form from the states of motion $A(p)\,A(-p)$ and $A(-p)\,A(p)$ the symmetric combination, i.e., the phase space must be divided by 2!. If the two particles are nonidentical A and B, then (1.8) gives states of motion $A(p)\,B(-p)$, $A(-p)\,B(p)$, $B(p)\,A(-p)$, and $B(-p)\,A(p)$. The last two states are physically the same as the first two, so again the phase space must be divided by 2! to avoid double counting. Note that this still leaves us with twice as many AB states as AA states, as one would expect.

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Simple Dual-Resonance Model for Inclusive Reactions*

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We write down a dual expression for the differential cross section in two-body reactions for inclusive production of a single particle with definite momentum. The formula is similar to the usual five-point amplitude but where the range of integration has changed. We show that it describes both limiting fragmentation and pionization. It furthermore shows approximate factorization as a function of $p_\perp^2$ and $p_\parallel^2$. The asymptotic behavior in $p_\perp^2$ is universal. We also generalize the formula for inclusive reactions where n particles are detected.

I. INTRODUCTION

Recently there has been considerable interest in the study of single-particle distributions in high-energy reactions. Several theoretical properties were predicted both from the parton model[1,2] and from the multiperipheral model.[3]

Some of the interesting features that have come out both from these models and from experiment are[4]: (i) The distribution in longitudinal momenta approaches a finite limit both in the lab frame (limiting fragmentation[1]) and in the c.m. frame (pionization). (ii) In the limiting-fragmentation region the differential cross section can be written approximately as

$$p_0 d\sigma/d^3p = f(p_\parallel^L)G(p_\perp^2), \tag{1}$$

where $G(p_\perp^2)$ is a universal function[4] (a decreasing exponential). (iii) In the limit of fast fragments, one should recover Regge behavior.

In this paper we want to show how the Veneziano amplitude can be used to derive an explicit formula for the differential cross sections which demonstrates all the properties stated above and which shows, furthermore, the implications of duality for these particular processes. However, we would like to stress that we are not explaining the existence of a finite limiting distribution because, as is well known,[2,3] this necessitates a model for the Pomeranchukon. We will rather adjust *ad hoc* the intercept of trajectories with the quantum numbers of the vacuum so as to get Pomeranchukon-like behavior. As the explicit dependence on the Pomeranchukon factors out, we argue in the final section that different models should provide similar results.

The qualitative features of the model reproduce the experimental results quite well in spite of the fact that we are mistreating (as usual) unitarity and spin. The only meaningful qualitative disagreement of the model with experiment occurs at large $p_\perp^2$ where the model gives a cross section falling off too fast. This confirms the expectation that unitarity corrections should be important at large angles.

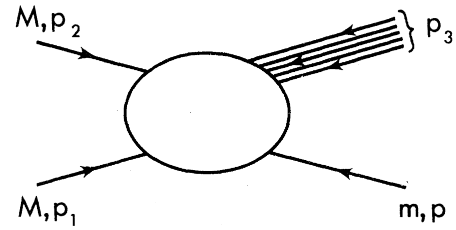

FIG. 1. Inclusive reaction. (m,p) is the detected particle.

The paper is organized as follows: In Sec. II we will derive the expression for the differential cross section. In Sec. III we will obtain the asymptotic behavior in $p_\perp^2$ and the pionization limit. In Sec. IV we will present other possible models and generalizations and a discussion of the results; in particular, we will discuss the irrelevance of the concept of limiting temperature and the counting of final states.

II. DERIVATION OF THE FORMULA

The process that we are considering is shown in Fig. 1. We are supposed to integrate over phase space the square of the modulus of the amplitude for all the particles $\{p_3\}$ that we do not detect. The method consists in using the six-point dual amplitude to do this integration[5] as shown in Fig. 2.

In doing this we are supposing that the complete set of states over which we are summing can be simulated by resonances. This is incompatible with the absence of exotic states (Harari-Freund ansatz). We will come back to this point in Sec. IV.

