to fail. This then prompted the search for a frame. Diebold and Poirier[14] have noted that the prediction for $\sigma_\perp$ fails for any quantization axis in the scattering plane ($\sigma_\perp$ is invariant to rotations about the y axis, so it is untouched by the DH rotation). Mannheim and Maor[15] made the proposal to take the quantization axis out of the plane and use the y axis, the transversity frame. It is amusing to note that Eqs. (1) are then satisfied, though we regard this as coincidental.

If the present data do not change, then the experimental status of Eqs. (1) is not too encouraging. A search for other $\pi^+\pi^-$ partial waves in the ρ region in the process $\pi^- p \to \pi^+\pi^- n$ is being conducted. Dar[16] has noted that such waves would mainly affect ρ^H_{1-1} and hence not spoil the good agreement of σ_{un}. It is perhaps encouraging to note that in the process $\gamma p \to \pi^+\pi^- p$ there definitely are other partial waves in the ρ mass region.[17]

Before conceding Eqs. (1) as an experimental failure we present two possible escape clauses (for highly optimistic readers). The first relies on duality. According to duality the ρ and Δ bands in $\pi^- p \to \pi^+\pi^- n$ are dual. Thus to parametrize the Dalitz plot as an incoherent sum of Breit-Wigner resonances may commit double counting and hence affect the right-hand side of Eqs. (1). Unfortunately, hadron physics has not yet reached the state where we can make a definitive statement on this subject.[18] The other way out relies on our knowledge of soft-pion physics. We have to correct to hard pions when we study processes in which the pions have high momenta. So maybe we should also have to make a "hard-ρ" correction, especially if the meson is relativistic. Again we cannot make a definitive (i.e., model-independent) statement.

Though polarized photoproduction of pions is the most studied process experimentally, there are a few other examples we can discuss as well. They are $\gamma N \to VN$ and $\gamma N \to \pi\Delta$. In $\gamma N \to VN$ we connect first to $VN \to VN$ and then to $\pi N \to \pi N$, using the quark model or some symmetry scheme. For unpolarized photons the predictions are satisfied very well in the helicity frame for cross sections.[19] The density matrix of the outgoing V is also well fitted in both unpolarized[9] and polarized[20] photoproduction. As noted by Gilman *et al.*[21] and by Mannheim,[22] diffraction scattering conserves s-channel helicity in the reactions $\gamma N \to VN$, $VN \to VN$, and $\pi N \to \pi N$ to leading order in s. Consequently there is only one (helicity-independent) amplitude common to the whole three processes, which then has to be the amplitude for VDM. Thus for diffraction there is anyway no need to look for a frame, and the extension to polarized photons gives no new information and just has to work. Finally for $\gamma N \to \pi\Delta$ we connect to $VN \to \pi\Delta$ and then to $\pi N \to V\Delta$, an s-u crossing property which depends on the signature of the exchanged Regge trajectories. Using the KSF prescription, Gotsman[10] was able to incorporate this property correctly, and so a helicity-frame prediction again works for the unpolarized data he analyzed. There have as yet been no tests for polarized $\gamma N \to \pi\Delta$ or for photoproduction with polarized targets. If the situation in $\gamma N \to \pi N$ is genuine, then these other tests will presumably also fail.

IV. NO-FRAME PHILOSOPHY

Our program now is to study VDM for invariants to see whether we can understand the above successes and failures from that standpoint. Our philosophy is that we have to make a continuation from $k^2=m_V^2$ to $k^2=0$ and the only way in which we can meaningfully do this is by using off-shell amplitudes and appealing to current conservation. We shall begin by reformulating a study of $A_1\to\rho\pi$ which we made in Ref. 3. We write the vertex $A_1\to J_\mu\pi$ as

$$\langle A_1|J_\mu|\pi\rangle=\epsilon_\mu^{A_1}G_E(k^2)+[(\epsilon^{A_1}\cdot k)q_\mu-(k\cdot q)\epsilon_\mu^{A_1}]G_M(k^2)+(\epsilon^{A_1}\cdot k)k_\mu G_{\rm off}(k^2),\qquad(2)$$

