Baryon Currents in the Fermi-Yang Model*†

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All possible baryon currents consisting of trilinear local products of anticommuting nucleon fields are constructed and classified with respect to $SU(2)\otimes SU(2)$. In contrast to the quark-model baryon currents investigated by Hwa and Nuyts, the Fermi-Yang model (nucleon field) baryon currents may be isospin $\frac{3}{2}$ as well as $\frac{1}{2}$. Commutators of the baryon currents with axial charges are investigated, as well as anticommutators of baryon currents with the Hermitian conjugates of their own divergences. Although isospin mixing does occur for some of the axial-charge–baryon-current commutators, it is found that those terms in the commutators carrying the appropriate isospin are always proportional to the baryon current being commuted. For an isospin-$\frac{1}{2}$ baryon current $B^\mu(x)$, it is found that $\delta(x_0-y_0)\{B^\mu(x),\partial^\nu B_\nu{}^\dagger(y)\}=\delta(x-y)\mathcal{O}(\mu,x)$ + (Schwinger term), where $\langle 0|\mathcal{O}(\mu,x)|0\rangle=0$. This property of $\mathcal{O}(\mu,x)$ is equivalent to a spectral-function sum rule, the single-particle approximation of which relates the positive- and negative-parity spin-$\frac{1}{2}$, isospin-$\frac{1}{2}$ nucleon resonances.

I. INTRODUCTION

USEFUL results have been obtained from the current-algebra commutation relations under the assumption that the vector and axial-vector currents, along with their divergences, serve as interpolating fields for all mesons having the appropriate quantum numbers. A possible extension of this scheme consists of postulating that baryons as well may be directly incorporated into current algebra via the existence of underlying "baryon currents" or fields, each of which couples to all eligible baryons. Algebraic relationships in the form of equal-time commutators and anticommutators of these new objects with themselves and with the more established quantities of conventional current algebra may then be examined for their verifiable consequences. Investigations along these lines have been carried out by a number of authors.[1–5] One such approach, proposed by Hwa and Nuyts,[1] is to define a baryon current, denoted by $B^\mu(x)$, which transforms relativistically as the direct product of a four-vector and Dirac spinor. In addition, $B^\mu(x)$ is to have definite isospin and strangeness. Hence we have a situation in which $B^\mu(x)$ and $\partial_\mu B^\mu(x)$ serve as interpolating fields for all spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ baryons having the proper internal quantum numbers, establishing to some extent an analogy with the axial-vector current.

Clearly the analogy is far from complete. For example the charges associated with $B^\mu(x)$ cannot be group generators, since as fermionlike quantities they do not obey commutation relations. Likewise, there is nothing to indicate that $B^\mu(x)$ could be put to use as an ingredient of a Hamiltonian. It is precisely this lack of interpretation for $B^\mu(x)$, other than that of interpolating field, which makes it more difficult to work with than the vector and axial-vector currents.

A natural place to begin in the investigation of baryon currents is to construct them, using basic constituents about which something is known or assumed. In this way it may be possible to establish algebraic properties of the baryon currents which are of more general significance than the specific model used in the construction.

This paper is primarily concerned with carrying out such a program for nonstrange baryon currents, using as constituents nucleon fields obeying canonical anticommutation relations. A corresponding construction using anticommuting quark fields has been performed by Hwa and Nuyts.[1] As will be seen, the results for the quark and Fermi-Yang models are a good deal different in many respects. On the other hand, generalizations that are satisfied in both models have that much more chance of being valid.

