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Off-Mass-Shell Veneziano-Type Amplitudes and π, K Form Factors

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The Veneziano-type amplitudes are constructed for the off-mass-shell scalar and vector amplitudes for the π-K system. These amplitudes are consistent with the constraints due to the current algebra and the model of Gell-Mann, Oakes, and Renner, and contain no free parameters. The soft-meson limits of these amplitudes are the π and K electromagnetic form factors, and the K_{l3} and K_{l4} form factors. All these form factors agree with the experiments except that the ratio $f_-(0)/f_+(0)$ remains small in the K_{l3} form factors.

I. INTRODUCTION AND SUMMARY

Recently, several authors[1-4] have discussed whether the dual-resonance model of the Veneziano type[5-7] could be made consistent with the current algebra[8] and the model of Gell-Mann, Oakes, and Renner.[9] For the π-K system, McKay *et al.*[1] constructed an amplitude which contains three satellite terms and is consistent with the current-algebra constraints,[10,11] using the recipe due to Lovelace[6] to extrapolate off the mass shell. However, it is more natural[12] in the dual-resonance model that the off-mass-shell amplitude include all the series of poles that correspond to the pseudoscalar daughters. This is in fact the case in the amplitude proposed by Csikor.[3] In our previous study,[4] we constructed an amplitude of the Csikor type which is consistent with the model of Gell-Mann, Oakes, and Renner[9] and also with current algebra,[10,11] with only one satellite term added.[13]

The purpose of the present paper is to extend our previous study[4] to the amplitude in which all the external pions and kaons are set off the mass shell. We construct in Sec. II an amplitude of this type which is consistent with current algebra[10,11] and the model of Gell-Mann, Oakes, and Renner.[9] The resultant amplitude is given by Eq. (5).

The purpose of the present paper is also to extend in Sec. II the above construction to the vector amplitude given by (7) in which an axial-vector current appears in addition to the divergences of axial-vector currents. This construction of the vector amplitude is based upon the relation (9) which relates this vector amplitude to the scalar amplitude discussed in Sec. II. This relation assumes that all the terms in (4) are individually covariant. We can then deduce the vector amplitude using the Csikor recipe[3] in the form of (12) which is suitable[12,14] in the case of spin 1. The result is given by (11)–(13). This vector amplitude contains no free parameters.

The significance of these vector amplitudes for the π-K system is that their soft-meson limits are the π and K electromagnetic form factors and the K_{l3} and K_{l4} form factors, which are discussed in Sec. IV. The main features of these form factors can be summarized as follows:

(1) The electromagnetic form factors satisfy the normalization conditions (17), without an explicit use of the conservation of the electromagnetic current. These form factors are well approximated by

$$f_\pi(\nu)=f_K^V(\nu)\cong 3f_K^S(\nu)\cong\Gamma(\tfrac{1}{2}+\delta+\nu)/\Gamma(\tfrac{1}{2}+\delta)\,, \quad (1)$$

when $|\nu|\lesssim 1$, where ν is the momentum transfer squared times the universal slope of meson trajectories, which is chosen here as $1/2(m_\rho^2-m_\pi^2)$, and $\delta=m_\pi^2/2(m_\rho^2-m_\pi^2)$. In (1), V and S denote the isovector and isoscalar components, respectively. The error involved in (1) is at most a few percent at $|\nu|\simeq 1$. The charge radii are $r_\pi=0.604$ fm, $r_{K^+}=0.493$ fm, and $r_{K^0}=0.349$ fm, whereas the experiment[15] reports $r_\pi=0.70\pm0.20$ fm.

(2) The K_{l3} form factors are given by (20) and (21), which are very smooth functions of ν in spite of their appearance. We find $f_+(0)=0.977$, $f_-(0)=-0.008$, and $\lambda_+=0.021$. The experiment[16] indicates $f_-(0)/f_+(0)=-0.31\pm0.0074$.

