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Note on Extreme-Energy Collisions*

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Within the framework of limiting fragmentation without pionization, the average multiplicity of hadron collisions is shown to be finite as the incident energy approaches infinity.

An interesting and elegant hypothesis of limiting fragmentation has been proposed by Benecke, Chou, Yang, and Yen (BCYY).[1] They emphasize the importance of the target system and the projectile system, and argue against the pionization process[2] in extreme-energy collisions. All the outgoing particles are considered as fragments of either the projectile or the target. The target (projectile) fragments are assumed to have finite energies and to approach limiting distributions in the target (projectile) system as the incident energy $E_{\rm inc}\to\infty$. These authors believe that the average multiplicity will be divergent as $E_{\rm inc}\to\infty$. Nevertheless, in their framework we shall see by a simple argument that the average multiplicity and the total invariant mass of the projectile or the target fragments must not diverge.

First of all, we would like to clarify the meaning of "fragments" and of "pionization particles." They are defined by classifying the outgoing particles after the hadron collision into two groups. However, there are two such classifications at present. One classification defines the fragments as the particles having finite momenta in the target (or projectile) system as the incident energy $E_{\rm inc}\to\infty$, and defines pionization as all remaining particles; while the other one defines pionization as the particles having finite momenta in the c.m. system as $E_{\rm inc}\to\infty$ and defines fragments as all remaining particles. We note that the pionization particles in the former definition do not necessarily have finite momenta in the c.m. system, and the fragments in the latter definition do not necessarily have finite momenta in the target (or projectile) system. Thus, if one defines fragments according to the former definition and pionization particles according to the latter definition, then the classification is model dependent and may be incomplete.

If one takes the former definition and assumes the absence of the pionization process at extreme energies, the picture which emerges is simple and intuitively clear. On the other hand, if one defines the fragments according to the latter definition and argues against the presence of the pionization process at extreme energies, then it would be difficult to verify such a picture experimentally, since data are always obtained in the target system, and those particles with energies $\propto (E_{\rm inc})^{1/2}$ (i.e., pionization particles) and for instance, particles with energies $\propto (E_{\rm inc})^{k/2}$, where $k<1$ (i.e., fragments), are difficult to distinguish experimentally when the incident energy $E_{\rm inc}$ is extremely high. Furthermore, such a picture of fragmentation is not intuitively clear because of the "wide energy spectrum" of the fragments. As stated in Table I of BCYY,[1] the target fragments have laboratory momenta of "O(1 GeV)" as $E_{\rm inc}\to\infty$, so that the second definition seems irrelevant in our discussion.

According to the BCYY hypothesis of limiting fragmentation,[1] any fragment i in an extreme-energy collision of hadrons must carry an energy $\xi_i\sqrt{s}$ measured in the c.m. system, where $s=-(p_\lambda^{(\rm proj)}+p_\lambda^{(\rm targ)})^2$ is the square of the total c.m. energy and ξ_i is a constant ($0<\xi_i<1$ for all i). It is easy to see that the average multiplicity must be finite[3] as $s\to\infty$ by considering the conservation of energy in the c.m. system $\sqrt{s}=\sum_i \xi_i\sqrt{s}$ and noting the requirement $1>\xi_i\geqslant\xi_{\min}>0$ for all i as $s\to\infty$. Thus, without the knowledge of the limiting distribution $\rho_1(\mathbf{p})$ in the target system T or the projectile system P, we can immediately prove that the relation

$$\int \rho_1{}^{(k)}(\mathbf{p})d^3\mathbf{p}=\sigma_{\rm tot}\bar{n}^{(k)}<\infty \qquad (k=T,P;\ \sigma_{\rm tot}<\infty,\ s\to\infty) \quad (1)$$

must hold in the BCYY framework, where $\bar{n}^{(T)}$ ($\bar{n}^{(P)}$) denotes the average multiplicity of particle with mass m emitted from the target (projectile). Similarly, we have

$$\int \rho_2{}^{(k)}(\mathbf{p}_1,\mathbf{p}_2)d^3p_2<\infty \quad (k=T,P;\ s\to\infty) \qquad (2)$$

for the two-particle limiting distribution function $\rho_2(\mathbf{p}_1,\mathbf{p}_2)$.[1] One can also easily show the total invariant mass M^* of the target fragments or of the projectile fragments must be strictly finite in the limit $s\to\infty$. If the integrals (1) and (2) diverge, then almost all of the fragments do not have finite momenta in either the target or the projectile system in the limit $s\to\infty$.

