Why Do Neutrinos Produce More W's Than Muons?*

F. A. BERENDS AND GEOFFREY B. WEST†
Cambridge Electron Accelerator, Harvard University, Cambridge, Massachusetts 02138
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A simple model is used to expose the mechanism which makes W production by neutrinos at least an order of magnitude larger than the production by muons. This difference was noted by us in a full calculation of the muon-induced process. Using the model, it is noticed that the reaction $\mu^-+n\to p+W^-+\mu^-$ does not suffer from this damping effect.

QUITE recently we presented a calculation of the total cross section for the production of W mesons via the muon-induced reaction[1,2]

$$\mu^-+p\to p+W^-+\nu_\mu. \tag{I}$$

The results of this calculation showed that this cross section was at least an order of magnitude *smaller* than that for the analogous neutrino-induced reaction

$$\nu_\mu+p\to\mu^-+p+W^+. \tag{II}$$

This latter process has been calculated by several different authors.[3,4] In Table I we exhibit the calculated cross section for both of these processes for some values of the lab energy and of the mass of the W (we have here used the results of a calculation by Wu *et al.*[3]). In our paper we pointed out that, although this large difference may at first appear rather surprising, it can, in fact, be rather easily understood by an examination of the characteristics of the Feynman graphs contributing to each process. Since writing that paper we have received several inquiries concerning the reasons for this difference. Since our original explanation was so brief, we are publishing this addendum in the hope of clarifying the problem. Below we present an explicit calculation of (I) and (II) using a simple model which has the important characteristics of the real calculation. We shall also take the opportunity of making some brief remarks about a related muon-induced W reaction which could prove to be experimentally interesting:

$$\mu^-+n\to p+W^-+\mu^-. \tag{III}$$

To lowest order in both the electromagnetic and weak couplings, the Feynman graphs contributing to processes (I) and (II) are those shown in Figs. 1 and 2, respectively. The crucial point to notice is that *no* diagram of the type in Fig. 1(a) occurs in the neutrino reaction. In terms of the equivalent photon reaction this is the only graph which has a direct-channel pole, the others being of the exchange-channel type. For these latter graphs it is possible to approach the corresponding pole (if the particle is light, as in the case of the muon), whereas in the former case this is not so since the invariant mass must be at least that of the W boson (M_W). Hence the neutrino reaction benefits from the relative smallness of the muon mass (m_μ), whereas the muon process does not. In the actual calculation this will reflect itself in the fact that Fig. 1(a) has no dependence on the photon-lepton c.m. scattering angle (θ_W).

In order to see this more explicitly, let us consider a simple model where all the particles (except the photon) are scalar. Let us further simplify the calculation by using a simplified covariant form of the Weizsäcker-Williams (WW) approximation.[5,6] In this approximation the process is described by real photon production modified by an equivalent virtual photon spectrum. Such a procedure is completely analogous to the simplest type of peripheral calculation in strong-interaction physics. If this approximation is used in our original calculations, then the general characteristics of the results, such as the dependence of the cross section on the muon lab energy and on M_W, are reproduced. There is, however, some overestimation in the absolute value of the cross section, in particular, at

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† Present address: Department of Physics, Stanford University, Stanford, Calif. 94305.

[1] F. A. Berends and G. B. West, Phys. Rev. D **1**, 122 (1970).

[2] Our results have since been confirmed by J. Reiff, Nucl. Phys. **B23**, 387 (1970), and by R. W. Brown, A. K. Mann, and J. Smith, Phys. Rev. Letters **25**, 257 (1970). The latter authors also evaluated reaction (II) and noted that for larger M_W the difference between reactions (I) and (II) can easily become two orders of magnitude. We also gather from A. Zepeda that he has checked our analytical expressions. We would like to thank him for pointing out some misprints in the text.

[3] T. D. Lee, P. Markstein, and C. N. Yang, Phys. Rev. Letters **7**, 429 (1961); J. S. Bell and M. Veltman, Phys. Letters **5**, 94 (1963); **5**, 151 (1963); G. von Gehlen, Nuovo Cimento **30**, 859 (1963); A. C. T. Wu, C.-P. Yang, K. Fuchel, and S. Heller, Phys. Rev. Letters **12**, 57 (1964); A. C. T. Wu and C.-P. Yang, Phys. Rev. D **1**, 3180 (1970).