(3) The K_{l4} form factors are given by (23)–(25). For the process $K^-\to\pi^++\pi^-+e+\nu$, we find $|F_1|=|F_2|=6.38$ and $|F_3|=1.97$ at the point where the lepton pair assumes the maximum energy, whereas we find $|F_1|=|F_2|=8.12$ and $|F_3|=0$ at the minimum point. The experimental data[17] appear to in-

dicate that $|F_1|=5.7\pm0.4$ and $|F_2|=7.5\pm1.1$, assuming $f_+(0)\sin\theta_C=0.21\pm0.01$.[16]

In summary, all the above form factors are consistent with the experiments except that $f_-(0)/f_+(0)$ remains small for the K_{l3} form factors.

II. SCALAR AMPLITUDE

Let us consider the off-mass-shell amplitude for the process

$$\pi^+(k)+K^-(p)\to\pi^+(-q)+K^-(-p'),$$

which is defined by

$$G(k,q,p,p')=\iiint dxdydz\, e^{-ikx-iqy-ipz}\langle 0|T\{\partial A^a(x),\partial A^b(y),\partial A^c(z),\partial A^d(0)\}|0\rangle, \tag{2}$$

where a, b, c, and d denote π^-, π^+, K^+, and K^-, respectively, and the four-momenta satisfy $k+q+p+p'=0$. We assume that the amplitude (2) is a scalar. The scattering amplitude on the mass shell can be obtained by taking the limits with respect to all the four-momenta, such as

$$\lim_{k^2\to m_a{}^2}\left(\frac{\sqrt{2}\,i}{F_a}\right)\left(\frac{m_a{}^2-k^2}{m_a{}^2}\right)G(k,q,p,p'), \tag{3}$$

where $\langle 0|\partial A^a|a\rangle=m_a{}^2F_a/\sqrt{2}$. The scalar amplitude (2) has the soft-meson limit given by

$$\begin{aligned}G(k,q,p,0)=&-\iint dxdy\, e^{-ikx-iqy}\langle 0|T\{[F^d,\partial A^c(0)],\partial A^a(x),\partial A^b(y)\}|0\rangle\\&-\iint dxdz\, e^{-ikx-ipz}\langle 0|T\{[F^d,\partial A^b(0)],\partial A^a(x),\partial A^c(z)\}|0\rangle\\&-\iint dydz\, e^{-iqy-ipz}\langle 0|T\{[F^d,\partial A^a(0)],\partial A^b(y),\partial A^c(z)\}|0\rangle,\end{aligned} \tag{4}$$

where F^d is the space integral of $A_0^d(0,\vec{x})$. If we assume, as usual, that the equal-time commutators in (4) belong to $1\oplus 8$, the last integral vanishes in (4). We assume that all the terms are individually scalars in (4).

The Veneziano-type expression for the scalar amplitude (2) that we study in this paper is given by

$$\begin{aligned}G(k,q,p,p')=&\,g[\Gamma(-\alpha_\pi(q^2))+\Gamma(-\alpha_K(p^2))][\Gamma(-\alpha_\pi(k^2))+\Gamma(-\alpha_K(p'^2))]\Gamma(-\alpha_\pi(q^2)-\alpha_K(p^2))\Gamma(-\alpha_\pi(k^2)-\alpha_K(p'^2))\\&\times\{B_1^{11}(s,t)+Q[\alpha_\pi(q^2)+\alpha_K(p^2)][\alpha_\pi(k^2)+\alpha_K(p'^2)]B_2^{11}(s,t)\},\end{aligned} \tag{5}$$

with

$$B_N^{11}(s,t)=\Gamma(1-\alpha_{K^*}(s))\Gamma(1-\alpha_\rho(t))/\Gamma(N-\alpha_{K^*}(s)-\alpha_\rho(t)),$$

where s, t, and u are $(k+p)^2$, $(k+q)^2$, and $(k+p')^2$, respectively, all the trajectories are linear with the universal slope $\alpha'=1/2(m_\rho^2-m_\pi^2)$, and g and Q are the parameters.