where $\epsilon_\mu^{A_1}$ is the polarization vector of the A_1 with momentum q_μ and mass m, and k_μ is the momentum of the current. We have introduced the "electric" and "magnetic" form factors $G_E(k^2)$ and $G_M(k^2)$ and a third form factor $G_{\rm off}(k^2)$ which only contributes off shell, i.e., does not appear in physical $A_1\to\rho\pi$, $A_1\to\gamma\pi$. We use the terms electric and magnetic in the sense that they respectively correspond to couplings of the A_μ and $F_{\mu\nu}$ type, and note that only the magnetic term is divergenceless. Current conservation now gives

$$(\epsilon^{A_1}\cdot k)[G_E(k^2)+k^2G_{\rm off}(k^2)]=0.\qquad(3)$$

In the A_1 rest frame, $\epsilon^{A_1}\cdot k=0$ for the A_1 in helicity one, and in helicity zero $\epsilon^{A_1}\cdot k\sim m^2-m_\pi^2$ as $k^2\to 0$. $G_{\rm off}(k^2)$ surely has no pole at $k^2=0$, so we obtain

$$(m^2-m_\pi^2)G_E(0)=0.\qquad(4)$$

Thus for unequal masses, $G_E(0)=0$. We note that in the unequal-mass case, the point $k_\mu=0$ (where the form factor becomes the charge) is unphysical, and so the physical interpretation of Eq. (4) is that we cannot build an electric charge out of unequal-mass states.[23] The magnetic coupling is independent of this mass-difference effect and so is allowed. Of course, for equal masses Eq. (4) is satisfied trivially, so that no constraint may be obtained. We note in passing however that this is the reason why in photoproduction each individual electric Born diagram is not gauge-invariant (only the sum is). We illustrate this remark by considering the process $\gamma X\to YZ$ with the arbitrary X, Y, and Z being spinless for simplicity. Let $\epsilon_\mu^\gamma V^\mu$ be the gauge-invariant electric γXX vertex. The amplitude associated with X exchange is

$$\epsilon_\mu^\gamma T^\mu=\frac{\epsilon_\mu^\gamma V^\mu g_{XYZ}}{s-m_X^2}.\qquad(5)$$

Now though $k_\mu V^\mu$ vanishes when the intermediate X is on-shell, it vanishes as $(s-m_X^2)$. This zero is then cancelled by the pole in the propagator leaving $k_\mu T^\mu$ finite.

So far we have only used current conservation. We now turn to VDM, and assume that $G_i(k^2)$ satisfy unsubtracted dispersion relations (UDR) in k^2 with ρ dominance. In fact it is sufficient to require UDR for $G_E(k^2)$. Because of the kinematics $G_M(k^2)$ and $G_{\rm off}(k^2)$ have anyway a better large k^2 behavior. Thus either by dispersing or by using the current-field identity we have

$$G_E(k^2)=\frac{g_{\gamma\rho}m_\rho^2}{m_\rho^2-k^2}G_E^{(\rho)},\qquad(6)$$

where $G_E^{(\rho)}$, the residue at the ρ pole, is the on-shell $A_1\rho\pi$ electric coupling. Introducing the on-shell $A_1\gamma\pi$ electric coupling $G_E^{(\gamma)}$, we then obtain $G_E^{(\gamma)}=g_{\gamma\rho}G_E^{(\rho)}$ by smoothness, and hence finally conclude that $G_E^{(\rho)}=0$, using Eq. (4). Thus we see that we are able to extend the range of applicability of ρ universality beyond diagonal matrix elements. Of course, we should stress that we have no constraint in the event of a subtraction being required. The helicity amplitudes of $A_1\to\rho\pi$ are related to $G_i^{(\rho)}$ by

$$\begin{aligned}g(\lambda_{A_1}=\lambda_\rho=1)&=g_1=G_E^{(\rho)}-\tfrac{1}{2}[m^2+m_\rho^2-m_\pi^2]G_M^{(\rho)},\\ g(\lambda_{A_1}=\lambda_\rho=0)&=g_0=\frac{E_\rho}{m_\rho}G_E^{(\rho)}-mm_\rho G_M^{(\rho)},\end{aligned}\qquad(7)$$

where everything is defined in the A_1 rest frame with quantization axis along the decay direction of motion. Thus, using the condition $G_E^{(\rho)}=0$ we are able to express g_0 in terms of g_1, viz.

