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Λd Scattering as a Probe of the ΛN Core Potential*

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Nonlocal separable (NLS), S-wave, spin-dependent, central potentials and a Faddeev formalism are used to solve the Λd scattering problem. The NN interaction is represented by an attractive NLS potential. Each of the ΛN interactions is represented by the sum of an attractive and a repulsive (core) NLS potential. Three different forms are used for this core potential, including no core at all. For a given ΛN spin each ΛN potential is adjusted to give the same low-energy scattering parameters, and the potentials with a core are adjusted to give the same phase shift over a wide energy range, as that given by a phenomenological local potential with a hard core. The ΛN wave functions and off-shell transition matrices are compared to see how each reflects the presence of the ΛN core potential. For incident Λ laboratory momentum in the range 100–300 MeV/c the Λd cross sections are reduced by (13–25)%, and they do depend on the off-shell behavior of the ΛN amplitude.

I. INTRODUCTION

This paper studies a nonrelativistic quantum-mechanical system of three hadrons, interacting via a sum of two-body potentials. The three-body system consists of a Λ hyperon, a neutron, and a proton. The theoretical investigation of such a multiparticle system may provide a tool for probing the two-body ΛN potential, i.e., it may yield information which cannot be obtained directly from two-body calculations and experiments. In the present work, low-energy Λ-deuteron scattering ing is investigated to determine whether Λd scattering cross sections are sensitive to short-range behavior of the ΛN interactions.

In the system being studied, each of the three particles is a spin-$\frac{1}{2}$ fermion. The Λ is taken to have mass $m_\Lambda = 1115.4$ MeV, strangeness -1, and 0 isospin. The nucleons are treated as identical particles of average mass $m_N = 938.9$ MeV, strangeness 0, and isospin $\frac{1}{2}$, coupled to form a state of 0 isospin. Since the deuteron is an isospin singlet, the three-body $\Lambda + n + p$ system is taken to be in the total isospin-0 state. The spin state of this system is either a doublet ($J = \frac{1}{2}$) or a quartet ($J = \frac{3}{2}$). The potential between each pair of particles is taken to be an S-wave, spin-dependent, central potential, so that the nucleon-nucleon (NN) potential is the $^3S_1$ potential and the ΛN potential is either a triplet, $^3S_1$, or a singlet, $^1S_0$, potential.

The ΛN interaction is, in fact, a two-channel process, $\Lambda N \to (\Lambda N, \Sigma N)$. The ΛN elastic-scattering problem below the Σ threshold (about 78 MeV), may be solved using a two-channel formalism. The Σ channel may be eliminated from the problem leaving a one-channel ΛN problem, with an effective potential that is energy-dependent and nonlocal. For energies near the Λ-channel threshold, the energy dependence of this effective potential should be negligible. For phenomenological analysis of the low-energy ΛN interaction, it has been customary to use a simple local shape for the remaining Λ potential. The result is that until recently[1] the low-energy ΛN interaction has been mostly analyzed using the same sort of potentials as were used to analyze the low-energy NN interaction. A one-channel ΛN potential is used throughout this work.

Since boson-exchange potential models have been quite successful in reproducing the extensive NN data, many authors have analyzed the hyperon-nucleon (YN) interactions by different meson-theoretic models.[2-5] The YN potential found in this manner has a number of undetermined parameters in it. The assumption of some symmetry model (e.g., SU_3), provides some relations among the coupling constants, while the rest of the parameters are treated as search parameters to reproduce the low-energy YN scattering parameters. These latter parameters have been determined (within rather wide limits) by the analysis of light hypernuclear binding energies, using a phenomenological single-channel YN potential with a hard core,[6-9] and by low-energy scattering data available at the time.

However, because of the possibility of constructing a number of different meson-theoretic potentials (by assuming different exchange mechanism and different symmetry models), which all produce the "same" Λp low-energy data, more accurate "tests" are highly desirable to eliminate all but one of these candidates for "the" ΛN potential. One obstacle to carrying out the "tests" is the arbitrary manner in which the ΛN potential is cut off at short distances, i.e., where a many-body problem is represented by just one parameter, the hard-core radius. The hard core was first introduced in NN interactions, as the simplest method of displaying our ignorance of the short-range behavior of the interactions that produced the experimental data. However, recent work on NN interactions has shown that a soft core, a supersoft core, or a nonlocal core is at least as good a representation of the true short-range behavior.[10-12] Therefore, it is desirable to have a "test" that is sensitive to the short-range behavior of the ΛN potential, i.e., an experiment that for a given meson-exchange model, and a given symmetry model, could distinguish among several meson-theoretic potentials with differing short-range behavior, but which all reproduce the same low-energy Λp scattering parameters.

The obvious way to investigate the short-range behavior of a potential is to do scattering experiments from very low up to fairly high energies. The partial-wave amplitudes with orbital angular momentum greater than zero will be insensitive to the short-range behavior. The S-wave amplitude will sense the potential over its whole range and the sensitivity to the short-range behavior of the potential increases with increase in the energy. Thus, by separating out the S-wave part of the cross section, one could see the effect of the short-range part of the potential.