To get an idea of how the construction should proceed, it is of interest to consider the meson currents. It is well known[6] that commutators of vector and axial-vector densities constructed from bilinear products of the $\mathcal{P}$ and $\mathfrak{N}$ quark fields reproduce the $SU(2)\otimes SU(2)$ algebra of currents when canonical anticommutation relations are assumed. These densities are given by

$$V^a{}_\mu(x)=\bar{q}(x)\tfrac{1}{2}\tau^a\gamma_\mu q(x) \tag{1}$$

and

$$A^a{}_\mu(x)=\bar{q}(x)\tfrac{1}{2}\tau^a\gamma_\mu\gamma^5 q(x)\,. \tag{2}$$

If the quark fields are simply replaced by nucleon fields in these expressions and canonical anticommutation relations are assumed for nucleon fields, the resultant quantities also clearly satisfy $SU(2)\otimes SU(2)$. We require that the baryon currents as well be constructed in terms of local products of the nucleon fields, enabling all equal-time commutators of baryon and meson currents to be calculable from the canonical anticommutation relations.

We are interested in the case where the spin of $B^\mu(x)$, like that of the axial-vector current, may be reduced by

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[1] R. C. Hwa and J. Nuyts, Phys. Rev. **151**, 1215 (1966).
[2] T. K. Kuo, Phys. Rev. **165**, 1708 (1968).
[3] M. Sugawara, Phys. Rev. **172**, 1423 (1968).
[4] J. Rothleitner, Nucl. Phys. **B3**, 89 (1967).
[5] M. Sugawara and J. W. Meyer, Phys. Rev. **174**, 1709 (1968).
[6] See, e.g., S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968), p. 20.

one unit only by taking the divergence. This is done by eliminating superfluous components via the condition

$$\gamma_\mu B^\mu(x)=0. \quad (3)$$

Equation (3) is also imposed in the quark-field construction.

In Sec. II we carry out in detail, up to arbitrary constants, the construction of all baryon currents in the Fermi-Yang model. In Sec. III these constants are specified so that the resulting baryon currents transform according to definite representations of $SU(2)\otimes SU(2)$. Section IV is devoted to a comparison with the quark-model results, while in Sec. V equal-time anticommutators are considered. Finally, in Sec. VI the main results are discussed and a possible application involving an equal-time anticommutator is described.

II. CONSTRUCTION OF BARYON CURRENTS

Our object is to construct the most general baryon currents having certain specified properties, the first of which is that they consist of sums of local products of nucleon fields. We confine out attention to trilinear local products, this being the simplest possibility in view of (3). The currents are to carry unit baryon number, so they must equal sums of products of two nucleon and one antinucleon field. The constituent nucleon fields are denoted by $\psi_{eu}(x)$, with e the isospin and u the Dirac index. An isospin-$\frac{1}{2}$ baryon current is similarly denoted by $B^\mu{}_{eu}(x)$.

Consider first the most general isospin-$\frac{1}{2}$ baryon current. It may be written in the form

$$B^\mu{}_{eu}=K^\mu(br;cs;dt;eu)\,\mathcal{A}(\bar{\psi}_{br}\psi_{cs}\psi_{dt}), \quad (4)$$

where repeated indices are summed over. The symbol $\mathcal{A}$ signifies that the product $\bar{\psi}_{br}\psi_{cs}\psi_{dt}$ is totally antisymmetrized, this being done to eliminate ambiguity in the ordering of the field operators. From (4) it is seen that there is no loss in generality caused by requiring

$$K^\mu(br;cs;dt;eu)=-K^\mu(br;dt;cs;eu). \quad (5)$$

Since they form complete sets in their respective spaces, the Pauli spin matrices plus the identity matrix and the 16 Dirac Γ matrices may be used in an expansion of K^μ according to

$$K^\mu(br;cs;dt;eu)=A^\mu{}_{\alpha\beta;EF}\tau^\alpha{}_{bc}\tau^\beta{}_{ed}\Gamma^E{}_{rs}\Gamma^F{}_{ut}, \quad (6)$$

where the indices of the Pauli matrices, α and β, each range from 0 through 3 with $\tau^0\equiv 1$. In keeping with the usual convention, Latin superscripts on the Pauli matrices assume only the values 1, 2, or 3. To simplify matters, an exterior-product notation is used in which (6), for example, would become

$$K^\mu=A^\mu{}_{\alpha\beta;EF}(\tau^\alpha\otimes\tau^\beta)(\Gamma^E\otimes\Gamma^F). \quad (7)$$