We begin therefore by writing

$$p_0 d\sigma/d^3p = \frac{G}{(p_1\cdot p_2)^2 - p_1^2 p_2^2}\sum_n \int_{-\infty}^{\infty} T_n(p_1p_2p_3p)T_n^*(p_1p_2p_3p)\delta^4(p_1+p_2+p_3+p)\delta(p_3^2-M_n^2)d^4p_3\,, \tag{2}$$

where $d\sigma$ is the differential cross section for the production of a particle with three-momentum $\vec{p}$. The summation is over all possible resonances and T_n is the amplitude for producing the nth resonance and the detected particle. Then

$$T_n T_n{}^*\delta(p_3^2 - M_n^2) = \frac{1}{\pi}\,\mathrm{Im}_{p_3^2} B_6(pp_1p_2 - p_2 - p_1 - p), \tag{3}$$

where we have assumed that only the (s,t) term contributes in T_n (this will correspond to the physical case $p+\bar{p}\to\pi+$ anything) and

$$\begin{aligned} B_6(pp_1p_2-p_2-p_1-p) = \int_0^1\int_0^1\int_0^1 & du\,dv\,du'(uu')^{-\alpha(p_1+p_2)-1}v^{-\alpha(p+p_1+p_2)-1} \\ & \times\left[\frac{(1-u)(1-u')}{(1-vu)(1-vu')}\right]^{-\alpha(p+p_1)-1}\left[\frac{(1-vu)(1-vu')}{(1-vuu')^2}\right]^{-\alpha(M^2)-2} \\ & \times\left[\frac{(1-v)(1-uu'v)}{(1-uv)(1-u'v)}\right]^{-\alpha_2(0)-1}(1-uu'v)^{-\alpha_1(0)-3}\left[\frac{u(1-u)(1-vu')}{u'(1-u')(1-vu)}\right]^{i\epsilon} \end{aligned} \tag{4}$$

Several points are worth mentioning in this formula. (a) The trajectories $\alpha_1(0)$ and $\alpha_2(0)$ always have the quantum numbers of the vacuum. Therefore we can put $\alpha_i(0)\approx 1$.[6] (b) The trajectory $\alpha(M^2)$ has always the quantum number of particle 1; therefore $\alpha(M^2)\approx 0$ (or an integer in the case in which the external particle is excited). (c) It is a valid representation only for all invariants negative. When continuing analytically to positive values we have to remember that exponents of factors that include u or u' have to be con-

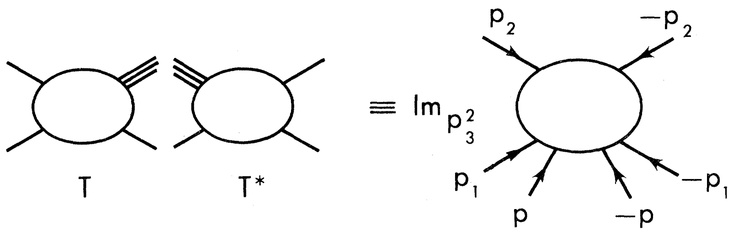

FIG. 2. Cutkosky's generalized unitarity relating the six-point function and the differential cross section for inclusive experiments.

tinued on opposite sides of the cuts. This is formally symbolized by the last factor in (4).

There are two different limiting distributions: (a) $p_\parallel$ finite in the lab system (or distribution in $x = p_\parallel^{\text{c.m.}}/p_1^{\text{c.m.}}$ for $x \geqslant 0$),[2,7] and (b) $p_\parallel$ finite in the c.m. frame (pionization, or emission of wee[4] partons). The limits of the relevant invariants are given in Table I.

For case (a) we obtain the limiting distribution by changing uu' and v into $e^{\rho u}e^{\rho u'}e^{\rho(1-u-u')}$, and integrating in the region $\rho \approx 0$. We find

$$p_0 s\, d\sigma/d^3p = \frac{G}{\pi}\,\mathrm{Im}_{p_3{}^2}\left\{\Gamma(-\alpha_1(0))\int_0^1 du'\int_0^{1-u'} du\left[\left(1-\frac{1}{u}\right)\left(1-\frac{1}{u'}\right)\right]^{-\alpha(p+p_1)+1}\right.$$
$$\left.\times[(1-u)(1-u')]^{-\alpha(M^2)-2}\left(\frac{1-u-u'}{(1-u)(1-u')}\right)^{-\alpha_2(0)-1}[-p_3{}^2(1-u-u')-s_1u-s_2u']^{\alpha_1(0)}\right\} . \tag{5}$$

