Study of Chiral $SU(2)\otimes SU(2)$ Current-Algebra Models. II. Phenomenological Lagrangians*

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(Received 1 February 1971)

The hard-pion effective Lagrangians of Arnowitt *et al.* are derived from a Lagrangian invariant under local chiral gauge transformations of the Yang-Mills type, except for a vector-meson mass term, which leads to the field-current identity, and a σ term, which leads to partial conservation of axial-vector current. The effect of other chiral-invariant terms, which break the local gauge invariance, is discussed.

I. INTRODUCTION

The hard-pion current-algebra technique[1-4] was invented in order to imbed the content of current algebra[5] and partial conservation of axial-vector current (PCAC) in a phenomenological theory of vertex functions and scattering amplitudes. In the formulation of Schnitzer and Weinberg,[1] single-particle dominance of N-point functions is assumed, and certain postulates concerning the smoothness of the so-called "primitive" N-point functions (which involve only the spin-1 particles) are introduced. The corresponding N-point functions which involve the pion field are then deduced using PCAC and generalized Ward identities. In a previous work,[6] we examined this formulation and found that other models of this type, with *a priori* equally plausible smoothness assumptions, could be introduced.

Arnowitt *et al.*,[2-4] on the other hand, formulate the method in terms of an effective Lagrangian, to be used for calculations only in the tree approximation, which incorporates the single-particle-dominance assumption in the field-current identities[7,8]

$$V_\mu^A = g_V \rho_\mu^A, \tag{1}$$

$$A_\mu^A = g_A a_\mu^A - F_\pi \partial_\mu \phi_\pi^A, \tag{2}$$

where V_μ^A and A_μ^A denote the isotopic vector and axial-vector currents, ρ_μ^A, a_μ^A, and ϕ_π^A are (renormalized) field operators for the ρ meson, A_1 meson, and pion, respectively, g_V and g_A are parameters, and F_π is the pion decay constant. The interaction terms in the Lagrangian are constrained by the requirement that the current-algebra, conservation-of-vector-current (CVC), and PCAC conditions are satisfied with the currents *defined* by Eqs. (1) and (2) – included is the requirement that the commutators between time and space components of the currents contain only c-number Schwinger terms. Additional assumptions, e.g., concerning the structure of the σ commutator

$$[A_0^A, \phi_\pi^B] \equiv i\sigma_{AB}, \tag{3}$$

are also conveniently expressed in this formalism.

Other authors[9-11] have introduced phenomenological Lagrangians which are chiral-symmetric apart from certain well-defined symmetry-breaking terms, which lead to predictions similar (but not identical) to those of the hard-pion technique. In particular, the field-current identity is satisfied by constructing a Lagrangian invariant under local chiral gauge transformations of the Yang-Mills type,[12] except for a mass term for the gauge fields. This leads to the field-current identity in the well-known way,[7,8,11,13] and the resulting currents satisfy a current algebra with c-number Schwinger terms (also known as an algebra of fields[8]). The chiral symmetry is broken by a σ term as in the original model of Gell-Mann and Lévy[14]; this guarantees PCAC.

We show here that this procedure, properly generalized, leads to the general answer to the demands of hard-pion current algebra.[15] To be precise, the hard-pion current-algebra effective Lagrangian can be obtained from a Lagrangian

$$\mathfrak{L} = \mathfrak{L}_0 + \tfrac{1}{2} m_0^2 (B_\mu^A B_\mu^A + B_{5\mu}^A B_{5\mu}^A) + F_\pi m_\pi^2 \sigma, \tag{4}$$

where $\mathfrak{L}_0$ is invariant under local chiral gauge transformations, B_μ^A, $B_{5\mu}^A$ are the vector and axial-vector gauge fields, and σ is an isoscalar spin-0 field, whose commutator with the axial charge Q_5^A is given by

$$[Q_5^A, \sigma] = -i\phi_\pi^A \tag{5}$$

to ensure PCAC.