Upon surveying the available tensors, it is easy to show that B^μ is an isospin-$\frac{1}{2}$ object if and only if

$$A^\mu{}_{\alpha\beta;EF}(\tau^\alpha\otimes\tau^\beta)=C^\mu{}_{EF}(1\otimes 1)+D^\mu{}_{EF}(\tau^k\otimes\tau^k), \quad (8)$$

where $C^\mu{}_{EF}$ and $D^\mu{}_{EF}$ are arbitrary constants. That there are two such constants simply reflects the fact that isospin $\frac{1}{2}$ occurs twice in the Kronecker product $\frac{1}{2}\otimes\frac{1}{2}\otimes\frac{1}{2}$. We require that $B^\mu(x)$ transform with respect to parity according to[7]

$$\mathcal{P}B^\mu(\mathbf{x},t)\mathcal{P}^{-1}=\gamma^0 B_\mu(-\mathbf{x},t). \quad (9)$$

The condition that B^μ transform relativistically as the direct product of a spinor and four-vector along with our other space-time requirements as expressed by (1) and (9) may be shown to be equivalent to the relation

$$A^\mu{}_{\alpha\beta;EF}\Gamma^E\otimes\Gamma^F =f_{\alpha\beta}\gamma_\nu\otimes\omega^{\nu\mu}+k_{\alpha\beta}\gamma_\nu\gamma^5\otimes\omega^{\nu\mu}\gamma^5+h_{\alpha\beta}\sigma_{\nu\rho}\otimes\sigma^{\nu\rho}\gamma^\mu, \quad (10)$$

with $f_{\alpha\beta}$, $k_{\alpha\beta}$, and $h_{\alpha\beta}$ arbitrary constants, and where

$$\omega^{\nu\mu}\equiv 3ig^{\nu\mu}+\sigma^{\nu\mu}. \quad (11)$$

Combining (8) and (10), we may express $A^\mu{}_{\alpha\beta;EF}$ in terms of six arbitrary constants as

$$\begin{aligned}A^\mu{}_{\alpha\beta;EF}(\tau^\alpha\otimes\tau^\beta)(\Gamma^E\otimes\Gamma^F) &=(1\otimes 1)(c_1\gamma_\nu\otimes\omega^{\nu\mu}+c_2\gamma_\nu\gamma^5\otimes\omega^{\nu\mu}\gamma^5+ic_3\sigma^{\nu\rho}\otimes\sigma_{\nu\rho}\gamma^\mu)\\ &\quad+(\tau^k\otimes\tau^k)(d_1\gamma_\nu\otimes\omega^{\nu\mu}+d_2\gamma_\nu\gamma^5\otimes\omega^{\nu\mu}\gamma^5\\ &\quad+id_3\sigma^{\nu\rho}\otimes\sigma_{\nu\rho}\gamma^\mu). \quad (12)\end{aligned}$$

Let M and N be two $n\times n$ matrices; define a transposition operator P_{24} by

$$(P_{24}(M\otimes N))_{abcd}=(M\otimes N)_{adcb}, \quad (13)$$

where

$$(M\otimes N)_{abcd}=M_{ab}N_{cd}. \quad (14)$$

Now the completeness relations[8] for the Pauli matrices and the Dirac Γ matrices are given respectively by

$$2P_{24}(1\otimes 1)=1\otimes 1+\tau^k\otimes\tau^k \quad (15)$$

and

$$4P_{24}(1\otimes 1)=1\otimes 1+\gamma_\mu\otimes\gamma^\mu +\tfrac{1}{2}\sigma_{\mu\nu}\otimes\sigma^{\mu\nu}-\gamma_\mu\gamma^5\otimes\gamma^\mu\gamma^5+\gamma^5\otimes\gamma^5. \quad (16)$$