There are several difficulties with the procedure just outlined above. First, the above argument assumes a local potential, when in fact there is no reason (other than convenience of the physicist) to assume a local short-range potential. Second, if the YN energy is high enough for the cross section to be sensitive to the short-range behavior of the potential, the ΣN channel is either open – thus compounding the experimental difficulties – or the energy is close to the ΣN threshold, so that the effects of this closed channel cannot be ignored in the theoretical analysis.[13,14]

These difficulties may be eliminated by limiting the YN energy range to energies that are (1) below the ΣN threshold, so that no real Σ's are present; (2) far below the ΣN threshold, so that the effect of the closed ΣN channel may be easily taken into account in the theoretical analysis (in which the YN potential is reduced to a single-channel ΛN potential); or (3) near the ΛN threshold, so that only S waves need to be used. The experimental situations which satisfy these conditions are (in order of decreasing theoretical cleanliness): (a) Λn and Λp bound states; (b) Λp and Λn low-energy scattering; (c) Λpn and Λnn bound states; (d) Λd low-energy scattering; (e) other light hypernuclear bound states; (f) low-energy Λ-nucleus scattering from light nuclei other than the deuteron. Now, the first experiment (a) is eliminated since no ΛN bound state exists. The nonexistence of free neutrons means Λn scattering is impossible and low-energy Λp scattering does not contain enough information to determine anything more than the Λp scattering lengths and effective ranges, so (b) is eliminated. The only experimentally determined ΛNN bound state is the isospin-0, spin-$\frac{1}{2}$ hypertriton, ${}_{\Lambda}H^3$, in which B_Λ, the binding energy of the Λ, is determined to be[15] 0.06 ± 0.06 MeV. This number is too small and too ill-determined to be of use in the determination of the short-range behavior of the ΛN potential. Therefore, (c) is also eliminated. Of the rest, (d) looks most promising for two reasons. First, the neglected three-body ΛNN forces, which may be significant, should have a smaller effect here than in problems with more than two nucleons present. Second, theoretical techniques have been developed in the last six years which allow, in certain cases, for the exact solution of three-particle problems.[16]

All the above is a motivation for calculating low-energy Λd scattering cross sections for several single-channel ΛN potentials which all give the same ΛN scattering lengths and effective ranges, but differ in their short-range behavior in that they give different ΛN cross sections at higher energies (which, of course, cannot be measured since the model breaks down at higher energies). If the calculated Λd cross sections are not measurably different, it would not be fruitful to do low-energy Λd scattering experiments. If they are different, useful information can be found from such experiments.

It now comes to the question of carrying out the calculations described directly above. In particular, what is the form to be used for the NN and ΛN potentials, and what kind of three-body formalism is to be used? The method employed here is a multiple-scattering formalism pioneered by Faddeev.[16] In addition, a potential of separable (or sum of separable) form is used for each two-body interaction.

Similar work has been done on the Λd scattering problem in the quartet spin state,[17] and also on the bound state of three nucleons,[18] which is significantly different from the ΛNN problem so that no conclusion can be drawn from it. In the study of Λd scattering, here, the two-body ΛN potentials used were investigated more thoroughly than in Ref. 17, in that the off-shell amplitudes, the poles in the scattering amplitudes, and the actual wave functions generated by these amplitudes (with different shapes of core potentials) were calculated. Both spin singlet and triplet ΛN potentials were used, and by setting the same core parameters for the two spin states, fewer potential parameters were needed than in the previous work.[17,19] Here, the three-body calculations were expanded to include doublet Λd scattering, and in the total three-body scattering cross sections both doublet and quartet spin states were included, so that the results obtained could be compared with experiments that do not involve a polarized Λ or deuteron.

Recent works by other authors[20-22] yield a number of ways to represent a local potential amplitude by a sum of separable terms. So far, these developments have been used in three-body problems where the total three-body energy, E, is negative (or used in the three-nucleon problem where it makes sense to consider the on-shell behavior of the two-body amplitude at energies much higher than are relevant to the one-channel model of the ΛN interaction). For such energies the kernels of the integral equations involved do not have the singularities that are present when E is positive, such as in Λd scattering above the deuteron breakup threshold.

In the following analysis of two-body systems, a sum of nonlocal separable potentials, instead of a local potential, is used. For the connection between nonlocal interactions and the local potentials (i.e., the experimental data), it is required that the separable potential reproduce the same on-energy-shell behavior as that of a local potential (at least for energies up to the order of 80 MeV). A phenomenological local (PL) potential, rather than a meson-theoretic potential, is used here. For the purpose of testing the short-range behavior of the ΛN potential, the former choice is as good as the latter, and the simple shape of the PL potential means that the dependence of the phase shift on energy for any given hard-core radius, scattering length, and effective range, can be easily calculated.

II. TWO-BODY INTERACTIONS

A. Local Potentials

The shape of the two-body, spin-dependent, charge-independent local potentials, $V_L(r)$, used here for the phenomenological model is an exponential well plus a hard core,

$$V_L(r)=\begin{cases} -V_0 e^{-\eta(r-d)}, & r \geq d \\ \infty, & r<d. \end{cases} \tag{1}$$

In the above equation, V_0 is the strength parameter, η is the range parameter, and d is the radius of the hard core. This was the shape used by Herndon and Tang (HT) in their very extensive analysis of light hypernuclei.[9] It is the HT results which are used as the "experimental results" to which the NLS potentials given below are fitted.