The Lagrangian $\mathfrak{L}_0$ can be quite complicated. We construct here the complete Lagrangian required to evaluate the two-point and three-point functions associated with chiral $SU(2)\otimes SU(2)$, and indicate how to proceed further. This construction is simpler than that of Arnowitt *et al.*[2-4]; it also provides a basis for understanding possible simplifying assumptions. It can also be generalized to

$SU(3)\otimes SU(3)$ [or $U(3)\otimes U(3)$], where nonlinear realizations[11] of the algebra may be relevant (although there now seem to be enough 0^+ mesons to complete a nonet), and, in any case, there are a number of open questions.[15]

An additional chiral-invariant term, which breaks the local gauge invariance (and hence the field-current identity), can be added to the Lagrangian; the coupling parameter in this term corresponds to the extra parameter in the "linear models" considered in our previous work.[6] The presence of this term may be observable experimentally, as indicated below. This provides a serious test of the field-current identities.

II. CHIRAL LAGRANGIANS, FIELD-CURRENT IDENTITY, AND PCAC

Consider a group of local gauge transformations

$$U=\exp[-i\alpha_A(x)Q^A] \tag{6}$$

generated by charges Q^A which satisfy commutation rules

$$[Q^A, Q^B]=if_{ABC}Q^C, \tag{7}$$

where, for the present, axial charges are included in the Q^A without special notation. Under infinitesimal transformations, ordinary fields transform according to[16]

$$\Phi_b(x)\to\Phi_b(x)+iK^A_{bc}\,\alpha_A(x)\Phi_c(x), \tag{8}$$

where the matrices $\underline{K}^A=(K^A_{bc})$ form a representation (in general reducible) of the algebra of the generators,

$$[\underline{K}^A, \underline{K}^B]=if_{ABC}\underline{K}^C, \tag{9}$$

vector gauge fields $B^A_\mu(x)$ transform according to

$$B^A_\mu(x)\to B^A_\mu(x)-f_{ABC}\alpha_B(x)B^C_\mu(x)+g^{-1}\partial_\mu\alpha_A(x), \tag{10}$$

with g an arbitrary parameter. The corresponding field strengths $F^A_{\mu\nu}(x)$, defined by

$$F^A_{\mu\nu}(x)=\partial_\mu B^A_\nu(x)-\partial_\nu B^A_\mu(x)+gf_{ABC}B^B_\mu(x)B^C_\nu(x), \tag{11}$$

transform simply according to

$$F^A_{\mu\nu}(x)\to F^A_{\mu\nu}(x)-f_{ABC}\alpha_B(x)F^C_{\mu\nu}(x). \tag{12}$$

The covariant derivatives of the ordinary fields, defined by

$$D_\mu\phi_b(x)\equiv\partial_\mu\phi_b(x)-igK^A_{bc}B^A_\mu(x)\phi_c(x), \tag{13}$$

also transform simply, according to

$$D_\mu\phi_b(x)\to D_\mu\phi_b(x)+iK^A_{bc}\alpha_A(x)D_\mu\phi_c(x). \tag{14}$$

Consider a Lagrangian

$$\mathfrak{L}=\mathfrak{L}(B^A_\mu, F^A_{\mu\nu}, \phi_b, D_\mu\phi_b) \tag{15}$$

describing these fields (specifically excluded is the appearance of $\partial_\mu B^A_\mu$ in the Lagrangian). The standard currents $J^A(x)$ are defined by

$$J^A_\mu(x)=-iK^A_{bc}\frac{\delta\mathfrak{L}}{\delta[\partial_\mu\phi_b(x)]}\phi_c(x) - f_{ABC}\frac{\delta\mathfrak{L}}{\delta[\partial_\mu B^B_\nu(x)]}B^C_\nu(x). \tag{16}$$

These currents generate the charges

$$Q^A(t)=\int_{x_0=t}J^A_0(\vec{x}, t)\,d^3x \tag{17}$$

and, as a consequence of the canonical equal-time commutation relations, the time components satisfy the local current algebra

$$[J^A_0(\vec{x}), J^B_0(\vec{y})]=if_{ABC}J^C_0(\vec{x})\delta(\vec{x}-\vec{y}). \tag{18}$$