From (15) and (16) it follows that

$$P_{24}(\tau^k\otimes\tau^k)=\tfrac{3}{2}(1\otimes 1)-\tfrac{1}{2}(\tau^k\otimes\tau^k), \quad (17)$$

$$P_{24}(\gamma_\nu\otimes\omega^{\nu\mu})=\tfrac{1}{2}\gamma_\nu\otimes\omega^{\nu\mu}+\tfrac{1}{2}\gamma_\nu\gamma^5\otimes\omega^{\nu\mu}\gamma^5 +\tfrac{1}{2}i\sigma_{\nu\rho}\otimes\sigma^{\nu\rho}\gamma^\mu, \quad (18)$$

$$P_{24}(\gamma_\nu\gamma^5\otimes\omega^{\nu\mu}\gamma^5)=\tfrac{1}{2}\gamma_\nu\otimes\omega^{\nu\mu}+\tfrac{1}{2}\gamma_\nu\gamma^5\otimes\omega^{\nu\mu}\gamma^5 -\tfrac{1}{2}i\sigma_{\nu\rho}\otimes\sigma^{\nu\rho}\gamma^\mu, \quad (19)$$

$$P_{24}(i\sigma_{\nu\rho}\otimes\sigma^{\nu\rho}\gamma^\mu)=\gamma_\nu\otimes\omega^{\nu\mu}-\gamma_\nu\gamma^5\otimes\omega^{\nu\mu}\gamma^5. \quad (20)$$

Equation (5) implies that

$$A^\mu{}_{\alpha\beta;EF}[P_{24}(\tau^\alpha\otimes\tau^\beta)][P_{24}(\Gamma^E\otimes\Gamma^F)] =-A^\mu{}_{\alpha\beta;EF}(\tau^\alpha\otimes\tau^\beta)(\Gamma^E\otimes\Gamma^F). \quad (21)$$

[7] All conventions are identical with those used in J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[8] Equations (15) and (16) follow directly from the orthogonality property of irreducible unitary group representations. See, e.g., M. Hamermesh, *Group Theory* (Addison-Wesley, Reading, Mass., 1962), p. 102.

From (21) we find that

$$c_1=-2d_3-d_2, \quad (22)$$

$$c_2=2d_3-d_1, \quad (23)$$

$$c_3=d_2-d_1+d_3. \quad (24)$$

Hence there are three linearly independent isospin-$\frac{1}{2}$ baryon currents. By adding terms whose coefficients are symmetric with respect to P_{24} (that is, terms that make no contribution to the baryon current), these three currents may be expressed much more simply in terms of a different set of constants as

$$A^{\mu}{}_{\alpha\beta;\,EF}(\tau^{\alpha}\otimes\tau^{\beta})(\Gamma^E\otimes\Gamma^F) =(\tau^k\otimes\tau^k)(e_1\gamma_\nu\otimes\omega^{\nu\mu}+e_2\gamma_\nu\gamma^5\otimes\omega^{\nu\mu}\gamma^5 +ie_3\sigma_{\nu\rho}\otimes\sigma^{\nu\rho}\gamma^{\mu}). \quad (25)$$

We denote possible isospin-$\frac{3}{2}$ baryon currents by

$$B^{d\mu}(x), \quad d=1,2,3. \quad (26)$$

This current is taken to transform under rotations in isospin space as the direct product of an isovector and isospinor. Hence its commutator with an isospin generator is given by

$$[Q^a,B^{d\mu}]=(i\epsilon^{adc}-\tfrac{1}{2}\tau^a\delta^{dc})B^{c\mu}. \quad (27)$$

Suppose also that the subsidiary condition

$$\tau^d B^{d\mu}=0 \quad (28)$$

is satisfied. Using (27) and (28) it is straightforward to verify that

$$[Q^a,[Q^a,B^{d\mu}]]=\tfrac{3}{2}(\tfrac{3}{2}+1)B^{d\mu}. \quad (29)$$

Hence $B^{d\mu}$ is a pure isospin-$\frac{3}{2}$ object.