In the Λd system both NN and ΛN forces are present. The NN force is to a very good approximation charge-symmetric (CS), but the ΛN force seems to contain an appreciable charge-symmetry-breaking (CSB) part.[9] Since the Λ isospin is zero and the deuteron is also an isospin singlet, the CSB part of the HT potentials plays no role and is dropped from the interaction Hamiltonian in the very beginning. The low-energy scattering parameters, the scattering length a and effective range r_0, are determined from the CS part of the interaction. From Ref. 9, these values are found to be[23]

$$a=-1.66\ \text{F}, \quad r_0=3.15\ \text{F} \tag{2}$$

for spin one, and

$$a=-2.09\ \text{F}, \quad r_0=2.85\ \text{F} \tag{3}$$

for spin zero.

The S-wave Schrödinger equation with $V_L(r)$ as given in Eq. (1) is readily solved.[24] With the scattering parameters given in Eqs. (2) and (3) and the same hard-core radius as used by HT the other

TABLE I. Local potential parameters.

Potential	Spin	d (F)	η (F^{-1})	V_0 (MeV)	$\delta(e)$ in degrees at e in MeV 20	40	60	80	e_c (MeV)
ΛN LHC	1	0.45	3.728	573.4	23.7	16.4	9.21	2.76	89.2
	0			599.5	27.3	19.6	11.7	4.97	96.4
ΛN LNC	1	0.00	1.9	110.6	27.2	25.1	23.0	21.4	...
	0			125.4	30.0	28.2	25.8	23.9	...
NN LHC	1	0.45	2.735	549.26	67.9	47.9	35.0	25.3	156.4

potential parameters may be found. The results are shown in Table I for these ΛN local hard-core (LHC) potentials. The phase shifts for these potentials at center-of-mass energies $e = 20$, 40, 60, and 80 MeV are also given, as is the energy $e = e_c$ for which each of these potentials gives a zero phase shift.

For purposes of comparison, local no-core (LNC) ΛN-potential parameters are also shown in Table I. These LNC parameters are found with the use of the potential in Eq. (1) with $d = 0$ and the stipulation that each LNC potential yield the same values of a and r_0 as the corresponding LHC potential.

The ${}^3S_1$ *np* low-energy parameters are obtained from the Tang and Herndon charge-symmetric potential[25] with the same shape as $V_L(r)$ given in Eq. (1), by a procedure similar to that used in the ΛN case. The low-energy parameters used in the nonlocal *NN* interaction are the ${}^3S_1$ scattering length $a_N = 5.3858$ F and the deuteron binding energy $E_D = 2.225$ MeV. The *NN*-potential parameters are also shown in Table I.

B. Nonlocal Potentials

A characteristic feature of all the phenomenological ΛN potentials developed from analysis of the light hypernuclei is the behavior of the phase shift with energy, namely, a positive phase shift at low energies that eventually goes through zero at higher energies. This behavior was set by using local potentials with repulsive cores, analogous to *NN* potentials. However, a single-term NLS potential produces no repulsion for short distances,[26] so that its phase shift does not change sign. The simplest form of NLS potential which could reproduce the above behavior, is a sum of two NLS potentials with one repulsive term representing the short-range and one attractive term representing the long-range behavior of the potential, that is, in a momentum-space representation,

$$V(k, k') = \lambda_1 v_1(k) v_1(k') + \lambda_2 v_2(k) v_2(k') . \qquad (4)$$

The method of numerical solution of the Faddeev equations for scattering problems ($E > 0$) necessitated the use of some simple analytic forms for the shapes of the separable terms $v_1(k)$ and $v_2(k)$. Thus, a one-parameter shape is chosen for each of these potentials with the range parameter β_i^{-1} being spin-dependent. A two-body potential in any given spin channel, then, has four unknown parameters: β_1^{-1}, β_2^{-1}, λ_1, and λ_2. By convention, the term with subscript 1 refers to the long-range, attractive part of the potential, and the term with subscript 2 refers to the short-range, repulsive part of the potential. In other words, the following inequalities must hold:

$$\beta_2^{-1} < \beta_1^{-1}, \quad \lambda_1 < 0, \quad \lambda_2 > 0 .$$

The potential shapes used are such as to yield three forms for the ΛN potential. One form is the potential designated YY, in which both $v_1(k)$ and $v_2(k)$ have the Yamaguchi shape[27] $v_1(k) = 1/(k^2 + \beta_1^2)$. Another is the potential designated YT in which $v_1(k)$ is a Yamaguchi shape, but the core $v_2(k)$ has the shape first used by Tabakin,[28] $v_2(k) = k^2/(k^2 + \beta_2^2)^2$. The third form, Y0, is a pure Yamaguchi shape, i.e., $\lambda_2 = 0$.

In order to obtain the required strong repulsion at short distances for a sum of two NLS potentials of the above types (with $\lambda_2 \neq 0$), the inequality $\lambda_2 > |\lambda_1|$ must hold. With this condition the Born amplitude for S-wave ΛN scattering is positive.

It was previously shown[29] that the low-energy Λd cross sections are insensitive to the existence of a core in the *np* potential. Hence, the *NN* potential considered here is a purely attractive, no-core potential of Yamaguchi shape. The parameters of the ${}^3S_1$ *NN* potential,

$$\beta_{NN}^{-1} = 0.6952 \text{ F}$$

and

$$\lambda_{NN} = -3326.5(2\pi)^3 \text{ MeV}^2 ,$$

are determined from the deuteron binding energy and the *NN* scattering length given above.