The first factor of (5), which consists of two Γ functions, is necessary because (5) must reduce as $p'\to 0$ to the first two integrals in (4). The second factor of (5), which consists also of two Γ functions, is necessary because of the limit of (5) as either $p\to 0$ or $q\to 0$. The subsequent two factors of (5), each of which consists of one Γ function, are needed to ensure that (5) exhibits the necessary poles as all the four-momenta approach the mass shell. The rest of (5) consists of the simplest Veneziano-type amplitude and a satellite term which is needed[4] to make (5) consistent with the Adler-Weisberger relation.[11] The specific satellite term in (5) is required to make (5) compatible with the model of Gell-Mann, Oakes, and Renner.[9] In particular, it is necessary that this satellite term vanish on the mass shell corresponding to π and K. We add that (5) is consistent with the Adler condition[10] since the relations

$$\begin{aligned}&\alpha_\rho(s)=\tfrac{1}{2}+\alpha_\pi(s),\\&\alpha_{K^*}(s)=\tfrac{1}{2}+\alpha_K(s),\\&\alpha_\varphi(s)=\tfrac{1}{2}+2\alpha_K(s)-\alpha_\pi(s)\end{aligned} \tag{6}$$

are assumed for the trajectories.

The amplitudes for the processes $\pi\pi\to\pi\pi$ and $KK\to KK$ have similar expressions. These amplitudes satisfy crossing symmetry even off the mass shell if we symmetrize or antisymmetrize them with respect to s, t, or u only. The resulting scattering amplitudes on the mass shell contain eight parameters which are the three g's and three Q's, and also F_π and F_K that will come out via (3) and a similar expression for K. As was shown in our previous work,[4] the factorization and the Adler-Weisberger relation[11] give seven conditions on these eight parameters. Therefore, they are all determined except for the over-all factors which are also fixed since the pion decay rate determines F_π as 93 MeV. Thus, the above amplitudes are completely determined.

III. VECTOR AMPLITUDE

We discuss here the Veneziano-type expression for the amplitude defined by

$$G_\mu(k,q,p,p') = \iiint dxdydz\, e^{-ikx-iqy-ipz}\langle 0|T\{\partial A^a(x), \partial A^b(y), \partial A^c(z), A^d_\mu(0)\}|0\rangle\,, \tag{7}$$

which describes the process $\pi^+(k)+K^-(p)\to\pi^+(-q)+K_A^-(-p')$ when all the four-momenta are on the mass shell. We assume that the amplitude (7) is a vector. This vector amplitude is related to the scalar amplitude by

$$p'_\mu G^\mu = -iG - i\iiint dxdydz\, e^{-ikx-iqy-ip'z}\delta(z_0)\langle 0|T\{[A^d_0(z), \partial A^c(0)], \partial A^a(x), \partial A^b(y)\}|0\rangle$$

$$-i\iiint dxdydz\, e^{-ikx-ipy-ip'z}\delta(z_0)\langle 0|T\{[A^d_0(z), \partial A^b(0)], \partial A^a(x), \partial A^c(y)\}|0\rangle\,, \tag{8}$$

where the term that contains $[A^d_0(z), \partial A^a(0)]$ has been dropped for the reason mentioned earlier concerning the soft-meson limit (4). Since the two integrals in (8) contain $\delta(z_0)$, they become independent of p' in the frame where $\vec{p}\,'=0$. Moreover, in this frame these integrals are exactly of the forms of the first two integrals in (4). Therefore, as long as the two integrals in (4) are individually scalars, we can infer that

$$p'_\mu G^\mu(k,q,p,p') = iG(k,q,p,p') + iG_1(k,q,-k-q,0) + iG_2(k,-k-p,p,0)\,, \tag{9}$$

where G_1 and G_2 stand, respectively, for the first two integrals in (4). As $p'\to 0$, the right-hand side of (9) vanishes, as it should, in spite of the fact that the individual terms do not necessarily vanish.

For our purpose of evaluating the π,K form factors, we have only to find the Veneziano-type expression for the vector amplitude in which two of the mesons are on the mass shell. We denote it by $G_\mu(k,q|p,p')$, where the mesons with the four-momenta k and q are on the mass shell. When the Veneziano-type expression (5) is assumed for G, the relation (9) becomes

$$p'_\mu G^\mu(k,q|p,p') = (2ig/\alpha'^2 F_\pi{}^2 M_\pi{}^4)\{\Gamma(-\alpha_K(p'^2))\Gamma(-\alpha_K(p^2))[B_1^{11}(s,t) + Q\alpha_K(p'^2)\alpha_K(p^2)B_2^{11}(s,t)]$$

$$-\Gamma(-\alpha_K(0))\Gamma(1-\alpha_\rho(t))\Gamma(1-\alpha_{K^*}(m_\pi{}^2))(1-Q\alpha_K(0))\}\,. \tag{10}$$