Currents $j^A_\mu(x)$ can also be defined, according to the prescription of Gell-Mann and Lévy,[14] as the response of the Lagrangian to a local gauge transformation,

$$j^A_\mu(x)=-\frac{\delta\mathfrak{L}}{\delta[\partial_\mu\alpha_A(x)]}. \tag{19}$$

To understand the relation between these currents, consider the Lagrangian first as a function of the fields and their ordinary derivatives, and then as a function of B^A_μ, $F^A_{\mu\nu}$, ϕ_b, $D_\mu\phi_b$. Denote functional differentiation of $\mathfrak{L}$ in the first instance by δ, in the second by Δ. Evidently

$$\frac{1}{2}\frac{\delta\mathfrak{L}}{\delta(\partial_\mu B^A_\nu)}=\frac{\Delta\mathfrak{L}}{\Delta F^A_{\mu\nu}}, \tag{20}$$

$$\frac{\delta\mathfrak{L}}{\delta B^A_\mu}=\frac{\Delta\mathfrak{L}}{\Delta B^A_\mu}+gJ^A_\mu, \tag{21}$$

$$\frac{\delta\mathfrak{L}}{\delta\phi_b}=\frac{\Delta\mathfrak{L}}{\Delta\phi_b}, \tag{22}$$

$$\frac{\delta\mathfrak{L}}{\delta(\partial_\mu\phi_b)}=\frac{\Delta\mathfrak{L}}{\Delta(D_\mu\phi_b)}, \tag{23}$$

where, in Eq. (21), J^A_μ is the standard current introduced in Eq. (17). The Gell-Mann–Lévy current is then given simply by

$$j^A_\mu=-\frac{1}{g}\frac{\Delta\mathfrak{L}}{\Delta B^A_\mu}. \tag{24}$$

The equations of motion for the gauge fields can be written then as

$$2\partial_\mu\frac{\Delta\mathfrak{L}}{\Delta F^A_{\mu\nu}}+gj^A_\mu=gJ^A_\mu. \tag{25}$$

The time components of these equations involve, apart from j^A_0, only canonical variables, and a careful manipulation of δ functions shows that the

$j_0^A(x)$ satisfy the same local equal-time algebra as the $J_0^A(x)$,

$$[j_0^A(\vec{x}), j_0^B(\vec{y})] = if_{ABC} j_0^C(\vec{x})\delta(\vec{x}-\vec{y}). \tag{26}$$

The field-current identity requires

$$\frac{\Delta \mathcal{L}}{\Delta B_\mu^A} = m_0{}^2 B_\mu^A, \tag{27}$$

whence the only explicit dependence of the Lagrangian on the gauge fields B_μ^A must be contained in a mass term,[17]

$$\mathcal{L}_1 = \tfrac{1}{2} m_0{}^2 B_\mu^A B_\mu^A. \tag{28}$$

If this is the case, then also the commutators of the $j_0^A(x)$ with the space components $j_k^A(x)$ at equal times contain only c-number Schwinger terms, and the space components commute with each other.[8,18]

In order to break chiral symmetry, the Lagrangian must contain a term $\mathcal{L}_{\rm SB}$ (which depends only on $F_{\mu\nu}^A$, ϕ_b, and $D_\mu\phi_b$); PCAC requires

$$[Q_5^A, \mathcal{L}_{\rm SB}] = -iF_\pi m_\pi{}^2 \phi_\pi^A, \tag{29}$$

where axial charges and a pion field are now introduced explicitly. On the other hand,

$$[Q_5^A, \phi_\pi^B] = i\sigma\delta_{AB} + \ldots, \tag{30}$$

where the omitted terms have isotopic spin 2. Once it is assumed that these terms are not present, then the identification

$$\mathcal{L}_{\rm SB} = F_\pi m_\pi{}^2 \sigma, \tag{31}$$

with $\langle {\rm vac}|\sigma|{\rm vac}\rangle = F_\pi$, can always be made within the framework of single-particle dominance, since the matrix element of σ between the vacuum and a single-particle state can be chosen at will by adjusting parameters in the symmetric part of the Lagrangian.