$$\frac{g_1}{g_0}=\frac{m^2+m_\rho^2-m_\pi^2}{2mm_\rho},\qquad(8)$$

which agrees reasonably well with the available data.[3] Thus Eq. (8) provides a direct test of ρ dominance of $G_E(k^2)$ without any need to perform an experiment involving a photon at all. A test of ρ dominance of $G_M(k^2)$ is then provided by comparing the $A_1\to\rho\pi$ and $A_1\to\gamma\pi$ rates.

The reason we have presented this analysis is to bring out the difference between electric- and magnetic-type couplings on the one hand and longitudinal and transverse on the other. In the special cases of $\rho\to\pi\pi$ and $\omega\to\pi\pi\pi$, we note that $\rho\to\pi\pi$ is electric and longitudinal, whereas $\omega\to\pi\pi\pi$ is magnetic and transverse. There is thus a tendency to exchange these terms and to speak loosely about electric when one means longitudinal and vice versa. However, in these particular processes the helicities of the ρ and of the ω are determined by conservation of angular momentum and parity only, independent of whether the current is conserved (i.e., $K^* \to K\pi$, $K^* \to K\pi\pi$ have the same kinematics). We see from our example of $A_1 \to \rho\pi$ that in more general configurations, g_1 and g_0 may be written as combinations of $G_E^{(\rho)}$ and $G_M^{(\rho)}$. Thus the terms longitudinal and electric are not interchangeable, and we see that we should take (almost take, actually) as our definition of vector dominance

$$f(\gamma + B \to C + D) = g_{\gamma V} f(V + B \to C + D), \tag{9}$$

where the V is coupled magnetically, rather than where the V is coupled transversely. We write "almost" as this is not quite complete, because states mass-degenerate with the external states may solve Eq. (4) without requiring $G_E(0)$ to vanish, and hence contribute electrically as intermediate states in Eq. (9). However, the prescription to use the off-shell amplitudes is complete. Thus, we resolve the frame problem by saying that we do not look for a frame where the vector-meson is transverse, since this is not how the current couples. The concepts of electric and magnetic are better defined than longitudinal and transverse (from the viewpoint of Lorentz invariance), are not ambiguous, and most importantly, may be continued to $k^2 = m_V^2$ without difficulty.

The extension of this analysis to 4-point functions is immediate, though it does raise one new feature. Here k^2, s, and t are present and so we hope that we can disperse in k^2 with s and t fixed. (It should be possible to ascertain if this is legitimate by contracting the current and one other state and studying the matrix element of a current commutator between states of infinite momentum. Then we may even obtain smoothness via locality. We are not aware that such a construction has been made in the literature). We shall therefore assume that it is valid to fix s and t, and study first the hypothetical process $J_\mu + \pi_2 \to \sigma_3 + \pi_4$ where σ is a 0^+ meson. The T matrix of the process is given by

$$T = \epsilon_\mu(k)[k^\mu B_1(k^2) + p_2^\mu B_2(k^2) + p_3^\mu B_3(k^2)], \tag{10}$$

where $B_i(k^2)$ are the equivalent of the corresponding Ball amplitudes of the process $J_\mu + N_2 \to N_3 + \pi_4$.[4] Current conservation then gives

$$2k^2 B_1 + (s - m_2^2 - k^2)B_2 + (k^2 + m_3^2 - t)B_3 = 0. \tag{11}$$

We may thus eliminate one of the invariants, say B_3, so that

$$T^e = [(\epsilon \cdot k)(t - m_3^2 - k^2) + 2(\epsilon \cdot p_3)k^2]A_1(k^2) + [(\epsilon \cdot p_2)(t - m_3^2 - k^2) + (\epsilon \cdot p_3)(s - m_2^2 - k^2)]A_2(k^2), \tag{12}$$

where we have introduced

$$A_1(k^2) = \frac{B_1(k^2)}{t - m_3^2 - k^2}, \quad A_2(k^2) = \frac{B_2(k^2)}{t - m_3^2 - k^2}, \tag{13}$$