Notice that we have distinguished different s variables so as to be able to make the correct analytic continuation. [Furthermore, this will turn out to be useful in discussing later the addition of a (ut) term to T_n.] For negative $p_3{}^2$, s_1, and s_2 the last bracket is always positive and therefore we do not get any imaginary part. For $p_3{}^2$ positive and small (s_1 and s_2 still negative), the region of integration R that contributes to the imaginary part is

$$(p_3{}^2-s_1)u+(p_3{}^2-s_2)u'-p_3{}^2 \leqslant 0, \quad u \geqslant 0, \quad u' \geqslant 0.$$

The discontinuity is therefore equal to[8]

$$2i\sin\pi\alpha(0)\,\Gamma(-\alpha_1(0))\iint_R du\,du'\left[\left(1-\frac{1}{u}\right)\left(1-\frac{1}{u'}\right)\right]^{\alpha(p+p_1)+1}[(1-u)(1-u')]^{-\alpha(M^2)-2}$$
$$\times\left[\frac{1-u-u'}{(1-u)(1-u')}\right]^{-\alpha_2(0)-1}[p_3{}^2(1-u-u')+us_1+u's_2]^{\alpha_1(0)} . \tag{6}$$

To continue this expression analytically, we make still another change of variables so that the range of integration does not change:

$$\bar{u} = \frac{p_3{}^2-s_1}{p_3{}^2}\,u = \lambda_1 u, \quad \bar{u}' = \frac{p_3{}^2-s_2}{p_3{}^2}\,u' = \lambda_2 u' . \tag{7}$$

We finally obtain

$$p_0 s\, d\sigma/d^3p = \frac{G}{\pi\Gamma(\alpha_1(0)+1)}(p_3{}^2)^{\alpha_1(0)}(\lambda_1\lambda_2)^{-1}\int_0^1 d\bar{u}'\int_0^{1-\bar{u}'} d\bar{u}\left[\left(1-\frac{\lambda_1}{\bar{u}}\right)\left(1-\frac{\lambda_2}{\bar{u}'}\right)\right]^{\alpha(p+p_1)+1}$$
$$\times\left[\left(1-\frac{\bar{u}}{\lambda_1}\right)\left(1-\frac{\bar{u}'}{\lambda_2}\right)\right]^{-\alpha(M^2)-2}\left[\frac{\lambda_1\lambda_2-\lambda_2\bar{u}-\lambda_1\bar{u}'}{(\lambda_1-\bar{u})(\lambda_2-\bar{u}')}\right]^{-\alpha_2(0)-1}(1-\bar{u}-\bar{u}')^{\alpha_1(0)}. \tag{8}$$

TABLE I. Limit of Mandelstam invariants in the regions of (a) limiting fragmentation and (b) pionization.

	$(p+p_1)^2$	$(p+p_2)^2$	$(p_1+p_2)^2$	$(p+p_1+p_2)^2 = M^{*2}$
(a)	$\frac{1-x}{x}(M^2x-m^2)-\frac{p_\perp{}^2}{x}$	$-xs$	s	$(1-x)s+O(\sqrt{s})$
(b)	$-[(p_\parallel{}^2+p_\perp{}^2+m^2)^{1/2}-p_\parallel]\sqrt{s}$	$-[(p_\parallel{}^2+p_\perp{}^2+m^2)^{1/2}+p_\parallel]\sqrt{s}$	s	$s-2(m^2+p_\parallel{}^2+p_\perp{}^2)^{1/2}\sqrt{s}$

We see that we will obtain a limiting distribution if $\alpha_1(0)=1$.[9] From now on, for the applications, we will suppose that $\alpha_1(0)=1$, $\alpha(M^2)=0$ and, unless there is ambiguity in the definition, we will write $\lambda_1=\lambda_2=\lambda$.

Formula (8) can be understood as the residue of the Pomeranchukon. As such we expect it to have duality properties.[10] In fact, by choosing variables $x=u/(1-u)$ and $y=u'/(1-u')$ we can write it down as a five-point dual amplitude. However, a crucial difference is that now the range of integration is limited by the lines $x=0$, $y=0$ and the hyperbola

$$\frac{(1-x)(1-x')}{1-xx'}=\frac{-s}{p_3^{\,2}-s}. \tag{9}$$

The right-hand side of this equation can be easily seen to be the product of all lines that are dual to the channel $p_3^{\,2}$. Therefore the net effect of (9) is that these lines do not reach zero on the boundary. This is of course consistent with duality. We would like to suggest that these features are more general than our derivation and are rather insensitive to the way we treat the Pomeranchukon.

