Multiplicity in Very High-Energy Particle-Nucleus Collisions*

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We investigate the broad features of final multiplicity in high-energy inelastic particle-nucleus collisions. The A dependence of multiplicity is rather sensitive to the nature of the high-energy particle-nucleon reaction. In particular, we investigate consequences of the particle-nucleon reaction being of multiperipheral type.

IT has become clear that at very high energies one will no longer be able to measure all momenta of all particles. In addition, the number of two- and three-particle correlations is so large that one will not be able to analyze the data through the study of these correlations. Instead one will have to work with quantities such as partial distributions or multiplicities. It has recently been suggested[1] that measurements on multiplicities in nuclear scattering may provide a useful tool for determining the nature of the dominant high-energy primary reactions. The basic idea behind this proposal is that the particle-nucleus amplitudes will depend in a relatively straightforward way on the particle-particle amplitudes, so that by measuring the former, one may hope to derive information about the latter. In addition, by measuring multiparticle production in nuclei, one can obtain information about the state in which the particles are produced by observing their interaction with nucleons farther downstream from the one on which they were created. High-energy production data on large nuclei is now being collected in a cosmic-ray experiment,[2] and more will become available soon from the National Accelerator Laboratory (NAL). It is the purpose of this brief note to exhibit some of the most striking predictions for this type of process. We shall be concerned throughout with deriving general results which can be expected in such reactions, rather than in making hard theoretical predictions on the basis of particular detailed models. Hence our results should be taken as indicating the broad features to be expected in multiparticle production from nuclei at high energies, rather than an investigation of details of such processes.

In discussing processes of this sort, there are basically two types of problems which must be handled. These are (1) those concerned with the behavior of the nucleus during the interaction, and (2) those concerned with models for the production of multiparticle states at high energies.

For our nuclear model, we choose to represent the nucleus as a spherical absorbing medium of uniform density whose radius is picked to match those measured in electron scattering experiment.[3] Thus we restrict our attention to those reactions in which the nucleus does not become excited or fragmented, i.e., to coherent scattering. In practice, this means that we look only at small forward angles, or at momentum transfers less than or on the order of the nuclear Fermi momentum. This, in turn, implies that our calculations will be best for those reactions in which the energy of the incident particle is sufficiently high so that the longitudinal momentum transfer associated with the creation of a high-mass final state is small. While there is some incoherent scattering even in the small-t region, it can be taken into account by standard techniques.[4]

There are two types of primary reactions we would like to use as extremes to illustrate what one can learn from the A dependence of high-energy coherent-scattering experiments. The first is pure isobar production without decay within the nucleus—a criterion obeyed by hadron resonances above 100 GeV. Here the number of particles scattering in the nucleus remains at one even though that particle may be an excited state of the projectile. The only possible increase in multiplicity is due to higher isobar production with each collision and the concommitant increase in the number of stable decay products of that higher isobar. The second type of primary scattering event is the truly inelastic reaction of the multiperipheral type.[5] In this case, the number of particles available to rescatter within the nucleus increases with each collision, so that an increase in multiplicity can now arise from the products of the initial multiperipheral interaction initiating reactions of their own, building up a cascade within the nucleus. Of course the dominant reaction at high energy may be intermediate between these two extremes. Nevertheless it is worthwhile to study the extreme cases in order to delineate the possibilities offered by nuclear scattering.

To understand this technique better we must first look more closely at multiple scattering in nuclei. Let us consider the multiple scattering of a single high-energy particle in a nucleus. We assume for con-

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[1] L. W. Jones, in Proceedings of the International Conference on Expectations for Particle Reactions at the New Accelerators, University of Wisconsin, 1970 (unpublished); and University of Michigan report (unpublished).
[2] K. N. Erickson, Ph.D. thesis, University of Michigan, 1970 (unpublished); L. W. Jones *et al.*, University of Michigan Report No. 03028-2-T, 1969 (unpublished).
[3] R. Herman and R. Hofstadter, *High Energy Electron Scattering Tables* (Stanford U. P., Stanford, 1960).
[4] J. S. Trefil, Nucl. Phys. **B11**, 330 (1969).
[5] K. A. Ter-Martirosyan, Nucl. Phys. **68**, 591 (1965); see also F. Zachariasen and G. Zweig, Phys. Rev. **160**, 1322 (1967).

