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Analysis of Spin Correlations in the β Decay of the Λ Hyperon*†

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Recent experimental data on the β decay of the Λ are analyzed. Integrated expressions for the measured quantities in terms of form factors are given up to second order in the parameter $\beta=(m_\Lambda-m_p)/m_\Lambda$. The sensitivity of these experimental quantities as functions of specific form factors is studied in detail and exhibited in a form which is useful for the analysis of present experiments and for the planning of future ones. Within the framework of $V-A$ theory, the possibility is explored that the data require the presence of an induced pseudotensor form factor. Qualitative discussions of the q^2 dependence of some form factors and of possible admixtures of scalar and tensor interactions are also given.

I. INTRODUCTION

New experimental evidence on the β decay of the Λ hyperon[1,2] is currently becoming available. In comparison with neutron β decay, Λ β decay

$$\Lambda \to p + e^- + \bar{\nu}$$

differs by the change in strangeness ($\Delta S \neq 0$) and the non-negligible magnitude of the Q value. Because our knowledge of $\Delta S \neq 0$ hyperon semileptonic decays is so limited, the analysis of this evidence is particularly interesting. We want to obtain information that might improve our picture of $\Delta S \neq 0$ decays.

In a previous paper,[3] which we shall refer to as I, a preliminary analysis was given. It was seen there that new complications might arise; in particular, some sum rules were introduced which allowed us to see that a violation of time-reversal invariance in Λ β decay could not alone bring better agreement with experiments, and that instead, second-class currents might be required.

In the present paper we assume time-reversal invariance and neglect radiative corrections. We study the possible restrictions that the Λ β-decay data impose on second-class currents, on the q^2 variation of some form factors, and on possible admixtures of scalar and tensor interactions to the $V-A$ picture. We compare the restrictions imposed by the Λ β decay alone with the predictions of Cabibbo's model, which relates the β decays of all the hyperons, and whose experimental support had previously come only from partial decay rates and electron-neutrino correlations. In Sec. II, we give expressions for the measured quantities, namely, the partial decay rate, the electron-neutrino correlation, and the electron, neutrino, and proton-Λ spin asymmetries, in terms of form factors integrated over the range of the invariant momentum transfer squared, q^2. We then study qualitatively the restrictions that the experimental data impose on the integrated form factors. In Sec. III, we assume that no second-class currents[4] are present and we make a detailed comparison with the predictions of Cabibbo's model.[5] In Sec. IV, we consider the influence of the q^2 dependence of some form factors. In Sec. V, we give a detailed study of the restrictions that the experimental data impose on second-class currents. In Sec. VI, we show how sensitive our results are to the actual value of the ν asymmetry, which is likely to improve in precision in the near future. Finally, in Sec. VII, we give a qualitative discussion of possible admixtures of scalar or tensor interactions to the $V-A$ theory.

II. EXPRESSIONS FOR THE PARTIAL DECAY RATE, THE e-ν CORRELATION, AND THE ASYMMETRIES

We assume that the interaction responsible for Λ β decay is of the current-current form, with both vector and axial-vector contributions,

$$\mathcal{L}_{\text{int}} = (G/\sqrt{2})\, j^h_\mu j^l_\mu + \text{H.c.} \tag{1}$$

The matrix element of the vector part of the hadronic current can be written as

$$\langle p | j^{h,V}_\mu | \Lambda \rangle = \left(\frac{m_p m_\Lambda}{E_p E_\Lambda}\right)^{1/2} \bar{u}(p)\left[f_1(q^2)\gamma_\mu + f_2(q^2)\frac{\sigma_{\mu\nu}}{m_\Lambda}q_\nu + f_3(q^2)\frac{q_\mu}{m_\Lambda}\right]u(\Lambda) \tag{2}$$

and for the axial-vector part as

$$\langle p | j^{h,A}_\mu | \Lambda \rangle = \left(\frac{m_p m_\Lambda}{E_p E_\Lambda}\right)^{1/2} \bar{u}(p)\left[g_1(q^2)\gamma_\mu + g_2(q^2)\frac{\sigma_{\mu\nu}}{m_\Lambda}q_\nu + g_3(q^2)\frac{q_\mu}{m_\Lambda}\right]\gamma_5 u(\Lambda), \tag{3}$$

where $q^2 = (p_\Lambda - p_p)^2 = (p_e + p_\nu)^2$ is the invariant four-momentum transfer and has a maximum value of $(0.16\ \mathrm{BeV})^2$. Our conventions for the γ matrices and the metric are those of Ref. 6.

The lepton current has the matrix element

$$\langle e | j_\mu^1 | \nu \rangle = \bar{u}(e)\gamma_\mu(1+\gamma_5)u(\nu)\,. \tag{4}$$

The form factors $f_3(q^2)$ and $g_2(q^2)$ correspond to second-class currents.[3] Since the mass of the electron is very small compared to m_Λ, we can neglect it. This means that $f_3(q^2)$ and $g_3(q^2)$ can be ignored because they appear multiplied by m_e in the transition amplitude. We are then left with four form factors.

In the Λ rest frame, the transition rate can be expressed in the form[6]

$$d\omega = \frac{G^2}{(2\pi)^5}\frac{m_p}{m_\Lambda}\frac{e^2\nu^3}{E-e}|\mathfrak{M}|^2\, de\, d\Omega_e\, d\Omega_\nu\,, \tag{5}$$

with $|\mathfrak{M}|^2$ given by

$$|\mathfrak{M}|^2 = \xi[1 + a\hat{e}\cdot\hat{\nu} + A\hat{e}\cdot\langle\vec{\sigma}_\Lambda\rangle + B\hat{\nu}\cdot\langle\vec{\sigma}_\Lambda\rangle + A'(\hat{e}\cdot\hat{\nu})(\hat{e}\cdot\langle\vec{\sigma}_\Lambda\rangle) + B'(\hat{e}\cdot\hat{\nu})(\hat{\nu}\cdot\langle\vec{\sigma}_\Lambda\rangle) + D\langle\vec{\sigma}_\Lambda\rangle\cdot\hat{e}\times\hat{\nu}]\,. \tag{6}$$

e is the electron energy and E is its maximum value,

$$E = \frac{{m_\Lambda}^2 - {m_p}^2}{2m_\Lambda}\,. \tag{7}$$

ν is the neutrino energy, which, by energy-momentum conservation, is a function of e and $\hat{e}\cdot\hat{\nu}$,

$$\nu = \frac{E-e}{1-(e/m_\Lambda)(1-\hat{e}\cdot\hat{\nu})}\,. \tag{8}$$

$\hat{e}$ and $\hat{\nu}$ are unit vectors along the directions of the electron and neutrino, respectively. $\langle\vec{\sigma}_\Lambda\rangle$ is the polarization of the Λ, G^2 is an over-all coupling constant, which is chosen to be the μ-decay coupling constant, and the proportionality factors are absorbed into the f's and g's. The coefficients in Eq. (6) are linear functions of e and ν, and quadratic functions of the form factors.

If the q^2 dependence of the form factors is either neglected or parametrized, say, linearly, then the decay rate, Eq. (5), can be integrated over angles and energy. Our interest lies mainly in calculating expressions that can be compared readily to the experimental numbers. What is usually measured in hyperon leptonic decays is the total decay rate, or branching ratio, the electron-neutrino correlation, and the electron, neutrino, and proton asymmetries.

