Evidence for an Isotensor Electromagnetic Current in Pion Photoproduction*

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An unambiguous and model-independent test for the presence of isotensor terms in pion photoproduction is suggested. We also show how the conventional theory of photoproduction can be generalized to include such a term, leaving the successful predictions for photoproduction on protons unchanged. Analysis of the available data on π^- photoproduction from neutrons yields evidence for striking effects due to such a term. Further experiments are discussed and, in particular, the importance of π^- radiative capture experiments, thus avoiding the use of a deuterium target, is stressed.

I. INTRODUCTION

BOTH the isospin and $SU(3)$ transformation properties of the electromagnetic current are usually assumed to be identical to those of the charge

$$Q=I_3+\tfrac{1}{2}Y. \quad (1)$$

In particular, under isospin transformations it is assumed to transform as the sum of an isoscalar and the third component of an isovector ($\Delta I=0$, 1 rule).[1] The lack of experimental evidence for this assumption was first pointed out by Grishin *et al.*[2] and by Dombey and Kabir.[3] In these and subsequent papers[4–6] a number of tests have been suggested and discussed, and a particularly interesting source of speculation has been the roles such terms may play in accounting for a number of problems in η decay.[5] However, the question has remained no nearer to resolution, despite its obvious importance with regard to ideas on symmetry breaking. We note in this respect that the two most widely used viewpoints on broken symmetries—i.e., theories in which the weak and electromagnetic currents are members of an octet obeying the usual commutation rules, and quark models in which the current couples directly to individual quarks—both require the conventional transformation laws to be exact. In this paper we wish to present evidence of appreciable effects in pion photoproduction due to the presence of an isotensor term in the electromagnetic current, in contradiction to the above rule.

In an earlier paper,[4] one of us suggested the study of single-pion photoproduction on neutrons in the region of the $\Delta(1236)$ resonance as a particularly useful way of investigating this problem. The main part of this paper will be concerned with the analysis of this reaction in general, and of the recent data on π^- production[7,8] in particular. In Sec. II we show how the presence or absence of isotensor terms can be tested for in a model-independent way, and in Sec. III we develop a detailed model for single-pion photoproduction in this region in the presence of isotensor terms which is a direct and simple generalization of the usual successful theory for photoproduction on protons. Going on to analyze the available data in Sec. IV, we find evidence for large effects due to such a term, and we finally indicate how its characteristics can be further confirmed.

II. ISOTENSOR TERMS IN $\gamma N \rightleftarrows \pi N$: GENERAL CHARACTERISTICS

Throughout this paper we ignore the possibility of T noninvariance so that the reactions

$$\gamma+N\rightleftarrows\pi+N \quad (2)$$

are identical. If in these reactions we abandon the usual assumptions on the isospin content of the electromagnetic current, then in addition to the usual (a) isoscalar amplitude A^0 leading to the $I=\frac{1}{2}$ final πN state, and (b) isovector amplitudes A^1, A^3 leading to the $I=\frac{1}{2}$, $\frac{3}{2}$ final states, we also have (c) an isotensor amplitude A^2 leading to the $I=\frac{3}{2}$ final state. The amplitudes for the observed processes are

$$\tfrac{3}{2}\sqrt{2}A(\gamma p\rightarrow\pi^+n)=3A^0+A^1+(\sqrt{\tfrac{3}{5}})A^2-A^3, \quad (3a)$$

$$\tfrac{3}{2}\sqrt{2}A(\gamma n\rightarrow\pi^-p)=3A^0-A^1+(\sqrt{\tfrac{3}{5}})A^2+A^3, \quad (3b)$$

$$3A(\gamma p\rightarrow\pi^0p)=3A^0+A^1-(2\sqrt{\tfrac{3}{5}})A^2+2A^3, \quad (3c)$$

$$3A(\gamma n\rightarrow\pi^0n)=-3A^0+A^1+(2\sqrt{\tfrac{3}{5}})A^2+2A^3. \quad (3d)$$

It is also useful to introduce the amplitudes for photoexciting the $I=\frac{3}{2}$ state on protons and neutrons,

$$_pA^3=(\sqrt{\tfrac{2}{3}})[A^3-(\sqrt{\tfrac{3}{5}})A^2], \quad (4a)$$

$$_nA^3=(\sqrt{\tfrac{2}{3}})[A^3+(\sqrt{\tfrac{3}{5}})A^2], \quad (4b)$$

* Research supported in part by the U. S. Atomic Energy Commission.