[4] Note that an analytic expression for (II) can be obtained from our work in Ref. 1 (if one is willing to neglect the muon mass) by using the substitution rule together with CP invariance and by reevaluating the θ_W integration. An explicit example of this is illustrated by Eqs. (1) and (2).

[5] V. N. Gribov, V. A. Kolkunov, R. B. Okun', and V. M. Shekhter, Zh. Eksperim. i Teor. Fiz. **41**, 1839 (1961) [Soviet Phys. JETP **14**, 1308 (1962)]. We actually used a simplified version of this in which K^2 is set equal to zero in some kinematic factors. This allows us to perform the K^2 integration trivially.

[6] Since we completed the body of this work, papers by V. V. Solov'ev and I. S. Tsukerman, Zh. Eksperim. i Teor. Fiz. **42**, 1252 (1962) [Soviet Phys. JETP **15**, 868 (1962)] and by H. Überall, Phys. Rev. **133**, B444 (1964) came to our attention. They also calculated these two processes using the WW approximation of Ref. 5. However, they do not concentrate on the *difference* between the two processes, although its large magnitude is present in their results and is mentioned in the latter paper.

TABLE I. Values of $\sigma^{(\mu)}$ and $\sigma^{(\nu)}$ from Refs. 1 and 3, respectively; also shown is a comparison of the ratio $\sigma^{(\nu)}/\sigma^{(\mu)}$ with the analogous R calculated from a scalar model. Units used here are 10^{-38} cm^2.

E_{lab} (GeV)	4	6	8	10	M_W (GeV)
$\sigma^{(\nu)}$	4.50	12.1	20.2	28.1	1
$\sigma^{(\mu)}$	1.13×10^{-1}	3.55×10^{-1}	6.78×10^{-1}	1.054	
$\sigma^{(\nu)}/\sigma^{(\mu)}$	40	34	30	27	
R	26	23	21	20	
$\sigma^{(\nu)}$	1.83×10^{-1}	2.19	5.70	9.88	1.5
$\sigma^{(\mu)}$	3.56×10^{-3}	3.4×10^{-2}	9.60×10^{-2}	1.84×10^{-1}	
$\sigma^{(\nu)}/\sigma^{(\mu)}$	51	64	59	54	
R	61	46	38	34	

low energies and large W masses; however, since we are only interested in ratios this will not be important. Our motivation in making these gross approximations is to expose in the simplest possible manner the basic mechanism for the difference between reactions (I) and (II).

We shall employ the same notation that we used in Ref. 1; this is exemplified in the figures. For the muon case (I) one can easily calculate the square of the invariant matrix element:

$$|M^{(\mu)}|^2=4\left[\frac{-m_\mu^2}{(s-m_\mu^2)^2}+\frac{-M_W^2}{(t-M_W^2)^2}-\frac{s+t}{(s-m_\mu^2)(t-M_W^2)}\right], \quad (1)$$

where $s\equiv(Q+K_2)^2$, $t\equiv(Q-K)^2$, and $u\equiv(Q-K_1)^2$.

A similar calculation for the neutrino process (II) or, more simply, $s\leftrightarrow u$ crossing, leads to

$$|M^{(\nu)}|^2=4\left[\frac{-m_\mu^2}{(u-m_\mu^2)^2}+\frac{-M_W^2}{(t-M_W^2)^2}+\frac{(s-M_W^2-m_\mu^2)}{(u-m_\mu^2)(t-M_W^2)}\right]. \quad (2)$$

Now when θ_W ($\equiv\cos^{-1}\mathbf{k}\cdot\mathbf{q}$ in the c.m. system) approaches 180°, $u\rightarrow0$ and Eq. (2) is greatly enhanced by the first term. On the other hand, the first term in Eq. (1) has no angular dependence so, since $s\geqslant M_W^2$, it can never become very large.

The various angular integrations can be performed straightforwardly to give the following expression for the total cross sections:

$$\sigma^{(\mu,\nu)}=\left(\frac{\alpha G_W}{p_1m_\mu}\right)^2\left(\frac{1}{8\pi}\right)\int_{M_W}^{(E-M)}dW\left[1+\left(\frac{L\cdot K_1}{K\cdot K_1}\right)^2\right]\times X^{(\mu,\nu)}(W)\ln\left(\frac{K^{2(+)}}{K^{2(-)}}\right), \quad (3)$$

