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Transition Radiation from Magnetic Monopoles

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Transition radiation from Dirac magnetic monopoles is calculated. The image picture is applied and compared with the rigorous solution of Maxwell's equations utilizing the symmetry between magnetic and electronic charges. Possible experimental applications are discussed in searching for monopoles in accelerators and cosmic radiation.

I. INTRODUCTION

Ever since Dirac first advanced his theory of the magnetic monopole,[1] experimental searches have been continuously conducted in cosmic radiation, with accelerators, and even on the lunar surface. To date, all these investigations have yielded negative results.[2-7] These experiments all depend for their success on Dirac's result that the magnetic charge g is related to the electronic charge e by the equation

$$g=\frac{n}{2}\frac{\hbar c}{e}, \tag{1}$$

where n is an integer. It follows immediately that processes characterized by photon emission from magnetic monopoles will be enhanced over similar processes involving unit electronic charge by a factor of

$$\frac{g^2/\hbar c}{\alpha}=\frac{1}{\alpha^2}\frac{n^2}{4}=4692n^2. \tag{2}$$

From Eq. (2) it is clear that ionization losses in matter, Čerenkov emission, or any other radiation from monopoles should be several orders of magnitude larger than the equivalent process for a particle with unit electronic charge. Detection systems are therefore designed to utilize this enhancement. Tracks in emulsion should show the heavy ionizing properties of monopoles; Čerenkov counters should also display characteristically large signals from them. In this paper, the radiation from a monopole crossing an interface between two media, the so-called transition radiation, is calculated. This transition radiation could be potentially useful as an experimental detection technique since the same enhancement factor occurs as in Eq. (2). Further, since the total intensity of transition radiation is linear with the Lorentz factor γ, it provides a measure of the monopole mass if it is coupled with an independent energy measurement.

Before the calculation of transition radiation is presented, a brief review of ionization loss and Čerenkov emission will be given.

A. Ionization Loss

To estimate the energy loss of fast monopoles to atomic electrons, a classical approach suffices. The transverse impulse imparted to an electron by a monopole is given by

$$\Delta\vec{p}_\perp=e\int_{-\infty}^{\infty}(\vec{\beta}\times\vec{B})_\perp dt=e\int_{-\infty}^{\infty}(\vec{\beta}\times\vec{B}_\perp)dt$$
$$=e\hat{n}_\perp\beta\int_{-\infty}^{\infty}B_\perp dt=\frac{e\hat{n}_\perp\beta}{2\pi b}\int\vec{B}\cdot d\vec{S}=\frac{2eg}{b}\hat{n}_\perp, \tag{3}$$

where $\perp$ refers to the plane normal to the monopole path, and the surface integral is taken over a cylindrical surface, with the monopole path as the axis and containing the electron. In accord with the usual procedure, the average energy loss is given by[8]

$$-\frac{dE}{dx}=N_e\left(\frac{2eg}{c}\right)^2\frac{\pi}{m_e}\ln\left(\frac{m_e v^2\gamma}{J}\right), \tag{4}$$

where γ is the Lorentz factor, J is the mean binding energy of the electron, and N_e is the electron density. More detailed calculations modify Eq. (4) mainly be incorporating the density effect.[9] The important point to note about Eq. (4) is the insensitivity to monopole velocity, particularly for $\beta<<1$. This is not, of course, the case with high-Z particles.

B. Čerenkov Radiation and Bremsstrahlung

It is possible to define a vector potential in the case of magnetic poles via the relation

$$\vec{B}=\nabla\times\vec{A}^{(m)}. \tag{5}$$

However, we also have the equation

$$\nabla\cdot\vec{B}=4\pi g. \tag{6}$$

Clearly, Eq. (5) cannot be quite true. In fact, as shown by Dirac,[1] it is sufficient for (5) to fail on a line (L) emanating from the pole position to infinity. Therefore, Eq. (5) must be replaced by

$$\nabla\times\vec{A}^{(m)}=\vec{B}+\vec{B}^{(f)}, \tag{5'}$$

where

$$B = g\vec{r}/r^3 \tag{7}$$

and

$$\vec{B}^{(f)} = f(\vec{r})\hat{\tau}(\vec{r}), \tag{8}$$

subject to the equation

$$\nabla \cdot \vec{B}^{(f)} = -4\pi g. \tag{9}$$

In Eq. (8), $f(\vec{r})$ is a function that is zero everywhere except on the curve L and has a δ-function singularity there in order to ensure the validity of Eq. (9); $\hat{\tau}(\vec{r})$ is the tangent vector to the curve L. The Coulomb gauge condition may be imposed on $\vec{A}^{(m)}$. Therefore, Eq. (5′) may be reduced by taking the curl to

$$\nabla^2\vec{A}^{(m)} = -\nabla \times \vec{B}^{(f)}, \tag{10}$$

which has the solution

$$\vec{A}^{(m)} = \frac{1}{4\pi}\int \frac{\nabla' \times \vec{B}^{(f)}}{R} d^3r'. \tag{11}$$

Equation (11) can now be transformed to a sum of a surface integral, which may be dropped, and a volume integration. The volume integral may then be broken up into a tube surrounding L and the rest of space. Only the tube contributes, and, from Eq. (9), $\vec{A}^{(m)}$ takes the form

$$\vec{A}^{(m)} = -g\int_L d\vec{r}' \times \nabla\left(\frac{1}{R}\right). \tag{12}$$

A displacement of L to L' implies a change in $\vec{A}^{(m)}$ that is proportional to the gradient of the solid angle subtended by the loop LUL'. Therefore the wave function of an electron in the presence of a stationary monopole will suffer a gauge transformation

$$\psi \to \psi e^{i(ge/c\hbar)\Omega}. \tag{13}$$

However, encircling either L or L' implies a change in ψ given by

$$\psi \to \psi e^{\pm i(ge4\pi/c\hbar)} \tag{14}$$

and the single valuedness of ψ implies the Dirac quantization condition, Eq. (1).