which are the FNW invariants for electroproduction.[5] We note that A_1 and A_2 have a kinematic singularity at $k^2 = t - m_3^2$ and as such cannot be considered as candidates for dispersion relations in k^2 with fixed t. The photoproduction amplitude (CGLN)[6] is then given by

$$T^\gamma = [(\epsilon \cdot p_2)(t - m_3^2) + (\epsilon \cdot p_3)(s - m_2^2)]A_2^\gamma, \tag{14}$$

where

$$A_2^\gamma = \frac{B_2(k^2 = 0)}{t - m_3^2}. \tag{15}$$

We note that A_2^γ does not have a kinematic singularity at $t = m_3^2$. This is because current conservation [Eq. (11)] forces B_2 to have a kinematic zero at $t = m_3^2$ at $k^2 = 0$ only, leaving A_2^γ finite. There is a lot of confusion in the literature over this point. We note only that it is necessary to distinguish between kinematic singularities in the variable k^2 and in the variable t. Also the kinematic factor $(t - m_3^2)^{-1}$ in Eq. (15) bears no relation to the Born term associated with σ exchange as this factor is obtained for all charge states of the σ including charge zero. Finally, the amplitude for $V + \pi_2 \to \sigma_3 + \pi_4$ is given by

$$T^V = \epsilon_\mu(k)[p_2^\mu B_2^V + p_3^\mu B_3^V]. \tag{16}$$

We thus see that T^e, T^γ, and T^V all have a different kinematic structure, so that we cannot immediately apply VDM to them. The only remaining candidate is $B_i(k^2)$ which we know to be free of kinematic singularities in k^2.[4] Thus we take as our definition of VDM

$$B_i(k^2) = \frac{m_V^2 g_{\gamma V}}{m_V^2 - k^2} B_i^V, \tag{17}$$

so that from current conservation we find that at $k^2 = 0$,

$$(s - m_2^2)B_2^V + (m_3^2 - t)B_3^V = 0. \tag{18}$$

This implies that

$$T^V = \frac{B_2^V}{(t - m_3^2)}[(\epsilon \cdot p_2)(t - m_3^2) + (\epsilon \cdot p_3)(s - m_2^2)]. \tag{19}$$

We note that Eq. (19) is not Eq. (14) as it also contains zero-helicity vector mesons and is on the vector-meson mass shell and not the photon mass shell. However, our construction admits of a mass continuation, i.e.,

$$\lim_{m_V^2\to 0} g_{\gamma V}T^V = T^\gamma\,, \tag{20}$$

which is what we set out to find. We now calculate the s-channel helicity amplitudes and find

$$\begin{aligned} g_1^\gamma &= -q\sin\theta_\gamma\, B_3^\gamma/\sqrt{2}\,,\\ g_1^V &= -q\sin\theta_V\, B_3^V/\sqrt{2}\,,\\ g_0^V &= \frac{m_V B_3^V Y(s,t)}{2(s-m_2{}^2)[s-(m_V-m_2)^2]^{1/2}[s-(m_V+m_2)^2]^{1/2}} \end{aligned} \tag{21}$$

where

$$q=[s-(m_3-m_4)^2]^{1/2}[s-(m_3+m_4)^2]^{1/2}/2s^{1/2}$$

and

$$\begin{aligned} Y(s,t) = {}& t(m_V{}^2-m_2{}^2-3s)-s^2+s(m_V{}^2+m_3{}^2+2m_4{}^2)\\ &-m_V{}^2(m_2{}^2+m_3{}^2)+m_2{}^2(m_2{}^2+3m_3{}^2-m_4{}^2)\,. \end{aligned}$$

Here θ_i $(i=\gamma, V)$ is the s-channel center-of-mass scattering angle. Thus VDM in the form $B_3^\gamma = g_{\gamma V}B_3^V$ does not yield $g_1^\gamma = g_{\gamma V}g_1^V$ except in the two limits $m_V^2\to 0$ or $s\to\infty$. In particular, as $s\to\infty$,

$$\begin{aligned} g_1^V &\to -B_3^V(-t)^{1/2}/\sqrt{2},\\ g_0^V &\to -B_3^V m_V/2\,, \end{aligned} \tag{22}$$

and we see that g_0^V does not go to zero. Nor will it go to zero in any other frame. That it stays up is a consequence of the fact that the physical vector meson is not massless. Thus our approach takes into account specifically that $m_V^2\neq 0$, and shows that it is not meaningful to search for a longitudinal projection scheme. We find that at arbitrary s we may relate g_1^γ to g_1^V (though not in any frame at all), but only as $s\to\infty$ do we obtain VDM in the helicity frame. Also we gain something in that we may relate g_0^V to g_1^V.