The decomposition of the axial-vector gauge field into a term proportional to an A_1 field, describing a spin-1^+ particle, and a term proportional to the derivative of the pion field follows in a natural way from the structure of the Lagrangian, as shown in Sec. III.

III. GENERAL CHIRAL $SU(2)\otimes SU(2)$ LAGRANGIAN

We begin with spin-0 fields π^A and s which transform under chiral $SU(2)\otimes SU(2)$ according to the $(\frac{1}{2}, \frac{1}{2})$ representation,

$$[Q^A, \pi^B] = i\epsilon_{ABC}\pi^C, \tag{32}$$

$$[Q_5^A, \pi^B] = -is\delta_{AB}, \tag{33}$$

$$[Q^A, s] = 0, \tag{34}$$

$$[Q_5^A, s] = i\pi^A, \tag{35}$$

with chiral-invariant

$$Q \equiv \pi^A\pi^A + ss \tag{36}$$

and a set of vector and axial-vector gauge fields B_μ^A, $B_{5\mu}^A$ with covariant fields

$$F_{\mu\nu}^A = \partial_\mu B_\nu^A - \partial_\nu B_\mu^A + g\epsilon_{ABC}(B_\mu^B B_\nu^C + B_{5\mu}^A B_{5\nu}^B), \tag{37}$$

$$F_{5\mu\nu}^A = \partial_\mu B_{5\nu}^A - \partial_\nu B_{5\mu}^A + g\epsilon_{ABC}(B_{5\mu}^B B_\nu^C + B_\mu^B B_{5\nu}^C). \tag{38}$$

(The relation of these fields to the particles will become clear presently.) The covariant derivatives of the spin-0 fields are given by[10,11]

$$\Delta_\mu \pi^A = \partial_\mu \pi^A + g\epsilon_{ABC} B_\mu^B \pi^C - gB_{5\mu}^A s, \tag{39}$$

$$\Delta_\mu s = \partial_\mu s + gB_{5\mu}^A \pi^A. \tag{40}$$

In order to satisfy the field-current identities and PCAC, the Lagrangian describing these fields must have the form

$$\mathcal{L} = \mathcal{L}_0 + \tfrac{1}{2} m_0{}^2 (B_\mu^A B_\mu^A + B_{5\mu}^A B_{5\mu}^A) + \tilde{F} m_\pi{}^2 s, \tag{41}$$

where $\mathcal{L}_0$ is invariant under local chiral gauge transformations. $\mathcal{L}_0$ has the general structure

$$\begin{aligned}\mathcal{L}_0 = &-\tfrac{1}{4}(F_{\mu\nu}^A F_{\mu\nu}^A + F_{5\mu\nu}^A F_{5\mu\nu}^A)(1+r_1 Q) - \tfrac{1}{4} r_2[(F_{\mu\nu}^A F_{\mu\nu}^B - F_{5\mu\nu}^A F_{5\mu\nu}^B)\pi^A\pi^B - F_{\mu\nu}^A F_{\mu\nu}^A \pi^B\pi^B - 2\epsilon_{ABC} F_{\mu\nu}^A F_{5\mu\nu}^B \pi^C s - F_{5\mu\nu}^A F_{5\mu\nu}^A ss]\\ &+ \tfrac{1}{2}[(\Delta_\mu \pi^A)^2 + (\Delta_\mu s)^2](1+\lambda_1 Q) + \tfrac{1}{2}\lambda_2(\pi^A \Delta_\mu \pi^A + s\Delta_\mu s)^2 - \tfrac{1}{2}\mu^2 Q + \tfrac{1}{4}\lambda Q^2\\ &+ \kappa(\epsilon_{ABC} F_{\mu\nu}^A \Delta_\mu \pi^B \Delta_\nu \pi^C + 2F_{5\mu\nu}^A \Delta_\mu \pi^A \Delta_\nu s)(1+\alpha_1 Q)\\ &+ \kappa\alpha_2[\epsilon_{ABC} F_{\mu\nu}^A \Delta_\mu \pi^B \pi^C + F_{5\mu\nu}^A(\Delta_\mu \pi^A s - \pi^A \Delta_\mu s)](\pi^D \Delta_\nu \pi^D + s\Delta_\nu s) + h\epsilon_{ABC}(F_{\mu\tau}^A F_{\tau\nu}^B + 3F_{5\mu\tau}^A F_{5\tau\nu}^B)F_{\mu\nu}^C(1+\beta_1 Q)\\ &+ h\beta_2[\epsilon_{ABC} F_{\mu\nu}^A (F_{\nu\lambda}^B F_{\lambda\mu}^D + 2F_{5\nu\lambda}^B F_{5\lambda\mu}^D)\pi^C\pi^D + 2(F_{\mu\nu}^A F_{\nu\lambda}^B + F_{5\mu\nu}^A F_{5\nu\lambda}^B)F_{5\lambda\mu}^A \pi^B s - \epsilon_{ABC} F_{\mu\nu}^A F_{5\nu\lambda}^B F_{5\lambda\mu}^C ss] + \cdots,\end{aligned} \tag{42}$$