The solution for an arbitrarily moving magnetic charge has been obtained by Dirac as a generalization of Eq. (12) for Minkowski space.[10] The generalization is most easily obtained by noting that the standard Liénard-Wiechert potential A_μ^{LW} reduces to $1/R$ in the rest frame of the pole. Dirac expresses A_μ^{LW} in terms of an integral over the particle worldline of the product $2\delta(R_\mu R_\mu)$ and the particle four-velocity. The generalization of Eq. (12) is immediate and results in a double integral over the sheet that is swept out in space-time by the space like curve L, which extends from $-\infty$ to the world point of the particle. The result obtained by Dirac is

$$A_\nu^{(m)}(x) = 2g\epsilon_{\nu\lambda\rho\sigma}\iint d\tau_0 d\tau_1 \frac{\partial y_\lambda}{\partial \tau_0}\frac{\partial y_\rho}{\partial \tau_1}\frac{\partial \delta((x-y)^2)}{\partial x_\sigma}, \tag{15}$$

where $y_\mu(\tau_0, \tau_1)$ specifies a point on the sheet, τ_0 parametrizes the particle worldline, and τ_1 parametrizes the curve L for fixed τ_0. It is then straightforward to show that Eq. (15) leads to an expression for the dual electromagnetic field tensor in terms of the Liénard-Wiechert potential scaled by g rather than e. The replacement of the electromagnetic field tensor by its dual for the solution when a magnetic charge replaces the electric charge is equivalent to the replacements

$$\vec{E} \to \vec{H}, \tag{16a}$$

$$\vec{H} \to -\vec{E}, \tag{16b}$$

$$e \to g. \tag{16c}$$

It follows that if both electric and magnetic charges are present, the field equations are invariant under (16a)–(16c) and the additional equations

$$g \to -e, \tag{17a}$$

$$\vec{v}_g, \vec{x}_g \to \vec{v}_e, \vec{x}_e, \tag{17b}$$

$$\vec{v}_e, \vec{x}_e \to \vec{v}_g, \vec{x}_g. \tag{17c}$$

If material media are present then an additional equation is

$$\epsilon \to \mu, \tag{18a}$$

$$\mu \to \epsilon. \tag{18b}$$

Equations (16)–(18) imply that the linear boundary conditions at infinity and at interfaces go into themselves. Therefore, to solve a problem with magnetic charges where the particle motion is given, all that is required are the solution of the electric-charge problem and the utilization of (16)–(18). It is immediately clear from this that the Čerenkov radiation from a monopole will be larger than that from a unit electronic charge by a factor of $\frac{1}{4}n^2/\alpha^2$. In addition, the polarization will be rotated 90° with respect to that of Čerenkov radiation from an electric charge since $\vec{E} \to \vec{H}$ and $\vec{E}\cdot\vec{H} = 0$. The same remarks also apply to bremsstrahlung from a monopole, with one important modification.

Since bremsstrahlung arises from accelerations caused by interactions of moving particles with atomic matter, and since atomic matter consists of electronic charges, this enhancement factor will be multiplied by an additional factor of $(nM_e/2\alpha M_g)^2$. If a particle carries both electronic and magnetic charge, it is possible to show that the energy loss whether by ionization, Čerenkov radiation, bremsstrahlung, or transition radiation is given by the independent sum of the electronic and magnetic charge contributions. This can be seen from the following argument.

From the linearity of Maxwell's equations modified to incorporate magnetic charge, the fields can be written

$$\vec{E} = \vec{E}^{(e)} + \vec{E}^{(m)} = e\vec{E}^{(1)} + g\vec{E}^{(2)} , \tag{19a}$$

$$\vec{H} = \vec{H}^{(e)} + \vec{H}^{(m)} = e\vec{H}^{(1)} + g\vec{H}^{(2)} , \tag{19b}$$

where (e) and (m) represent fields from an electronic or magnetic charge alone. The symmetry implied by Eqs. (16)–(18) yields

$$\vec{E}^{(e)} \to \vec{H}^{(m)} \Rightarrow \vec{E}^{(1)} = \vec{H}^{(2)} , \tag{20a}$$

$$\vec{H}^{(e)} \to -\vec{E}^{(m)} \Rightarrow \vec{H}^{(1)} = -\vec{E}^{(2)} . \tag{20b}$$

It follows immediately from (19) and (20) that terms proportional to eg in the Poynting vector and energy density will vanish and that, therefore, the energy loss dw is additive. Thus, only the electronic problem need be solved, dw calculated, Eqs. (16)–(18) applied, and the results added to treat the general case of electronic and magnetic charges.