venience that, regardless of the type of individual reaction, the original projectile remains more or less intact and retains its high energy. We also assume that the total cross section is a constant. (We shall later assume for simplicity that it is the same constant for all particles.) We call this leading particle the primary. In an isobar-type reaction the primary is the projectile or its excited state. In a multiperipheral-type reaction the primary is the particle at the top of the multiperipheral chain (this is discussed in more detail below). In our model, the primary propagates through the nucleus with a mean free path ζ determined by the number density of nucleons and the total primary-nucleon cross section σ_T, according to

$$1/\zeta=\sigma_T\times(\text{number density of nucleons}). \quad (1)$$

For a nucleus of a given radius R, we can find the probability $P(I)$ for the occurrence of I collisions by integrating over both the path length available for collision at a given impact parameter b and the impact parameter. This relative probability (unnormalized) is given by

$$P(I)=\pi\int_0^{2R}\eta d\eta\int_0^{\eta}dz_1\int_{z_1}^{\eta}dz_2\cdots\int_{z_{I-1}}^{\eta}dz_I \times(1/\zeta)^I\exp(-\eta/\zeta)$$
$$=\pi\int_0^{2R}\eta d\eta(\eta/\zeta)^I\exp(-\eta/\zeta)/I!, \quad (2)$$

where $\eta=(R^2-b^2)^{1/2}$ is the total distance in nuclear matter traversed at a given impact parameter b. This form is easily derived by noting that the probability of a particle travelling a distance z in an absorbing medium without a collision is just $e^{-z/\zeta}$, and the probability of at least one collision between z and $z+dz$ is just $1-e^{-dz/\zeta}\approx dz/\zeta$, so that the probability of having collisions at $z_1, z_2, \ldots, z_I$ and then getting out of the nucleus with no further interactions is just

$$(\zeta)^{-I}\exp(-z_1/\zeta)\exp[-(z_2-z_1)/\zeta]\cdots \times\exp[-(\eta-z_I)/\zeta]$$

which, after some trivial cancellations, leads directly to Eq. (2). We note in passing that $P(I)$ is closely related to the incomplete gamma function,[6] and that the average number of collisions is just

$$\bar{I}=\sum_{I=0}^{\infty}IP(I)\Big/\sum_{I=0}^{\infty}P(I)=\frac{4}{3}\frac{R}{\zeta}. \quad (3)$$

For our calculation, we shall choose $\sigma_T=40$ mb, a value appropriate for a proton primary (and hence the value most useful for analyzing cosmic-ray data). This corresponds to $\zeta=1.83$ F. $P(I)$, probably normalized, is given for this value of ζ and various nuclei in Fig. 1. As one might expect, for larger nuclei the distribution is flatter and the peak in the distribution occurs at larger values of I.

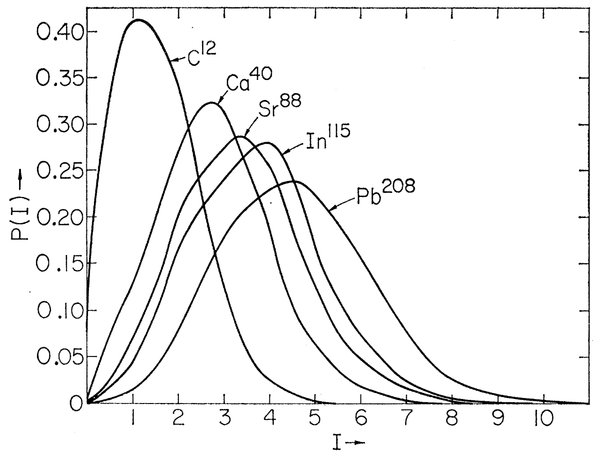

FIG. 1. Probability $P(I)$ for I primary collisions in various nuclei. The mean free path is determined by $\sigma_T=40$ mb.