[1] K. M. Watson, Phys. Rev. 85, 852 (1952).
[2] V. G. Grishin, V. L. Lyuboshitz, V. I. Ogievetskii, and M. I. Podgoretski, Yadern. Fiz. 4, 126 (1966) [Soviet J. Nucl. Phys. 4, 40 (1967)].
[3] N. Dombey and P. K. Kabir, Phys. Rev. Letters 17, 730 (1966).
[4] G. Shaw, Nucl. Phys. B3, 338 (1967).
[5] M. Veltman and J. Yellin, Phys. Rev. 154, 1469 (1967); S. L. Adler, Phys. Rev. Letters 18, 519 (1967); 18, 1036 (1967).
[6] E. E. Bergmann, Phys. Rev. 172, 1441 (1968); B. Gittelmann and W. Schmidt, *ibid.* 175, 1998 (1968); R. J. Blin-Stoyle, Phys. Rev. Letters 23, 535 (1969).
[7] Aachen-Berlin-Bonn-Hamburg-Heidelberg-München Collaboration, Nucl. Phys. B8, 535 (1968); H. Butenschon, DESY Report No. R1-70/1, 1970 (unpublished). We use the results reported in this more recent analysis.
[8] Pavia-Rome-Frascati-Napoli Collaboration, Nuovo Cimento Letters 3, 697 (1970).

which in the absence of isotensor terms are equal. Clearly in order to separate A^2 and A^3, neutron data are necessary in addition to the already plentiful proton data. For the partial-wave amplitudes we shall follow the notation of Chew *et al.*[9] (hereafter referred to as CGLN). Thus $E_{l\pm}{}^i$ ($M_{l\pm}{}^i$) corresponds to an electric (magnetic) multipole transition to the πN final state with $J=l\pm\frac{1}{2}$. Assuming unitarity and T invariance, the Watson theorem[10] guarantees that the phases of these amplitudes are given by the corresponding πN phase shifts, provided only that multipion production and other inelastic processes are negligible. This latter is an excellent approximation for all waves up to laboratory photon energies $E_\gamma \sim 550$ MeV (pion laboratory kinetic energy $T_\pi \sim 400$ MeV), except for p_{11}, for which it is only good up to $E_\gamma \sim 400$ MeV ($T_\pi \sim 250$ MeV).[11] The resonance position is at $E_\gamma = 350$ MeV ($T_\pi = 200$ MeV).

Let us now consider how the presence of an isotensor term, or rather an isotensor excitation of the $\Delta(1236)$ resonance, can be detected in an unambiguous and model-independent way. To do this it is convenient to consider the difference of the total cross sections for reactions (3a) and (3b) as a function of energy over the resonance region.

$$\Delta'(W) = (k/q)[\sigma_t(\gamma n \to \pi^- p) - \sigma_t(\gamma p \to \pi^+ n)], \quad (5)$$

where q and k are c.m. pion and photon momenta, respectively.[12] Noting that the only multipoles which can give rise to rapid energy variations in this region are those leading to excitation of the resonance and, separating off terms involving these from the other slowly varying terms, we have

$$\Delta'(W) = (64\pi/9)(\sqrt{\tfrac{3}{5}}) \operatorname{Re}[(\sqrt{15}) M_{1+}{}^0(W) M_{1+}{}^{3*}(W) - M_{1+}{}^1(W) M_{1+}{}^{2*}(W) + M_{1+}{}^2(W) M_{1+}{}^{3*}(W)] + \text{slowly varying terms}. \quad (6)$$

Here we have neglected the electric quadrupole excitation E_{1+} which is experimentally extremely small on protons.[13] If it were not small it could be separated off also and the following argument would go through completely unchanged, applying to it as well as to the magnetic excitation.

As we have said, the Watson theorem guarantees that the phases of the multipoles are given by the corresponding πN phases in this region. Hence, in contrast to the resonant $M_{1+}{}^{2,3}$ amplitudes, $M_{1+}{}^{0,1}$ will be real and slowly varying in this region. In the absence of any isotensor $M_{1+}{}^2$ transition, the only source of rapid energy variation in (6) is the first term, which will look therefore like the real part of a resonance (and in fact with present estimates of $M_{1+}{}^0$ will be rather small). In particular there is no possibility of a dip or peak. On the other hand, if $M_{1+}{}^2$ is nonzero, by Watson's theorem the interference term $\operatorname{Re} M_{1+}{}^2 M_{1+}{}^{3*}$ is necessarily either purely constructive or descructive, and a dip or peak will necessarily result. Such a structure in $\Delta'(W)$ is therefore a completely unambiguous sign of an isotensor term.

Similar arguments can also be given for the total cross sections for π^0 photoproduction. However, as we shall see, resonance production in this case is completely dominant and the background contribution is very small compared to the sort of effect we shall find in the π^- data, so that arguments of the above type are not necessary.

Thus the study of the total cross sections provides a straightforward and model-independent method of testing for isotensor excitationof the resonance. However, the discussion of differential-cross-section and polarization data is more complex owing to interference effects between different partial waves, and it is therefore necessary to discuss the theory of photoproduction in this region in a little more detail, which we do in Sec. III.

III. ISOTENSOR TERMS IN $\gamma N \rightleftarrows \pi N$: DETAILED THEORY

One advantage of studying these reactions is that photoproduction on protons has been studied in great detail experimentally. Further, the conventional approach to this problem, initiated by CGLN[9] and developed with increasing refinement by several authors,[14–16] has proved remarkably successful in understanding these data. Clearly if this success is to be retained, isotensor terms can only be introduced in a rather restricted way. In this section we reconsider the usual picture without making any assumptions on the isospin nature of the current, and show (a) that the ambiguity introduced is essentially entirely in the M_{1+} multipole, (b) that results essentially identical to those of the usual model for photoproduction on protons can be reproduced even in the presence of quite large isotensor terms, and that therefore (c) their presence or absence can only be determined by a study of the neutron data.