II. THEORY OF TRANSITION RADIATION

A. Method of Images

The calculation of transition radiation was first performed by Frank and Ginsburg.[11] This calculation was concerned with the radiation in the optical and suboptical frequency range emitted backward from the interface of two dielectric media which was traversed normally by a particle of velocity $\vec{v}$ and charge e. In the very-low-frequency range, a very simple picture of transition radiation ensues when one of the media is a perfect conductor and $v/c \ll 1$. In this case, the radiation can be considered to be that produced by the annihilation of the charge with its image when the particle enters the conductor. Since the field is perfectly shielded from the space behind the conductor after the charge has entered, the Fourier component of the radiation field is given by

$$\vec{H}_\omega = \frac{1}{2\pi c} \vec{A} \times \frac{\vec{R}_0}{R_0} , \tag{21}$$

where $\vec{A}$ is the vector potential of the charge and the image at $t<0$, and it is assumed that $\vec{A} \to 0$ immediately upon entry of the charge into the conductor. $\vec{R}_0$ is a position vector from the point of entry into the conductor to an observation point in the wave zone. This is a good approximation for low frequencies. The low-frequency approximation will be valid if $\beta \ll 1$. For extreme relativistic velocities $\beta \sim 1$, it is generally not valid to consider a material a perfect conductor since high-frequency field components ($\omega \geq \omega_{\rm opt}$) are important.

The vector potential of charge and image is given by the following expression valid in the wave zone:

$$\vec{A} \doteq \frac{1}{cR_0} \left[\frac{e\vec{v}}{1 - \vec{R}_0 \cdot \vec{v}/R_0 c} + \frac{e\vec{v}}{1 + \vec{R}_0 \cdot \vec{v}/R_0 c} \right] \doteq \frac{2e\vec{v}}{cR_0} . \tag{22}$$

The radiation spectral density is now

$$\frac{dw_e}{d\Omega d\omega} = c\,|H_\omega|^2 R_0^{\,2} = \frac{e^2 v^2}{\pi^2 c^3} \sin^2\theta , \tag{23}$$

which, after integrating over the half-space behind the foil, yields

$$\frac{dw_e}{d\omega} = \frac{4e^2 v^2}{3\pi c^3} = \tfrac{4}{3}\alpha\beta^2 , \tag{24}$$

where the last equality holds if ω is measured in energy units.

It is impossible to construct a simple picture of transition radiation in terms of the annihilation of magnetic charges. The reason for this is that magnetic fields do not image in the same way as electric fields. To calculated transition radiation from monopoles for the perfect-conductor situation, it is convenient to utilize the symmetry principle discussed in Sec. I. The important point is that the dual electromagnetic field tensor was related in the usual way to the standard Liénard-Wiechert potential $\vec{A}^{\rm LW}$ scaled by g rather than e. Therefore the following equations obtain for the case of a monopole entering a perfect conductor that shields the exterior from any field once the monopole has entered (no time-varying fields are allowed for the moving monopole in a perfect conductor, which means that no fields will penetrate to the exterior):

$$\vec{E}_\omega = \frac{1}{2\pi c} \vec{A}^{\rm LW} \times \frac{\vec{R}_0}{R_0}, \tag{25}$$

$$\frac{dw_g}{d\Omega d\omega} = c\,|\vec{E}_\omega|^2 R_0^{\,2} . \tag{26}$$

The problem of computing $\vec{A}^{\rm LW}$ is solved by noting that since H_z and E_t are zero at the boundary, a solution in terms of the monopole and an image monopole with the *same* g can be written as follows:

$$\vec{A}^{\rm LW} \doteq \frac{g\vec{v}}{cR_0} \left[\frac{1}{1 - \vec{R}_0 \cdot \vec{v}/R_0 c} - \frac{1}{1 + \vec{R}_0 \cdot \vec{v}/R_0 c} \right] \doteq \frac{2g\beta\vec{v}}{cR_0} \cos\theta . \tag{27}$$

This yields for the spectral densities

$$\frac{dw_g}{d\Omega d\omega}=\frac{g^2v^2\beta^2}{\pi^2c^3}\sin^2\theta\cos^2\theta\,, \tag{28}$$

$$\frac{dw_g}{d\omega}=\frac{4g^2v^2\beta^2}{15\pi c^3}=\frac{1}{15}\frac{n^2}{\alpha}\beta^4\,, \tag{29}$$

in which the last equality holds when ω is measured in energy units. As expected, the polarization for monopole radiation is rotated 90° with respect to that from an electronic charge. Moreover, these expressions hold for the radiation emitted in the forward direction when the particles emerge from a conductor.

Although Eqs. (23), (24), (28), and (29) illustrate the differences in transition radiation from electronic as compared with magnetic charges, neither type of radiation amounts to much energy since $\beta<1$ and $\omega\sim 0$. There might be some advantage in the utilization of a low-frequency ($\omega<\omega_{\rm IR}$) transition radiator for particles with nonrelativistic velocities, but a more promising approach is to consider $\beta\to 1$ and $\omega\geqslant\omega_{\rm opt}$. In this case, the image picture is not realistic or convenient either for magnetic or electric charges. Instead, it is most convenient to solve the general boundary value problem for the electronic charge case by utilizing Maxwell's equations supplemented by the constitutive relations $\vec{D}=\epsilon\vec{E}$, $\vec{B}=\mu\vec{H}$ and then applying the symmetry relations (16)–(18). The case of conducting media is treated simply by choosing an appropriate form for ϵ, viz., $\epsilon=\epsilon_0-i4\pi\sigma/\omega$. The ideal conductor case is derived by taking the limit $\epsilon\to -i\infty$. It is important to note that in applying the symmetry conditions (16)–(18), care must be taken to observe the following order. First solve the general problem for arbitrary ϵ, μ, then apply (16)–(18), and finally replace ϵ and μ by appropiate functions of ω and medium properties. If this order is not followed, errors will result. In Mergelian's calculation[12] the final result obtained is

$$\frac{dw_g}{d\omega}=\frac{g^2}{\pi c}\left(\frac{\sqrt{\mu}-1}{\sqrt{\mu}+1}\right)\left(\ln\frac{2}{1-\beta}-1\right). \tag{30}$$