where Q is the invariant introduced above in Eq. (36).

We have included all terms which give rise to effective two-field and three-field couplings even in the presence of symmetry breaking, when

$$\langle \text{vac}|s|\text{vac}\rangle \equiv s_0 \neq 0 \tag{43}$$

(and we have retained the appropriate number of terms in the expansion of the arbitrary functions of the invariant Q, which can appear as factors multiplying the various couplings). Additional terms for four-field and higher couplings can be included in a straightforward way, using the isomorphism between $SU(2)\otimes SU(2)$ and $SO(4)$ [and, in particular, the correspondences between (π^A, s) and components of a four-vector, and between $(F^A_{\mu\nu}, F^A_{5\mu\nu})$ and components of an antisymmetric second-rank tensor] to generate chiral-invariant couplings. In general, it is necessary to include terms which involve the product of $N+2$ fields, two of which are from the spin-0 multiplet, in order to generate all the effective N-field couplings in the presence of symmetry breaking.

To interpret the fields in terms of particles, introduce renormalized fields ϕ^A_π and $\bar{\sigma}$ for the pion and the σ meson, ρ^A_μ and a^A_μ for the ρ and A_1 mesons, with

$$\pi^A = Z_\pi^{1/2}\phi^A_\pi, \tag{44}$$

$$s = s_0 + Z_\sigma^{1/2}\bar{\sigma}, \tag{45}$$

$$B^A_\mu = Z_V^{1/2}\rho^A_\mu, \tag{46}$$

$$B^A_{5\mu} = Z_A^{1/2}a^A_\mu + \eta D_\mu \phi^A_\pi, \tag{47}$$

where

$$D_\mu \phi^A_\pi \equiv \partial_\mu \phi^A_\pi + g\epsilon_{ABC}B^B_\mu \phi^C_\pi . \tag{48}$$

[This definition of a^A_μ and the one implicit in Eq. (2) differ only by a point transformation.] Let

$$\rho^A_{\mu\nu} \equiv \partial_\mu \rho^A_\nu - \partial_\nu \rho^A_\mu, \tag{49}$$

$$a^A_{\mu\nu} \equiv \partial_\mu a^A_\nu - \partial_\nu a^A_\mu . \tag{50}$$

The parameters appearing here are determined by (i) elimination of terms linear in $\bar{\sigma}$ from the Lagrangian, which requires

$$\tilde{F} m_\pi^2 = (\mu^2 - \lambda s_0^2)s_0, \tag{51}$$

and (ii) reduction of the kinetic energy terms to the standard form

$$\mathcal{L}_{\text{kin}} = -\tfrac{1}{4}(\rho^A_{\mu\nu}\rho^A_{\mu\nu} + a^A_{\mu\nu}a^A_{\mu\nu}) + \tfrac{1}{2}(\partial_\mu \phi^A_\pi \partial_\mu \phi^A_\pi + \partial_\mu \bar{\sigma}\partial_\mu \bar{\sigma}), \tag{52}$$