The e-ν correlation is defined as

$$\alpha_{e\nu} = 2\,\frac{N(\theta_{e\nu} < \frac{1}{2}\pi) - N(\theta_{e\nu} > \frac{1}{2}\pi)}{N(\theta_{e\nu} < \frac{1}{2}\pi) + N(\theta_{e\nu} > \frac{1}{2}\pi)}\,,$$

where $N(\theta_{e\nu} < \frac{1}{2}\pi)$ is the number of e-ν pairs that make an angle $\theta_{e\nu}$ smaller than 90°, and the electron asymmetry is defined as

$$\alpha_e = 2\,\frac{N(\theta_e < \frac{1}{2}\pi) - N(\theta_e > \frac{1}{2}\pi)}{N(\theta_e < \frac{1}{2}\pi) + N(\theta_e > \frac{1}{2}\pi)}\,,$$

where θ_e is the angle between the e direction and the polarization of the Λ. α_ν and α_p are defined similarly.

We have calculated[7,8] expressions for the measured quantities, ignoring the q^2 dependence of the form factors and keeping terms proportional up to β^2, with β defined as

$$\beta = (m_\Lambda - m_p)/m_\Lambda\,,$$

$$R = G^2\frac{\Delta m^5}{60\pi^3}[(1-\tfrac{3}{2}\beta+\tfrac{6}{7}\beta^2)|f_1|^2 + (\tfrac{4}{7}\beta^2)|f_2|^2 + (3-\tfrac{9}{2}\beta+\tfrac{12}{7}\beta^2)|g_1|^2 + (\tfrac{12}{7}\beta^2)|g_2|^2 + (\tfrac{6}{7}\beta^2)\mathrm{Re}f_1f_2^* + (-4\beta+6\beta^2)\mathrm{Re}g_1g_2^*]\,, \tag{9}$$

$$R\times\alpha_{e\nu} = G^2\frac{\Delta m^5}{60\pi^3}[(1-\tfrac{5}{2}\beta+\tfrac{11}{7}\beta^2)|f_1|^2 + (-\tfrac{2}{7}\beta^2)|f_2|^2 + (-1-\tfrac{3}{2}\beta+\tfrac{25}{7}\beta^2)|g_1|^2 + (-2\beta^2)|g_2|^2 + (-\tfrac{2}{7}\beta^2)\mathrm{Re}f_1f_2^* + (4\beta-2\beta^2)\mathrm{Re}g_1g_2^*]\,, \tag{10}$$

$$R\times\alpha_e = G^2\frac{\Delta m^5}{60\pi^3}[(-\tfrac{1}{3}\beta+\tfrac{3}{14}\beta^2)|f_1|^2 + (-\tfrac{4}{21}\beta^2)|f_2{}^2| + (-2+\tfrac{8}{3}\beta-\tfrac{9}{14}\beta^2)|g_1|^2 + (-\tfrac{4}{3}\beta^2)|g_2|^2 + (-\tfrac{2}{3}\beta+\tfrac{14}{21}\beta^2)\mathrm{Re}f_1f_2^* + (2-\tfrac{11}{3}\beta+\tfrac{15}{7}\beta^2)\mathrm{Re}f_1g_1^* + (-\tfrac{2}{3}\beta+\tfrac{32}{21}\beta^2)\mathrm{Re}f_1g_2^* + (-\tfrac{2}{3}\beta+\tfrac{32}{21}\beta^2)\mathrm{Re}f_2g_1^* + (\tfrac{16}{21}\beta^2)\mathrm{Re}f_2g_2^* + (\tfrac{10}{3}\beta-\tfrac{94}{21}\beta^2)\mathrm{Re}g_1g_2^*]\,, \tag{11}$$

$$R\times\alpha_\nu = G^2\frac{\Delta m^5}{60\pi^3}[(\tfrac{1}{3}\beta-\tfrac{3}{14}\beta^2)|f_1|^2 + (\tfrac{4}{21}\beta^2)|f_2{}^2| + (2-\tfrac{8}{3}\beta+\tfrac{9}{14}\beta^2)|g_1|^2 + (\tfrac{4}{3}\beta^2)|g_2|^2 + (\tfrac{2}{3}\beta-\tfrac{14}{21}\beta^2)\mathrm{Re}f_1f_2^* + (2-\tfrac{11}{3}\beta+\tfrac{15}{7}\beta^2)\mathrm{Re}f_1g_1^* + (-\tfrac{2}{3}\beta+\tfrac{32}{21}\beta^2)\mathrm{Re}f_1g_2^* + (-\tfrac{2}{3}\beta+\tfrac{32}{21}\beta^2)\mathrm{Re}f_2g_1^* + (\tfrac{16}{21}\beta^2)\mathrm{Re}f_2g_2^* + (-\tfrac{10}{3}\beta+\tfrac{94}{21}\beta^2)\mathrm{Re}g_1g_2^*] \tag{12}$$

$$R\times\alpha_p = G^2\frac{\Delta m^5}{60\pi^3}\,\tfrac{5}{2}[(-1+\tfrac{11}{6}\beta-\beta^2)\mathrm{Re}f_1g_1^* + (\tfrac{1}{3}\beta-\tfrac{5}{6}\beta^2)\mathrm{Re}f_1g_2^* + (\tfrac{2}{3}\beta-\tfrac{7}{6}\beta^2)\mathrm{Re}f_2g_1^* + (-\tfrac{2}{3}\beta^2)\mathrm{Re}f_2g_2^*]\,, \tag{13}$$

TABLE I. Experimental data on $\Lambda\ \beta$ decay.

R ($\sec^{-1}$)	$\alpha_{e\nu}$	α_e	α_ν	α_p
$(3.35\pm 0.14)\times 10^6$	-0.01 ± 0.08	0.13 ± 0.07	0.74 ± 0.16	-0.51 ± 0.08
Refs. 2,12	Refs. 1,2,13–17	Refs. 1,2,15,18	Refs. 1,15	Refs. 1,2

where $\Delta m = m_\Lambda - m_p$.

Of these expressions, Eq. (9) for the rate was given by Desai[9] without the second-class terms; his and our common terms agree. In Eqs. (9)–(13) the polarization of the decaying Λ is not shown explicitly; therefore, these expressions must be compared with data that have been normalized to unit polarization. Notice that Eqs. (11) and (12) satisfy a theorem given by Weinberg.[10] In our case in which m_e is neglected, this theorem says that when the e and ν variables are interchanged, the f and g interference terms are symmetric, while the others are antisymmetric.

An important remark is that g_2 contributes to some of these expressions through the terms $\mathrm{Re}\, g_1 g_2^*$ with coefficients comparable to those of f_1^2. A similar remark had already been made for the electron energy spectrum and the partial decay rate.[11]

In order to develop some feeling for the restrictions that the experimental numbers, given in Table I,[12–18] impose on the form factors, we have plotted in Figs. 1–8 the e-ν correlation and the three asymmetries as functions of g_1/f_1 for different values of f_2/f_1 and g_2/f_1.

When g_2/f_1 is zero, Figs. 1–4 show the following:

(a) $\alpha_{e\nu}$ is symmetric in g_1/f_1 and favors values of g_1/f_1 in a fairly narrow range around ± 0.70, within 0.10. It has little sensitivity to f_2/f_1.

(b) α_e favors positive g_1/f_1. The bounds from $\alpha_{e\nu}$ now indicate that for negative f_2/f_1, g_1/f_1 has two allowed ranges, one around 0.8 and another one around 0.1. Positive values of f_2/f_1 are within the error bars if f_2/f_1 is not too close to +2.

(c) α_ν favors g_1/f_1 around 0.13, within 0.15, for positive f_2/f_1.

(d) α_p for negative f_2/f_1 favors g_1/f_1 around 0.25 quite narrowly, within 0.10 or in a wide range towards values past 1.0. For positive f_2/f_1, values around 0.6 are preferred.