The problem with Eq. (30) is that for $\omega\geqslant\omega_{\rm opt}$, $\mu=1$, and therefore $dw_g/d\omega\sim 0$.[13] Certainly a particle that interacts as strongly with the photon field as a monopole should give a large burst of transition radiation at relativistic velocities. Presumably, the error involved was an inappropriate application of the symmetry principle. Note that the result presented in Ref. 12 for total energy of a magnetic and electronic charge does not possess the appropriate symmetry. In the following derivation, the procedure of Garibian[14] will be followed for the general case of ϵ, $\mu\neq 1$, and the final result then obtained by application of the symmetries (16)–(18).

B. Rigorous Solution of Maxwell's Equations

The problem is to obtain the solution of Maxwell's equations for two regions, 1, 2, with a particle with charge e and velocity $\vec{v}$ crossing the boundary plane $z=0$ at $t=0$. The first step is to resolve the relevant fields into Fourier components with respect to time. This approach of solving Maxwell's equations follows that of Fermi.[9]

$$(\vec{E},\vec{H},\vec{D},\vec{B})=\int d\omega\, e^{-i\omega t}(\vec{E}_\omega,\vec{H}_\omega,\vec{D}_\omega,\vec{B}_\omega)\,. \tag{31}$$

The relations between $\vec{E}$ and $\vec{D}$, and $\vec{B}$ and $\vec{H}$ take the form

$$\begin{aligned}\vec{D}_\omega&=\epsilon(\omega)\vec{E}_\omega\,,\\ \vec{B}_\omega&=\mu(\omega)\vec{H}_\omega\,.\end{aligned} \tag{32}$$

Setting the origin of the coordinate system at the point at which the particle traverses the boundary and noting that the particle current density is given by

$$\vec{j}=e\vec{v}\delta(\vec{x}-\vec{v}t) \tag{33}$$

yields Maxwell's equations for regions 1, 2 as

$$\nabla\times\vec{H}_{\omega 1,2}=\frac{-i\omega}{c}\epsilon_{1,2}\vec{E}_{\omega 1,2}+\frac{2e\hat{n}_z}{c}e^{i(\omega/v)z}\delta(\vec{\rho})\,, \tag{34}$$

$$\nabla\times\vec{E}_{\omega 1,2}=\frac{i\omega}{c}\mu_{1,2}\vec{H}_{\omega 1,2}\,, \tag{35}$$

$$\nabla\cdot\vec{E}_{\omega 1,2}=\frac{2e}{v\epsilon_{1,2}}\delta(\vec{\rho})e^{i(\omega/v)z}\,, \tag{36}$$

$$\nabla\cdot\vec{H}_{\omega 1,2}=0\,, \tag{37}$$

where $\vec{x}=\vec{\rho}+z\hat{n}_z$.

Next, the field vectors are resolved into Fourier components with respect to the transverse displacement vector $\vec{\rho}$:

$$(\vec{E}_\omega,\vec{H}_\omega,\vec{D}_\omega,\vec{B}_\omega)=\int d^2\kappa\, e^{i\vec{\kappa}\cdot\vec{\rho}}[\vec{E}_\omega(\vec{\kappa},z),\vec{H}_\omega,\vec{D}_\omega,\vec{B}_\omega]\,. \tag{38}$$

Equations (34)–(37) become

$$\left[i\vec{\kappa}+\hat{n}_z\frac{\partial}{\partial z}\right]\times\vec{H}_{\omega 1,2}=\frac{-i\omega\epsilon_{1,2}}{c}\vec{E}_{\omega 1,2}+\frac{e\hat{n}_z}{2\pi^2c}e^{i(\omega/v)z}\,, \tag{39}$$

$$\left[i\vec{\kappa}+\hat{n}_z\frac{\partial}{\partial z}\right]\times\vec{E}_{\omega 1,2}=\frac{i\omega}{c}\mu_{1,2}\vec{H}_{1,2}\,, \tag{40}$$

$$\left[i\vec{\kappa}+\hat{n}_z\frac{\partial}{\partial z}\right]\cdot\vec{E}_{\omega 1,2}=\frac{e}{2\pi^2v\epsilon_{1,2}}e^{i(\omega/v)z}\,, \tag{41}$$

$$\left[i\vec{\kappa}+\hat{n}_z\frac{\partial}{\partial z}\right]\cdot\vec{H}_{\omega 1,2}=0\,. \tag{42}$$

The general solution of Eqs. (39)–(42) can be written as the sum of a particular solution (particle field), denoted by the superscript p, and a homogeneous solution (the radiation field), denoted by a prime. It is clear that the particle fields must be of the form

$$(\vec{E}^p_\omega, \vec{D}^p_\omega, \vec{H}^p_\omega, \vec{B}^p_\omega) = (\vec{e}^p_\omega, \vec{d}^p_\omega, \vec{h}^p_\omega, \vec{b}^p_\omega)e^{i(\omega/v)z}. \tag{43}$$

The vector $\vec{k}$ (wave vector for the particle field) is defined as follows:

$$\vec{k} = \vec{\kappa} + \frac{\omega}{v}\hat{n}_z. \tag{44}$$

By utilization of (43) and (44), Eqs. (39)–(42) are transformed into a set of algebraic equations for $\vec{e}^p_\omega, \ldots$, etc. These are readily solved with the results

$$\vec{h}^p_{\omega 1,2} = \frac{ie}{2\pi^2 c}\frac{\vec{\kappa}\times\hat{n}_z}{k^2 - \chi_{1,2}\omega^2/c^2}, \tag{45}$$

$$\vec{e}^p_{\omega 1,2} = \frac{-ie}{2\pi^2\epsilon_{1,2}v}\frac{\vec{\kappa} + (\omega/v)(1-\beta^2\chi_{1,2})\hat{n}_z}{k^2 - \chi_{1,2}\omega^2/c^2}, \tag{46}$$

where $\chi = \epsilon\mu$.