To find the Veneziano-type expression for G_μ that satisfies (10), we first write G_μ as

$$G_\mu(k,q|p,p') = \frac{2ig}{\alpha' F_\pi{}^2 M_\pi{}^4}[\{2A^1_{\mu\nu}k^\nu + 2B^1_{\mu\nu}q^\nu + C^1 p_\mu{}'\} + Q\{2A^2_{\mu\nu}k^\nu + 2B^2_{\mu\nu}q^\nu + C^2 p'_\mu\}]\,, \tag{11}$$

where the first and second curly brackets correspond to the simplest Veneziano term and a satellite term, respectively [cf. Eq. (5)]. We require for simplicity that the p'^2 dependence of $A^N_{\mu\nu}$ and $B^N_{\mu\nu}$ in (11) with $N=1$ and 2 be given entirely by

$$D^N_{\mu\nu}(p') = \sum_{n=N}^{\infty}\frac{\beta^N_n}{p'^2-m_n{}^2}\left(g_{\mu\nu} - \frac{p'_\mu p'_\nu}{m_n{}^2}\right), \qquad \sum_{n=N}^{\infty}\frac{\beta^N_n}{p'^2-m_n{}^2} = \Gamma(N-\alpha_K(p'^2))\,. \tag{12}$$

The factor (12) is the modification of the Csikor recipe[3] which is suitable[12,14] in the case of spin 1. The Veneziano-type expression for G_μ is then uniquely determined as

$$\begin{aligned}
A^N_{\mu\nu} &= [N-1-\alpha_K(0)]^{-1}\Gamma(N-1-\alpha_K(p^2))D^N_{\mu\nu}(p')B^{11}_N(s,t)\Psi^N_K(p^2, 2kp')\,,\\
B^N_{\mu\nu} &= [N-1-\alpha_K(0)]^{-1}D^N_{\mu\nu}(p')\Gamma(1-\alpha_{K^*}(s))\Psi^1_{K^*}(m_\pi{}^2+2qp', 2qp')\,,\\
C^N &= \Gamma(N-1-\alpha_K(p^2))\Gamma(N-1-\alpha_K(p'^2))B^{11}_N(s,t)\Psi^N_K(p'^2, p'^2)\\
&\quad + \Gamma(N-1-\alpha_K(0))\Gamma(1-\alpha_\rho(t))\Gamma(1-\alpha_{K^*}(m_\pi{}^2))\Psi^1_{K^*}(m_\pi{}^2+2qp', -p'^2)\,,
\end{aligned} \tag{13}$$

where

$$\Psi_a^N(x,y)=\left[1-\frac{\Gamma(N-1-\alpha_a(x)+\alpha'y)}{\Gamma(N-1-\alpha_a(x))}\right]\Big/\alpha'y\,.$$

The above Veneziano-type amplitude contains no free parameter. This amplitude is consistent with the current algebra since it satisfies (9), which is due to the current algebra. It is also consistent with the model of Gell-Mann, Oakes, and Renner since all the scalar amplitudes in (9) are consistent with this model. The above construction on the Veneziano-type amplitude can easily be extended to all the vector amplitudes of the π-K system.

IV. FORM FACTORS

The current algebra implies that the soft-meson limits of the vector amplitudes defined by (7) and (11) are nothing but the π, K form factors. The electromagnetic form factors are given by

$$f_\pi(\nu)(k-q)_\mu=G_\mu\begin{pmatrix}\pi^- & \pi^+ & \big| & \pi^+ & \pi^- \\ k & q & \big| & 0 & p'\end{pmatrix}, \quad (14a)$$

$$=2G_\mu\begin{pmatrix}\pi^- & \pi^+ & \big| & K^+ & K^- \\ k & q & \big| & 0 & p'\end{pmatrix}, \quad (14b)$$

$$f_K^V(\nu)(k-q)_\mu=2G_\mu\begin{pmatrix}K^- & K^+ & \big| & \pi^+ & \pi^- \\ k & q & \big| & 0 & p'\end{pmatrix} \quad (14c)$$