III. BEHAVIOR IN TRANSVERSE MOMENTUM AND PIONIZATION

Let us now find out the asymptotic behavior in $p_\perp^{\,2}$ [$p_\parallel^{\,2}\gg p_\perp^{\,2}\gg(\alpha')^{-1}$]. We have to consider in (8) the region $u\approx u'\approx\frac{1}{2}$. Then using standard methods we find

$$p_0 d\sigma/d^3p=\frac{G}{128\sqrt{\pi}}(1-x^2)^3\left[\frac{4x}{(x+1)^2}\right]^{-\alpha_2(0)+1}\times p_\perp^{\,-5}\exp\left[-2\frac{p_\perp^{\,2}}{x}\ln\left(\frac{1+x}{1-x}\right)\right]. \tag{10}$$

This formula will have a finite limit for $x\to 0$ (pionization) if again $\alpha_2(0)=1$.[11] Now the other Pomeranchukon trajectory becomes relevant, and the residue can be depicted as shown in Fig. 3.

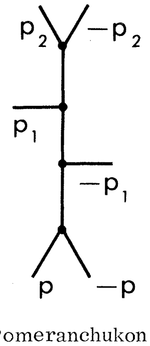

FIG. 3. Double Pomeranchukon exchange contributing to pionization from the end of the chain.

For small (but not wee[4]) x, this expression demonstrates the experimentally observed factorization of the dependence in $p_\parallel^{\,2}$ and $p_\perp^{\,2}$ [in the form (1)]. In fact for x varying between 0 and 0.62 the expression $(1/2x)\ln[(1+x)/(1-x)]$ changes from 1 to 1.17. Comparing this formula with the experimental fit of Bali *et al.*,[12] it is obvious that in general the damping predicted is too strong. Therefore for large $p_\perp^{\,2}$ we expect important corrections from other diagrams. In the subasymptotic region, perhaps a better formula is obtained if we replace $p_\perp^{\,2}$ by $[\alpha(t)+1]x$. Then, because of the factor $p_\perp^{\,-5}$, factorization will be broken and we will not have a universal $p_\perp^{\,2}$ dependence. (However, $\langle p_\perp\rangle$ will change slowly, from around 140 MeV for pions to 500 MeV for very large masses.)

For $x\to 1$ we should recover Regge behavior. In fact we obtain[5]

$$p_0 d\sigma/d^3p=\frac{G[\Gamma(-\alpha(t))]^2}{2\pi\Gamma(2-2\alpha(t))}\left(\frac{s}{p_3^{\,2}}\right)^{2\alpha(t)}p_3^{\,2}. \tag{11}$$

The limiting distribution in the c.m. system is also easily calculated [$\alpha_2(0)=1$]; we obtain

$$p_0 d\sigma/d^3p=\frac{G}{\pi}\int_0^1 du\int_0^{1-u}du'\left(\frac{2E}{\sqrt{s}}+u+u'\right)^{-2}\times(u+u'-1)\exp\left[-2E(E-p_\parallel)\left(\frac{1}{u}+\frac{1}{u'}\right)\right], \tag{12}$$

where

$$E=(p_\parallel^{\,2}+p_\perp^{\,2}+m^2)^{1/2}.$$

Notice that the asymptotic behavior in $p_\perp^{\,2}$ ($p_\perp^{\,2}\gg p_\parallel^{\,2}, \alpha'^{\,-1}$) is steeper than before (it goes like $e^{-8p_\perp^{\,2}}$).

Finally we want to discuss the case where the T_n also contain (tu) and (us) terms. When we square them, we will find both diagonal (st, st) and nondiagonal (st, ut) contributions. Furthermore, the excited missing tower of particles can have both signatures, and therefore we have to consider the possibility of joining with a twist. In this way one can count up to 12 different six-point functions. However, a quick inspection shows that most of them go to zero exponentially, so that in fact for fixed $x>0$, we only have to consider the following permutations:

$$B_6(pp_1p_2-p_2-p_1-p),$$
$$B_6(pp_1p_2-p_2-p-p_1),$$
$$B_6(p_1pp_2-p_2pp_1). \tag{13}$$

Formula (8) remains unchanged if the definitions for λ_1 and λ_2 are modified accordingly (replacing s by u).