The above-mentioned approach is based on the use of fixed-t dispersion relations for the four invariant amplitudes $A_i(s,t,u)$, where we follow the conventional CGLN notation throughout. Assuming C invariance, these amplitudes have simple crossing properties, and the isotensor amplitudes have the same crossing rela-

[9] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

[10] K. M. Watson, Phys. Rev. **95**, 228 (1954).

[11] See, e.g., C. Lovelace, in *Proceeding of the Heidelberg International Conference on Elementary Particles, 1967*, edited by H. Filthuth (Interscience, New York, 1968).

[12] Note that $\Delta'(W)$ differs from $\Delta(W)$ defined in A. I. Sanda and G. Shaw, Phys. Rev. Letters **24**, 1310 (1970), by the kinematical factor k/q.

[13] A. Donnachie and G. Shaw, Nucl. Phys. **87**, 556 (1967).

[14] W. Schmidt, Z. Physik **182**, 76 (1964).

[15] A. Donnachie and G. Shaw, Ann. Phys. (N.Y.) **37**,333 (1966).

[16] F. A. Berends, A. Donnachie, and D. L. Weaver, Nucl. Phys. **B4**, 54 (1968).

tions as the isoscalar amplitudes, i.e.,

$$A_i(s,t,u)=\xi_i A_i^2(u,t,s)\,, \tag{7a}$$

where

$$\xi_i=+1 \qquad (i=1,3,4) \tag{7b}$$

$$=-1 \qquad (i=2)\,. \tag{7c}$$

Thus the dispersion relations for these amplitudes will be

$$A_i^2(s,t,u)=\frac{1}{\pi}\int_{s_0}^{\infty}ds'\,\mathrm{Im}A_i^2(s',t)\left(\frac{1}{s'-s}+\xi_i\frac{1}{s'-u}\right), \tag{8}$$

from which a set of equations for the multipole amplitudes $M_{l\pm}^2$, $E_{l\pm}^2$ can be derived, identical to those for the isosclar multipoles[15,16] $M_{l\pm}^0$, $E_{l\pm}^0$ except that there are no Born terms in this case.

Now in order to determine the real parts of the amplitudes, provided there are no subtractions, we need only a model for the absorptive part. The dominant terms in the latter are shown in Fig. 1, where Fig. 1(c) is meant to signify that the imaginary parts in the dispersion integrals are dominated by the excitation of the resonance. The question of subtractions is easily dealt with since PCAC and gauge invariance lead to the result that the threshold amplitudes are given by the Born terms alone to *second* order in μ/m, i.e., the subtractions are zero to this order.[17] In the threshold region the experimental data on both π^+ production and the ratio of π^- to π^+ production are in excellent agreement with this, and π^0 production is very small as predicted.[13,17]

What effect will the presence of $\Delta I=2$ terms have on the above terms? The only way in which an isotensor term can enter these diagrams is in the excitation of the resonance, i.e., in the $\Delta N\gamma$ coupling. In its presence the coupling on protons and neutrons will be different. In its absence, the same.

This resonance excitation can occur through both the M_{1+} and E_{1+} multipoles in general. However, on protons it is well known to proceed almost entirely through the magnetic dipole amplitude M_{1+}, the electric quadrupole amplitude being very small (${}_pE_{1+}^3$ is about $-0.03\ {}_pM_{1+}^3$ at resonance[13]) and showing no sign of any resonance peak in the imaginary part.[13,16] We shall assume this is so on neutrons also, and consider only ${}_NM_{1+}^3$ (i.e., ${}_pM_{1+}^3$, ${}_nM_{1+}^3$) excitations for the moment. Let us consider the relations between these amplitudes and the corresponding scattering amplitude $f_{1+}^3(W)$ in simple resonance models. Writing the usual resonance formulas the result

$$(kq)^{1/2}\,{}_NM_{1+}^3(W)=\frac{[{}_N\Gamma_\gamma(k)\Gamma(q)]^{1/2}}{2(W_r-W)-i\Gamma(q)}$$
$$=\left(\frac{{}_N\Gamma_\gamma(k)}{\Gamma(q)}\right)^{1/2}qf_{1+}^3(W) \tag{9}$$

[17] G. W. Gaffney, Phys. Rev. **161**, 1599 (1967).