Thus, when $g_2 = 0$, we expect the following situation: $\alpha_{e\nu}$ selects g_1/f_1 narrowly around 0.7. In α_e this range of g_1/f_1 selects f_2/f_1 negative. Then this range of f_2/f_1 in α_p selects g_1/f_1 narrowly around 0.3. α_ν shows a marked preference for g_1/f_1 around 0.4. We can see that $\alpha_{e\nu}$ and α_ν oppose each other and that α_p aided by α_e also oppose $\alpha_{e\nu}$. Then two different values for g_1/f_1 are implied by the four correlations, one around 0.7 and another around 0.30. If future experiments confirm the experimental values we are using, and only the error bars are reduced, then this effect would turn into a real ambiguity.

When g_2/f_1 is allowed to be different from zero, then Figs. 5–8 together with Figs. 1–4 show the following:

(a) Either $\alpha_{e\nu}$ induces g_1/f_1 to move from around 1.3 to smaller values, say 0.3, as g_2/f_1 varies from 2 to −2, or it induces g_1/f_1 to move from small negative to large negative values as g_2/f_1 varies from 2 to −2. It has little sensitivity to f_2/f_1.

(b) α_e induces in g_1/f_1 behavior similar to that which $\alpha_{e\nu}$ induces, except that the range of g_1/f_1, which was previously negative, is now a rather narrow range from small positive to small negative values. This asymmetry is sensitive to f_2/f_1, favoring negative values of f_2/f_1 as g_2/f_1 goes to −2.

(c) α_ν clearly selects positive values of g_1/f_1 in a fairly narrow range, say between 0.15 and 0.5, as g_2 goes from −2 to 2. It shows little sensitivity to f_2/f_1.

(d) α_p also selects positive values of g_1/f_1 only. Here the value of f_2/f_1 does not matter. For $g_2/f_1 = 2$ there are two ranges for g_1/f_1, still not very sensitive to f_2/f_1, but as g_2/f_1 tends toward −2, f_2/f_1 imposes rather strict bounds on g_1/f_1; specifically, for f_2/f_1 from −1 to 2, g_1/f_1 is either very small (around 0.10) or quite large.

Thus, the ν asymmetry requires smaller values of g_1/f_1, around 0.2. With this restriction the e-ν correlation selects large negative values of g_2/f_1, around −2, and then the p asymmetry requires f_2/f_1 to be between −2 and −1. Lastly, the e asymmetry confirms these choices. We can see that in this case, when the restriction of null g_2 is dropped, there does seem to be a common solution to the combined requirements of the three asymmetries and the e-ν correlation on the form factors.

III. CABIBBO'S MODEL AND $\Lambda\ \beta$ DECAY

This model consists of the following assumptions.[5]

(i) The interaction is $V-A$ and the vector current belongs to the same SU_3 octet as the electromagnetic current does.

(ii) The axial-vector current belongs to another SU_3 octet.

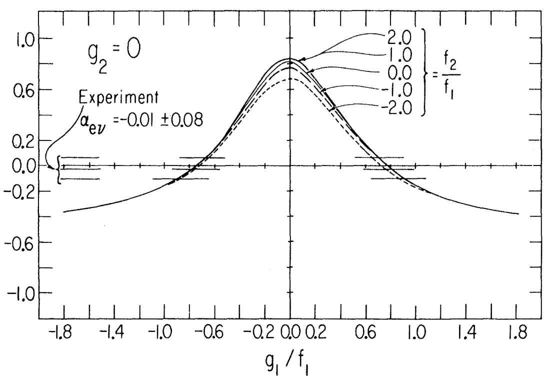

Fig. 1. Electron-neutrino correlation as a function of g_1/f_1 and f_2/f_1 when $g_2 = 0$.

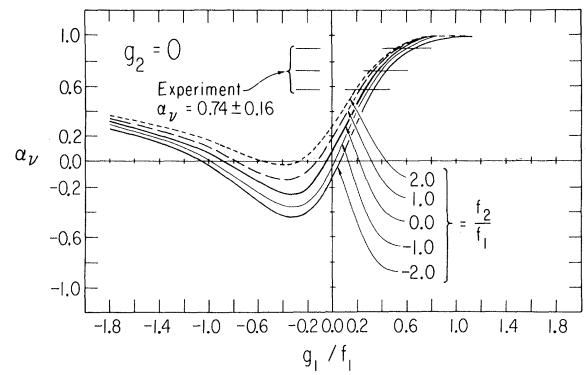

Fig. 3. Neutrino asymmetry as a function of g_1/f_1 and f_2/f_1 when $g_2 = 0$.

(iii) The universality of the leptonic weak coupling constant is modified through the introduction of an angle parameter θ in the hadronic current as follows:

$$j_\mu^h = \cos\theta\, j_\mu^{\Delta S=0} + \sin\theta\, j_\mu^{\Delta S\neq 0}\,.$$

Assumption (iii) is designed to parametrize the observed depression in the transition rates of the $\Delta S \neq 0$ relative to the $\Delta S = 0$ semileptonic decays. Assumption (i) is the extension of CVC to $\Delta S \neq 0$ processes, and in the limit of exact SU_3 asymmetry, the vector $\Delta S \neq 0$ current would be conserved. Assumption (ii) provides a framework for relating the properties of the axial-vector current in different processes.

The spirit of the model is that the symmetry limit gives a good approximation to nature, that is, that the symmetry-breaking effects are small. This is supported to some extent by the Ademollo-Gatto theorem.[19] Still, it is interesting to see how accurate the predictions of Cabibbo's model are, since any small deviation could serve as a guide to incorporate symmetry breaking into the model.

There is no theoretical basis for evaluation of the angle parameter; its value has been determined by fits to the experimental data of many hyperon decays. One is faced with several possible interpretations[20] of this value. Either θ is a pure weak effect and its value will not be affected by symmetry breaking due to strong interactions, or θ will have to be refitted once a breaking mechanism is introduced. To avoid this ambiguity we prefer not to give our results using the angle parameter explicitly, but instead to give them using the form factors directly.

In the past, the experimental data on hyperon decay have consisted mainly of transition rates and some correlations. Such data have been compared with Cabibbo's model by making certain assumptions in order to reduce the number of parameters used in the fits to the data. These assumptions are that the q^2 dependence of the form factors can be neglected or taken small and parametrized linearly, that, as mentioned above, the matrix elements do not differ much from their symmetry value, and that no second-class-current contribution is substantial. In an SU_3 model the g_2 contribu-

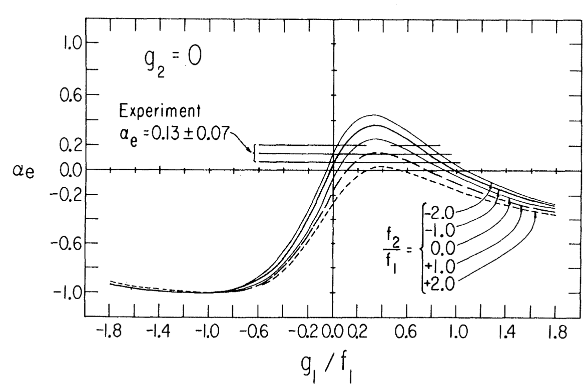

Fig. 2. Electron asymmetry as a function of g_1/f_1 and f_2/f_1 when $g_2 = 0$.