We can now use these distributions to make predictions based on our two models for the nature of the primary scatterings. For an isobar model, the final multiplicity is just the multiplicity from the decay of the isobar produced at the Ith collision weighted by $P(I)$. This will result in a multiplicity not very different from that observed on a proton.

In a multiperipheral model the prediction is more striking but more difficult to calculate accurately. We use the simplest multiperipheral model that is consistent with a constant total cross section. It will be the high-energy data that reveals whether the appropriate stringent kinematical constraints (all subenergies large, all momentum transfers small) must be satisfied. We assume that the characteristics of the collisions are as if these constraints do hold. With this model, the distribution for producing N particles in a primary collision is flat for N from 2 to $N_{\max}\sim\ln s$. The particle which emerges from the top of the multiperipheral chain retains almost all of the energy of the projectile. This particle we call a primary or first-level particle. The particles in the chain have successively lower energies, the energy of the ith such particle being approximately β^i times the energy of the primary. β depends on N, the number of particles in the chain, in such a way that the last particle on the chain remains at rest in the lab. For the coherent nuclear reactions we are studying this last particle in just the target nucleon. We call the second particle in the chain a secondary, or second-level particle, and similarly for other lower-level particles. The reason the multiperipheral reaction is interesting to us is that both the primary and the lower-level particles can cascade.[7] The difficulty arises as follows. When

[6] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun (U. S. Department of Commerce, National Bureau of Standards, Washington, D. C., 1964), Applied Math. Sec. 6, Chap. 6.5, p. 260, formula 6.5.1.

[7] Lower-level particles follow the primary particle through the nucleus. One may ask whether the fact that the primary's going ahead has changed the nucleus to the extent that scattering of lower-level particles is affected. This problem has been investigated in Glauber theory and the effect is found to be very small [see J. S. Trefil, Phys. Rev. Letters **23**, 1075 (1969)].

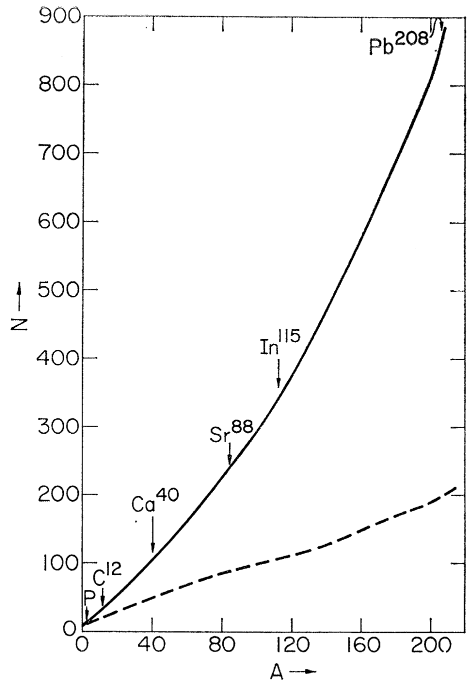

FIG. 2. Total number of final particles versus A, using an average number of collisions per particle and an average number of particles produced per collisions. As described in the text, we take $\bar{N}=10$, $\beta=1$, and $\sigma_T=40$ mb. The solid curve which rises slightly faster than linearly represents chaining in all possible levels, and the dashed curve represents chaining through the third level.

the primary produces an N-particle chain, the energy of the secondary is reduced by a factor $\beta(N)$. This secondary can itself have a subsequent multiperipheral collision with a probability for producing N' particles which is flat out to $N'_{\max}$. $N'_{\max}$ now depends on $\beta(N)$, namely, $N'_{\max}=N_{\max}-\beta(N)$. Thus the multiplicity distribution arising from a *given* sequence of collisions is already very complicated. In addition, the multidimensional probability distribution for collisions other than just primary collisions, in analogy with Fig. 1, is difficult to calculate. Expressions analogous to Eq. (2) for a given number of particles produced at an arbitrary collision are straightforward to write down, but numerical calculation is not feasible, and complete treatment of the multiplicity distribution on nuclei when the primary reaction is multiperipheral is very complicated.