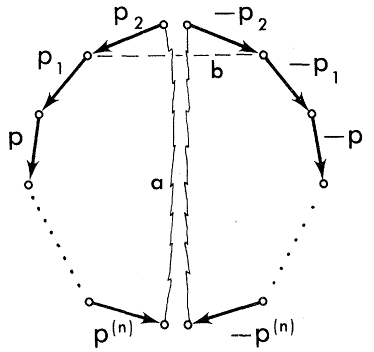

FIG. 4. Generalization to n detected particles.

For pionization we have to add those diagrams obtained from interchanging the momenta of the initial and final particles. The calculations follow the same pattern as for the considered example.

IV. GENERALIZATION TO N DETECTED PARTICLES; DISCUSSION

From the duality properties that we have discussed in the form (8), we can easily guess an expression for the differential cross section for detecting N particles with definite p_L in an inclusive reaction.

We consider the dual diagram shown in Fig. 4. The broken line a indicates that we have already taken the imaginary part in that channel. Furthermore, we are interested in the asymptotic behavior in $(p_2+p_1)^2=s$ and $(p_2+p_1+p+p'+\cdots p^{(n)})^2=M^{*2}$; the only contribution will come from the Regge trajectory b. To find the residue of this Regge pole, we write a formula which has all the factors of the corresponding $(2n+1)$-point function. Choosing as independent variables lines that do not cross a, the region of integration R will turn out to be bounded by

$$\begin{aligned} &u_i=0, \quad i=1\cdots n \\ &u_i'=0, \quad i=1\cdots n \qquad\qquad (14)\\ &g=\pi u_{ii'}=s/(s-M^{*2}), \end{aligned}$$

where g = product of all lines that cross a.

In our derivation we made use of a kind of generalized optical theorem. In general it is true that through crossing and unitarity, the differential cross section for inclusive experiments can be derived by taking the imaginary part of an amplitude. The paradoxical situation is that we have chosen a non-unitary amplitude to do that. Therefore, we should not try to analyze in detail the final states of the model. We should rather consider it as some kind of Born approximation (but for the cross section) and then add loops so as to get a totally unitary amplitude. Then for each different topological dual surface[13] (specified by ϵ = orientability, p = number of tori or number of projective planes, and w = number of windows and ways of distributing external lines among them), there will be an infinite class of diagrams obtained by just adding loops to the original one. The addition of loops can be argued away by saying that it will just renormalize the trajectories, and then in average we will recover the term without loops. (However, see below for a discussion on this point.) That is not the case for the different topological surfaces. For example, if we do not add the diagram shown in Fig. 5 (particles emitted in the middle of the chain), we predict precisely zero for the production of forward π^+ in π^-p collisions (except for exotic states).[14] We have calculated some higher-order diagrams, such as the ones shown in Fig. 6,[15] to leading order in $\ln s$. The conclusions are that if the model for the Pomeranchukon is a factorizable one [for instance a Regge pole or the so-called Pomeranchukon of Fig. 6(b)[16]], then the formula obtained is essentially the same.

Finally, we would like to mention a surprising theoretical consequence. The limiting temperature of Fubini and Veneziano[17] does not seem to play any role in the distribution in $p_\perp$. This could mean one of two things: (a) Thermodynamic equilibrium is not reached during the collision. The identification of the limiting temperature with Hagedorn's fit would then be pure coincidence. This is ugly but possible. (b) Our model does not describe thermodynamic equilibrium, which could be connected

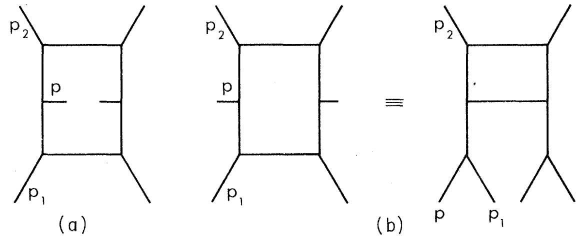

FIG. 5. Examples of diagrams to be added.