The homogeneous equations are transformed into a wave equation of the form

$$\left(\frac{d^2}{dz^2} + \lambda_{1,2}{}^2\right)(\vec{E}'_\omega, \vec{H}'_\omega) = 0, \tag{47}$$

where

$$\lambda^2 = \frac{\omega^2}{c^2}\chi - \kappa^2. \tag{48}$$

The homogeneous solutions are

$$\vec{H}'_{\omega 1,2} = \vec{h}'_{\omega 1,2}e^{\pm i\lambda_{1,2}z}, \tag{49}$$

$$\vec{e}'_{\omega 1,2} = \frac{-c}{\omega\epsilon_{1,2}}(\vec{\kappa} \pm \lambda_{1,2}\hat{n}_z)\times\vec{h}'_{\omega 1,2}, \tag{50}$$

where there must be only reflected waves in region 1 (minus sign) and transmitted waves in region 2 (plus sign).

The homogeneous solutions are obtained by utilization of continuity of $\vec{B}_z$, $\vec{D}_z$, $\vec{H}_z\times n_z$, and $\vec{E}\times\hat{n}_z$ at $z=0$. This is most readily done by resolving the homogeneous fields $\vec{h}'_\omega$, $\vec{e}'_\omega$ along the three orthogonal vectors $\hat{n}_z$, $\vec{\kappa}$, and $\vec{\kappa}\times\hat{n}_z$. By using the fact that

$$(\vec{\kappa} - \lambda_1\hat{n}_z)\cdot\vec{h}'_{\omega 1} = 0, \tag{51}$$

$$(\vec{\kappa} + \lambda_2\hat{n}_z)\cdot\vec{h}'_{\omega 2} = 0, \tag{52}$$

and noting that the continuity of D_z and $\vec{H}\times\hat{n}_z$ yields redundant equations when combined with (45) and (46), the homogeneous solutions can be obtained in a straightforward manner. The results are

$$\vec{h}'_{\omega 1} = \frac{ie\epsilon_1\omega}{2\pi^2 cv}\frac{\eta_1}{\zeta}(\kappa\times\hat{n}_z), \tag{53}$$

$$\vec{h}'_{\omega 2} = \frac{-ie\epsilon_2\omega}{2\pi^2 cv}\frac{\eta_2}{\zeta}(\vec{\kappa}\times\hat{n}_z), \tag{54}$$

$$\vec{e}'_{\omega 1} = \frac{ie\kappa^2}{2\pi^2 v}\frac{\eta_1}{\zeta}\hat{n}_z + \frac{ie\lambda_1}{2\pi^2 v}\frac{\eta_1}{\zeta}\vec{\kappa}, \tag{55}$$

$$\vec{e}'_{\omega 2} = \frac{ie}{2\pi^2 v}\kappa^2\frac{\eta_2}{\zeta}\hat{n}_z - \frac{ie\lambda_2}{2\pi^2 v}\frac{\eta_2}{\zeta}\vec{\kappa}, \tag{56}$$

where

$$\zeta = \lambda_2\epsilon_1 + \epsilon_2\lambda_1, \tag{57}$$

$$\eta_1 = \frac{\epsilon_2/\epsilon_1 - (v/\omega)\lambda_2}{k^2 - \chi_1\omega^2/c^2} + \frac{-1 + (v/\omega)\lambda_2}{k^2 - \chi_2\omega^2/c^2}, \tag{58}$$

$$\eta_2 = \frac{\epsilon_1/\epsilon_2 + (v/\omega)\lambda_1}{k^2 - \chi_2\omega^2/c^2} - \frac{(v/\omega)\lambda_1 + 1}{k^2 - \chi_1\omega^2/c^2}. \tag{59}$$

In cylindrical coordinates the Poynting vector in the direction of the radiation field wave vector is given by

$$S_R = \frac{c}{4\pi}(H'_\varphi E'_z\sin\theta + H'_\varphi E'_\rho\cos\theta), \tag{60}$$

where the components E'_φ and H'_ρ will be seen to be zero. $E'_{1\rho}(\vec{x}, t)$ will be calculated in the wave zone first:

$$E'_{1\rho}(\vec{x}, t) = \frac{ie}{2\pi^2 v}\iiint d^2\kappa\,\kappa\cos\Phi\,d\omega\frac{\lambda_1\eta_1}{\zeta}\times e^{i(\kappa\rho\cos\Phi - i\lambda_1 z - i\omega t)}, \tag{61}$$

where an axis in $\vec{\kappa}$ space is aligned in the $\vec{\rho}$ direction. With a standard representation of the Bessel function, Eq. (61) becomes

$$E'_{1\rho} = \frac{-e}{\pi v}\iint\frac{\kappa^2\lambda_1\eta_1}{\zeta}J_1(\rho\kappa)d\kappa d\omega\,e^{-i\lambda_1 z - i\omega t}, \tag{62}$$