It may be shown that $B^{d\mu}$ is determined up to a constant. A possible representation is

$$A^{d\mu}{}_{\alpha\beta;\,EF}(\tau^{\alpha}\otimes\tau^{\beta})(\Gamma^E\otimes\Gamma^F) =e(i\epsilon^{dbc}\tau^b\otimes\tau^c-2\tau^d\otimes 1)(\gamma_\nu\otimes\omega^{\nu\mu}). \quad (30)$$

III. $SU(2)\otimes SU(2)$ TRANSFORMATION PROPERTIES

Four linearly independent baryon currents have been derived. We now further specify the arbitrary constants that appear in the construction in such a way that each baryon current belongs to a definite representation $(I_1,I_2)\oplus(I_2,I_1)$ of $SU(2)\otimes SU(2)$. From the canonical anticommutation relations it follows that the commutator of an axial charge with a constituent field is given by

$$[Q^a{}_5(x_0),\psi(x)]=-\tfrac{1}{2}\tau^a\gamma^5\psi(x), \quad (31)$$

i.e., ψ belongs to a $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ representation of $SU(2)\otimes SU(2)$. By commuting an arbitrary axial charge first with $B^{d\mu}$ and then with the resulting expression, it is straightforward using (31) to verify that

$$\gamma^5[Q^a{}_5(x_0),B^{d\mu}(x)]=-\tfrac{1}{3}(i\epsilon^{adc}-\tfrac{1}{2}\tau^a\delta^{dc})B^{c\mu}(x) +(k_1/3k_2)(2\delta^{ad}+i\epsilon^{adc}\tau^c)B^{\mu}(x) \quad (32)$$

and

$$\gamma^5[Q^a{}_5(x_0),B^{\mu}(x)]=\tfrac{5}{6}\tau^a B^{\mu}(x)+(4k_2/3k_1)B^{a\mu}(x), \quad (33)$$

where $B^{d\mu}$ and B^{μ} are particular linear combinations of the derived currents given in terms of arbitrary constants k_1 and k_2 as

$$B^{d\mu}(x)=k_1 i\epsilon^{dab}\{V^a{}_\nu(x),\omega^{\nu\mu}\tau^b\psi(x)\} -2k_1\{V^d{}_\nu(x),\omega^{\nu\mu}\psi(x)\}, \quad (34)$$

$$B^{\mu}(x)=k_2\{V^a{}_\nu(x),\omega^{\nu\mu}\tau^a\psi(x)\} -k_2\{A^a{}_\nu(x),\omega^{\nu\mu}\gamma^5\tau^a\psi(x)\}. \quad (35)$$

Here

$$V^a{}_\nu(x)=\bar{\psi}(x)\tfrac{1}{2}\tau^a\gamma_\nu\psi(x) \quad (36)$$

and

$$A^a{}_\nu(x)=\bar{\psi}(x)\tfrac{1}{2}\tau^a\gamma_\nu\gamma^5\psi(x). \quad (37)$$

In obtaining (34) and (35) we have used the identity

$$\mathcal{A}(\bar{\psi}_{br}(x)\psi_{cs}(x)\psi_{dt}(x))=\tfrac{1}{2}\{\mathcal{A}(\bar{\psi}_{br}(x)\psi_{cs}(x)),\psi_{dt}(x)\}. \quad (38)$$

For the two linearly independent currents that remain it may be shown that

$$[Q^a{}_5(x_0),B'^{\mu}(x)]=-\tfrac{1}{2}\tau^a\gamma^5 B'^{\mu}(x), \quad (39)$$

where

$$B'^{\mu}(x)=k_3\{V^a{}_\nu(x),\omega^{\nu\mu}\tau^a\psi(x)\} +k_4\{A^a{}_\nu(x),\omega^{\nu\mu}\gamma^5\tau^a\psi(x)\} +i(k_4-k_3)\{T^a{}_{\nu\beta}(x),\tau^a\sigma^{\nu\beta}\psi(x)\} \quad (40)$$

and

$$T^a{}_{\nu\beta}(x)=\bar{\psi}(x)\tfrac{1}{2}\tau^a\sigma_{\nu\beta}\psi(x). \quad (41)$$