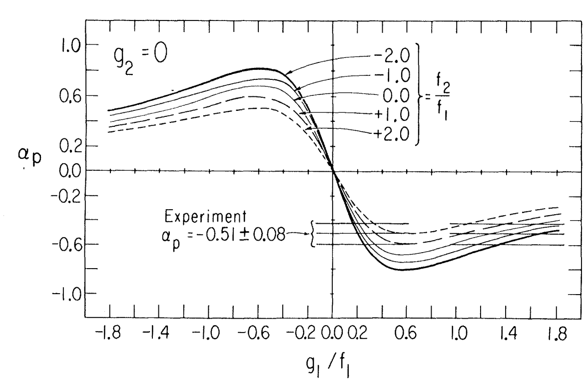

Fig. 4. Proton asymmetry as a function of g_1/f_1 and f_2/f_1 when $g_2 = 0$.

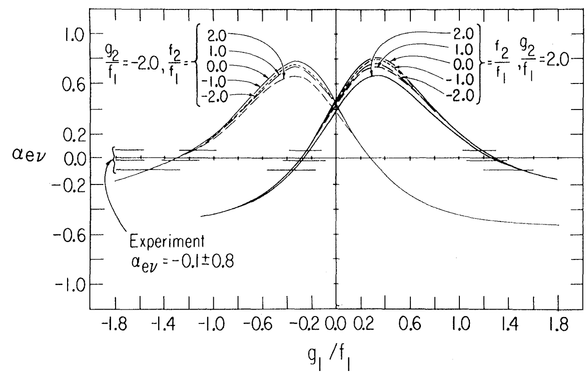

Fig. 5. Electron-neutrino correlation as a function of $g_1/f_1, f_2/f_1$, and g_2/f_1.

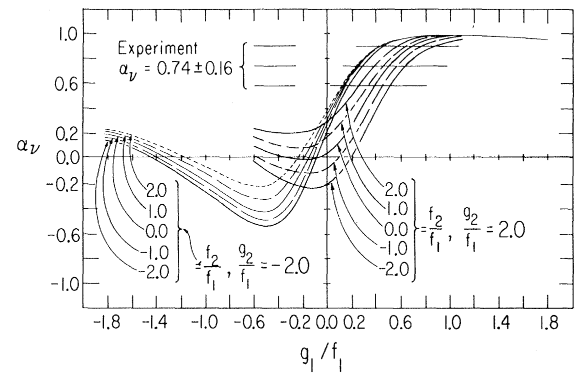

Fig. 7. Neutrino asymmetry as a function of g_1/f_1, f_2/f_1, and g_2/f_1.

tion is zero in the symmetry limit. The hadronic part of the transition amplitude for Λ β decay then is[21]

$$\langle p|j_\mu^h|\Lambda\rangle = (\tfrac{3}{2})^{1/2}\sin\theta\,\bar{u}(p)\Big[\gamma_\mu + \frac{\mu_p}{2m_\Lambda}\sigma_{\mu\nu}q^\nu + (F+\tfrac{1}{3}D)\gamma_\mu\gamma_5\Big]u(\Lambda)\,, \quad (14)$$

where μ_p is the proton magnetic moment, $(\frac{3}{2})^{1/2}$ is a Clebsch-Gordan coefficient, and F and D represent the mixture of symmetric and antisymmetric octet in the reduced matrix element of the axial-vector current. The relations to the form factors of Eqs. (9)–(13) are

$$f_1 = (\tfrac{3}{2})^{1/2}\sin\theta\,, \quad (15)$$

$$f_2 = (\tfrac{3}{2})^{1/2}\sin\theta\,\tfrac{1}{2}\mu_p\,, \quad (16)$$

$$g_1 = (\tfrac{3}{2})^{1/2}\sin\theta\,(F+\tfrac{1}{3}D)\,. \quad (17)$$

Similar expressions obtain for the other hyperon decays again in terms of θ, F, and D.[21] These parameters are then determined by fits to the relevant data, and Eqs. (15)–(17) give us the values of the form factors. The predictions of Cabibbo's model for Λ β decay are given[22] in Table II, together with the corresponding values for the asymmetries, the e-ν correlation, and the partial decay rate.

Using only the data on Λ β decay, we have performed a χ^2 fit with f_1, f_2, and g_1 as parameters, assuming that their q^2 dependence can be ignored, that no g_2 is present, and that radiative corrections can also be ignored.

The fitted values we obtained are given in Table III, normalized to the μ-decay coupling constant.

The minimum of χ^2 is 4.94, corresponding to a probability of 9%. The values of the asymmetries, and the rate, for Λ β decay, calculated with the fitted values of the form factors are also shown in Table III. We see that the somewhat large χ^2_{min} obtained, corresponds to the fact that the values of the e, ν, and p asymmetries are off their experimental counterparts by 1 standard deviation each. In Fig. 9 the contours corresponding to 1 standard deviation, in the sense of χ^2 changing by one from its minimum, are drawn.

Comparing Tables II and III we see that the Cabibbo-model predictions and our fit agree. The large value of χ^2_{min} of our fit can be traced to the

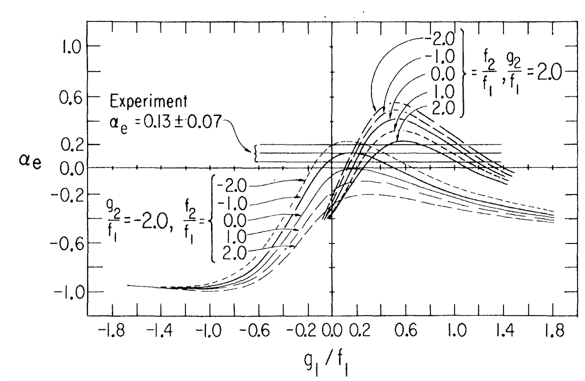

Fig. 6. Electron asymmetry as a function of g_1/f_1, f_2/f_1, and g_2/f_1.

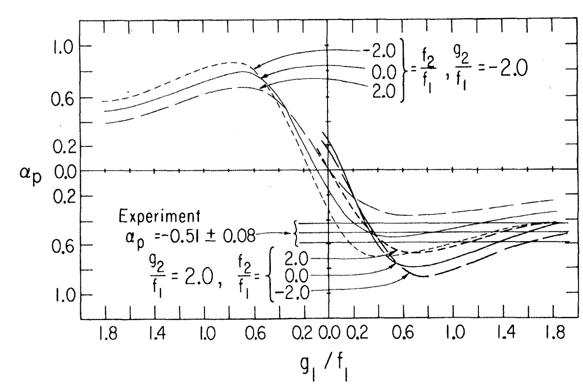

Fig. 8. Proton asymmetry as a function of g_1/f_1, f_2/f_1, and g_2/f_1.

TABLE II. Predictions of Cabibbo's model for $\Lambda\ \beta$ decay.

$f_1=0.29$	$R=3.31\times10^6\ \text{sec}^{-1}$
$f_2=0.29$	$\alpha_{e\nu}=0.02$
$g_1=0.21$	$\alpha_e=0.03$
$g_1/f_1=0.72$	$\alpha_\nu=0.98$
$g_2=0.0$	$\alpha_p=-0.59$

opposing tendencies of $\alpha_{e\nu}$ and the α_e, α_ν pair, as discussed in Sec. II. And this, in turn, conforms with the fact that the predictions of Cabibbo's model for the three asymmetries are each 1 standard deviation off their experimental counterparts.