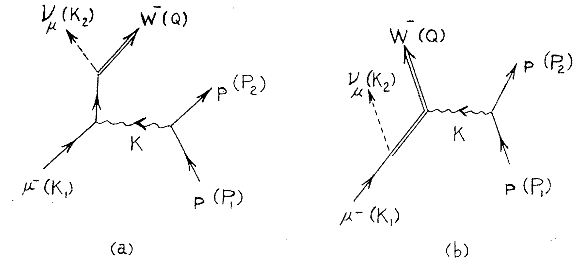

FIG. 1. Lowest-order contributions to reaction I.

where $L\equiv P_1+P_2$ and $W\equiv\sqrt{s}$; E is the total c.m. energy for the over-all process and p_1 is the 3-momentum of the proton in the system where the muon is at rest.[7] The quantities $X(W)$ are defined as follows:

$$X^{(\mu)}(W)=\frac{q}{k^2}\left[\left(\frac{W^2+M_W^2}{W^2-M_W^2}\right)\ln\left(\frac{W}{M_W}\right)-1\right] \quad (4)$$

and

$$X^{(\nu)}(W)=\frac{q}{k^2}\left\{\frac{(W^2-M_W^2-m_\mu^2)}{2qW}\times\ln\left[\frac{(k_{20}+q)(q_0+q)}{m_\mu M_W}\right]-1\right\}$$

$$\approx\frac{q}{k^2}\left[\ln\left(\frac{2q}{m_\mu}\right)+\ln\left(\frac{W}{M_W}\right)-1\right], \quad (5)$$

where q and k are the 3-momenta of the W and the photon and k_{20} and q_0 are the energies of the neutrino and W, respectively, evaluated in the lepton-photon c.m. system of each particular process. Equation (3) is the analog of Eq. (20) of Ref. 1 and the reader may refer to that for the precise expressions for the kinematic quantities as well as for the limits on K^2 [$K^{2(\pm)}$].[8] We should point out that the G_W used in this model should not be confused with the G_W used in Ref. 1 since, in this case, it has the dimensions of a mass. However, again since we are only interested in the ratio $R\equiv\sigma^{(\nu)}/\sigma^{(\mu)}$, this is not relevant.

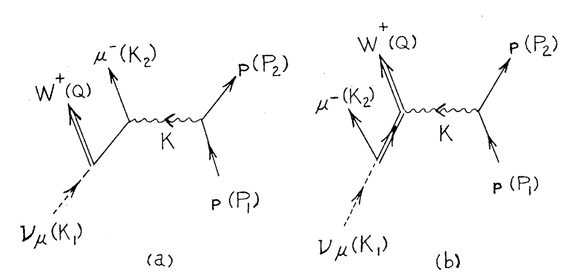

FIG. 2. Lowest-order contributions to reaction II.

[7] This is the case for σ^μ. For σ^ν one should consider the quantity p_1m_ν, which equals $\frac{1}{2}(E^2-m_p^2)$, as can be seen by applying the limit $m_\nu\rightarrow0$ to the formal definition of p_1m_ν.

[8] Actually there is a misprint in these limits. The quantities B and C should read $B=-(E^2+M^2-m^2)(E^2+M^2-S)/2E^2+2M^2$, $C=M^2(S-m^2)^2/E^2$. We would like to thank Dr. Zepeda for pointing this out to us.

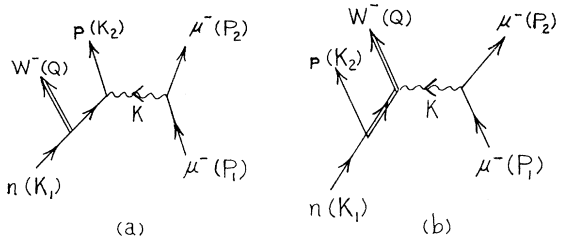

FIG. 3. Lowest-order contributions to reaction III.

From Eqs. (4) and (5) one can estimate the ratio R and show in a rather simple fashion why it is expected to be large. If m_μ is neglected, one can write

$$\frac{X^{(\nu)}(W)}{X^{(\mu)}(W)}=\left[\ln\left(\frac{W^2-M_W{}^2}{m_\mu W}\right)+\ln\left(\frac{W}{M_W}\right)-1\right]\Bigg/ \left[\left(\frac{W^2+M_W{}^2}{W^2-M_W{}^2}\right)\ln\left(\frac{W}{M_W}\right)-1\right]. \quad (6)$$