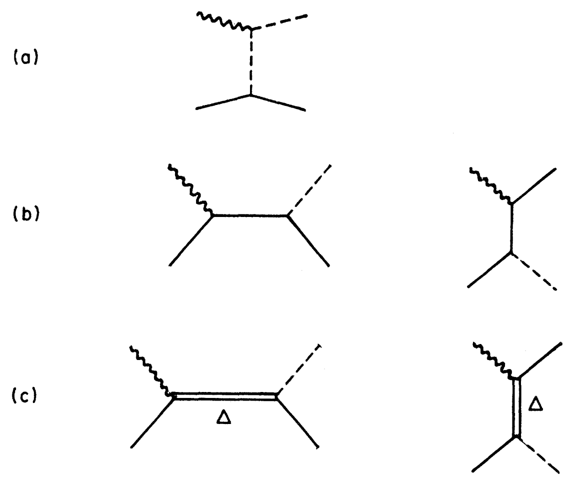

FIG. 1. Principal contributions to the low-energy absorptive parts.

is obtained. ${}_N\Gamma_\gamma(k)$ and $\Gamma(q)$ are the radiative and pionic decay widths of the resonance, and we have approximated the total width by $\Gamma(q)$ alone. If we now take simple relativistic Breit-Wigner forms for these widths:

$$\Gamma(q)=\gamma q^3\,,\quad {}_N\Gamma_\gamma(k)=\gamma_N k^3\,, \tag{10}$$

then the relation

$${}_NM_{1+}^3(W)=\left(\frac{\gamma_N}{\gamma}\right)^{1/2}\frac{k}{q}f_{1+}^3(W) \tag{11}$$

results. One could try to improve this by using more complex forms for the widths, for example, the Layson form.[18] If, however, for protons we take

$$(\gamma_p/\gamma)^{1/2}=\mu/2f\,, \tag{12}$$

where μ and f are the nucleon magnetic moment and pion-nucleon coupling constant, respectively, the well-known static model result is obtained, which, if the experimental $f_{1+}^3(W)$ is used, is in excellent agreement with the proton data in the resonance region.

It is now clear how to introduce isotensor terms into this model. While retaining the usual amplitude for photoexcitation on protons (see below), one simply allows for the possibility of a different radiative width on neutrons. To do this we introduce the (necessarily real) parameter x, defined by

$${}_nM_{1+}^3(W)=(1+x)\ {}_pM_{1+}^3(W) \tag{13}$$

or

$${}_n\Gamma_\gamma(k)=(1+x)^2\ {}_p\Gamma_\gamma(k)\,. \tag{14}$$

Finally, before turning to the other waves, we note that an alternative discussion of these multipoles can be given in terms of the integral equations mentioned above. For a discussion of these, and in particular of the ambiguities in their solution, we refer to the Appendix.

[18] M. Gell-Mann and K. Watson, Ann. Rev. Nucl. Sci. **4**, 219 (1954); W. Layson, Nuovo Cimento **27**, 724 (1963).

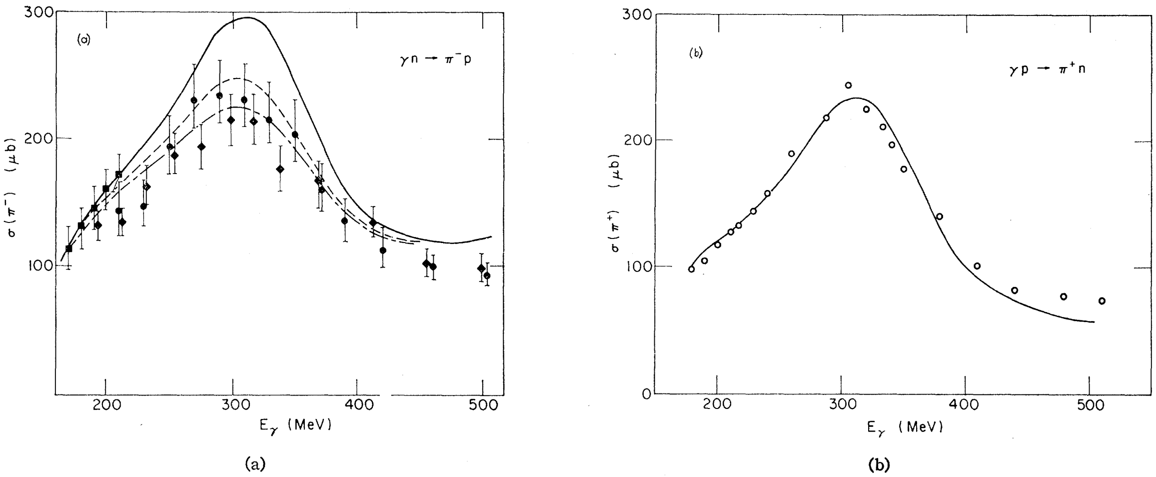

FIG. 2. (a) Total cross section for reaction (3b) as a function of energy. Solid line $x=0$ (no isotensor), dashed line $x=-0.2$, dash-dot line $x=-0.3$. For discussion of the data, see text. (b) Total cross section for reaction (3a) as a function of energy. Data points are from Walker (Ref. 25).