C. Numerical Results

The parameters of the ΛN spin singlet and triplet potentials for both the YY and YT forms are determined by imposing the conditions that (1) in each spin channel the separable potential gives the same scattering length and effective range as the LHC potential; (2) the core parameters (β_2 and λ_2) are the same in the spin-0 and spin-1 channels; (3) in each channel the NLS-potential phase shift $\delta(e)$ goes through zero at the same energy, $e = e_c$, as does the phase shift of the LHC potential (this insures that the nonlocal phase shift be very close to the local phase shift for all lesser energies).

All the parameters of the ΛN spin-0 and spin-1 separable potentials are determined from the above conditions with no free parameters left and, in fact, the fitting of the parameters is done with only one more parameter – the core strength – than in present in the LHC potential. The values of the parameters for different shapes of separable potential and different low-energy local parameters are given in Table II.

For the Y0 form only condition (1) is imposed. The resulting parameters are also shown in Table II.

Figure 1 shows the ΛN ${}^3S_1$ phase shift as a func-

TABLE II. ΛN NLS-potential parameters.

NLS pot.	Spin 0 $1/\beta_1$ (F)	$1/\beta_2$ (F)	$-\lambda_1$ [$\mathrm{MeV}^2/(20\pi)^3$]	λ_2	Spin 1 $1/\beta_1$ (F)	$1/\beta_2$ (F)	$-\lambda_2$ [$\mathrm{MeV}^2/(20\pi)^3$]	λ_2
Y0	0.6669	···	1.570	···	0.6792	···	1.339	···
YT	0.4923	0.4355	5.201	91.23	0.5205	0.4355	3.867	91.23
YY	0.3801	0.2225	90.07	1055	0.3826	0.2225	83.81	1055

tion of energy for the Y0, YT, YY, LNC, and LHC potentials. The YT, YY, and LHC phase shifts are within 0.2 deg of each other over the range shown. Any difference that shows up in the Λd cross sections, calculated in the next section, when the YY or YT ΛN potential is used rather than the Y0 potential, is expected to be less than the difference obtained if the LHC and LNC potentials were used instead.

As a result of the procedure carried out here a number of different ΛN potentials are obtained, Y0, YY, and YT, all of which give the same low-energy parameters a and r_0 as the LHC potential, and two of which (YY and YT) are equivalent to the LHC potential in that they give the same phase shift over a wide energy range (up to $e \approx 100$ MeV). This in itself is a useful result in that the set of two-body potentials obtained could now be used in the calculation of light hypernuclear binding energies (possibly along with a three-body ΛNN potential) as an alternative method for investigating the short-range behavior of the ΛN potential. In particular the ${}_{\Lambda}H^3$ binding energy could be calculated for all these different NLS potentials. From previous work,[30,31] all of the ΛN potentials used here are expected to give values for this energy in agreement with the experimental value given above.

Further comparison of the short-range behavior of the Y0, YT, and YY potentials may be carried out by investigating the radial wave function $u(r) = r\psi(r)$, the off-shell scattering amplitude, and the poles in the scattering amplitudes.

At a center-of-mass energy of 20 MeV the behavior of the ΛN spin-1 radial wave function $u(r)$ for the NLS, LHC, and LNC potentials is given

