Eikonal Model for Large-Angle np and pp Elastic Scattering*

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An eikonal model is described for large-angle np and pp elastic scattering at medium energies. Each amplitude is parametrized as a sum of one-boson exchanges modulated by the multiple virtual emission of soft neutral $I=0$ and $I=1$ vector mesons. Fits to the data are presented.

A MODEL for large-angle np and pp elastic scattering at medium energies is presented, based upon the field-theoretic damping mechanism provided by the exchange of an arbitrary number of soft, virtual, neutral, vector mesons (SVNVM) between all pairs of external nucleon lines in each process. The model is an extension of previous work by Fried and Gaisser[1] and Gaisser,[2] in which it was shown that SVNVM exchange may describe the qualitative features of the nucleon electromagnetic form factors and of pp elastic scattering at high energies and large momentum transfers. We here consider NN scattering at somewhat lower energies, between roughly 3 and 7 GeV/c (incident laboratory momentum). This region is particularly important, for it seems to mark the transition between the characteristics of low-energy and high-energy NN scattering, as suggested by the changes in the forward and backward peaking structure.[3] In this region, straightforward extensions of the methods of low-energy potential scattering are no longer suitable,[4,5] while the simplifying asymptotic features of high-energy scattering are not yet dominant.[6]

The basic idea underlying this elementary model is that a "hard" Born-approximation contribution, causing a large-angle scattering, is damped in a multiplicative way by multiple SVNVM exchange. This hard part may be regarded as being analogous to a generalized potential; but in contrast to potential scattering and some low-energy dispersion methods, unitarity is incorporated not by the iteration of the generalized potential, but rather by picturing elastic scattering as resulting from the shadow of inelastic processes, and by approximating the effect of the latter by states with an arbitrary number of massive SVNVM. Charge exchange in np scattering is assumed to arise from the hard part alone.[7]

Using the insight gained by the extensive analysis of low-energy NN data,[5] the hard part of each amplitude is written as the sum of $I=1$ and $I=0$ pseudoscalar, vector, and scalar mesons, taken to be the π, η, ρ, ω, δ, and ϵ, with masses of 140, 550, 765, 782, 960, and 700 MeV, respectively[8]; the coupling of the ϕ mesons to nucleons has been assumed small and has been neglected. The coupling strengths are considered as parameters to be determined from the data. For the SVNVM we take into account both $I=1$ and $I=0$, which may be thought of as soft ρ_0 and ω exchange. With the techniques and approximations of Ref. 1 used to sum the SVNVM exchanges, our expressions for the unpolarized np and pp differential cross sections are

$$\frac{d\sigma_{np}}{dt}=B_{np}(s,t)e^{F_{np}(s,t)},\quad \frac{d\sigma_{pp}}{dt}=B_{pp}(s,t)e^{F_{pp}(s,t)},\quad (1)$$

where

$$F_{np}(s,t)=4(\gamma_0+\gamma_1)F(t)+4(\gamma_0-\gamma_1)[F(u)-F(4m^2-s)],\quad (2)$$
$$F_{pp}(s,t)=4(\gamma_0+\gamma_1)[F(t)+F(u)-F(4m^2-s)].$$

Here

$$F(t)=1-\frac{2x+1}{[x(x+1)]^{1/2}}\ln[x^{1/2}+(x+1)^{1/2}],$$

with $x=t/4m^2$ and $s+t+u=4m^2$. The parameters γ_0 and γ_1 are proportional to the effective coupling strengths of the $I=0$ and $I=1$ SVNVM. The functions $B_{np}(s,t)$ and $B_{pp}(s,t)$ are the unpolarized differential cross sections arising from the Born terms alone,[9]

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[1] H. Fried and T. K. Gaisser, Phys. Rev. **179**, 1491 (1969).

[2] T. K. Gaisser, Phys. Rev. D **2**, 1337 (1970).

[3] J. Cox *et al.*, Phys. Rev. Letters **21**, 641 (1968); **21**, 654 (1968). We have taken the np data from these papers, and the pp data of A. R. Clyde, LRL Report No. UCRL-16275, 1966 (unpublished); C. M. Ankenbrandt *et al.*, Phys. Rev. **170**, 1223 (1968); and C. W. Akerlof *et al.*, *ibid.* **159**, 1138 (1968).