It now remains to integrate over the above results in the absorptive parts to give, together with the Born terms, the real parts for the other multipoles. Their small imaginary parts can then be obtained using the Watson theorem. The results will, of course, now be a function of the parameter x. However, by explicit calculation we find that for the range of x that we will need to consider ($x\cong-0.2$), the changes in other multipoles from the normal case ($x=0$) are essentially negligible. Thus, for example, in π^- production, the change in E_{0+}, which is the only nonresonant multipole to make an appreciable contribution to the total cross section, the change is less than 0.3%. For $M_{1+}{}^1$ and other small multipoles, the changes are about 2%,

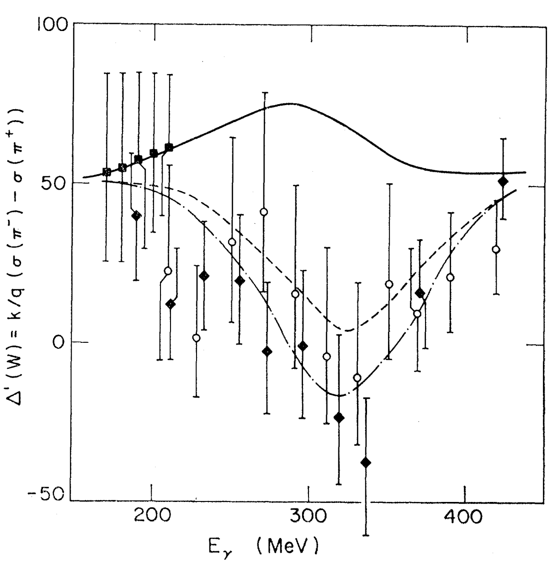

FIG. 3. The function $\Delta'(W)$ defined in Eq. (5) as a function of energy. Solid line $x=0$, dashed line $x=-0.2$, dash-dot line $x=-0.3$. For discussion of the data, see text.

which produces negligible effects compared to the change in the dominant ${}_nM_{1+}{}^3$ amplitude. We thus conclude that these effects can be neglected, and the conclusions (a)–(c) stated at the beginning of this section follow.

It now remains to determine the value of x from a study of the neutron data, which we go on to do in Sec. IV. First, however, we summarize the detailed values of the multipoles which we use.

(a) ${}_pM_{1+}{}^3(W)$: Here we use the solution of the fully relativistic integral equation given by Berends *et al.*[16] In the resonance region this is in close agreement with both Eqs. (11), (12), and experiment. Thus, normalizing the prediction to one at resonance, values of 1.01 ± 0.03 from polarized-photon measurements on π^+ photoproduction[19] and 1.01 ± 0.02 from the total cross section for π^0 photoproduction[13] are obtained, and a fit by Schwela and Weizel[20] gives 1.03 (all evaluated at the resonance position $E_\gamma=350$ MeV). In the region below resonance, below 300 MeV, there is evidence that ${}_pM_{1+}{}^3$ should be larger than predicted by a few percent,[21] but since our analysis is centered on the resonance, this need not concern us at the present.

(b) ${}_nM_{1+}{}^3(W)$: This is then given by Eq. (13).

(c) $M_{1-}{}^0(W)$, $M_{1-}{}^1(W)$: As we have pointed out in Sec. II, the Watson theorem breaks down at comparatively low energies for these multipoles (at $E_\gamma\sim400$ MeV), owing to the noticeable inelasticity in the p_{11} wave already apparent at these energies.[11] It is thus not possible to calculate these multipoles reliably by the dispersion method, and in discussing differential-cross-section and polarization measurements it will be

[19] M. Grilli, M. Nigro, and E. Schiavuta, Nuovo Cimento **49**, 326 (1967).

[20] D. Schwela and R. Weizel, Z. Physik **221**, 71 (1969).

[21] F. A. Berends and A. Donnachie, Phys. Letters **30B**, 555 (1970).

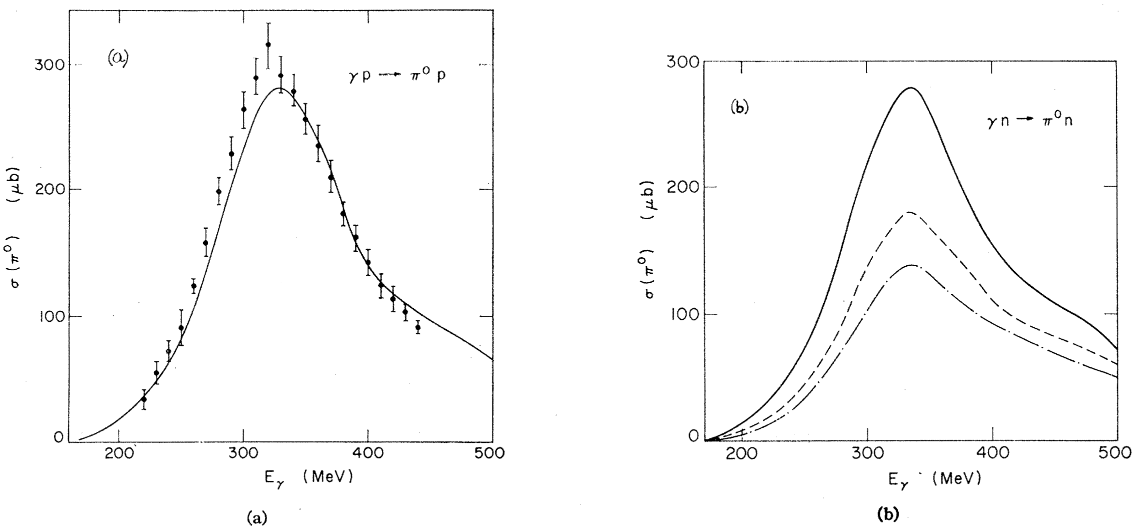

FIG. 4. (a) Prediction of the total cross section for reaction (3c). Data points are from Fischer *et al.* (Ref. 26). (b) Prediction of the total cross-section for reaction (3d). Solid line $x=0$, dashed line $x=-0.2$, dash-dot line $x=-0.3$.