To complete the analysis we look at $J_\mu+\pi_2\to\pi_3+\pi_4$ Here there is only one amplitude which is already conserved, i.e.,

$$T=\epsilon_{\mu\nu\sigma\tau}\epsilon^\mu(k)p_2^\nu p_3^\sigma p_4^\tau B(k^2) \tag{23}$$

and this process is purely transverse. The helicity amplitudes are given by

$$g_1^i=\varphi_i(s,t)^{1/2}B^i/2\sqrt{2} \quad (i=\gamma,V)\,, \tag{24}$$

where $\varphi(s,t)$ is the Kibble function. Thus again only in the limit $s\to\infty$ does VDM for invariants give VDM in the helicity frame. So even when there is no other frame available except the helicity frame we still should not apply VDM to helicity amplitudes except at large s.

The extension of our analysis to $J_\mu+N_2\to N_3+\pi_4$ is direct and has already been discussed in the literature,[2,24-27] with sometimes varying predictions. However, all of these authors realized that a no-subtraction philosophy enables us to recover Eqs. (1) in the helicity frame in the limit $s\to\infty$. Meiere[26] in particular has stressed that the failure of Eqs. (1) experimentally entails the need for subtractions. On the whole, the aim of these authors was to see if the invariant-amplitude approach could give a clue as to the correct frame. Our approach is that we have to use invariant amplitudes as this is the only kinematically consistent procedure. We should point out that if the Ball amplitudes require subtractions there are still predictions (depending on how we fix the subtraction constants), and then even at large s we do not recover the helicity-frame VDM (or VDM in any other frame either). Cho and Sakurai[28] have noted that in the electric Born model they necessarily obtain the helicity frame at large s. (Dar[16,27] has also obtained a similar result using the absorption model). This lead Cho and Sakurai to suspect that the choice of frame was a dynamical problem. As they noted in Ref. 2, the $B_i(k^2)$ amplitudes in the electric Born model are independent of k^2, and as we have seen this property itself is sufficient to guarantee the helicity frame.

So where does VDM stand experimentally? The predictions of Eqs. (1) are still bad. Such a situation forces us to the need for subtractions in some of the Ball amplitudes. Further, the subtraction constants must be related in such a way that they cancel in $\sigma_{\rm un}$. It is nice to conjecture that the subtractions are required in the 4-point function because they are required in the vertices of 3-point functions related to intermediate states in which the coupling is electric. Cho and Sakurai[2] have noted that the other class of predictions, i.e., those which express g_0 in terms of g_1, seem to be working well in $\pi N\to\rho N$. However, they only investigated the small-t region where some of the elements of the spin-density matrix of the outgoing ρ are fixed by angular momentum constraints, so it is not immediately clear how well the predictions are being tested. At the moment, the experimental situation is such that in some cases we need subtractions and in others ($\gamma N\to VN$) we seem not to need them.[29] Thus dynamically the state of the vector-dominance model is still obscure.

Note Added in Proof

A point which we had not discussed in the paper is a difficulty inherent in any analysis based on the use of invariant amplitudes, namely their intrinsic arbitrariness. Different choices for the set of invariants assumed to be suitable for VDM give differing physical predictions. We illustrate this remark for our example of $A_1 \to \rho\pi$ by considering instead of the set of invariants defined in Eq. (2), the set used in Ref. 3, i.e.,

$$\langle A_1 | J_\mu | \pi \rangle = \epsilon_\mu^{A_1} A(k^2) + (\epsilon^{A_1} \cdot k) q_\mu B(k^2) + (\epsilon^{A_1} \cdot k) k_\mu G_{\text{off}}(k^2) . \qquad (25)$$