IV. QUARK-MODEL BARYON CURRENTS

As mentioned previously, an analogous construction of baryon currents in which trilinear local products of anticommuting quarks are utilized has already been performed by Hwa and Nuyts.[1] Since there are no $\bar{q}$'s involved in these currents, the limits imposed by statistics are much more severe than in the Fermi-Yang model, with the result that only a single nonstrange baryon current can be produced. It is of isospin $\frac{1}{2}$. Since it is unique, it follows immediately that its commutator with axial charges is identical (up to a possible sign difference) with (39).

V. ANTICOMMUTATION RELATIONS

Equal-time anticommutators such as

$$\delta(x_0-y_0)\{B^{\mu}(x),B^{\nu\dagger}(y)\} \quad (42)$$

or

$$\delta(x_0-y_0)\{B^{\mu}(x),\partial_\nu B^{\nu\dagger}(y)\} \quad (43)$$

are also of interest. The anticommutator (42) has been studied[1] in the quark model. By using the canonical anticommutation relations to evaluate it, one obtains a lengthy sum of products of two meson current densities. Results of a generally similar nature follow from the baryon currents found in this paper. Meaningful consequences of this anticommutator have not been found, and it will not be discussed further.

For the isospin-$\frac{1}{2}$ baryon currents of the Fermi-Yang model it is possible to get some idea of the form the anticommutator (43) may take. In this situation $\psi^\dagger(x)$ and $\partial_\nu B^{\nu\dagger}(x)$ have the same quantum numbers. Of course, the form of $\partial_\nu B^{\nu\dagger}(x)$ cannot be determined without knowing the field equations; however, we make the assumption that, for purposes of calculating this anticommutator, $\partial_\nu B^{\nu\dagger}(x)$ may be replaced with $\psi^\dagger$. Upon doing this and applying the canonical anticommutation relations, one sees that the anticommutator is linear in meson currents.

If $B^\mu(x)$ is chosen as in (35), one obtains

$$\delta(x_0-y_0)\{B^\mu(x),\partial_\nu B^{\nu\dagger}(y)\} =2k_2\delta(x-y)[V^a{}_\nu(x)\tau^a\omega^{\nu\mu}-A^a{}_\nu(x)\tau^a\omega^{\nu\mu}\gamma^5 +\tfrac{1}{2}iT^a{}_{\nu\rho}(x)\tau^a\sigma^{\nu\rho}\gamma^\mu-\tfrac{3}{2}iT_{\nu\rho}(x)\sigma^{\nu\rho}\gamma^\mu]+\mathrm{ST}. \quad (44)$$

On the other hand, if the form (40) is used for $B^\mu(x)$ the result is

$$\delta(x_0-y_0)\{B^\mu(x),\partial_\nu B^{\nu\dagger}(y)\} =\tfrac{3}{2}\delta(x-y)[(k_3+k_4)V^a{}_\nu(x)\tau^a\omega^{\nu\mu} +(k_3+k_4)A^a{}_\nu(x)\tau^a\omega^{\nu\mu}\gamma^5+i(k_4-k_3)T^a{}_{\nu\rho}(x)\tau^a\sigma^{\nu\rho}\gamma^\mu] +\tfrac{3}{2}\delta(x-y)[(k_3-3k_4)V_\nu(x)\omega^{\nu\mu}+(k_4-3k_3)A_\nu(x)\omega^{\nu\mu}\gamma^5 +i(k_4-k_3)T_{\nu\rho}(x)\sigma^{\nu\rho}\gamma^\mu]+\mathrm{ST}, \quad (45)$$

where $A_\nu(x)$, $V_\nu(x)$, and $T_{\nu\rho}(x)$ are isoscalar analogs of $A^a{}_\nu(x)$, $V^a{}_\nu(x)$, and $T^a{}_{\nu\rho}(x)$:

$$A_\nu(x)=\tfrac{1}{4}[\bar\psi(x),\gamma_\nu\gamma^5\psi(x)], \quad (46)$$

$$V_\nu(x)=\tfrac{1}{4}[\bar\psi(x),\gamma_\nu\psi(x)], \quad (47)$$

$$T_{\nu\rho}(x)=\tfrac{1}{4}[\bar\psi(x),\sigma_{\nu\rho}\psi(x)]. \quad (48)$$

Possible Schwinger terms have been allowed for in (44) and (45). The anticommutators (44) and (45) have an interesting feature in common:

$$\delta(x_0-y_0)\langle 0|\{B^\mu(x),\partial_\nu B^{\nu\dagger}(y)\}|0\rangle=\langle 0|\mathrm{ST}|0\rangle. \quad (49)$$

In addition, it may be noted that for $k_3=k_4$ the non-Schwinger term $I=1$ part of the anticommutator (45) assumes the model-independent form

$$\delta(x_0-y_0)\{B^\mu(x),\partial_\nu B^{\nu\dagger}(y)\}_{I=1} =3k_3\delta(x-y)[V^a{}_\nu(x)\tau^a\omega^{\nu\mu}+A^a{}_\nu(x)\tau^a\omega^{\nu\mu}\gamma^5]. \quad (50)$$

By model independent we mean that the relation in question includes only baryon currents and the $SU(2)\otimes SU(2)$ currents.

It is important to note that (49) does not depend critically on $\partial_\nu B^{\nu\dagger}$ being set equal to $\psi^\dagger$ for purposes of calculating anticommutators. All that is required is that those parts of $\delta(x_0-y_0)\{B^\mu(x),\partial_\nu B^{\nu\dagger}(y)\}$ proportional to $\delta(x-y)$ contain no noncovariant factors of the form $g^{\mu 0}$. Equation (3) will then guarantee that (49) is true.

VI. DISCUSSION AND APPLICATIONS

An item of obvious interest in working with baryon interpolating fields within the framework of current algebra is the way in which they commute with axial charges. The simplest assumption, and one commonly made in the literature,[9] is that a baryon current or field $B(x)$ of isospin I_B yields itself when commuted with an axial charge, i.e.,

$$[Q^a{}_5(x_0),B(x)]=\pm t^a\gamma^5 B(x). \quad (51)$$

Here t^a is the $(2I_B+1)\times(2I_B+1)$ matrix representative of the ath isospin generator. Equation (51) is satisfied for the two constructed isospin-$\frac{1}{2}$ baryon currents represented by (40) independently of the values of k_3 and k_4, as well as by the baryon current constructed in the quark model. On the other hand, (51) must be modified slightly to accommodate the remaining two baryon currents, as evidenced by (32) and (33). This is to be expected, since there is nothing to prevent the construction of representations of $SU(2)\otimes SU(2)$ other than those of the form $(I_B,0)\oplus(0,I_B)$. Nevertheless, the commutators (32) and (33) still have the property that those parts diagonal in isospin are proportional to the particular baryon current being commuted. In the absence of any principle which tells us which current(s) to keep and which to discard, we seek a generalization which is valid for all of them. For any of the constructed currents in either the quark or Fermi-Yang models it is true that

$$t^a[Q^a{}_5(x_0),B^\mu(x)]=r_B\gamma^5 B^\mu(x), \quad (52)$$

with r_B a numerical constant. Some applications of (52) will be discussed elsewhere.[10]

We now discuss a simple application of the anticommutator (49). Writing down a Lehman-Källén spectral representation,[11] it is straightforward to show that (49) is equivalent to

$$\int_0^\infty dM^2\rho_{1/2}(M^2)\frac{\partial}{\partial x_\mu}[\Delta^+(x,M)-\Delta^-(x,M)]\Big|_{x_0=0}=0, \quad (53)$$

where

$$\Delta^\pm(x,M)=\frac{1}{(2\pi)^3}\int d^4p\, e^{\mp ipx}\theta(p_0)\delta(p^2-M^2). \quad (54)$$