necessary to treat them as free parameters (see below). However, the total cross sections are insensitive to this multipole, so that as long as these are discussed this ambiguity is unimportant. In this paper we shall use, initially, the results of the calculation of Donnachie and Shaw[15] in which the Born and resonance terms only are included.[22]

(d) Other multipoles: All other multipoles up to and including f waves are retained with their values taken from the analysis of Berends *et al.*[16] In this calculation the (small) contributions to the dispersion integrals from the multlpoles other than $M_{1+}{}^3$ have also been included, not just ${}_NM_{1+}{}^3$ as outlined above.

IV. COMPARISON WITH EXPERIMENT

A. Total Cross-Section Data

In Fig. 2(a) we plot the predicted cross section for reaction (3b),

$$\gamma+n\rightarrow\pi^-+p\,,$$

for the values $x=0$, -0.2, and -0.3. The data are taken from three sources: (a) In the threshold region the theoretical predictions, which depend very little on x, are in excellent agreement with both the π^+ differential cross section[13,16] and with the experimental ratios of π^- to π^+ production obtained from deuterium measurements[23] (shown by squares). The points shown here have been estimated from these data. (b) Results of the ABBHHM collaboration[7] (shown by circles); this is an experiment on deuterium which covers the range 0.2–1.5 GeV photon laboratory energy. (c) Results of the PRFN collaboration[8] (shown by diamonds); this is also a bubble-chamber experiment on deuterium, covering the range 0.18–1.0 GeV. In both experiments the results are extracted from the deuterium data using the spectator model. However, we note that at a slightly higher energy than that which we are concerned with, 520 MeV, the results of these experiments have been confirmed by a measurement of the inverse radiative capture process ($\pi^-p\rightarrow\gamma_n$).[24] It is of course of great importance to continue these measurements to lower energies (Sec. IV C). However, for the moment we compare to the deuteron data and, as can be seen in Fig. 2(a), the negative x values are clearly favored.

For π^+ production, as pointed out in Sec. III, the model reduces to the usual one, and in particular agrees rather well with the total cross-section data[25] as is shown in Fig. 2(b).

In Fig. 3 we have plotted the quantity $\Delta'(W)$ using the same π^- data as above. The characteristic dip which results in a model-independent way from the presence of isotensor resonance excitations as shown above is clearly evident in both our model and the data.

The predicted total cross sections for π^0 production on protons and neutrons are shown in Figs. 4(a) and 4(b), respectively, where the data in the former case are taken from the recent experiment of Fischer *et al.*[26] We remind the reader that these curves are to some extent in error below 300 MeV owing to the error in ${}_pM_{1+}{}^3$ in this region noted in Sec. III. This does not apply in the resonance region 300–400 MeV, however, and since in this region the resonance dominates almost

[22] This is preferred to the more detailed calculation of Berends *et al.* (Ref. 16) since the latter authors include effects due to a large photoexcitation on neutrons of the $N(1400)$ resonance. This does not seem compatible with more recent data (Ref. 24).

[23] M. Bazin and J. Pine, Phys. Rev. **132**, 2735 (1963); J. P. Burg and R. K. Walker, *ibid.* **132**, 447 (1963).

[24] P. A. Berardo *et al.*, Phys. Rev. Letters **24**, 419 (1970).

[25] R. L. Walker, in Proceedings of the Fourth International Symposium on Electron and Photon Interactions at High Energies, Daresbury, 1969, edited by D. W. Braden (unpublished).

[26] G. Fischer *et al.*, Nucl. Phys. **B16**, 93 (1970).

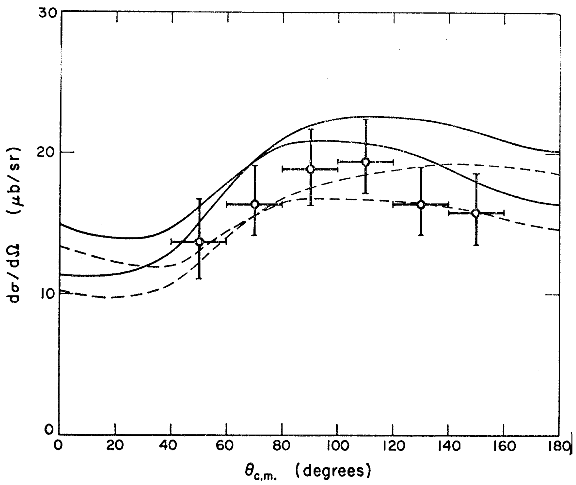

FIG. 5. The differential cross section for reaction (3b) at the resonance position ($E_\gamma=350$ MeV) for $x=0$, -0.2 and the maximum and minimum values of M_{-1} (see text). Data points are from Ref. 7. Solid lines $x=0$, dashed lines $x=-0.2$.

completely, the presence of an isotensor term leads to very large effects, as can be seen. Further, since the contribution of all other partial waves is only 4 or 5 μb, the interpretation is trivially unambiguous.