IV. q^2 DEPENDENCE OF THE FORM FACTORS

The experimental data do not allow us to make a thorough check of the q^2 dependence of the $\Lambda\ \beta$-decay form factors, so, once again, in the expectation that such a dependence is small, we approximate it by a linear form as follows:

$$f_1(q^2)=f_1(0)+\frac{q^2}{{m_\Lambda}^2}\lambda_1^f,$$

$$g_1(q^2)=g_1(0)+\frac{q^2}{{m_\Lambda}^2}\lambda_1^g.$$

In this form the q^2 dependence contribution is of order β^2, so we can handle this contribution consistently only if the slopes λ_1^f and λ_1^g are small; otherwise our formulas (9)–(13) would have to be corrected to higher order in β and the q^2 dependence of f_2 and g_2 should be considered as well. With these restrictions in mind, Eqs. (9)–(13) gain the terms (9′)–(13′), respectively:

$$R\text{:}\quad G^2\frac{\Delta m^5}{60\pi^3}(\tfrac{4}{7}\beta^2)(f_1\lambda_1^f+5g_1\lambda_1^g)\,, \tag{9′}$$

$$R\times\alpha_{e\nu}\text{:}\quad G^2\frac{\Delta m^5}{60\pi^3}(\tfrac{24}{7}\beta^2)(-g_1\lambda_1^g)\,, \tag{10′}$$

$$R\times\alpha_e\text{:}\quad G^2\frac{\Delta m^5}{60\pi^3}(\tfrac{4}{7}\beta^2)(g_1\lambda_1^f+f_1\lambda_1^g-4g_1\lambda_1^g)\,, \tag{11′}$$

$$R\times\alpha_\nu\text{:}\quad G^2\frac{\Delta m^5}{60\pi^3}(\tfrac{4}{7}\beta^2)(g_1\lambda_1^f+f_1\lambda_1^g+4g_1\lambda_1^g)\,, \tag{12′}$$

$$R\times\alpha_p\text{:}\quad G^2\frac{\Delta m^5}{60\pi^3}(-\tfrac{5}{6}\beta^2)(f_1\lambda_1^g+g_1\lambda_1^f)\,, \tag{13′}$$

TABLE III. Fitted values of the form factors when $g_2=0$ and corresponding values of the measured quantities.

$f_1=0.30\pm0.02$	$\alpha_e=0.04$
$f_2=0.25\pm0.25$	$\alpha_\nu=0.97$
$g_1=0.21\pm0.01$	$\alpha_p=-0.60$
$g_1/f_1=0.70\pm0.06$	$\chi^2_{\min}=4.94$
$R=3.35\times10^6\ \text{sec}^{-1}$	Prob. = 9%
$\alpha_{e\nu}=0.04$	Degrees of freedom = 2

TABLE IV. Fitted values of the form factors and the slope of g_1 when $g_2=0$.

$f_1=0.35$	$\lambda_1^f=0$
$f_2=0.23$	$\lambda_1^g=4.54$
$g_1=0.12$	$\chi^2_{\min}=2.77$
$g_1/f_1=0.34$	Prob. = 10%
$g_2=0$	Degrees of freedom = 1

where $f_1(0)\equiv f_1$ and $g_1(0)\equiv g_1$.

Since there are only five experimental numbers, we made two fits, first taking λ_1^g zero and second taking λ_1^f zero. The only result of interest, with our restrictions, appears in Table IV, normalized to the μ-decay coupling constant. Since we discuss the q^2 contribution of the form factors qualitatively, we do not quote error bars.

For λ_1^f nonzero, no acceptable fit was found; only very large slopes were obtained.

The χ^2 found in Table IV is no appreciable improvement from the χ^2 found before, when no q^2 dependence was considered, but now the constant parts of the form factors differ from what we had before and the predictions of Cabibbo's model, especially the g_1/f_1 ratio.

For comparison one can estimate the slope of $g_1(q^2)$ assuming that its q^2 dependence is dominated by the nearest pole, which corresponds to ${m_{KA}}^2=1.54\ \text{BeV}^2$:

$$g_1(q^2)=\frac{g_1(0)}{1-q^2/{m_{KA}}^2}\simeq g_1(0)+\frac{g_1(0)}{{m_{KA}}^2}q^2\,.$$

The slope λ_1^g would then be given by

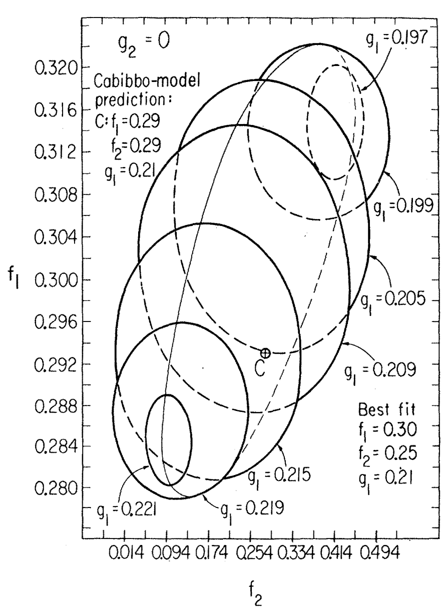

Fig. 9. Contours of $\chi^2=\chi^2_{\min}+1$ when $g_2=0$.

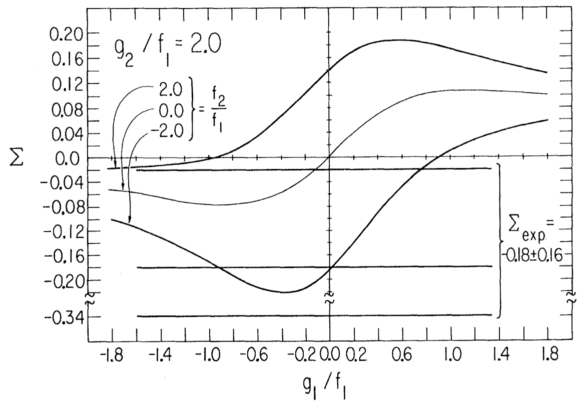

FIG. 10. Dependence of Σ on g_1/f_1 and f_2/f_1 when $g_2/f_1=2$.

$$\lambda_1^g=\frac{g_1(0)}{m_{KA}{}^2}m_\Lambda{}^2\,.$$

If the fitted value of $g_1(0)$ were used, then

$$\lambda_1^g=0.90\,,$$

which is rather small compared to the value in Table IV.

The experimental data are not adequate to determine the combined effect of the q^2 dependence of both form factors. But, by comparing Tables I and II and using Eqs. (11′)–(13′), we can see that for small slopes, such a combined effect can not bring better agreement between the three asymmetries predicted by Cabibbo's model and the experimental ones. Indeed, to match both the proton and the neutrino asymmetries, respectively, negative λ_1^g and λ_1^f are favored by Eqs. (12′) and (13′), while to match the electron asymmetries negative λ_1^g and positive λ_1^f are favored.

We can conclude then that a small q^2 variation of the form factors does not improve the agreement between Cabibbo's model and the Λ β-decay data.

V. SECOND-CLASS CURRENTS

We want to study the restrictions that the Λ β-decay data impose on second-class currents. They contribute, in the approximation we are using, through g_2. In this section we ignore the q^2 dependence of the form factors. In Sec. II, the different quantities measured experimentally were plotted as functions of the form factors, and it was qualitatively argued that a nonzero contribution of g_2 is favored by the data. We want to extend this kind of analysis by looking at two particular sum rules that were introduced in I.