which requires

$$Z_V^{-1} = 1 + r_1 s_0^2, \tag{53}$$

$$Z_A^{-1} = 1 + (r_1 - r_2)s_0^2, \tag{54}$$

$$Z_\sigma^{-1} = 1 + (\lambda_1 + \lambda_2)s_0^2, \tag{55}$$

$$Z_\pi^{-1} = (1 + \lambda_1 s_0^2)(1 - \eta g s_0)^2 + \eta^2 m_0^2, \tag{56}$$

and, in order to eliminate a term proportional to $a^A_\mu \partial_\mu \phi^A_\pi$,

$$\eta = \frac{(1 + \lambda_1 s_0^2)g s_0}{m_0^2 + (1 + \lambda_1 s_0^2)g^2 s_0^2} . \tag{57}$$

The particle masses are then given by

$$m_\rho^2 = Z_V m_0^2, \tag{58}$$

$$m_A^2 = Z_A[m_0^2 + (1 + \lambda_1 s_0^2)g^2 s_0^2], \tag{59}$$

$$m_\pi^2 = Z_\pi(\mu^2 - \lambda s_0^2), \tag{60}$$

$$m_\sigma^2 = Z_\sigma(\mu^2 - 3\lambda s_0^2), \tag{61}$$

and the constants g_V, g_A in the field-current identity are given by

$$g_V = -Z_V^{1/2} m_0^2/g, \tag{62}$$

$$g_A = -Z_A^{1/2} m_0^2/g, \tag{63}$$

whence $(g_A/g_V)^2 = Z_A/Z_V$. Since the parameter r_2 is arbitrary, (g_A/g_V) is arbitrary, so $g_A = g_V$ is *not* a general requirement of chiral Lagrangian models.

The pion decay constant is given by

$$F_\pi^2 = \frac{g_V^2}{m_\rho^2} - \frac{g_A^2}{m_A^2} \tag{64}$$

(Weinberg's first sum rule[19]). Then also

$$s_0 = Z_\pi^{1/2} F_\pi \tag{65}$$

and

$$\eta = \frac{1}{g s_0}\left(1 - \frac{m_\rho^2}{g_V^2}F_\pi^2\right) . \tag{66}$$

The three-point functions for this Lagrangian reduce to those given by Arnowitt *et al.*[2] We remark that the ρ-π-π and A_1-ρ-π vertices depend only on the single parameter

$$\bar{\kappa} \equiv -Z_\pi Z_V^{1/2} g_V (1 + \alpha_1 s_0^2)(1 - \eta g s_0)^2 \kappa \tag{67}$$

in addition to g_A, g_V, F_π, and the particle masses. This parameter is related to the parameter ξ of Ref. 6 by

$$2\bar{\kappa} = \frac{g_A^2}{m_A^2 F_\pi^2}\frac{m_\rho^2}{m_A^2}\left(\frac{g_A^2}{g_V^2} + 1 + \xi\right) . \tag{68}$$

The ρ-ρ-ρ and A_1-A_1-ρ vertices contain additional parameters (with some relations between them); the vertices involving the σ are similarly undetermined.

IV. DISCUSSION

The general Lagrangian $\mathcal{L}_0$ leads to a rather limited set of predictions; hence it is desirable to look for simplifying assumptions which restrict the parameters. The experimental data[20] on the colliding-beam reaction

$$e^+ + e^- \to \pi^+ + \pi^-$$

do not permit $\kappa = 0$, but it may be possible to eliminate the remaining anomalous couplings (so that $\lambda_1=\lambda_2=0$, $r_1'=r_2=0$, $\alpha_1=\alpha_2=0$, $\beta_1=\beta_2=0$), since $g_A=g_V$ (or $r_2=0$) is consistent with the analysis of Ref. 6, in view of the limitations both of the approximation and of the present experimental data on A_1 decay. The remaining parameters are not well determined by present data.