B. Differential Cross Sections and Polarizations

Data on the asymmetry from linearly polarized photons at 90° have been obtained for the π^- reaction in this region by Nishikawa *et al.*[27] This is especially useful in that this quantity is rather insensitive to changes in M_{1+},[28] and so can be used to estimate the value of M_{1-}. The predicted value[15] of the latter for π^- production is -0.69 (in units of $10^{-2}\ \mu^{-1}$) and is essentially constant over the resonance region. Fitting the asymmetry parameter at 350 and 390 MeV, values of -0.43 ± 0.38 and -0.52 ± 0.27 are obtained, giving a mean of -0.49 ± 0.22 in agreement with the prediction we have used. This results in a contribution to the total cross section of $5_{-4}^{+6}\ \mu$b.

The effect on the predictions for the differential cross section is, however, as we have said, more important. In Fig. 5 we show the differential cross section at 350 MeV for the largest and smallest M_{1-} values allowed above (-0.27, $-0.79\times10^{-2}\ \mu^{-1}$) for both $x=0$, -0.2. Clearly when more accurate differential-cross-section data become available, M_{1-} as well as M_{1+} should be obtained empirically from the data. As can be seen, the data are quite consistent with our model, but at their present level of accuracy add little further information to those already deduced from the total cross-section results.

C. Further Experiments

The rather striking evidence for isotensor terms already apparent in the rather meager data at present available makes even more clear the need for further experimental investigation of this area. However, the data we have used on the neutron reaction[7,8] were obtained from measurements on deuterium and are necessarily subject to some uncertainty on that account. As stressed in Ref. 3, a study of the radiative capture reaction

$$\pi^-+p\rightarrow\gamma+n$$

completely avoids these uncertainties. As we have noted, such an experiment has already been reported[24] at a somewhat higher energy, and at this energy ($E_\gamma=520$ MeV) confirms and refines the result of the above deuteron experiment.[7,8] It is of great importance to carry out such measurements over the region of the first resonance, both to avoid the ambiguity associated with deuterium and to improve the accuracy of the available data.

Finally, further experiments on photoproduction on deuterium in this region are also of importance, despite the above difficulty, both because of the very large effects predicted for π^0 production on neutrons, and also because of the added possibility of comparing the results for π^- photoproduction (3b) with the inverse radiative capture reaction, thus providing a direct test of T invariance where there is no known reason to suppose that T-violating effects would be suppressed.[29]

ADDENDUM I: RADIATIVE CAPTURE AT FORWARD ANGLES

Recently, some information on radiative capture in the region of the first resonance has become available.[30] This consists of eight points over the incident pion momentum range 220–380 MeV/c, the center of momentum angle varying between 20° and 40°. In fact, the angles used are too small to allow any firm conclusion to be drawn about the resonance excitation, as one may suspect from the fact that the data show no marked energy variation over the resonance region. There is also appreciable sensitivity to the little known M_{1-} multipole in this region (see Fig. 5). The authors state that their data agree well with the prediction of Berends *et al.*,[16] which corresponds to $x=0$. However, the value of M_{1-} used here is rather large, and if it is reduced to, for example, the value later favored by the same authors,[21] then the M_{1+} excitation would also have to be reduced to maintain the same success. For the purpose of comparison, we interpolated their data and obtained

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{c.m.}}=10.1\pm1.0\ \mu\text{b} \quad \text{at} \quad E_\gamma=350\ \text{MeV}, \quad \theta_{\text{c.m.}}=27°.$$

Thus, the result will depend strongly on the value of M_{1-} used; without more data at larger angles, no conclusion can yet be drawn.

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[27] T. Nishikawa *et al.*, Phys. Rev. Letters **21**, 1288 (1968).
[28] A change in x of 0.2 causes a change in the asymmetry of 0.02.
[29] N. Christ and T. D. Lee, Phys. Rev. **148**, 1520 (1966).
[30] J. Favier *et al.*, Phys. Letters **31B**, 609 (1970).

ADDENDUM II: $\gamma n \to \pi^- p$ FROM π^- TO π^+ RATIO

The existence of the isotensor component of the electromagnetic current was based on the $\gamma n \to \pi^- p$ cross section deduced from the data obtained using deuterium. A possible normalization error in deducing the neutron cross section has been our greatest concern. To resolve this ambiguity we are studying the existing π^--to-π^+-ratio data at the energy and angle where they are available. [The π^- to π^+ ratio near threshold (Ref. 23) has been considered in the text.] This work will be published in the near future. The preliminary result is that $\gamma n \to \pi^- p$ data obtained using deuterium are consistent with those obtained from the π^- to π^+ ratio at the resonance region.