VDM applied to $A(k^2)$, $B(k^2)$ yields[3]

$$\frac{g_1}{g_0} = \frac{2m(m^2 - m_\pi^2)}{m_\rho(3m^2 - m_\rho^2 + m_\pi^2)} , \qquad (26)$$

in contrast to the result of Eq. (8). This comes about because of the relation

$$G_E(k^2) = A(k^2) + \tfrac{1}{2}(m^2 + m_\rho^2 - m_\pi^2) B(k^2) + \tfrac{1}{2}(k^2 - m_\rho^2) B(k^2) , \qquad (27)$$

so that pole terms in one set of invariants induce nonsingular contributions to the dispersion relations for the other set of invariants. The whole of our dilemma is contained in the question of whether ρ dominance in the range $m_\rho^2 \geq k^2 \geq 0$ is more reliable for the set $G_E(k^2)$, $G_M(k^2)$ or for the set $A(k^2)$, $B(k^2)$, and this is a dynamical problem for which *a priori* we have no information. However, only the set $G_E(k^2)$, $G_M(k^2)$ has a straightforward physical interpretation, and since this is the set which provides ρ universality for both diagonal and off-diagonal matrix elements, we are inclined to the view that this is the correct set.

The strongest support for off-diagonal ρ universality is provided in an experiment of $\bar{p}n$ annihilation at rest into $\pi^-\pi^-\pi^+$. No ρ signal is seen at all,[30] and since the $\bar{p}n$ system must have the quantum numbers of a (heavy) pion, it accords with our selection rule $G_E^{(\rho)} = 0$. For the $A_1 \to \rho\pi$ decay, the predictions of Eqs. (8) and 26 are similar. Equation (8) gives $g_1/g_0 = 1.04$ and Eq.(26) gives 1.15, to be compared with a recent experimental value[31] of $g_1/g_0 = 0.89 \pm 0.07$. Our analysis may also be applied to the $B \to \omega\pi$ decay. Equation (8) gives $|g_0|^2 = 0.29$ and Eq. (26) gives $|g_0|^2 = 0.25$, to be compared with the most recent determination[32] of $|g_0|^2 = 0.06 \pm 0.10$. Thus the data on 1^+ decays are not conclusive enough to allow experiment to choose the best set of invariants. We suspect that our analysis goes wrong for the $B \to \omega\pi$ decay because here the neutral mode $B^0 \to \omega\pi^0$ is open ($A_1^0 \to \rho^0\pi^0$ is forbidden by charge-conjugation invariance), so that the requirement of the decoupling of the elctric charge becomes an empty statement as there is no electric charge.

The physical interpretation of universality above suggests that for the nucleon form factor VDM predictions should be made for the F's (Dirac and Pauli) rather than the G's (introduced by Hand, Miller, and Wilson[33]). Since we have demonstrated the difference between electric and longitudinal and between magnetic and transverse, it is perhaps unfortunate that these authors actually defined the G's as electric and magnetic since they are, respectively, longitudinal and transverse in the Breit frame.

Despite the above ambiguity in the choice of invariants, we find for our example of $A_1 \to \rho\pi$ that we recover both Eq. (20) and the relation $g_1^\gamma = g_{\gamma V} g_1^V$ for either set of invariants in the limit $m/m_V \to \infty$. Though we have not been able to prove it, we suspect that the continuation implied by Eq. (20) and the use of the helicity frame for VDM in the large s limit are independent of the choice of invariants, and test only the concept of vector dominance itself. Relations between g_1 and g_0 and $O(1/s)$ corrections to helicity frame VDM are sensitive to the specific choice of invariants used. A promising approach to avoid the arbitrariness in the choice of invariants has been proposed by Berlad and Eilam.[27] They use the unambiguous Poincaré reduced T-matrix elements and again find helicity-frame VDM in the large s limit.

We should also mention that we have possibly overemphasized the role of subtractions in this paper. More recently, we have investigated the role of contributions of other vector mesons to the dispersion relations.[34] Polarized photoproduction of pions is experimentally sensitive enough to measure any ρ, ρ' interference terms.

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