An alternative attempt at simplification is to insist on the absence of effective couplings of the type

$$\epsilon_{ABC}\rho^A_{\mu\nu}\partial_\mu\phi^B_\pi\partial_\mu\phi^C_\pi$$

and

$$a^A_{\mu\nu}\partial_\mu\phi^A_\pi\partial_\nu\overline{\sigma},$$

which lead to form factors which have undesirable behavior at large momentum transfer. It is debatable whether such restrictions on phenomenological Lagrangians, intended for use at low momentum transfer, are really warranted, but the absence of the first coupling is quite consistent with experiment,[20] and the A_1 decay seems to proceed through the ρ-π channel.

There are two derivative couplings of σ-π-π, and to eliminate both of them, it is necessary that

$$F_\pi^{\,2} < g_V^{\,2}/2m_\rho^{\,2}.$$

This is a strong restriction (the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation gives equality here), but not necessarily wrong. A weaker assumption, that the σ-π-π vertex function is independent of the off-shell σ mass, is consistent with a σ width ~ 200–300 MeV. However, in the absence of deeper theoretical motivation for this type of assumption, we defer further speculation along these lines.

It is interesting to consider the effect of adding to the Lagrangian of Eqs. (41) and (42) a term

$$\mathcal{L}'=f[\epsilon_{ABC}B^A_\mu\pi^B\Delta_\mu\pi^C+B^A_{5\mu}(\pi^A\Delta_\mu s-s\Delta_\mu\pi^A)], \qquad (69)$$

which is chiral-invariant, but not invariant under the local gauge transformations. Evidently the Gell-Mann–Lévy currents are given by

$$j^A_\mu=-\frac{m_0^{\,2}}{g}B^A_\mu-\frac{f}{g}\epsilon_{ABC}\pi^B\Delta_\mu\pi^C, \qquad (70)$$

$$j^A_{5\mu}=-\frac{m_0^{\,2}}{g}B^A_{5\mu}-\frac{f}{g}(\pi^A\Delta_\mu s-s\Delta_\mu\pi^A). \qquad (71)$$

The renormalization and diagonalization can be carried through as before, except that now

$$m_A^{\,2}=Z_A[m_0^{\,2}+(1+\lambda_1 s_0^{\,2})g^2s_0^{\,2}+2fgs_0^{\,2}], \qquad (72)$$

$$Z_\pi^{\,-1}=(1+\lambda_1 s_0^{\,2})(1-\eta g s_0)^2 -2\eta f s_0(1-\eta g s_0)+\eta^2 m_0^{\,2}, \qquad (73)$$

and the parameter η is given by

$$\eta=\frac{(1+\lambda_1 s_0^{\,2})gs_0+fs_0}{m_0^{\,2}+(1+\lambda_1 s_0^{\,2})g^2s_0^{\,2}+2fgs_0^{\,2}}. \qquad (74)$$

Consistency of the tree approximation requires that the coefficient of $\partial_\mu\phi^A_\pi$ in $j^A_{5\mu}$ be $-F_\pi$; explicit calculation shows this to be the case.

The currents of Eqs. (70) and (71) satisfy the local current algebra at equal times, and the space components of the currents commute; however, the Schwinger terms in the commutators of space and time components are no longer c-numbers. Weinberg's first spectral-function sum rule[19] cannot be verified explicitly, but if it is true, then the Ward identities of Refs. 1 and 6 will still be valid, and the smoothness assumptions appropriate to the linear models of Ref. 6 will be satisfied in the tree approximation; the extra parameter appearing in those models is related to the coupling constant f in Eq. (69). Tests for the absence or presence of such a coupling are necessary to validate the generalized Yang-Mills approach to chiral symmetry.[21,22]

ACKNOWLEDGMENTS

It is a pleasure to thank H. Pietschmann for his gracious hospitality at the University of Vienna, and R. Arnowitt for interesting discussions.

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*Supported in part by the National Science Foundation under Grant No. GP-12396, and by "Fonds zur Förderung der wissenschaftlichen Forschung."