which, in the wave zone, becomes

$$E'_{1\rho} = \frac{-e}{\pi v(2\pi R\sin\theta)^{1/2}}\iint\frac{\kappa^{3/2}\lambda_1\eta_1}{\zeta}\times(e^{Rf(\kappa)-3\pi i/4} + e^{R\varphi(\kappa)+3\pi i/4})d\kappa d\omega, \tag{63}$$

where the following replacements have been made:

$$z = -R\cos\theta,\quad \rho = R\sin\theta, \tag{64}$$

and the functions $f(\kappa)$ and $\varphi(\kappa)$ are defined by

$$f(\kappa) = i\kappa\sin\theta + i\lambda_1\cos\theta, \tag{65}$$

$$\varphi(\kappa) = -i\kappa\sin\theta + i\lambda_1\cos\theta. \tag{66}$$

The function $f(\kappa)$ has a saddle point in the complex κ plane at $\kappa_0 = (\omega/c)\chi_1{}^{1/2}\sin\theta$ and $\varphi(\kappa)$ has one at $\kappa'_0 = -(\omega/c)\chi_1{}^{1/2}\sin\theta$. In the integral involving $\varphi(\kappa)$, the saddle point lies nowhere near the path of integration, whereas the saddle point of $f(\kappa)$ can be intersected and a path of steepest descent traversed. Therefore, in the integral with φ, the replacement $d\kappa = d\varphi/\varphi'$ can be made, and by integration by parts it can be shown that this integral is $O(R^{-1/2})$ with respect to the first integral and may, therefore, be neglected.[15]

The dominant integral can be evaluated by the saddle-point method, with the result

$$E'_{1\rho}=\frac{e\beta^2}{\pi vR}\sin\theta\cos^2\theta\int d\omega\, e^{iR(\omega/c)\sqrt{\chi_1}-i\omega t}\chi_1^{3/2}\xi_1, \tag{67}$$

where

$$\xi_1=\frac{(\epsilon_2/\epsilon_1-1)[1+\beta(\chi_2-\chi_1\sin^2\theta)^{1/2}-\beta^2\chi_1]-\beta^2\epsilon_2\mu_1(\mu_2/\mu_1-1)}{[\epsilon_1(\chi_2-\chi_1\sin^2\theta)^{1/2}+\epsilon_2\sqrt{\chi_1}\cos\theta](1-\beta^2\chi_1\cos^2\theta)[1+\beta(\chi_2-\chi_1\sin^2\theta)^{1/2}]}. \tag{68}$$

The same procedure is utilized for the evaluation of the remaining field components, with the result

$$E'_{1z}=\frac{e\beta^2}{\pi vR}\sin^2\theta\cos\theta\int d\omega\, e^{iR(\omega/c)\sqrt{\chi_1}-i\omega t}\chi_1^{3/2}\xi_1, \tag{69}$$

$$H'_{1\varphi}=\frac{e\beta^2}{\pi vR}\sin\theta\cos\theta\int d\omega\, e^{iR(\omega/c)\sqrt{\chi_1}-i\omega t}\mu_1\epsilon_1^{2}\xi_1. \tag{70}$$

The total energy radiated into $d\Omega$ is now given by

$$\frac{dw_e}{d\Omega}=R^2\int_{-\infty}^{\infty}S_R(t)dt, \tag{71}$$

which yields

$$\frac{dw_e}{d\Omega d\omega}=\frac{e^2\beta^2}{\pi^2c}\sin^2\theta\cos^2\theta\,\mu_1^{5/2}\epsilon_1^{7/2}|\xi_1|^2. \tag{72}$$

For monopoles the result is

$$\frac{dw_g}{d\Omega d\omega}=\frac{g^2\beta^2}{\pi^2c}\sin^2\theta\cos^2\theta\,\mu_1^{7/2}\epsilon_1^{5/2}|\tilde{\xi}_1|^2, \tag{73}$$

where $\tilde{\xi}_1$ is derived from ξ_1 by interchanging μ and ϵ.

By putting $\mu_1=\epsilon_1=\mu_2=1$ and $\epsilon_2=\epsilon$, Eqs. (72) and (73) become

$$\frac{dw_e}{d\Omega d\omega}=\frac{e^2\beta^2}{\pi^2c}\sin^2\theta\cos^2\theta\left|\frac{(\epsilon-1)[1+\beta(\epsilon-\sin^2\theta)^{1/2}-\beta^2]}{(1-\beta^2\cos^2\theta)[(\epsilon-\sin^2\theta)^{1/2}+\epsilon\cos\theta][1+\beta(\epsilon-\sin^2\theta)^{1/2}]}\right|^2, \tag{72'}$$

$$\frac{dw_g}{d\Omega d\omega}=\frac{g^2\beta^6}{\pi^2c}\sin^2\theta\cos^2\theta\left|\frac{\epsilon-1}{(1-\beta^2\cos^2\theta)[(\epsilon-\sin^2\theta)^{1/2}+\cos\theta][1+\beta(\epsilon-\sin^2\theta)^{1/2}]}\right|^2. \tag{73'}$$

The following limiting cases are of interest:

$$\left(\frac{dw_e}{d\Omega d\omega}\right)_{\epsilon\to-i\infty}=\frac{e^2\beta^2\sin^2\theta}{\pi^2c(1-\beta^2\cos^2\theta)^2}, \tag{74}$$

$$\left(\frac{dw_g}{d\Omega d\omega}\right)_{\epsilon\to-i\infty}=\frac{g^2\beta^4\sin^2\theta\cos^2\theta}{\pi^2c(1-\beta^2\cos^2\theta)^2}. \tag{75}$$

In the nonrelativistic limit, Eqs. (74) and (75) reduce correctly to Eqs. (23) and (28). In fact, it is clear that Eqs. (74) and (75) are identical with the image results even for $\beta\sim1$. It is important to note that the image approximation yields a realistic picture only for the subinfrared frequencies (i.e., when $\sigma/\omega\gg1$).