$$=2\left[G_\mu\begin{pmatrix}K^+ & K^- & \big| & K^+ & K^- \\ k & q & \big| & 0 & p'\end{pmatrix} - G_\mu\begin{pmatrix}K^+ & K^- & \big| & K^0 & \bar K^0 \\ k & q & \big| & 0 & p'\end{pmatrix}\right], \quad (14d)$$

$$f_K^S(\nu)(k-q)_\mu=\tfrac{1}{3}\left[2G_\mu\begin{pmatrix}K^+ & K^- & \big| & K^+ & K^- \\ k & q & \big| & 0 & p'\end{pmatrix} + G_\mu\begin{pmatrix}K^+ & K^- & \big| & K^0 & \bar K^0 \\ k & q & \big| & 0 & p'\end{pmatrix}\right], \quad (14e)$$

where ν is the variable in (1), and V and S denote, respectively, the isovector and isoscalar components; G_μ is given by (7) and (11) except that the four-momenta belong to three mesons which are specified explicitly. Thus, the Veneziano-type amplitudes in the previous sections lead to the following form factors:

$$\begin{aligned}{}^\pi f_\pi(\nu)&={}^\pi f_K^V(\nu)=\bar g F_\pi^{\,2}E(\delta,0;\nu)\,,\\ {}^K f_\pi(\nu)&={}^K f_K^V(\nu)=\bar g F_K^{\,2}E(\Delta,D;\nu)\,,\\ f_K^S(\nu)&=\tfrac{1}{3}\bar g F_K^{\,2}[2E(\Delta,-D;\nu)+E(\Delta,D;\nu)]\,,\end{aligned}\quad (15)$$

with

$$\begin{aligned}E(x,y;\nu)&=x\Gamma(\tfrac{1}{2}+y)\Gamma(\tfrac{1}{2}+x-y-\nu)\\ &\times\frac{1}{\nu}\bigg[\Gamma(1+x-\nu)\left(2-\frac{\Gamma(x)}{\Gamma(x-\nu)}-\frac{\Gamma(\frac{1}{2}+y)}{\Gamma(\frac{1}{2}+y+\nu)}\right)\\ &+Q\left(\frac{x}{1+x}\right)\Gamma(2+x-\nu)\left(2-\frac{\Gamma(1+x)}{\Gamma(1+x-\nu)}-\frac{\Gamma(\frac{1}{2}+y)}{\Gamma(\frac{1}{2}+y+\nu)}\right)\bigg],\end{aligned}\quad (16)$$

where

$$\bar g=4ig/\alpha'^3F_\pi^{\,2}m_\pi^{\,4}F_K^{\,2}m_K^{\,4}\,,\quad \alpha'=1/2(m_\rho^{\,2}-m_\pi^{\,2})\,,$$

$$\delta=\alpha' m_\pi^{\,2}\,,\quad \Delta=\alpha' m_K^{\,2}\,,\quad D=\Delta-\delta\,,$$

and the left-hand superscript π or K implies whether (14a) and (14c), or (14b) and (14d), are used to evaluate the form factors.

Our first remark is that the soft-meson limits (14a)–(14e) do not contain terms proportional to $(k+q)_\mu$ and the above form factors satisfy the correct normalization conditions,

$${}^\pi f_\pi(0)={}^K f_\pi(0)={}^\pi f_K^V(0)={}^K f_K^V(0)=f_K^S(0)=1\,. \quad (17)$$

These are usually the consequences of the conservation of the electromagnetic current, which is, however, not used explicitly in the above derivation. In our case, the above consequences follow because our Veneziano-type amplitude satisfies (9), which is due to the current algebra.

Our second remark concerns the two form factors given by (14a) and (14b), respectively. The ratio of ${}^K f_\pi(\nu)$ to ${}^\pi f_\pi(\nu)$ is unity at $\nu=0$, but varies very slowly to ~0.97 as $-\nu$ becomes ~1. Of course, this ratio must be unity for all ν in order for the evaluation of the form factors to be self-consistent. We can attribute this small inconsistency to our choice of the factor $D_{\mu\nu}^N(p')$ given by (12), since we can eliminate it by replacing $D_{\mu\nu}^N(p')$ by $D_{\mu\nu}^N(p')/R(p'^2)$, where $R(\nu)$ is above ratio of the two form factors. In other words, this small breakdown of consistency is not a serious one.