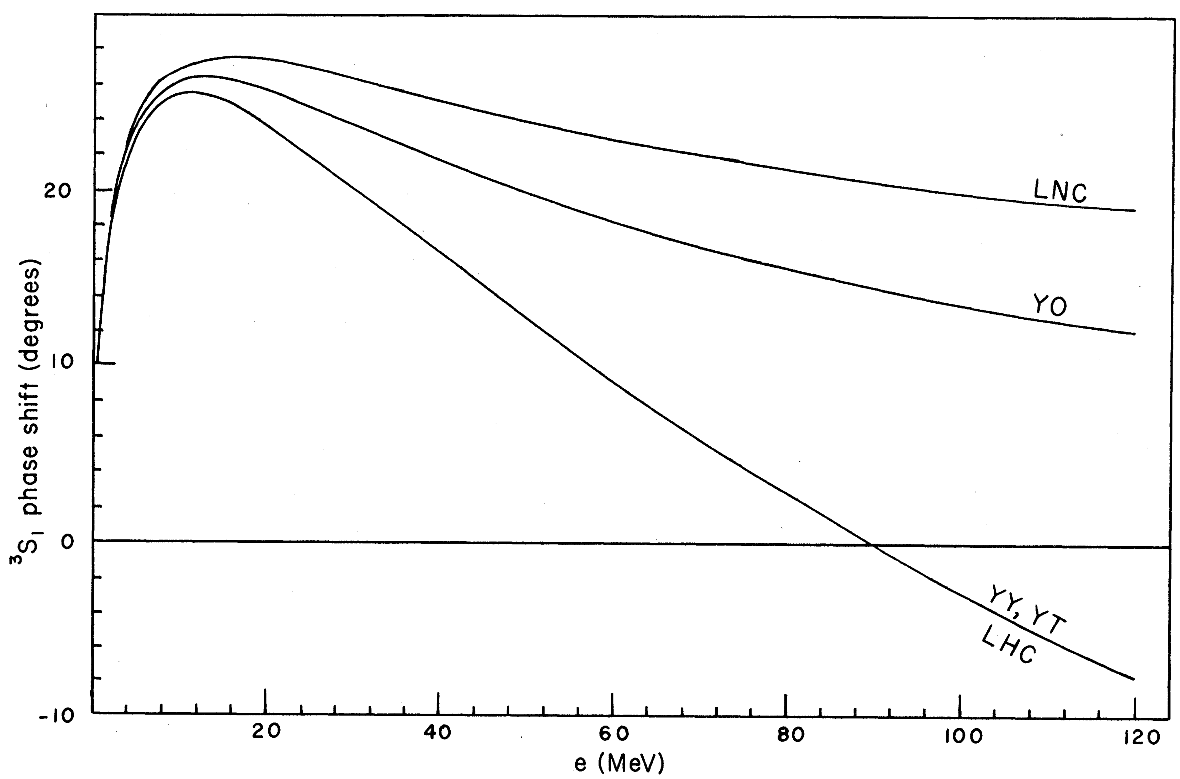

FIG. 1. The ΛN ${}^3S_1$ phase shift for the local no-core (LNC), the local hard-core (LHC), and the nonlocal separable potentials Y0, YT, and YY.

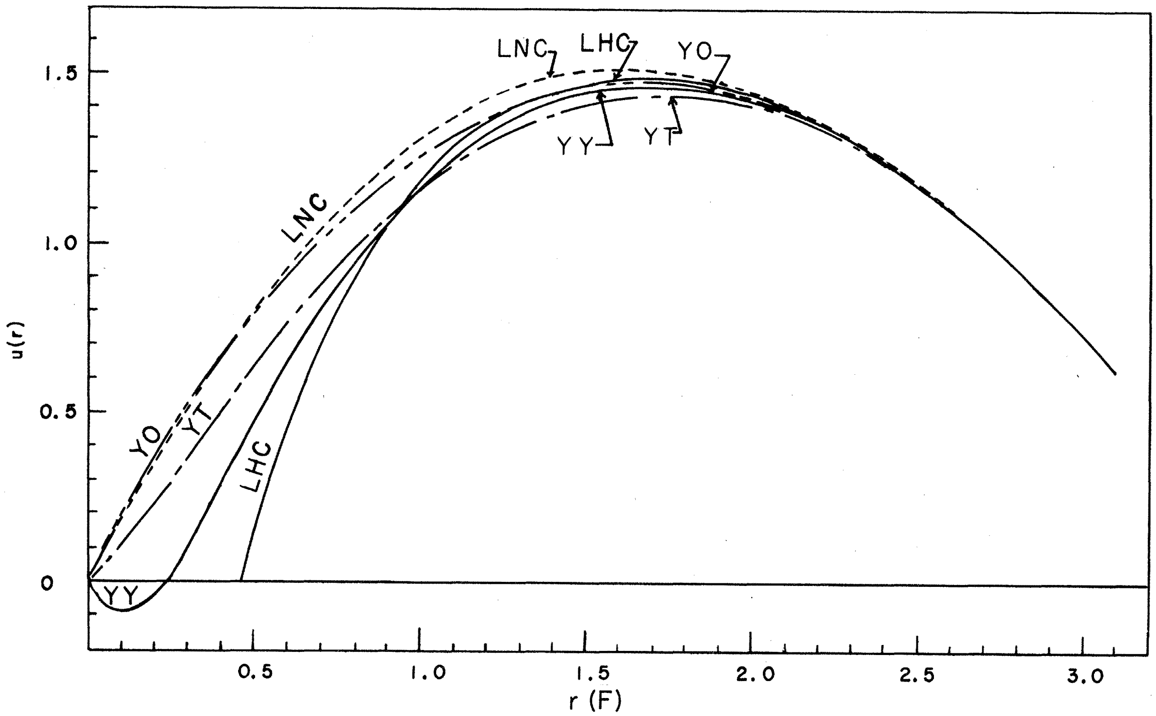

FIG. 2. The ΛN $^3S_1$ radial wave function $u(r) = r\psi(r)$ at a c.m. energy of 20 MeV for the local no-core (LNC), the local hard-core (LHC), and the nonlocal separable potentials Y0, YT, and YY. Only the LHC curve is shown for $r > 2.5$ F.

in Fig. 2. The same work carried out for other values of $e \leq 40$ MeV and for the spin-0 channel yielded results so similar to those shown in Fig. 2 that they are not presented here. An examination of Fig. 2 shows (a) the wave function of the potential Y0 quite closely resembles that of the purely attractive LNC local potential; (b) the potential YT looks more like the potential Y0 than does the potential YY, but it is repulsive at short distances $0 < r < 0.4$ F; (c) at a given energy the YT potential gives a wave function "equivalent" to that obtainable from a soft-core local potential, while the YY potential gives a wave function with a node. This very nonlocal behavior of the YY wave function shows up at such small r that its effect on low-energy Λd scatterings should be negligible.

Mongan[32] has shown that if the full off-energy-shell t matrix vanishes at infinite energy, the knowledge of the half-off-energy-shell t-matrix elements $t(e,k,k')$, where $k' \neq k = (2\mu e)^{1/2}$, in the scattering region (i.e., $e > 0$), plus the values of the bound-state form factors (for negative energies) are sufficient to determine the full off-energy-shell two-body t-matrix element $t(e,p,k')$, where $p \neq k$, at all energies, within a constant. Furthermore, it has been proven[33] that the function $F(k,k')$ defined by

$$F(k,k') = t(e,k,k')/t(e,k,k),$$

is a "real" function. Therefore, the full off-shell t-matrix element is completely determined by the real function $F(k,k')$ and the experimentally measured phase shifts. In order to compare the off-shell behavior of the potentials used in Sec. III (which, at sufficiently low energy, all have the same on-shell behavior), it is sufficient to study the real function $F(k,k')$.