There are, however, two distinct and, in a sense, orthogonal ways in which we can get an indication of the dependence of multiplicity on the atomic number A. (1) In the first approach we give up the idea of getting a distribution of multiplicities but concentrate on estimating the average multiplicity due to the cascading of many particles. (2) In the second approach we study the effect of nuclear scattering of the primary on the shape of the particle multiplicity distribution, but without trying to include the effect of cascading of lower-level particles.

(1) In order to estimate the average effect on multiplicity of the cascading of lower-level particles, we assume that a particle in the nucleus undergoes an average number of collisions ($\bar{I}$) in the nucleus and that each collision produces an average number of particle according to its energy.

The average number of primary collisions in a nucleus is taken directly from Eq. (3). $\bar{I}$ is a linear function of the nuclear radius R, or is approximately proportional to $A^{1/3}$. How can we estimate the average number of lower-level collisions? It is reasonable that particles produced in the primary collision would have an average number of collisions equal to $\bar{I}-1$ since one "average collision length" has already been traversed.[8] Similarly, particles in the second primary collision or in the first collision produced by lower-level particles of the first primary collision would have $I-2$ collisions on the average. In order to test this, we calculated the two-dimensional collision probability analogous to Eq. (2) for primary collisions and collisions of the secondary particle in the first primary collision. As expected, the average number of collisions for the secondary was approximately one less than the average number of collisions of the primary. We therefore take as a working rule that a particle produced after n collisions undergoes an average number of collisions equal to $\bar{I}-n$.

Next we study the number of particles produced per collision. Let the average number produced in the primary collision be $\bar{N}$. According to our previous remarks, the average number produced by the collision of a second-level particle would be $\bar{N}-\beta$ (note that since $\bar{N}=\frac{1}{2}N_{\max}$, this β equals one-half of our previous β); by a third-level particle, $\bar{N}-2\beta$, etc. We further make the reasonable assumption that β is independent of energy. This means that the collision of a third-level particle of a primary collision would produce the same number of particles, $\bar{N}-2\beta$, as the collision of a particle which is itself the secondary of a secondary produced in a primary collision. It is now clear how to count the average number of particles produced by the collision of any lower-level particle, whether this lower-level particle be the direct product of a primary collision or the result of cascading of other lower-level particles. We say that a particle is a universal jth-level particle if in a collision it produces $\bar{N}-(j-1)\beta$ particles on the average. We note that both $\bar{N}$ and β are quantities which must be determined by experiment.

With the above approaches to number of collisions and particles produced per collision, it is a straightforward matter of counting to compute the number of particles emerging from a given nucleus. Recall that

[8] The assumption is implicit here that all particles have the same mean free path. A refinement to this calculation which is rather simple to make is, for example, to assume that the primary particle is a proton ($\sigma_T=40$ mb, $\zeta_T=1.83$ F) and that all lower-level particles are pions ($\sigma_T=30$ mb, $\zeta_T=2.40$ F). The proton or pions would then have as many further collisions as the available remaining average path length would allow.

in the multiperipheral model the target particle remains at rest and does not emerge from the nucleus. Then, for $\bar{I}$ an integer and including the effect of cascading up to the Jth universal level, the total number of emerging particles is

$$N^J=1+(\bar{N}-2)\bar{I}+\sum_{j=1}^{J}(\bar{N}-2-j\beta)\times\sum_{l=1}^{j}\frac{1}{(l+1)!}\binom{j-1}{j-l}\prod_{k=1}^{l+1}(\bar{I}-k+1). \quad (4)$$

When $\bar{I}$ is not an integer, we consider only terms in the product over k which remain positive. We are then making an error (small in the cases we consider) which depends on $\bar{I}$ minus the largest integer in $\bar{I}$. We also constrain J such that each term in the sum over j remains positive.

In order to illustrate the effect, we calculate N^J from Eq. (3) for the $\bar{I}$ given by the arrows in Fig. 1, for $\bar{N}=10$, and for $\beta=1$. We put $J=9$, which is the last universal level that can conceivably rescatter in the nucleus. The results are shown in Fig. 2. Notice that N grows with A at a rate slightly faster than linear, a rather striking rise in multiplicity. It is perhaps worth noting that although we took $\bar{N}$ arbitrarily at some fixed energy, the rate of increase of $\bar{N}$ is $\sim\ln s$; therefore we would expect the nuclear effect also to increase as $\ln s$.