In this connection, it is interesting to consider the distribution of particles obtained in Feynman's parton model[4] using the concept of fragmentation. Suppose the

* Work supported by the National Research Council of Canada.
[1] J. Benecke, T. T. Chou, C. N. Yang, and E. Yen, Phys. Rev. 188, 2159 (1969).
[2] See, for example, H. Cheng and T. T. Wu, Phys. Rev. Letters 23, 1311 (1969); R. Murphy, Symposium on Multiparticle Production [ANL Report No. ANL/HEP 6909, 1968 (unpublished)].
[3] P. Heckman, McGill report 1970 (unpublished). Here, we shall present a much more simple argument.
[4] R. P. Feynman, in *High Energy Collisions*, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1969).

outgoing particles do not have "wee momenta" [i.e., $p=O(1\text{ GeV})$] in the c.m. system, but have momenta $\xi\sqrt{s}$ with $0<\epsilon\leqslant\xi<1$, then one has an average multiplicity $\bar{n}$ proportional to $\sim\ln\{\epsilon+[\epsilon^2+(p_\perp{}^2+m^2)/s]^{1/2}\}^{-1}$. By suitable choice of the parameters, $\bar{n}$ might be proportional to $\ln s$ when, say, $10<\sqrt{s}<10^5$ GeV and become constant when $\sqrt{s}\gg10^5$ GeV. This serves to illustrate the point that if one requires the integrals (1) and (2) to be divergent, then the original simplicity of the concept of limiting fragmentation will no longer hold.

Nevertheless, we note that the results (1) and (2) are by no means a serious objection to the BCYY framework in the sense that a large but finite multiplicity as $s\to\infty$ has not yet been excluded experimentally and that the BCYY framework could easily have a divergent multiplicity simply by assuming the presence of the pionization process at extreme energies.[2] At the present time, most people apparently believe that all multiplicities will become infinite[5] at infinite incident energies.

In conclusion, we would like to emphasize that the hypothesis of limiting distributions in a particular reference system is a sort of dynamical assumption in the sense that the "rate" of divergence of the average multiplicity is fixed. On the other hand, the number of kaons and pions has been measured in cosmic-ray experiments and we find, for example, in NN collisions at an energy $\sim10^3$ GeV in the laboratory the ratio K/π is quite different for the set of high-energy outgoing mesons ($K/\pi\approx0.1$) and the set of low-energy mesons ($K/\pi\approx0.7$).[6] This may not be easy to accommodate within the intuitive picture of fragmentation without pionization.

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[5] If the average multiplicity $\bar{n}$ is finite, then a dynamical explanation is needed of why it should be finite and what its upper bound should be. A finite $\bar{n}$ is unlikely from the view point of hadron democracy; and it is also apparently unlikely in the quark model or in the "parton" model of Feynman.

[6] See M. Koshiba, in *High Energy Collisions*, Ref. 4.

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Forward Photoproduction of a Charged Pion and $N(1520)$*

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The analysis of the reaction $\gamma p\to\pi^+N(1520)$ is considered and the influence of low-t theorems in the near forward direction is discussed. It is shown that present preliminary experimental results are not inconsistent with the dominance of the amplitudes whose forms near $t\approx\mu^2$ are determined by low-t theorems. Predictions are presented for $d\sigma/dt$ and the spin-density matrix $\rho_{\lambda\lambda'}$.

I. INTRODUCTION

THE success of the description of the high-energy photoproduction reactions $\gamma N\to\pi^\pm N$ and $\gamma N\to\pi^\pm\Delta$ in the near-forward direction by parts of the electric Born terms[1–3] suggests an investigation of the generality of the effect in other charged-pion photoproduction reactions. The most accessible of these is probably the reaction involving the "second nucleon resonance" $\gamma N\to\pi^\pm N(1520)$. In a recent paper by Campbell, Clark, and Horn,[3] hereafter called CCH, the success of the electric Born terms has been explained in part by low-t theorems which place reaction-dependent restrictions on the t dependence of the reaction in the near forward direction. In this note, the results obtained for $\gamma N\to\pi^\pm\Delta$ are extended to $\gamma N\to\pi^\pm N(1520)$.

II. KINEMATICAL STRUCTURE AND LOW-t THEOREMS

The kinematical details for $\gamma N\to\pi N(1520, J^P=\frac{3}{2}^-)$ may be generalized from those given for $\gamma N\to\pi\Delta(1238, J^P=\frac{3}{2}^+)$ in CCH by the appropriate inclusion of the factor $i\gamma_5$ in Eq. (7) of CCH. A set of amplitudes (B_i) is thereby defined and the application of the gauge-invariance condition yields low-t theorems for certain amplitudes which are identical in form to Eq. (11a) of CCH,

$$\lim_{t\to\mu^2}[B_4(s,t)k\cdot P+B_5(s,t)] = -\lim_{t\to\mu^2}B_2(s,t)k\cdot P=-efF_\pi. \quad (1)$$

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[1] For $\gamma N\to\pi N$, see B. Richter, in *Proceedings of the Third International Symposium on Electron and Photon Interactions at High Energies, Stanford Linear Accelerator Center, 1967* (Clearing House of Federal Scientific and Technical Information, Washington, D. C., 1968), p. 313; A. M. Boyarski, F. Bulos, W. Busza, R. Diebold, S. D. Eckland, G. E. Fischer, J. R. Rees, and B. Richter, Phys. Rev. Letters **20**, 300 (1968).

[2] For $\gamma N\to\pi\Delta$, see P. Stichel and M. Scholz, Nuovo Cimento **34**, 1381 (1965); A. M. Boyarski, R. Diebold, S. D. Ecklund, G. E. Fischer, Y. Murata, B. Richter, and W. S. C. Williams, Phys. Rev. Letters **22**, 148 (1969).

[3] J. A. Campbell, Robert Beck Clark, and D. Horn, Phys. Rev. D **2**, 217 (1970).