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APPENDIX: $M_{1+}{}^2$ AND $M_{1+}{}^3$ IN STATIC MODEL

The magnitude of ${}_pM_{1+}{}^3$ was first successfully discussed by CGLN[9] in the static model. Projecting out partial waves from the fixed-t relations, expanding in $\omega=(W-m)/m$, and retaining only the leading terms, they arrived at the equation

$$\frac{M_{1+}{}^3(\omega)}{kq}=\frac{2\mu f}{3\omega}+\frac{1}{\pi}\int_1^\infty d\omega'\frac{\mathrm{Im}M_{1+}{}^3(\omega')}{k'q'}\times\left(\frac{1}{\omega'-\omega}+\frac{1}{9(\omega'+\omega)}\right). \quad \text{(A1)}$$

The corresponding equation for the isotensor term is

$$\frac{M_{1+}{}^2(\omega)}{kq}=\frac{1}{\pi}\int_1^\infty d\omega'\frac{\mathrm{Im}M_{1+}{}^2(\omega')}{k'q'}\times\left(\frac{1}{\omega'-\omega}+\frac{1}{3(\omega'+\omega)}\right). \quad \text{(A2)}$$

Now in these equations the crossed terms are very small compared to the Born and direct terms. Further, the expansion parameter ω/m has the rather large value 0.32 at resonance. We therefore neglect the crossed terms and rewrite (A1) and (A2) in the form

$$\frac{{}_NM_{1+}{}^3(\omega)}{kq}=\frac{2\mu f}{3\omega}+\frac{1}{\pi}\int_1^\infty d\omega'\frac{\mathrm{Im}{}_NM_{1+}{}^3(\omega')}{k'q'(\omega'-\omega)}, \quad \text{(A3)}$$

where

$${}_nM_{1+}{}^3(\omega)-{}_pM_{1+}{}^3(\omega)=(2\sqrt{\tfrac{2}{5}})M_{1+}{}^2(\omega) \quad \text{(A4)}$$

and

$$\frac{M_{1+}{}^2(\omega)}{kq}=\frac{1}{\pi}\int_1^\infty d\omega'\frac{\mathrm{Im}M_{1+}{}^2(\omega')}{k'q'}\frac{1}{\omega'-\omega}. \quad \text{(A5)}$$

The analogous equation for the corresponding scattering amplitude $f_{1+}{}^3(\omega)$ is

$$\frac{f_{1+}{}^3(\omega)}{q^2}=\frac{4f^2}{3\omega}+\frac{1}{\pi}\int_1^\infty d\omega'\frac{\mathrm{Im}f_{1+}{}^3(\omega')}{q'^2}\frac{1}{\omega'-\omega}. \quad \text{(A6)}$$

CGLN[9] noted that (A3) and (A6) were proportional and proposed the proportional solution

$${}_pM_{1+}{}^3(\omega)=\frac{\mu k}{2fq}f_{1+}{}^3(\omega). \quad \text{(A7)}$$

If we assume that the phases of these multipoles are the same as the scattering phase at all energies and that this phase goes to zero at infinity, then the above solution is unique.[31] If, however, we assume a different but equally plausible boundary condition—that the phase goes asymptotically to π, for example—then (A7) is no longer unique since the corresponding homogeneous equation

$$\frac{{}_p\bar{M}_{1+}{}^3(\omega)}{kq}=\frac{1}{\pi}\int_1^\infty d\omega'\frac{\mathrm{Im}{}_p\bar{M}_{1+}{}^3(\omega')}{k'q'(\omega'-\omega)} \quad \text{(A8)}$$

now has nonzero solutions, any of which may be added. We retain (A7), however, since in the resonance region it approximately agrees with experiment.

Let us now turn to the equation for ${}_nM_{1+}{}^3(\omega)$, which is also satisfied by (A7). However, this solution is again ambiguous up to a solution of the corresponding homogeneous equation (A8), which is just the equation satisfied by the isotensor amplitude $M_{1+}{}^2(\omega)$ [Eq. (A5)], i.e., by the difference of ${}_pM_{1+}{}^3$ and ${}_nM_{1+}{}^3$. Again, this is not necessarily zero unless we make the arbitrary assumption that its phase goes asymptotically to zero. Thus, for example, the form (A7) also follows from the use of a relativistic Breit-Wigner form for the amplitudes [see Eqs. (9)–(12)], in which the phase goes asymptotically to π, and such a formula

$$(2\sqrt{\tfrac{2}{5}})M_{1+}{}^2(\omega)=x\frac{\mu k}{2fq}\frac{\gamma q^3}{2(\omega_r-\omega)-i\gamma q^3}=x\frac{\mu k}{2fq}f_{1+}{}^{(3)}(\omega)=x\,{}_pM_{1+}{}^3(\omega) \quad \text{(A9)}$$

is an exact solution of (A5) for any real x, so that the result of Eq. (13) is reproduced,

$${}_nM_{1+}{}^3(\omega)=(1+x)\,{}_pM_{1+}{}^3(\omega). \quad \text{(A10)}$$

Thus the empirical success of Eq. (A7) cannot be used as an argument against the presence of isotensor terms. Of course, the static model involves rather drastic approximations, but the general form and largest terms of the full equations are retained so that the general nature of the ambiguities which is our main concern here will remain unchanged.

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