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[16]We exclude here the possibility of nonlinear realizations. See Refs. 10 and 11 for a discussion of these.

[17]The argument given here is essentially that of Lee and Zumino (Ref. 13).

[18]It is an open question which of the currents j_μ^A or J_μ^A generate the electromagnetic and weak interactions. Only the j_μ^A lead to the appropriate single-particle poles in form factors when the Lagrangian is used in the tree approximation, but closed-loop diagrams must be considered in any event to treat the form factors for timelike momenta.

[19]S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

[20]The data are analyzed in M. T. Vaughn, Lett. Nuovo Cimento 2, 851 (1969); see also Ref. 6.

[21]The colliding-beam reaction $e^+ + e^- \to A_1^\pm + \pi^\pm$ is a promising test [see Ref. 6 and also M. T. Vaughn and P. J. Polito, Lett. Nuovo Cimento 1, 74 (1971)], although it involves a rather long extrapolation off the mass shell of the ρ.

[22]*Note added in proof.* After the completion of this work, our attention was drawn to the work of V. Ogievetsky and B. M. Zupnik [Nucl. Phys. B24, 612 (1970)], who consider a general chiral Lagrangian for the nonlinear σ model and propose a restriction on derivative couplings which corresponds to the smoothness assumptions of the standard hard-pion method (Refs. 1 and 2). This restriction does not eliminate the new parameters which appear when the σ field is an independent field.

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K_{l3}-Decay Form Factors and Asymptotic $SU(3)$ Symmetry*

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(Received 26 January 1971)

A relation between the K_{l3}-decay form factors and the pion electromagnetic form factor is derived by using only the hypothesis of asymptotic $SU(3)$ symmetry. This relation enables us to predict the complete behavior of the K_{l3}-decay form factors, once the behavior of the pion electromagnetic form factor is given. For the choice of the dipole formula, $(1-t^2/m_\rho^2)^{-2}$, for the pion form factor, we predict the following for the parameters of the K_{l3}-decay form factors: $\xi(0)\simeq -0.53$, $\lambda_+\simeq 0.023$, and $\lambda_-\simeq 0.0010$. If we extrapolate the result from the physical K_{l3}-decay region $0\le t\le (m_K - m_\pi)^2$ to include the region $0\le t\le (m_K+m_\pi)^2$, the presence of a dip in the K_{l3}-decay scalar form factor in this region is indicated. The extrapolated form factor satisfies approximately the soft-pion constraint given at $t=m_K^2$. The effect of particle mixing between the usual pseudoscalar nonet and the possible higher-lying pseudoscalar mesons is not considered.

I. INTRODUCTION

In the past years the K_{l3}-decay form factors have been intensively discussed.[1] We write the form factors with the obvious notation $[V_\mu^{K^-}(x) = V_\mu^4(x) - iV_\mu^5(x)]$

$$\langle \pi^0(\vec{p})|V_\mu^{K^-}(0)|K^+(\vec{k})\rangle = \frac{1}{(4p_0k_0)^{1/2}}\frac{1}{\sqrt{2}}[f_+(t)(k+p)_\mu + f_-(t)(k-p)_\mu]. \tag{1.1}$$

The ξ parameter is usually defined by

$$\xi(t) = f_-(t)/f_+(t),$$

where $t=-(k-p)^2$. The *scalar* form factor, $f(t)$, is defined by

$$f(t) = f_+(t) + \left(\frac{t}{m_K^2 - m_\pi^2}\right) f_-(t). \tag{1.2}$$

The usual parametrization of the $f_\pm(t)$ in the physical region is given by

$$f_\pm(t) = f_\pm(0)[1+\lambda_\pm t/m_\pi^2].$$

In terms of this scalar form factor the well-known soft-pion prediction[2] is written as $f(m_K^2) \simeq f_K/f_\pi$. Here f_K and f_π denote the form factors of the $K\to\mu+\bar{\nu}$ and $\pi\to\mu+\bar{\nu}$ decays. This prediction may be subject to some error due to the soft-pion limit involved. The slope of $f(t)$ at $t=0$ is given by