The two sum rules are defined in terms of the measured quantities as follows:

$$\Sigma=\frac{\langle\xi\rangle\{[1-\frac{1}{2}\beta(1+a)](\alpha_\nu-\alpha_e)-(1-a)\}}{\langle\xi\rangle(1+a)}\,,$$

$$\Pi=\frac{\langle\xi\rangle\{[1+\frac{1}{2}\beta(1-a)](\alpha_\nu+\alpha_e)\}}{\langle\xi\rangle(1+a)}\,,$$

where

$$a=\frac{\alpha_{e\nu}+\frac{3}{2}\beta}{1+\frac{1}{2}\beta\alpha_{e\nu}}\,,\quad \langle\xi\rangle=\frac{R}{1-\frac{1}{2}\beta a}\,,\quad\text{and } \beta=\frac{m_\Lambda-m_p}{m_\Lambda}\,.$$

These definitions are valid to first order in β, so we restrict the analysis of Σ and Π to this order. In terms of the form factors, the sum rules are

$$\langle\xi\rangle(1+a)\Sigma=\tfrac{2}{3}\beta(|f_1|^2+|g_1|^2+2\mathrm{Re}f_1f_2^*+2\mathrm{Re}g_1g_2^*),\qquad(18)$$

$$\langle\xi\rangle(1+a)\Pi=4\mathrm{Re}f_1g_1^*+\tfrac{4}{3}\beta\,\mathrm{Re}(2f_1g_1^*-g_1f_2^*-f_1g_2^*),\qquad(19)$$

where

$$\langle\xi\rangle(1+a)=2(1+\beta)(|f_1|^2+|g_1|^2)\,.$$

The experimental values of Σ and Π are

$$\Sigma=-0.18\pm0.16\,,\qquad(20)$$

$$\Pi=\ \ 0.75\pm0.12\,.\qquad(21)$$

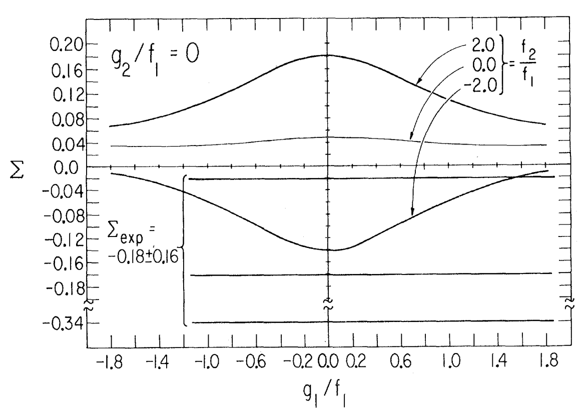

Fig. 11. Dependence of Σ on g_1/f_1 and f_2/f_1 when $g_2/f_1=0$.

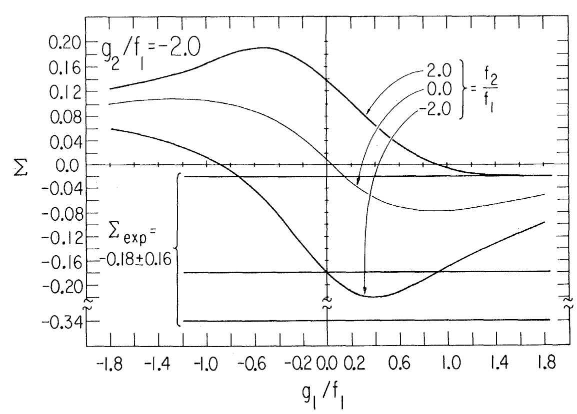

Fig. 12. Dependence of Σ on g_1/f_1 and f_2/f_1 when $g_2/f_1=-2$.

TABLE V. Fitted values for the form factors and corresponding values of the measured quantities when $g_2 \neq 0$.

$f_1 = 0.36 \pm 0.012$	$\alpha_e = 0.10$
$f_2/f_1 = -1.40^{+1.02}_{-1.19}$	$\alpha_\nu = 0.69$
$g_1/f_1 = 0.19^{+0.13}_{-0.14}$	$\alpha_p = -0.55$
$g_2/f_1 = -2.51^{+0.80}_{-0.84}$	$\chi^2_{\min} = 0.56$
$R = 3.35 \times 10^6\ \mathrm{sec}^{-1}$	Prob. =47%
$\alpha_{e\nu} = 0.00$	Degrees of freedom = 1

The usefulness of these sum rules lies in the fact that they are very sensitive to the form factors and that Σ is independent of any time-reversal-violating phase between axial-vector and vector form factors. They permit one to appreciate readily the second-class contribution favored by the experimental data. In Figs. 10–15 we have plotted Σ and Π for ranges of interest of the form factors. The experimental values for Σ and Π are also displayed.

One can easily see from these plots how the experimental value of Π forces g_1/f_1 to be positive and how for positive g_1/f_1 the experimental value of Σ favors g_2/f_1 and f_2/f_1 towards -2. It is clear that the main feature of Σ and Π is that they are so sensitive to changes in the different form factors that they reduce the masking effects of the experimental errors.

This discussion, together with the one of Sec. II, enables one to interpret more clearly the numerical evaluation of the form factor. We now wish to consider such an evaluation.

We have made a χ^2 fit to the five experimental numbers, using Eqs. (9)–(13), allowing g_2 to be present. The results are shown in Table V. The corresponding values of the rate and asymmetries are also shown.

We obtained a reasonable fit as was expected from the discussion of the various graphs. The higher probability is, of course, also reflected

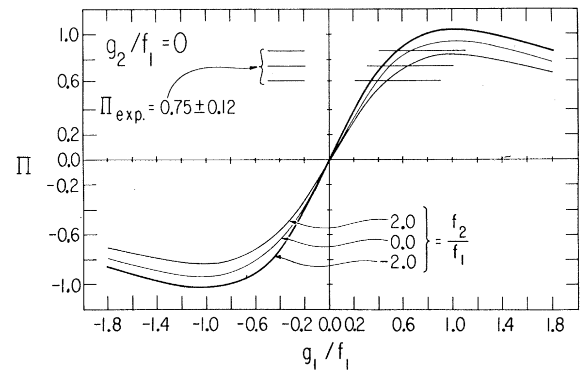

Fig. 14. Dependence of Π on g_1/f_1 and f_2/f_1 when $g_2/f_1 = 0$.

in the fact that the values of the asymmetries calculated with the fitted form factors lie quite close to the experimental ones, as can be seen by comparing Tables V and I.

Inasmuch as a scale transformation of the form factors changes the rate but not the correlations, the ratios f_2/f_1, g_1/f_1, and g_2/f_1 cannot depend on the rate and may be fitted to the correlations alone. The errors in the above ratios of form factors can be shown by drawing 1-standard-deviation contours in the sense that χ^2 increases by one from its minimum value. This is done in Fig. 16.

Since the fit we have obtained is a fairly good one, we see then that, restricting oneself to Λ β-decay data, the assumption that second-class currents are absent or present is crucial to the agreement between Cabibbo's model predictions and Λ β-decay data. The fitted values, given in Table V, show that f_1 does not change much from the value it has when $g_2 = 0$, but f_2 changes so much that if the vector current is still in an SU_3 octet, then its connection with the SU_3 properties of the electromagnetic current is masked by too large a symmetry breaking.

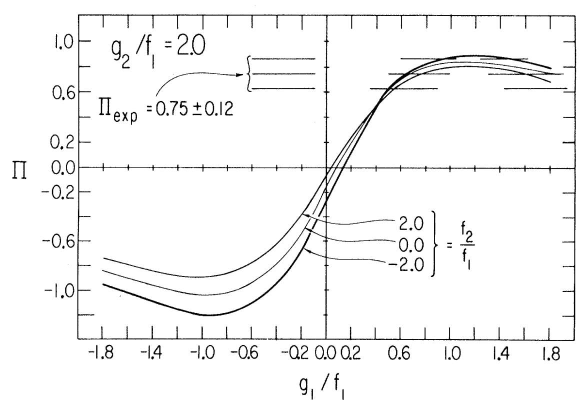

FIG. 13. Dependence of Π on g_1/f_1 and f_2/f_1 when $g_2/f_1 = 2$.