Here $\rho_{1/2}(M^2)$ is the spin-$\frac{1}{2}$ spectral function defined by

$$\theta(p^2)\theta(p_0)\rho_{1/2}(p^2)=(8/3)(2\pi)^3\sum_n \delta(p-p_n)(p_\mu p_\nu/p^2) \times\{\gamma_5,\langle 0|B^\mu(0)|n\rangle\langle n|\bar B^\nu(0)|0\rangle\}. \quad (55)$$

Equation (53) yields the spectral function sum rule

$$\int_0^\infty dM^2\rho_{1/2}(M^2)=0. \quad (56)$$

[9] See, e.g., A. M. Gleeson, Phys. Rev. **149**, 1242 (1966); K. Bardakci, *ibid.* **155**, 1788 (1967); and Ref. 5.
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Assuming that $\rho_{1/2}(M^2)$ is saturated by single-particle states, (56) becomes

$$\sum_k \epsilon_k C_k^2/M_k=0. \tag{57}$$

Here ϵ_k and M_k are respectively the intrinsic parity and mass of the kth isospin-$\frac{1}{2}$, spin-$\frac{1}{2}$ nucleon resonance. The C_k are real coupling constants, defined by

$$\langle 0|B^\mu(0)|\epsilon_k,M_k,p,s\rangle = i[(2\pi)^3 EM_k]^{-1/2}C_k\Gamma_k(4p^\mu/3M_k-\tfrac{1}{3}\gamma^\mu)u(p,s), \tag{58}$$

where $\Gamma_k=1$ for $\epsilon_k=1$, and $\Gamma_k=\gamma_5$ for $\epsilon_k=-1$. Taking viewpoints different from but compatible with ours, Sugawara[3] and Rothleitner[12] have independently obtained the sum rule

$$\sum_k \epsilon_k C_k^2 M_k=0. \tag{59}$$

Given the existence of the four nucleon resonances $P_{11}(940)$, $P_{11}(1470)$, $S_{11}(1535)$, and $S_{11}(1700)$, Genz[13] has given a very simple argument that a pair of sum rules differing by two powers of the mass as in (57) and (59) imply the existence of a P_{11} ($M\geqslant 1470$ MeV) resonance, which may be identified with the observed $P_{11}(1780)$.

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Light Cone as Seen in the Gluon Model

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To second order in a gluon model, the longitudinal electroproduction function respects scaling and thus provides a laboratory for the study of various light-cone theorems recently derived in the literature. In particular, the operator-Schwinger-term sum rule is upheld by a hitherto unsuspected singularity at ω $(\equiv -q^2/2\nu)=0$ which may obliterate any direct connection between light-cone singularities and electroproduction experiments. We also discuss the use of causal representations in the literature of deep inelastic processes emphasizing the delicacy of popular techniques.

I. INTRODUCTION

A GREAT deal of theoretical discussion has recently been focused on the electroproduction[1] and neutrino-production experiments. These experiments, in contrast to on-shell hadron-hadron scattering experiments, offer us information on the far-off-shell deep-inelastic region and thus provide a natural probe into the small-distance structure of the theory. Bjorken[2] has noted that the functions $\tilde{F}_i(\omega,q^2)$ approach finite limits $F_i(\omega)$ in the deep-inelastic region ($q^2\to\infty$, $\nu\to\infty$, $\omega=-q^2/2\nu$ finite) if the Callan-Gross integral[3] $\lim_{q^2\to\infty}\int d\omega\,\tilde{F}_2(\omega,q^2)$ exists and if certain well-studied pathologies are barred.[4] (Our notation is set forth in Appendix A.) This "scaling theorem" essentially imposes stringent smoothness constraints on hadron dynamics, which are typically not respected by perturbation theory. Translated into configuration space, these constraints limit the possible singularities of the electroproduction functions on the light cone.[5–11] In particular, Jackiw, Van Royen, and West[12] and others have shown that the commutator functions C_1 has the

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[12] See Refs. 6, 7, and 9, among others.