(2) As our second distinct method of studying the nuclear effects, we study the effect of many primary collisions on the number distribution. We have already stated that in single collisions the distribution is flat up to $N_{\max}$. Qualitatively we may see the effect as follows: for a given number of primary collisions, there are several ways one can have a given number of particles emerge, and the number of ways differs for different final numbers of particles. For the reasons discussed above, we make a quantitative calculation of the number distribution including only multiple collisions of the primary particle.

Let us first assume that there are I primary collisions. If there are n_i particles produced in the ith collision and $j_i=n_i-2$, recall that the number produced in I collisions is

$$N=\sum_{i=1}^{I} j_i+1\,.$$

If $p(n)$ represents the probability of producing n particles in a given collision,

$$p(n)=\begin{cases}1/N_{\max}, & n\leq N_{\max}\\ 0, & n>N_{\max}\end{cases} \quad (5)$$

then the probability that N particles emerge when there are I collisions is

$$p(N,I)=\sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\cdots\sum_{j_I=0}^{\infty}p(n_1)p(n_2)\cdots p(n_I)\delta_{\Sigma j_i+1,N}$$

$$=(1/N_{\max})^I\sum_{j_1=0}^{N_{\max}-2}\sum_{j_2=0}^{N_{\max}-2}\cdots\sum_{j_I=0}^{N_{\max}-2}\delta_{\Sigma j_i+1,N}\,.$$

If we define $J_{\max}=N_{\max}-2$, this can be written

$$p(N,I)=(1/N_{\max})^I\times\sum_{j_1=0}^{J_{\max}}\sum_{j_2=0}^{\min(J_{\max},N-1-j,0)}\cdots\sum_{j_I=0}^{\min(J_{\max},N-1-j_1\cdots j_{I-1},0)}1\,. \quad (6)$$

Finally, the probability of having N particles emerge, averaged over all numbers of primary collisions, is

$$P(N)=\sum_{I=0}^{\infty}P(N,I)P(I)\,, \quad (7)$$

where $P(I)$ is given by Eq. (2).

We have calculated $P(N)$ and compared it to $p(n)$ for C^{12}, Sr^{88}, and Pb^{208} in the cases $N_{\max}=5$, 10, and 20. (A detailed model for multiperipheral events will give some relations between $N_{\max}$ and the energy of the primary, but we do not wish to make any choice of the relationship here.) These functions are shown in Figs. 3(a)–3(c). Notice the departure from the flat multiperipheral distribution. This occurs because the number of collisions is smaller in the smaller nuclei, hence the distribution is more peaked at smaller N and falls off more rapidly at larger N. Also, for small $N_{\max}$ and small A we see a pronounced second maximum near $N_{\max}$. This is because in a single collision no more than $N_{\max}$ particles can be produced. The second peak represents the separation between the effect of single and double or higher collisions. The distribution is smoothed out for larger radii and larger $N_{\max}$.

We should note that including the effects of cascading lower-level particles would affect the distribution shape chiefly at larger N. This is because lower-level collisions cannot occur until inelastic primary reactions have occurred.

We would now like to make a few remarks about the limitations of this idea as we have presented it. We have stated that in order to apply our results it is necessary to stay within the coherent peak. For a large nucleus this peak may be very narrow. To the extent that transverse momentum is cut off at $\sim$ several hundred MeV at high energies, a substantial number of events may lie within this peak. However, the experimental difficulties may be very severe because the angle which the coherent peak subtends may force the experimenter to have to count particles which are essentially in the beam line.