To obtain the radiation in the forward direction, the replacement ϵ_1, $\mu_1\leftrightarrow\epsilon_2$, μ_2, $v\to-v$ is made in Eqs. (72) and (73). For the radiation in the forward direction emitted by the particles as it emerges into the vacuum, all that is required is the replacement of β by $-\beta$ in (72) and (73). This causes an enhancement of these expressions due to a decrease of denominators; this is especially important in the x-ray region, where $\epsilon\to1$. In particular, the effective cutoff frequencies beyond which the spectral density does not contribute to the total intensity in the forward direction is proportional to the particle energy and is approximately $\gamma\omega_p$, where ω_p is the plasma frequency of the medium. In the backward direction there is effectively no transition radiation above optical frequencies. It is noted that Eqs. (74) and (75) are invariant under the replacement $\beta\to-\beta$. These equations hold for frequencies only up to the infrared region. Therefore, for relativistic velocities the contribution of the image picture to the total energy is minuscule $[O(\ln\gamma)\omega_{IR}/\gamma\omega_p)]$. To utilize transition radiation from monopoles effectively as an experimental detection technique, forward radiation should be utilized via the relation

$$\frac{dw_g}{d\Omega d\omega}=\frac{g^2\beta^6}{\pi^2 c}\sin^2\theta\cos^2\theta\left|\frac{\epsilon-1}{(1-\beta^2\cos^2\theta)[(\epsilon-\sin^2\theta)^{1/2}+\cos\theta][1-\beta(\epsilon-\sin^2\theta)^{1/2}]}\right|^2. \qquad (76)$$

For $\beta\to 1$,

$$\frac{dw_g}{dw_e}\to\frac{g^2}{e^2}=\frac{n^2}{4\alpha^2}, \qquad (77)$$

as expected.

It is clear from the image picture that if the particles are obliquely incident there will be some variation in the radiation intensity. In fact, if the particle velocity is $\vec{v}_\perp + v\cos\psi\,\hat{n}_z$, the image velocity must be $\vec{v}_\perp - v\cos\psi\,\hat{n}_z$, where $\vec{v}_\perp$ is the velocity component perpendicular to the z axis and ψ is the angle between the z axis and $\vec{v}$. This will yield a factor of $\cos^2\psi$ to Eq. (24) and more complicated angle dependence in Eq. (28). However, for relativistic velocities the image picture is not relevant for the bulk of the radiation, and the intensity will be unchanged and the radiation in the forward direction will be concentrated in the particle direction.[16] Figure 1 shows the integration of (76) over solid angle for various γ values for the x-ray region above 1 keV.

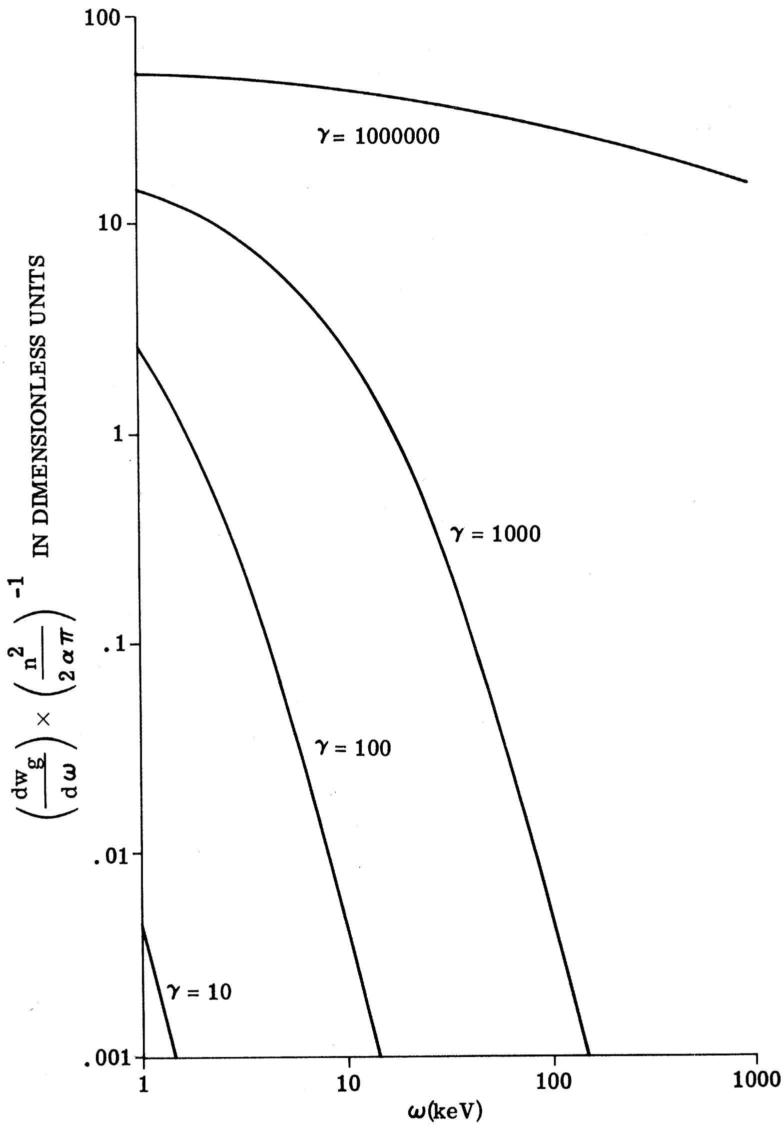

FIG. 1. Transition radiation spectrum from aluminum-vacuum interface.