In Fig. 3 the function $F(k,k')$ for the ΛN spin-1 channel at $e = k^2/2\mu = 20$ MeV is plotted against k' for each of the potentials Y0, YT, YY, LHC, and LNC. Similar graphs are obtained for spin-0 and other values of $e \leq 40$ MeV. Although the YY, YT, and LHC potentials have the same on-shell t matrix for $e \leq 100$ MeV, their off-shell t-matrix elements behave quite differently. On the basis of Fig. 3 the use of the YT potential in Λd scattering would tend to underrepresent the effect of a local hard core, while the use of the YY potential might overrepresent somewhat the effect of a local hard core. Combining this behavior with the "not attractive enough" representation of LNC by Y0 noted in Fig. 1, it appears that the difference between the use of the LNC rather than the LHC ΛN potential in Λd scattering should be well represented by the difference obtained in the use of the Y0 rather than the YY ΛN potential.

Since the singularities of the t matrix are among its most important properties, these are found

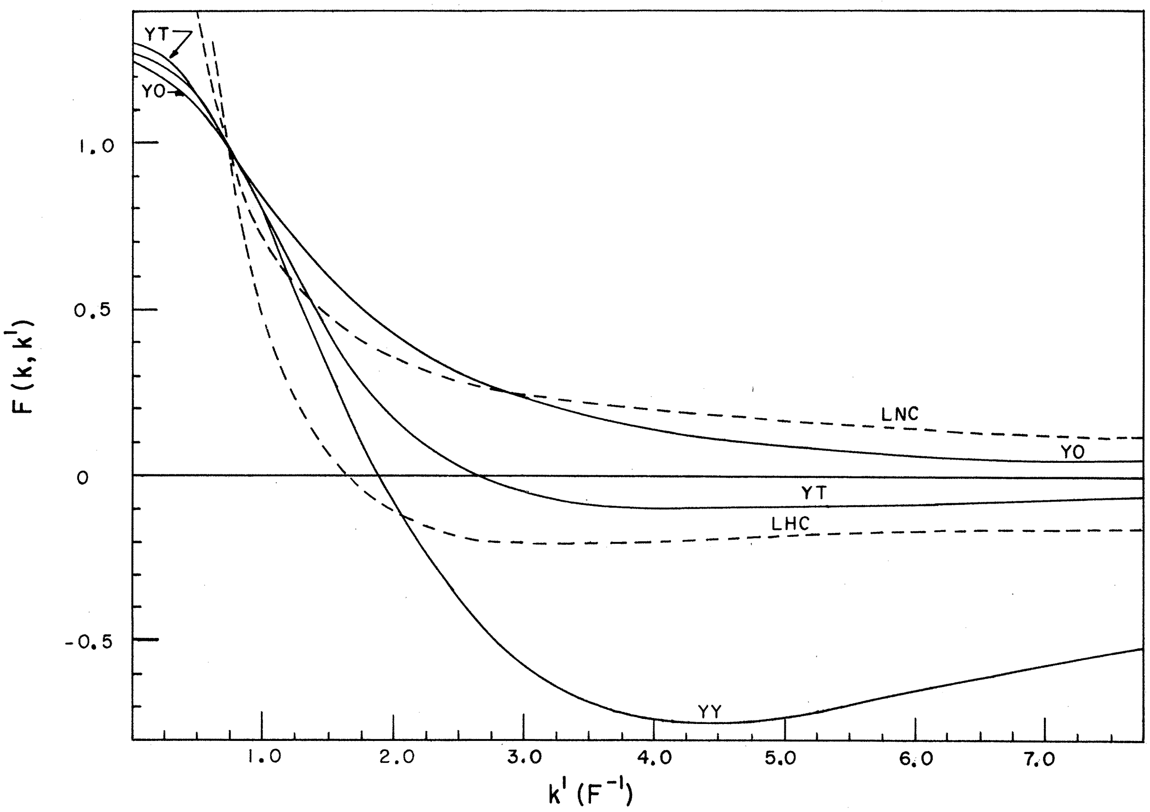

FIG. 3. The function $F(k,k')$ for the ΛN $^3S_1$ channel at a ΛN c.m. energy $\epsilon = k^2/2\mu = 20$ MeV for the local no-core (LNC), the local hard-core (LHC), and the nonlocal separable potentials Y0, YT, and YY.

and listed in Table III for the Y0, YT, and YY potentials. The range singularities $k = i\beta_i$ are also listed (the last two columns in this table). There is always a pole on the negative imaginary axis, close to where the anti-bound-state pole ($k = k_1$) of the local potential must be. The second pole on the negative imaginary axis ($k = k_2$) and all the rest of the singularities are for the most part far away compared to the anti-bound-state pole k_1, although in some cases these other poles (e.g., potential YT) are close enough to affect the phase shifts at the highest energies considered here; i.e., for $k = 300$ MeV/c.