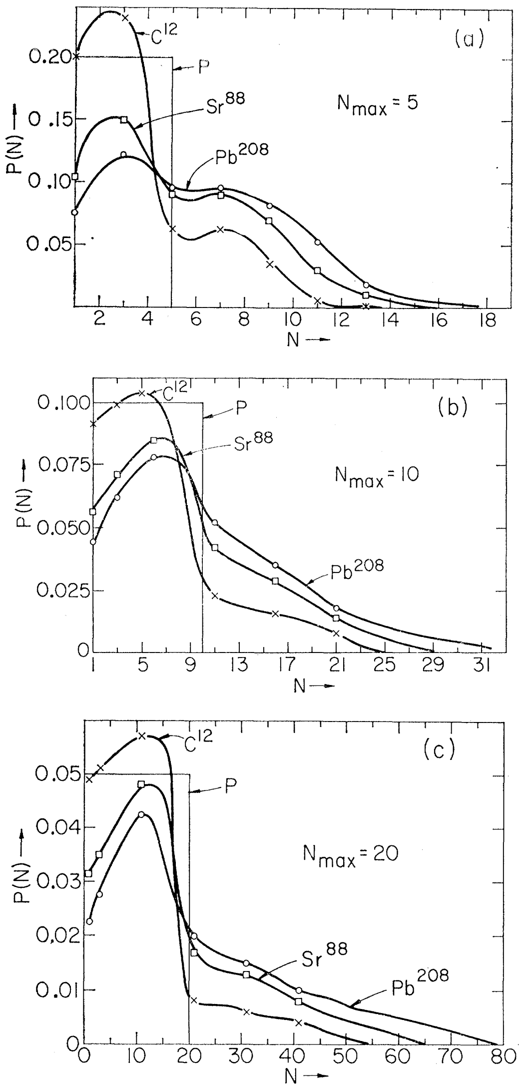

FIG. 3. Final particle distribution for three nuclei and three values of N_{max}. The square curve is the distribution of the fundamental multiparticle spectrum. These curves represent the effect of chaining the primary particle only.

In the sense that produced particles are further from the beam line, an experiment in the incoherent region is simpler to perform. A calculation of multiplicity in this region cannot be done by treating the nucleus as a dense medium. There is, however, a calculation which could be performed and which would be appropriate to this region. This is a Monte Carlo calculation in which the nucleus is treated as a collection of free nucleons. One could ask for the number of fast particles lying within a certain finite cone, given the nature of the primary reactions. In contrast to the remark made in Ref. 7, in such a Monte Carlo calculation the effect a fast particle would have on a slower particle following it through the nucleus would be considerable. This calculation, although somewhat more involved, is very similar to the old Monte Carlo shower calculations. There is a further practical reason why this Monte Carlo calculation should be performed. In the type of experiment we are talking about it will not be experimentally possible to decide on the final state of the nucleus. Furthermore, many of the outgoing final particles are neutral so that an estimate of the total transverse momentum of the final particles will not be easy to make with any accuracy, leading to uncertainty about whether one is within the coherent peak. Thus it would be more useful experimentally to have an estimate for the sum of coherent and incoherent cross section for fast outgoing particles within some finite forward cone.

Assuming again that we can stay within the coherent peak so that our model can be applied, there are cautionary remarks to be made, especially to the effect that our results for final multiplicity are overestimates. We assumed in our calculation of the average final multiplicity (Fig. 2) that particles at *every* possible level of the multiperipheral chain cascade. Since the energies of lower-level particles may be relatively low, this is an assumption that can be called into question. The dashed line in Fig. 2 shows the effect on average multiplicity—somewhat less striking—when only particles up to the third level on the multiperipheral chain are allowed to cascade.

We have made three further assumptions which should be examined further. The first is that the multiperipheral process in nuclear matter is the same as in vacuum. This is not an assumption which can be tested without recourse to nuclear physics, but it does not seem physically unreasonable. Secondly, we have assumed the primary retains sufficient energy to produce the same number of particles at each collision. It would be simple to modify our result to include decreasing multiplicity with each collision. Thirdly, the multiperipheral model we used is one which leads to a constant cross section. Recent data seem to indicate that σ_T may not be flat at high energies. If this is the case, the simple multiperipheral model we have used would have to be modified. Nevertheless, the general features we have discussed would remain, namely, nuclear-dependent sensitivity of the total number of final particles to the "inelasticity" of the primary reaction, and nuclear-dependent sensitivity of the shape of the final number distribution.

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