Since all other factors are essentially the same, one can expect that setting W to some "average" value in Eq. (6), say, $\frac{1}{2}E$, will lead to a reasonable estimate of R from this equation. For example, suppose we take an incident lab energy of 10 GeV; then $E\sim 4.5$ GeV, so it is reasonable to take 2 GeV as some average value for W in Eq. (6). We then find that, for $M_W=1$ GeV, $R\sim 15$. In Table I we have given the values for R calculated numerically from Eq. (3). From this table we see that at 10 GeV and $M_W=1$ GeV, $R=20$, indicating that using Eq. (6) as an estimate in this way is reasonable. Returning to Eq. (6), one can very easily understand two of the more interesting properties of R:

(i) For a given energy, R increases with M_W. The point is that R is enhanced not only because of the m_μ factor in the numerator of (6) but *also* because the denominator can actually become quite small. A straightforward differentiation of (6) with respect to $M_W{}^2$ shows that R should be a monotonically increasing function of $M_W{}^2$.

TABLE II. Values of the ratios $\sigma^{(\nu)}/\sigma^{(\mu)}$ and $\sigma^{(\nu)}/\sigma^{(n)}$ calculated from our scalar model for various high energies and W masses.

$E_{\rm lab}$ (GeV)	50	100	150	200	M_W (GeV)
$\sigma^{(\nu)}/\sigma^{(\mu)}$	30	28	27	26	2
$\sigma^{(\nu)}/\sigma^{(n)}$	4.6	4.5	4.5	4.5	
$\sigma^{(\nu)}/\sigma^{(\mu)}$	63	47	43	41	4
$\sigma^{(\nu)}/\sigma^{(n)}$	2.9	2.9	2.9	2.9	
$\sigma^{(\nu)}/\sigma^{(\mu)}$	968	140	93	78	8
$\sigma^{(\nu)}/\sigma^{(n)}$	1.9	1.9	2.0	2.0	

(ii) For a given M_W, R decreases with energy. Again, a straightforward differentiation of (6) does indeed confirm this (provided one is not too near threshold). Notice also that in the extreme asymptotic region, one expects $R\to 2$.[9]

These properties are all confirmed in a direct numerical evaluation of Eq. (3). The precise values of R are exhibited in Table I. We have therefore shown how this crude scalar model leads very naturally to a rather simple explanation for the general characteristics of R. Notice from the table that the actual values of R calculated with spins and without the WW approximation are only a factor of 2 larger than the estimate from our scalar model. In Table II some ratios for higher masses and energies are given.

As a final point we would like to make a few remarks concerning the process (III). To lowest order in the *strong* interactions (as well as the weak and electromagnetic), the two Feynman diagrams contributing here are analogous to those in the neutrino process (II); see Fig. 3. Although one does not know how to take proper account of the strong interactions other than to put in some phenomenological form factors, nevertheless one might hope that these graphs can at least give an order-of-magnitude estimate.[10] We have therefore calculated the total cross section for (III) [$\sigma^{(n)}$] using our scalar model. The results are presented in ratio form in Table II. As might be expected, this cross section lies somewhere between $\sigma^{(\mu)}$ and $\sigma^{(\nu)}$; however, it is rather encouraging that $\sigma^{(n)}$ is not significantly smaller than $\sigma^{(\nu)}$. Unfortunately, one cannot enhance this cross section by performing coherent production experiments from nuclei. Furthermore, the analogous process from proton targets[11] will be considerably smaller since the corresponding Feynman graphs have the characteristics of the muon-induced process (I); see Fig. 1. Nevertheless, we believe that this process may be more suitable for W production by muon beams than reaction (I).

A discussion on these matters with Professor F. E. Low is gratefully acknowledged.

[9] It should be noted that this value is the same as the ratio found by Solov'ev and Tsukerman (Ref. 6). The reason, however, is completely different. In the case where the spins are included, the energy behavior of $X^{\mu\,\nu}$ is different. Besides the terms that we have, there are extra terms behaving like q and $q\ln(s/M_W{}^2)$ which one would expect to dominate the asymptotic W behavior of X^μ and X^ν. However, this limit is approached relatively slowly because of a delicate cancellation. Furthermore, because of the weighting towards low W in the integrand of Eq. (3), the asymptotic behavior of $\sigma^{(\mu)}$ and $\sigma^{(\nu)}$ is reached even more slowly. Hence in the region of experimental interest the mechanism which we have discussed (namely, the approach to the μ pole) is still of great importance.

[10] Form factors may suppress these estimates.

[11] This process has been evaluated in the WW approximation by A. Weis and P. K. Kabir, Nucl. Phys. B4, 643 (1968).