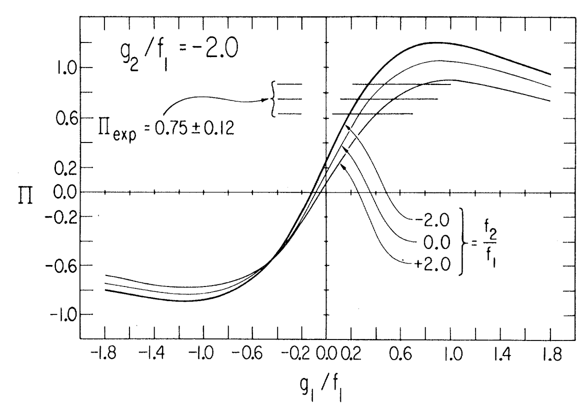

Fig. 15. Dependence of Π on g_1/f_1 and f_2/f_1 when $g_2/f_1 = -2$.

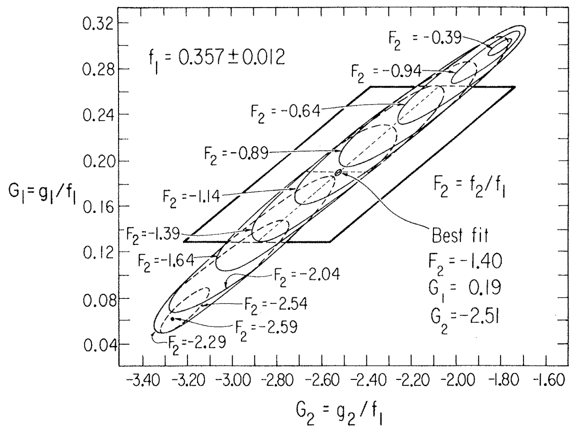

Fig. 16. Contours of $\chi^2 = \chi^2_{\min}+1$ when $g_2 \neq 0$.

The presence of g_2 can be made compatible with assumption (ii) of Sec. III only as a symmetry-breaking effect, but the fitted value of g_2 is so large compared to the one of g_1 that if this fit is taken seriously, then it might not be easily reconciled with assumption (ii), whose main usefulness is to give certain group properties to the axial-vector current. Since the g_2 term corresponds to an SU_3 symmetry-breaking term, we see then that Λ β-decay data favor its being a very large symmetry-breaking term.

VI. SENSITIVITY OF THE FIT TO THE VALUE OF α_ν

The ν asymmetry will be known more precisely in the near future. In order to appreciate how sensitive the fitting procedure is to the experimental value of the ν asymmetry, we have repeated the fitting assuming different values for α_ν, taking its error as the one corresponding to approximately 700 events. Table VI contains the results.

The main conclusion is that for ν asymmetry values very close to unity, the $g_2 \neq 0$ and $g_2 = 0$ fits are comparable, and as the ν asymmetry grows smaller, the $g_2 \neq 0$ fit gradually imposes itself upon the $g_2 = 0$ fit. Thus, the ν asymmetry provides interesting information concerning the possible existence of second-class currents.

VII. SCALAR AND TENSOR CONTRIBUTIONS

Another possibility we want to explore is the presence of scalar and tensor admixtures to the $V-A$ interaction. We only give a qualitative discussion, because the presently available data do not permit a definite settlement of the structure of the interaction that governs Λ β decay. We would nevertheless like to have some indications of the effects of scalar and tensor contributions. For our purpose then, it seems sufficient to consider these contributions to zeroth order, but, of course, better formulas[23] will be needed to make quantitative determinations.

We shall follow Ref. 24. We assume that the emitted neutrino can be considered a two-component object and that time-reversal invariance remains valid. The following terms must be added to the interaction Lagrangian, Eq. (1):

$$\mathcal{L}_{\rm int}: \ \frac{G}{\sqrt{2}} S^h S^l + \frac{G}{\sqrt{2}} T^h_{\lambda\mu} T^l_{\lambda\mu} + \text{H.c.} \qquad (1')$$

The matrix elements of the hadronic scalar and tensor parts of the interaction are

$$\langle p | S^h | \Lambda \rangle = f_S \left(\frac{m_\Lambda m_p}{E_\Lambda E_p}\right)^{1/2} \bar{u}(p) u(\Lambda) ,$$

$$\langle p | T^h_{\lambda\mu} | \Lambda \rangle = \frac{1}{\sqrt{2}} f_T \left(\frac{m_\Lambda m_p}{E_\Lambda E_p}\right)^{1/2} \bar{u}(p) \sigma_{\lambda\mu} u(\Lambda) ,$$

where all the induced terms are left out. The corresponding matrix elements of the leptonic parts are

$$\langle e | S^l | \nu \rangle = \bar{u}(e)(1+\gamma_5) u(\nu) ,$$

$$\langle e | T^l_{\lambda\mu} | \nu \rangle = (1/\sqrt{2}) \bar{u}(e) \sigma_{\lambda\mu} (1+\gamma_5) u(\nu) .$$

The following terms must be added to Eqs. (9)–(13):

TABLE VI. Dependence of the fits on α_ν.

f_1	f_2	g_1	g_2	g_1/f_1	$\chi^2_{\min}$	Degrees of freedom	Prob. (%)	α_ν
0.30	0.20	0.21	0.0	0.70	3.24	2	20	0.89 ± 0.10
0.34	−0.10	0.14	−0.50	0.40	1.58	1	21	0.89 ± 0.10
0.30	0.20	0.21	0.0	0.70	3.92	2	15	0.84 ± 0.10
0.35	−0.20	0.12	−0.61	0.35	1.21	1	26	0.84 ± 0.10
0.30	0.20	0.20	0.0	0.69	6.37	2	4 [a]	0.75 ± 0.10
0.35	−0.41	0.08	−0.80	0.24	0.65	1	40 [a]	0.75 ± 0.10
0.30	0.20	0.20	0.0	0.67	10.38	2	1	0.65 ± 0.10
0.36	−0.65	0.05	−1.00	0.14	0.24	1	64	0.65 ± 0.10
0.30	0.19	0.20	0.0	0.65	15.9	2	<1	0.56 ± 0.10
0.36	−0.85	0.02	−1.15	0.06	0.04	1	85	0.56 ± 0.10

[a] This is the result of previous sections modified by a smaller error in α_ν.

TABLE VII. Fitted values of the form factors allowing scalar or tensor admixtures.

f_1	f_2	g_1	g_2	f_s	f_T	χ^2_{min}	Degrees of freedom	Prob.
0.31	0.15	0.17	0	0.18	0	0.02	1	88%
0.21	1.10	0.13	0	0	0.16	0.16	1	69%

$$R: \quad G^2 \frac{\Delta m^5}{60\pi^3}(|f_S|^2 + 3|f_T|^2), \tag{9''}$$

$$R\times\alpha_{e\nu}: \quad G^2 \frac{\Delta m^5}{60\pi^3}(-|f_S|^2 + |f_T|^2), \tag{10''}$$

$$R\times\alpha_{e}: \quad G^2 \frac{\Delta m^5}{60\pi^3}(2|f_T|^2 + 2\mathrm{Re} f_S f_T^*), \tag{11''}$$

$$R\times\alpha_{\nu}: \quad G^2 \frac{\Delta m^5}{60\pi^3}(2|f_T|^2 - 2\mathrm{Re} f_S f_T^*), \tag{12''}$$

$$R\times\alpha_{p}: \quad -G^2 \frac{\Delta m^5}{60\pi^3}(\tfrac{5}{2}|f_T|^2). \tag{13''}$$

The sum rule Σ, which we introduced in Sec. V, was designed so that it would vanish to zeroth order in β. Scalar and tensor couplings do, however, give zeroth-order contributions:

$$\langle\xi\rangle(1+a)\Sigma = -4\,\mathrm{Re} f_S f_T^* - 2|f_S|^2 - 2|f_T|^2 + O(\beta), \tag{18'}$$

$$\langle\xi\rangle(1+a)\Pi = 4|f_T|^2 + 4\,\mathrm{Re} f_1 g_1^* + O(\beta). \tag{19'}$$

Comparing with Eqs. (18) and (20), we see that a negative value of Σ can easily occur as a result of a scalar or tensor admixture.