In spite of their appearances, the above form factors are very smooth functions of ν and are well approximated by (1). The π and K charge radii given in Sec. I are computed in terms of

$$r=\left(6\alpha'\left|\frac{\partial f(\nu)}{\partial\nu}\right|_{\nu=0}\right)^{1/2}. \quad (18)$$

The K_{l3} form factors, which are given by

$$2G_\mu\begin{pmatrix}\pi^- & K^- & \big| & \pi^+ & K^- \\ k & q & \big| & 0 & p'\end{pmatrix}=f_+(\nu)(k-q)_\mu+f_-(\nu)p'_\mu\,, \quad (19)$$

assume the expressions

$$f_+(\nu)=\bar{g}F_\pi F_K\delta\Gamma(\tfrac{1}{2})\Gamma(\tfrac{1}{2}+\Delta-\nu)\{\Gamma(1+\Delta-\nu)\times[\Psi(\Delta-\nu,\nu-D)-\Psi\tfrac{1}{2}+D,-\nu-D)]+Q[\Delta/(1+\Delta)][\cdots]\} \tag{20}$$

and

$$\begin{aligned}f_-(\nu)=\bar{g}F_\pi F_K\delta\Gamma(\tfrac{1}{2})\Gamma(\tfrac{1}{2}+\Delta-\nu)\{&-\Gamma(1+\Delta)\Psi(\Delta-\nu,\nu-D)\\ &\times[1+D\Psi(1+\Delta,-\nu)]+\Gamma(1+\Delta)\Psi(\tfrac{1}{2}+\nu,-\nu-D)\\ &\times[1-D\Psi(1+\Delta,-\nu)]+\Delta\Gamma(\delta)\Psi(\Delta-\nu,\nu)\\ &+\Gamma(1+\Delta)\Gamma(\tfrac{1}{2}-D)\Psi(\tfrac{1}{2}+\nu,-\nu)/\Gamma(\tfrac{1}{2})\\ &+Q[\Delta/(1+\Delta)][\cdots]\},\end{aligned} \tag{21}$$

where

$$\Psi(x,y)=\left[1-\frac{\Gamma(x+y)}{\Gamma(x)}\right]\Big/ y$$

and the square brackets with dots inside stand for all the preceding terms, except the Q terms, in the curly brackets, with Δ replaced by $1+\Delta$ in (20) and (21).

The K_{l4} form factors for the process $K^m(p)\to\pi^a(-k)+\pi^b(-q)+l+\nu$ are given by

$$\begin{aligned}G_\mu(k,q,p|p')&=\langle\pi^a(-k),\pi^b(-q)|A^l_\mu(0)|K^m(p)\rangle\\ &=i(1/\sqrt{2}m_K)[(k+q)_\mu F_1\\ &\quad+(k-q)_\mu F_2+p'_\mu F_3],\end{aligned} \tag{22}$$

where a,b and l,m are the isospin indices of the pion and kaon, respectively, and the F's can be decomposed as

$$\begin{aligned}F_1&=A_S\delta_{ab}\delta_{lm}-iA_A\epsilon_{abc}(\tau_c)_{lm},\\ F_2&=A'_A\delta_{ab}\delta_{lm}-iA'_S\epsilon_{abc}(\tau_c)_{lm},\\ F_3&=B_S\delta_{ab}\delta_{lm}-iB_A\epsilon_{abc}(\tau_c)_{lm}.\end{aligned} \tag{23}$$

In (23), those form factors with the subscript S or A are, respectively, symmetric or antisymmetric under the interchange of k and q. Our Veneziano-type amplitudes yield

$$\begin{aligned}A_S&=A'_S=\tfrac{1}{2}[S^2(s,t)+S^2(u,t)],\\ A_A&=A'_A=\tfrac{1}{2}[S^2(s,t)-S^2(u,t)],\\ B_S&=\tfrac{1}{2}[S^1(s,t)+S^1(u,t)],\\ B_A&=\tfrac{1}{2}[S^1(s,t)-S^1(u,t)],\end{aligned} \tag{24}$$

where

$$S^N(s,t)=\bar{g}m_KF_K\Gamma(\Delta-\nu+N-1)B^{11}_N(s,t). \tag{25}$$

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