III. Λd SCATTERING

A. Results and Conclusions

The use of the Faddeev formalism for this problem has been amply discussed in the literature.[17,19] The resulting set of coupled integral equations are made amenable to numerical solution, for energies above the threshold for deuteron break-up, by the use of contour rotation. All numerical work was performed at the USC Computer Center's IBM 360/65. The numerical results for the cross sections are estimated to be correct to within 1%.

Table IV exhibits the results for the S-wave Λd S-matrix element, $S_0 = e^{2i\delta_0}$, as well as the S-wave elastic (σ_{el}) and total (σ_{tot}) cross sections for both quartet (superscript Q) and doublet (superscript D) Λd scattering. Each of these is shown for different values of the incident Λ laboratory momentum p_Λ and for the three different shapes of the two-body ΛN potentials described above. In Table V the scattering cross sections with *all* the necessary partial waves are listed. The average total cross section is

TABLE III. Poles of the ΛN spin-1 amplitude in MeV/c.

NLS pot.	$-ik_1$	$-ik_2$	$k_{3,4}$	$k_{5,6}$	$-ik_7$	$-ik_8$
Y0	75.112	506	···	···	291	···
YT	75.797	386	$\pm 12 - i336$	$\pm 1147 - i718$	379	453
YY	75.737	854	$\pm 3246 - i937$	···	516	887

$$\bar{\sigma}_{tot} = \tfrac{2}{3}\sigma^Q_{tot} + \tfrac{1}{3}\sigma^D_{tot} \,. \qquad (5)$$

From Tables IV and V the following conclusions may be drawn:

(a) The short-range behavior of the ΛN potential does have an effect on low-energy Λd scattering. This is evident from a comparison of $\bar{\sigma}_{tot}$ for different core potentials at a given momentum p_Λ. This result is in agreement with the results obtained by Schick[17] for quartet Λd scattering, where a slightly different criterion was used to include the effect of short-range ΛN potentials.

(b) The effect of a ΛN repulsive core is always to reduce the size of the Λd cross section. This is evident from a comparison of the first row (the "no-core" values) of a given p_Λ, with the second and third rows (the "with-core" values). This effect was expected since the ΛN repulsive core reduces the ΛN phase shift. This reduction is always larger for the YY potential than for the YT potential (compare rows 2 and 3 for each p_Λ). This confirms the conclusion of Sec. II that the YY NLS shape looks more like a hard-core potential than does the YT potential. From the nature of the dependence of the momentum-space representations of the YY and YT potentials on k^2 this result seems reasonable. In other words, even though the deuteron is a loosely bound system and the ΛN interaction is not as strong as the np interaction, the Λd low-energy scatterings do depend on the two-body off-shell t-matrix elements.

(c) From comparison of Tables IV and V, almost the entire effect of the ΛN core appears in the S-wave Λd cross section. A possible explanation for this behavior is that the short-range ΛN repulsion is most effective in reducing the Λd cross section when it is present at lower energies as well as at higher energies, i.e., when the low-energy off-shell behavior of the ΛN t matrix looks most like that due to a local hard-core potential, in addition to the on-shell t matrix at higher energy reproducing the local hard-core potential phase shift. The Λd scattering with $L > 0$ would not, of course, be very sensitive to this low-energy effect.

(d) The size of the reduction of the Λd cross section depends upon the Λd center-of-mass energy E, i.e., upon p_Λ. Over the energy region investigated ($E \leq 25$ MeV), the percentage effect of the ΛN core is strongest at the highest energy. There are two competing effects here. First, the S-wave part of the cross section is proportionately larger at lower energies, and secondly, the percentage change in the S-wave part of the cross section is greater at higher energies. The second effect evidently dominates the first.

(e) The size of the reduction of the Λd cross section depends upon the Λd spin state. On a percentage basis, the ΛN core is much less effective in modifying the Λd doublet cross section than the Λd quartet cross section. In absolute terms, the ΛN core is somewhat less effective in the doublet state than in the quartet state. In Table IV compare column 3 and 5 with 4 and 6, respectively, for the no-core versus the with-core case.

To further study this effect for the doublet scattering, different combinations of core and no-core NLS potentials were inserted in the two-body singlet and triplet, or triplet and singlet, ΛN potentials, respectively. The results for $p_\Lambda = 200$

TABLE IV. S-wave Λd cross sections (in mb) and corresponding S-matrix elements.

p_Λ (MeV/c)	ΛN pot.	σ^Q_{el}	σ^D_{el}	σ^Q_{tot}	σ^D_{tot}	$\bar{\sigma}_{tot}$	S^D_0	S^Q_0
100	Y0	1197	1228	1203	1236	1214	$-0.986 - i0.021$	$-0.934 + i0.331$
	YT	1142	1228	1147	1236	1177	$-0.986 + i0.038$	$-0.843 + i0.552$
	YY	1099	1194	1107	1204	1139	$-0.930 + i0.310$	$-0.778 + i0.607$
150	Y0	418	440	452	482	462	$-0.742 + i0.385$	$-0.635 + i0.569$
	YT	389	433	416	474	435	$-0.713 + i0.449$	$-0.505 + i0.738$
	YY	372	410	397	482	412	$-0.597 + i0.643$	$-0.437 + i0.794$
200	Y0	174	185	203	221	209	$-0.418 + i0.605$	$-0.308 + i0.723$
	YT	154	177	178	211	189	$-0.356 + i0.661$	$-0.142 + i0.824$
	YY	147	164	168	190	175	$-0.219 + i0.787$	$-0.081 + i0.851$
250	Y0	79.7	86.0	101	111	104	$-0.117 + i0.693$	$-0.017 + i0.753$
	YT	65.6	77.8	81.3	102	88.0	$-0.019 + i0.723$	$+0.183 + i0.806$
	YY	62.2	70.3	76.3	87.4	80.0	$+0.122 + i0.802$	$+0.234 + i0.814$
300	Y0	39.1	42.7	54.0	59.9	55.9	$+0.133 + i0.696$	$+0.218 + i0.722$
	YT	28.5	35.1	38.2	51.0	42.4	$+0.262 + i0.686$	$+0.447 + i0.720$
	YY	26.7	30.9	35.3	41.4	37.3	$+0.401 + i0.732$	$+0.489 + i0.714$