We have performed a χ^2 fit to the experimental data using Eqs. (9)–(13) and (9″)–(13″), taking the scalar and tensor contributions one at a time. The results are shown in Table VII. No error bars are quoted, because our discussion is qualitative.

The fit improves noticeably from that obtained in Sec. III, as expected. The scalar admixture shows the interesting feature that the $V-A$ part remains close to what was obtained in previous fits and to the Cabibbo-model predictions. The price paid is a rather large scalar contribution. In contrast, the introduction of a tensor admixture favors a strong change in the f_2/f_1 ratio, too large to correspond to a small symmetry-breaking deviation from Cabibbo's model. For the purpose of comparison, the discussion of Ref. 25 about the contribution of S and T interactions to neutron β decay can be consulted. There it was found that the A and V interactions predominate with a probable maximum contribution of about 30% of S and T interactions.

VIII. FINAL REMARKS

We have studied the restrictions imposed by the Λ β-decay data under different assumptions. Our findings can be summarized as follows:

(a) There is a solution with $g_2 = 0$, near Cabibbo's predictions for g_1/f_1 and f_2/f_1, but with low probability because of deviations of the ν and e asymmetries from the Cabibbo values.

(b) Within the $V-A$ framework, there is another solution which has $g_2 \neq 0$, this time with high probability, but with values of g_1 and f_2 which deviate considerably from Cabibbo's predictions.

(c) The Λ β-decay data favor $g_2 \neq 0$ contributions that cannot be reconciled with small symmetry breaking, even when reasonable changes in the experimental value of the neutrino asymmetry are considered.

(d) Allowing some modifications to the pure $V-A$ picture, it is interesting that the deviations between the ν and e asymmetries and the SU_3 scheme are rather naturally accounted for by an added scalar interaction.

It should be kept in mind that our conclusions are valid inasmuch as radiative corrections can be neglected.

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[1]Argonne-Chicago-Ohio State-Washington University collaboration, paper submitted to the Fifteenth International Conference on High-Energy Physics, Kiev, 1970 (unpublished).

[2]CERN-Heidelberg collaboration, paper submitted to the Fifteenth International Conference on High-Energy Physics, Kiev, 1970 (unpublished).

[3]R. Oehme, R. Winston, and A. Garcia, Phys. Rev. D 3, 1618 (1971).

[4]S. Weinberg, Phys. Rev. 112, 1375 (1958); N. Cabibbo, Phys. Letters 12, 137 (1964).

[5]N. Cabibbo, Phys. Rev. Letters 10, 531 (1963); M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 706 (1960); M. Gell-Mann, Physics 1, 63 (1964).

[6]D. R. Harrington. Phys. Rev. 120, 1482 (1960); J. M. Watson and R. Winston, *ibid.* 181, 1907 (1969). These papers contain many further references.

[7]The expression for the proton asymmetry is due mainly to R. Winston; the author thanks him for his generosity.

[8]The very complete expressions of V. Linke, Nucl. Phys. B12, 669 (1969), should be consulted.

[9]P. S. Desai, Phys. Rev. 179, 1327 (1969).

[10]S. Weinberg, Phys. Rev. 115, 481 (1959).

[11]I. Bender, V. Linke, and H. J. Rothe, Z. Physik 212, 190 (1968).

[12]Barbaro-Galtieri *et al.*, Rev. Mod. Phys. 42, 87 (1970).

[13]J. E. Maloney and B. Sechi-Zorn, Phys. Rev. Letters 23, 425 (1969).

[14]R. J. Loveless, J. Canter, J. A. Cole, J. Lee-Franzini, and P. Franzini, Bull. Am. Phys. Soc. 14, 519 (1969).

[15]V. G. Lind, T. O. Binford, M. L. Good, and D. Stern, Phys. Rev. 135, B1483 (1964).

[16]C. Baglin *et al.*, Nuovo Cimento 35, 977 (1965).

[17]M. Baggett, N. Baggett, F. Eisele, H. Filthuth, H. Frehse, V. Hepp, R. Howard, E. Leitner, and G. Zech, Heidelberg report (unpublished).

[18]J. Barlow, J. M. Blair, G. Conforto, M. I. Ferrero, C. Rubbia, J. C. Sens, P. J. Duke, and A. K. Mann, Phys. Letters 18, 64 (1965).

[19]M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1964).

[20]R. Oehme and G. Segrè, Phys. Letters 11, 94 (1964).

[21]H. Filthuth, in *Proceedings of the Topical Conference on Weak Interactions, CERN, Geneva, 1969* (CERN, Geneva, 1969).

[22]Ebenhöh, F. Eisele, H. Filthuth, W. Föhlish, V. Hepp, E. Leitner, W. Presser, H. Schneider, T. Thouw, and G. Zech, in Proceedings of the Fifteenth International Conference on High-Energy Physics, Kiev, 1970 (unpublished); F. Eisele *et al.*, Z. Physik 225, 383 (1969).

[23]P. H. Frampton and W.-K. Tung, Phys. Rev. D 3, 1114 (1971); and W.-K. Tung, *ibid.* 3, 1678 (1971).

[24]J. D. Jackson, S. B. Treiman, and H. W. Wyld, Phys. Rev. 106, 517 (1957).

[25]M. T. Burgy, V. E. Krohn, T. B. Novey, G. R. Ringo, and V. L. Telegdi, Phys. Rev. 120, 1829 (1960).

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Remark on the Isospin Mass Differences

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The isospin splittings of the $\frac{1}{2}^+$ baryon octet are observed empirically to depend upon the (average) sum of charge and hypercharge or equivalently, upon the (average) $V_3 = -\frac{1}{2}(Q+Y)$ spin component. We construct a simple quark model compatible with this observation and then use this model to predict the $\frac{3}{2}^+$ decuplet isospin splittings. One obtains different decuplet predictions from the standard approach, enabling one eventually to test whether or not the baryon regularity is accidental. If nonaccidental, then support is given to recent conjectures that isospin splittings may be partially nonelectromagnetic.

Since the advent of unitary symmetry,[1] the mass splittings between isospin multiplets within an SU_3 multiplet have been understood as being due to a tensor which transforms as the hypercharge Y. Thus, ignoring electromagnetic and weak interactions, one can write an effective-mass Hamiltonian as

$$H_{\text{strong}} = \lambda_0 \mathrm{Tr}\overline{B}B + \lambda_1 \mathrm{Tr}\overline{B}YB + \lambda_2 \mathrm{Tr}\overline{B}BY + \lambda_3 \mathrm{Tr}\overline{B}YBY\,,$$

where B is the standard 3×3 baryon matrix and where the relatively small 27-plet contribution is usually set equal to zero. The splittings between members of an isospin multiplet have been thought to be purely electromagnetic in origin and thus due to a tensor which transforms as the charge Q. Thus one writes the isospin contribution as[2]

$$H_{\text{isospin}} = \delta_1' \mathrm{Tr}\overline{B}QB + \delta_2' \mathrm{Tr}\overline{B}BQ + \delta_3' \mathrm{Tr}\overline{B}QBQ\,,$$