[4] What is needed here is more in the line of a relativistic impact-parameter representation, and the present work may be thought of as a crude field-theoretic version of this.

[5] For a discussion of the extensive work done on one-boson-exchange models for low-energy NN scattering, see R. Bryan, Nucl. Phys. (to be published) and references therein.

[6] For a discussion of the asymptotic features on NN scattering, see T. T. Chou and C. N. Yang, Phys. Rev. **170**, 1591 (1968), and the earlier work of T. T. Wu and C. N. Yang, *ibid.* **137**, B708 (1965). For more phenomenological work on large-angle NN scattering, see R. Serber, Phys. Rev. Letters **10**, 357 (1963); A. D. Krisch, in *Lectures in Theoretical Physics*, edited by W. E. Britten *et al.* (Gordon and Breach, New York, 1967); N. P. Chang, Phys. Rev. **175**, 1741 (1968); and D. Weingarten, National Accelerator Laboratory report (unpublished).

[7] Our results suggest that multiple exchange of soft charged mesons is less important than that of neutrals, but a firm estimate has not yet been made.

[8] See, e.g., Particle Data Group, Rev. Mod. Phys. **42**, 87 (1970).

[9] These are evaluated using well-known methods. To save space, we do not give them here. Details of these and further results of our work may be found in H. Fried and K. Raman (unpublished).

written in terms of the parameters g_π^2, $g_{1\rho}^2$, $g_{2\rho}^2$, $g_{1\omega}^2$, g_η^2, g_ϵ^2, and g_δ^2, where $g_\pi^2=(1/4\pi)g_{\pi NN}^2$, etc. The Dirac and Pauli couplings of the ρ meson are given by $g_{1\rho}$ and $g_{2\rho}$; for the ω, the Pauli coupling is known from estimates from low-energy data[5] to be much smaller than the Dirac coupling, and will be neglected. We find it convenient to use the parameters $\gamma=\gamma_0+\gamma_1$ and $\xi=\gamma_1/\gamma_0$ instead of γ_0 and γ_1.

With these formulas, we have tried to fit the absolute magnitudes of the np and pp elastic cross sections near 90° (in the c.m. system), and the angular distributions for both processes in the large-angle region (between about 60° and 120°). Our main qualitative results are as follows. (a) If only single-pion exchange is used for the Born terms, the shapes of the np and pp angular distributions may be reproduced by choosing $\gamma=2.2$–2.4 and $\xi=0.05$, but the ratio $R=(d\sigma_{np}/d\sigma_{pp})_{90°}$ is then too small. At these energies, one may obtain $R\approx 1$ with pion exchange alone if the isoscalar SVNVM dominate $(\xi\to 0)$, but this produces np angular distributions