TABLE V. Λd cross sections in mb.

p_Λ (MeV/c)	ΛN pot.	σ_{el}^Q	σ_{el}^D	σ_{tot}^Q	σ_{tot}^D	$\bar{\sigma}_{tot}$
100	Y0	1201	1233	1208	1242	1219
	YT	1147	1234	1152	1242	1182
	YY	1104	1200	1112	1210	1145
150	Y0	432	458	474	511	486
	YT	403	451	438	502	459
	YY	387	428	419	471	436
200	Y0	194	210	243	271	252
	YT	174	202	217	261	232
	YY	168	190	207	240	218
250	Y0	99.9	111	146	168	153
	YT	85.4	102	124	155	134
	YY	82.6	95.8	120	143	134
300	Y0	56.0	63.5	95.6	111	101
	YT	44.1	54.3	76.0	96.9	83.0
	YY	42.9	51.0	73.5	89.1	78.7

MeV/c, with all necessary partial waves included, are listed in Table VI. Results for σ_{tot}^D and σ_{el}^D show that putting a core in both singlet and triplet ΛN potentials (Table V), does not produce an additive effect in the Λd (doublet) cross section. The result for having a core in both is only a little more than having a core in the singlet ΛN potential alone; i.e., a lot of cancellation – or destructive interference – must be taking place. Thus, when two ΛN interactions are contributing to the Λd cross section, the effect of inserting a core in both is about the same as inserting a core in only one. This is consistent with the result obtained in the beginning of this section (e), but does not shed more light on the possible reasons for this behavior.

(f) For a fixed set of potentials, S_0^D and S_0^Q, considered as vectors in the complex plane, rotate clockwise as the Λd energy increases; i.e., both doublet and quartet bound states exist in all cases, since $S_0 = e^{2i\delta_0}$ and the clockwise rotation of S_0 implies δ_0 is decreasing from π rather than increasing from zero. The doublet bound state is just the ${}_\Lambda H^3$ $J=\frac{1}{2}$ hypernucleus. The $J=\frac{3}{2}$ ${}_\Lambda H^3$ bound state has not been found experimentally. It has appeared in theoretical calculations before. In particular the HT local potentials to which the NLS potentials used here are fitted predict such a bound state with $B_\Lambda \leq 0.01$ MeV.

B. Remarks

In the incident Λ laboratory momentum range of 100–300 MeV/c, the Λd cross section is sensitive to the short-range behavior of the two-body ΛN potentials. The reduction in the cross section is (13–25)%, being larger for higher energies. Consider two meson theoretic potentials which have the same on-shell t-matrix behavior, but have different short-range behavior; e.g., one has a hard core and the other vanishes at short distances. Low-energy Λd scatterings might well be able to distinguish between the two, if experiments can be done within an accuracy of 25%. Presently, such accurate experiments are out of the question. However, for reasons mentioned in Sec. I (i.e., the cleanliness of the theory, etc.), low-energy Λd scattering is the most promising "test" for obtaining this information.

TABLE VI. Λd cross sections in mb at $p_\Lambda = 200$ MeV/c.

ΛN spin	ΛN pot. in D	ΛN pot. in Q	σ_{el}^Q	σ_{el}^D	σ_{tot}^Q	σ_{tot}^D	$\bar{\sigma}_{tot}$
0	YY	···	194	194	243	247	244
1	Y0	Y0					
0	Y0	···	168	202	207	257	223
1	YY	YY					
0	YT	···	194	204	243	264	253
1	Y0	Y0					
0	Y0	···	173	206	217	264	233
1	YT	YT					

Low-energy Λd-scattering experiments might be well worth trying if the factors which have been neglected do not seriously decrease the percentage change of the cross sections. These factors include (a) the existence of the second channel, and (b) the existence of the higher partial waves in the two-body potentials. These two factors are probably the most serious of all the other factors involved. There does not exist a good way to approximate the effect of these factors within the framework of the present calculations. Therefore calculations which include these factors should be carried out with some care.[34] Possible other flaws, such as the neglect of three-body ΛNN forces, are not expected to be serious. The answer to the question of whether the experiment is worth doing is not known at this time, but is probably closer to yes, depending on further calculation and refinements of experimental techniques. It should be emphasized that much better Λp scattering experiments and better values for the binding energy of the light hypernuclei, especially the hypertriton, are a necessity.

Other approximations to a hard-core potential should be investigated since the results do depend on the off-shell behavior of the ΛN amplitudes. Finally, the use of a method with a more realistic theoretical base for obtaining the equivalent separable t matrices is most desirable.

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