III. DISCUSSION OF RESULTS

A. Utilization of Transition Radiation for Detection Purposes

From the results derived above it is evident that the characteristically large burst of transition radiation produced when a magnetic monopole crosses intermedia interfaces (e.g., metallic foils) can provide a useful signature for detecting such particles if they exist in cosmic radiation or are produced either by accelerators or from cosmic-ray interactions with the earth's atmosphere. In particular, the linearity in γ of the transition radiation intensity in the x-ray region allows a relatively accurate mass measurement of a monopole if an independent energy measurement is made. Even in the optical or suboptical frequency region where the intensity depends only logarithmically on γ [e.g., integration of Eq. (75) over θ for a frequency band independent of γ], there will be enough photons produced from a few hundred foils for statistically meaningful γ measurements. In particular, the extensive work of Yuan *et al.*[17] and of Alikhanian *et al.*[18] on the development of a transition radiation detector for measuring the γ of relativistic particles indicates that an experiment to detect monopoles by means of transition radiation seems feasible. In particular, even for subrelativistic velocities, monopole transition radiation should be detectable. It is important to note that in any experiment to detect transition radiation from relativistic particles, the foil spacing and thickness must satisfy the formation zone conditions; otherwise the analysis of the radiation produced will be complicated by interference effects.[17]

B. Production of Monopoles in Accelerators and Possible Presence in Cosmic Radiation

If monopoles are produced in an accelerator such as the facility now under construction at the National Accelerator Laboratory, the effective γ at threshold for a heavy monopole produced in a p-p collision is given by

$$\gamma_g \doteq M_g/M + 1\,, \tag{78}$$

where M is the proton mass, and it is assumed that $M_g/M > 1$. For a colliding-ring facility such as the one under construction at CERN, $\gamma_g = 1$ at threshold. In cosmic-ray collisions, Eq. (78) holds for production at threshold. However, because of the cosmic magnetic fields, monopoles will presumably be accelerated to very high energies yielding, therefore, very large γ values.[19] Consequently, at NAL and in cosmic-radiation searches, monopoles should certainly yield large characteristic transition radiation signals (see Fig. 1); in fact, for cosmic radiation, with the probably very high γ values attained, transition radiation provides a good potential mass-determination or energy-determination technique. It is also possible that the production of free monopoles occurs with a reasonably high rate only far above threshold because of the very strong Coulomb-like attraction between g, $\bar{g}$ pairs.[20] In this case, assuming a diffraction-dissociation-type mechanism, the effective γ_g is given by

$$\gamma_g \doteq (M/M_g)\gamma_p\,, \tag{79}$$

where γ_p is E/M for the incoming proton beam. If, for example, it is necessary to supply a relative kinetic energy of $2M_g$ in order for the poles to escape their mutual attraction for a measurable production rate, $E > 4E_{\text{th}}$ and M_g could be as low as 4–5 GeV and would yet have escaped detection in accelerator and cosmic-ray experiments. This would yield $\gamma_g \sim 7$ at CERN and $\gamma_g \sim 30$–100 at NAL (the lower value corresponding to slow particles in the c.m. frame). To see how this might happen, the following line of reasoning may prove instructive. The first assumption is that the production of monopoles takes place in p-p collisions via the production and subsequent dissociation of a time like virtual photon. If the final-state interaction between $g, \bar{g}$ is neglected, the cross section can be estimated by

$$\frac{d\sigma(E,q^2)_{g,\bar{g}}}{dq^2} = K\frac{d\sigma(E,q^2)_{\mu^+,\mu^-}}{dq^2}\,, \tag{80}$$

where $d\sigma_{\mu^+,\mu^-}/dq^2$ is the differential cross section for producing a massive muon pair with a muon center-of-mass energy of $\sqrt{-q^2}$. Equation (80) takes into account the hadronic structure, and K is the ratio of the couplings $[(\gamma\to g,\bar{g})/(\gamma\to\mu^+,\mu^-)]^2$. If, following Goto,[21] these couplings are interpreted as dissociation probabilities, then $K_{\max} = 1/\alpha$. Recent theoretical and experimental work can be utilized to estimate $d\sigma_{\mu^+,\mu^-}/dq^2$ for various q^2 values.[22-24] For example, if $M_g \sim 5$ GeV then

$$\int_{q^2_{\min}}^{q^2_{\max}} dq^2 \frac{d\sigma_{g,\bar{g}}}{dq^2} \simeq 10^{-36}\ \text{cm}^2\,. \tag{81}$$

In taking account of the final-state interaction it is important to note that there will be strong competition for $g, \bar{g}$ free states from multiphoton $g, \bar{g}$ annihilation states. Also it is necessary to supply enough energy to overcome the strong mutual attractions of the monopole pair. Both of these factors could easily bring Eq. (81) down to nonmeasurable cross sections until the incoming proton energy gets large enough. In conclusion, if the object is to look for free monopoles then the search for fast monopoles by means of transition radiation is a feasible approach.

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