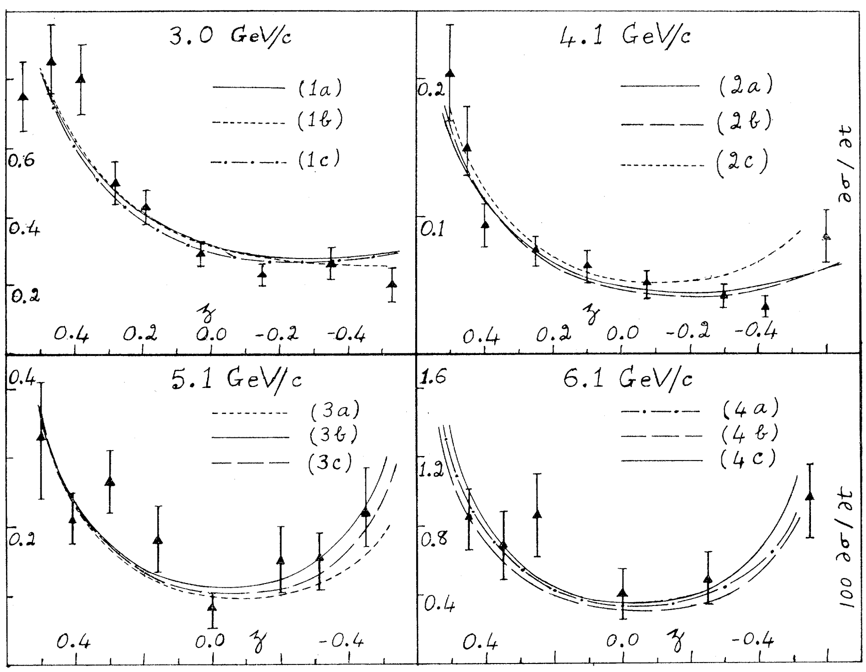

FIG. 1. np differential cross sections at 3.0, 4.1, 5.1, and 6.1 GeV/c. $\partial\sigma/\partial t$ is given here in mb/(GeV/c)^2. The parameters for the different curves are given in Table I.

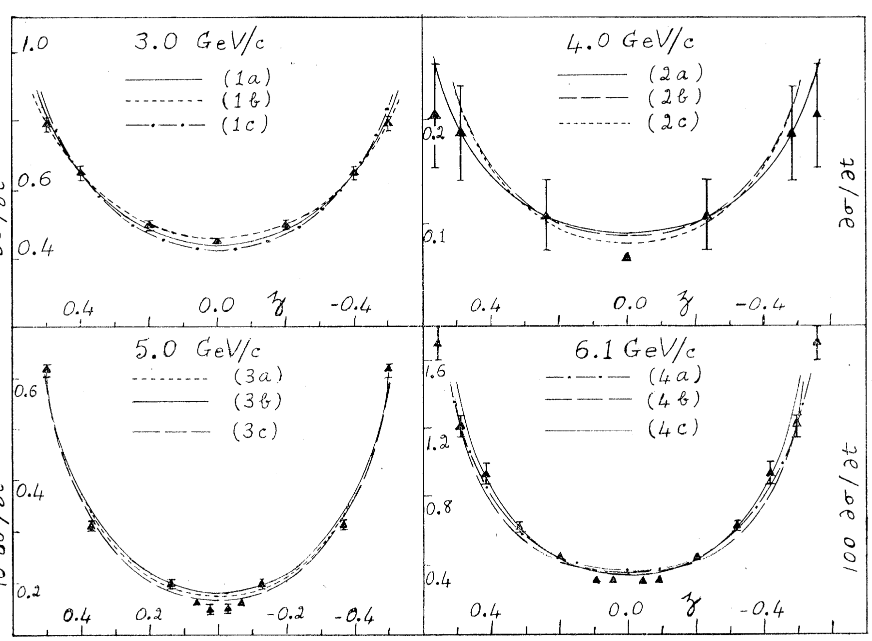

FIG. 2. pp differential cross sections at 3.0, 4.0, 5.0, and 6.1 GeV/c. $\partial\sigma/\partial t$ is given here in mb/(GeV/c)^2. The parameters for the different curves are given in Table I.

TABLE I. Parameters for the curves in Figs. 1 and 2.

Incident laboratory momentum (GeV/c)	Curve No.	γ	ξ	g_π^2	g_ϵ^2	g_δ^2	$g_{1\rho}^2$	$g_{1\omega}^2$	$g_{1\omega}^2/g_{1\rho}^2$
3.0	1*a*	2.3	0.06	5.2	2.0	1.1	0.20	0.40	2.0
	1*b*	2.3	0.06	5.8	2.2	1.0	0.22	0.18	0.8
	1*c*	2.4	0.05	5.0	1.9	1.0	0.19	0.57	3.0
4.0(pp), 4.1(np)	2*a*	2.3	0.06	5.8	2.2	1.0	0.22	0.22	1.0
	2*b*	2.3	0.05	3.8	1.47	1.47	0.15	0.85	5.8
	2*c*	2.4	0.05	5.4	2.0	0.92	0.20	0.6	3.0
5.0(pp), 5.1(np)	3*a*	2.3	0.04	4.1	0.86	0.86	0.27	0.7	2.5
	3*b*	2.4	0.04	4.6	0.0	0.96	0.48	0.48	1.0
	3*c*	2.4	0.04	4.1	0.86	0.86	0.27	0.7	2.5
6.1	4*a*	2.2	0.04	3.94	0.0	0.93	0.27	0.32	1.2
	4*b*	2.3	0.03	4.7	1.0	1.0	0.31	0.22	0.7
	4*c*	2.4	0.04	4.3	0.0	1.6	0.43	0.52	1.2