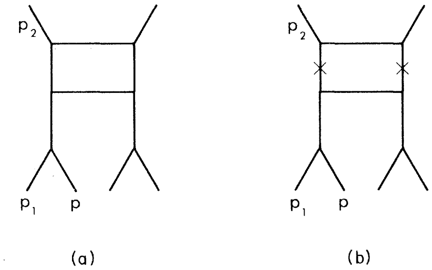

FIG. 6. Other models for a factorizable Pomeranchukon. These diagrams give essentially Eq. (8) (in leading order).

with the possibility of final-state interactions. But this means that the sum of all planar-loop diagrams should show thermodynamic equilibrium and will change their behavior in an essential way, i.e., even on the average, it could not be equal to the Born approximation. The other possibility – that planar diagrams can not account for thermodynamic equilibrium while nonplanar diagrams can – seems to be totally unreasonable. Therefore, it would be interesting to consider planar-loop corrections at large $p_\perp^2$ and see if they tend to give less damping.

Added note: M. Suzuki has mentioned to me the interesting possibility that by combining two or more differential cross sections, one can eliminate the contribution of one or both Pomeranchukons. Our method will be more applicable to the combination than to each of the differential cross sections.

Note Added in Proof

In a recently circulated preprint, DeTar *et al.*[18] have pointed out that there is some arbitrariness in the calculation of contributions coming from diagrams like $B_6(pp_1p_2-p_2-p_1-p)$. They consider phases in all invariants, and they choose them in such a way that all oscillating factors damp to zero. Our results are obtained with a different prescription, which we believe is correct *if* we are going to add all other loop diagrams before comparing with experiment, i.e., if we take the Veneziano amplitude as a Born term. However, we know that as a *model* to be compared with experiment, dual amplitudes should be averaged before calculating the cross section. In that case the prescription of DeTar *et al.* should be preferred. Accordingly a semiphenomenological factor $e^{k\alpha(t)}$ should be multiplied in (8) and (10) and (11) and a corresponding $e^{-k'\alpha(s)}$ in Eq. (12). [Therefore $B_6(pp_1p_2-p_2-p_1-p)$ does not contribute to pionization.[18,19]]

The asymptotic behavior in $p_\perp^2$ is still given by a formula like (10) (without any phenomenological cutoff) but originating in terms like $B_6(p_1pp_2-p_2-p-p_1)$.

ACKNOWLEDGMENTS

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[4]S. D. Drell, in *Proceedings of the International Conference on Expectations for Particle Reactions at the New Accelerators* (Physics Department, University of Wisconsin, Madison, Wis., 1970).

[5]In the context of an approximation scheme, this trick was used by L. N. Chang, P. G. O. Freund, and Y. Nambu, Phys. Rev. Letters 24, 628 (1970), to obtain Eq. (11) of the present paper. I thank K. Bardakci for calling my attention to this work.

[6]On the other hand, we are not claiming that our formula will describe diffractive dissociation, i.e., those cases where the Regge trajectory referred to in (iii) is the Pomeranchukon itself.

[7]There will, of course, be a limiting distribution in the target frame, too. This is related by crossing symmetry, and therefore from now on we assume $x \geq 0$.

[8]We are supposing that there is no Stokes's phenomenon.

[9]Compare with multiperipheral models, Ref. 3.

[10]Compare with H. Satz and K. Schilling, CERN Reports No. 1127 and 1148 (unpublished).

[11]Something similar is concluded in multiperipheral models, Ref. 3. Notice, however, that the way the second Pomeranchukon appears is completely different.

[12]N. Bali, L. S. Brown, R. D. Peccei, and A. Pignotti, Phys. Rev. Letters 25, 557 (1970), and Phys. Letters 33B, 175 (1970).

[13]K. Kikkawa, S. Klein, B. Sakita, and M. A. Virasoro, Phys. Rev. D 1, 3258 (1970).

[14]The multiperipheral model needs this diagram for pionization. This is not the case in this model. I thank A. Pignotti for interesting conversations concerning this point. In general, notice that the concept "emitted in the middle of the chain" is not duality-invariant [Fig. 5(b)].

[15]We have renormalized these diagrams following A. Neveu and J. Scherk, Phys. Rev. D 1, 2355 (1970), and we have assumed that we could take the limit on the negative axis and then rotate.

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[19]In this way we agree with Mueller's analysis. A. H. Mueller, Phys. Rev. D 2, 2963 (1970).