with too much backward peaking. (b) When ρ and ω contributions are included in the hard amplitude, the effective Pauli-Dirac coupling ratio of the ρ (and also ω) must be chosen very small; otherwise, R is much smaller than the observed value $R\approx 1$. Even then, it is not possible to get a good fit to the angular distributions, and we take this to indicate that there exist important Born contributions other than π, ρ, and ω. (c) The addition of an ϵ and an η contribution to the hard amplitude again tends to decrease R. To fit the data we find it necessary to include the exchange of an isovector, scalar (0^+) state, which we denote as the δ, with a suggested mass at about 960 MeV[8]; we can then fit the ratio R. The angular distributions at the lower energies require, in addition, the inclusion of an isoscalar, scalar meson which we take to be the ϵ at about 700 MeV. It turns out that the results are not sensitive to the parameters of the η exchange; for simplicity, we have here omitted the η and retained just the π, ρ, ω, δ, and ϵ hard exchanges.

The fits to the data[10] are shown in Figs. 1 and 2 and Table I. At lower energies in the range considered, the data can be approximately reproduced by the following ranges of parameters:

$$\begin{aligned} &\gamma=2.2\text{–}2.4, \quad \xi=0.05\text{–}0.06, \quad g_\pi^2=3.8\text{–}6.0, \\ &g_\delta^2=0.9\text{–}1.5, \quad g_\epsilon^2=1.5\text{–}2.3, \quad g_{1\rho}^2=0.15\text{–}0.2. \end{aligned} \tag{3}$$

At the higher energies, somewhat higher values of the ρ coupling and lower values of the ϵ coupling and of ξ are indicated: $g_{1\rho}^2=0.3$–0.5, $g_\epsilon^2=0$–1.0, and $\xi=0.03$–0.04. The large uncertainties in the np data prevent a good estimate of $g_{1\omega}^2$. The best fit for $g_{1\omega}^2$ is sensitive to the values chosen for ξ and γ; in most cases, fits can be obtained with $g_{1\omega}^2/g_{1\rho}^2=1.0$–$3.0$.

An important result indicated by these fits is that the soft vector-meson corrections are dominated by ω exchange, for the energies and momentum transfers considered here.[11] It is interesting that the $I=0/I=1$ vector-meson coupling ratio of the Born term is considerably lower than that of the SVNVM. This may be partly due to the situation that the Born part of the np charge-exchange amplitude receives a contribution from charged meson exchange, and the SVNVM do not. One may note that our values for the π, ρ, and ω couplings are smaller than those suggested by the low-energy data, an understandable effect which should occur if all vertex structure other than SVNVM exchange is omitted, as above.

We are grateful to R. Bryan for a useful discussion of one-boson-exchange models in low-energy NN scattering. During the writing of this paper, one of us (H.M.F.) was a guest at SLAC, and would like to thank S. Drell for extending the hospitality of the Theory Group.

[10] In these fits we have taken the Pauli coupling of the ρ to be negligibly small, for the reasons mentioned above. Owing to the large uncertainties in the np data at 7 GeV/c, we have not shown the curves for this energy.

[11] The parameter ξ could not have been determined from pp scattering alone. A calculation just completed by H. Goldberg at Northeastern University, using a soft model to fit the new data for single ω production in NN scattering, suggests that the SVNVM are principally ω. This is in accord with our results.

Erratum

Simple Relation between Cross Sections for Neutrino Scattering and Total Muon-Capture Rates by Nuclei, JOHN FRAZIER, C. W. KIM, AND MICHAEL RAM [Phys. Rev. D 1, 3168 (1970)]. The research of one of the authors (M. R.) was supported in part by grants from the Research Foundation of the State University of New York and from the National Science Foundation.