$$\begin{aligned}H^3(|\Delta S|=1)&=\frac{f^3\beta}{2\sqrt{6}}l(v_6+u_7)+(h\partial_\mu j_0^{\mu(+)})\\&=f^2\frac{\beta l}{2\sqrt{6}}u_7-f^2\frac{\beta\sqrt{3}a}{8}(j_0j_7^5+j_7^5j_0)\\&\qquad+(h\partial_\mu j_6^\mu)\,,\end{aligned}\qquad(23)$$

where the last terms—in brackets—do not affect the decay.

It is now possible to calculate various ratios for the amplitudes of CP-breaking interactions, from strong and weak interaction parameters.

It is important to notice that the addition of the H_0' counterterm (13) ensures that there is no $|\Delta S|=2$ interaction even in third order; such a term appears here only in fourth order, in agreement with the Okun-Pontecorvo argument.

As an immediate result, we also see that in third order, the $|\Delta I|=\frac{3}{2}$ amplitude is just 7% of the $|\Delta I|=\frac{1}{2}$ amplitude, so that the parameters $|\eta_{00}|$ and $|\eta_{+-}|$ should be equal to within $\sim$10%.

Finally we emphasize that the SNCVC assumption made in Ref. 1—namely, that the octet matrix elements of

$$\{j_ij_l\}_8,\ \{j_i^5j_l^5\}_8,\ \{j_0j_i\}\quad(i,l=1\cdots8)$$

are equal between states belonging to the lowest baryon octet—enabled us to obtain[1] the value $\sim\frac{1}{9}-\frac{1}{12}$ for $\sin\phi_V$. This explains our decision to retain this value, in spite of the fact that we used in this article a form of $H^{\rm NL}$ apparently somewhat different from the one used in Ref. 1, where $H^{\rm NL}\propto\{j_c,j_c^\dagger\}$ (j_c is the Cabibbo current).

Note that the only value depending on $\sin\phi_V$ which we get here, $A_{3/2}/A_{1/2}$, is in very good agreement with experiment.

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Vector-Meson Exchange Contributions to the $K_1\rightarrow\gamma\gamma$ Decay Rate*

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By assuming that (1) the decay $K_1\rightarrow\gamma\gamma$ is saturated by the 2π intermediate state, (2) the $K\pi\pi$ vertex is of the first-order weak interaction, and (3) the $\pi\pi\rightarrow\gamma\gamma$ process goes by exchanges of ρ, ω, and ϕ, the contributions to the decay rate of $K_1\rightarrow\gamma\gamma$ from the three exchanges are calculated. It is found that the contributions do not alter in any significant way the result of the perturbation calculation using the Born term for the $\pi\pi\rightarrow2\gamma$ reaction.

I. INTRODUCTION

At present there is no experimental information on the decay of the short-lived neutral kaon into two γ rays. In view of the possibility of observing effects due to CP violation in the decays $K\rightarrow2\gamma$, it is important to know the expected decay rate of $K_S^0\rightarrow2\gamma$. Previous calculations[1,2] have used the unitarity relation for $K_S^0\rightarrow2\gamma$ and assumed the two-pion state to be the dominant one. The decay rate for $K_S^0\rightarrow2\pi$ is known (we will in future write $K_1\rightarrow2\pi$ and assume CP conservation), so the model-dependent part of this calculation is the estimate of the $2\pi\rightarrow2\gamma$ amplitude, for which the Born approximation has been used. In this paper we improve this estimate by including the vector-meson exchanges. We find that they make a rather small correction to the results already published.[1,2]

In Sec. II we review briefly the results of the previous calculations and define our notation. Section III contains the details of the vector-meson exchange diagrams and our conclusions are given in Sec. IV.

II. PERTURBATION CALCULATION OF RATE OF $K_1\rightarrow\pi\pi\rightarrow\gamma\gamma$

In the decay mode $K_1\rightarrow\gamma\gamma$, the Lorentz-invariant matrix element $\mathfrak{M}$ that satisfies gauge invariance and Bose statistics can be written as[1]

$$\begin{aligned}\mathfrak{M}&=\tfrac{1}{2}\mathcal{H}(s)F_{\mu\nu}^{(1)}F^{\mu\nu(2)}\\&=\mathcal{H}(s)[(\epsilon_1\cdot\epsilon_2)(k_1\cdot k_2)-(\epsilon_1\cdot k_2)(\epsilon_2\cdot k_1)].\end{aligned}\qquad(1)$$

The superscripts refer to the two photons; the ϵ's and k's are their polarization and momentum four-vectors, respectively. $\mathcal{H}(s)$ in the rest frame of K_1 is just a function of the squared mass $s=M_K^2$ of the K_1 and is represented by the box in Fig. 1.

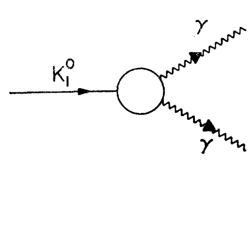

FIG. 1. Feynman diagram describing $K_1\rightarrow\gamma\gamma$.

* Work supported in part by U. S. AEC under Contract No. AT-(30-1)-3668B.

[1] V. Barger, Nuovo Cimento **32**, 128 (1964); H. Stern, *ibid.* **51A**, 197 (1967).

[2] B. R. Martin and E. de Rafael, Nucl. Phys. **B8**, 131 (1968).

By writing the unitarity relation for the amplitude $\mathfrak{M}$ and assuming CPT, it can be argued that the sum over the intermediate states is saturated by the 2π system only, within the order of approximation Ge^2. Hence for the imaginary part of the $K_1 \to 2\gamma$ amplitude we need models for the $K_1 \to 2\pi$ vertex and the $2\pi \to 2\gamma$ amplitude. For the $K\pi\pi$ vertex, we use the simple Lagrangian

$$\mathcal{L}_{\text{int}} = \lambda\psi_K \phi_\pi^\dagger \phi_\pi \tag{2}$$

and assume it goes by a first-order weak process. This means that we can calculate the coupling constant λ from the known decay rate of $K_1 \to \pi\pi$.[3]

For the $\pi\pi \to \gamma\gamma$ part, one usually employs the Lagrangian

$$\mathcal{L}_{\text{int}} = -ieA_\mu(x)[\phi_\pi(x)\overleftrightarrow{\partial}^\mu \phi_\pi^\dagger(x)] + e^2 A_\mu(x)A^\mu(x)\phi_\pi(x)\phi_\pi^\dagger(x)\,, \tag{3}$$

which holds only for charged pions, and then treats the process in Born approximation.[4] This ignores, however, any vector-meson contributions to the $\pi\pi \to 2\gamma$ vertex.

If at present we limit ourselves to the pion-exchange terms and call them $\mathfrak{M}_\pi$ (cf. Fig. 2) then, using the Feynman rules,[5] we have

$$\begin{aligned}\mathfrak{M}_\pi = i(\lambda_{K\pi^+\pi^-})\frac{e^2}{(2\pi)^4} &\times\left\{\int d^4k \frac{4(k\cdot\epsilon_1)(k-k_1)\cdot\epsilon_2}{(k^2-\mu^2)[(Q-k)^2-\mu^2][(k_1-k)^2-\mu^2]}\right.\\ &+\int d^4k \frac{4(k\cdot\epsilon_2)(k-k_2)\epsilon_1}{(k^2-\mu^2)[(Q-k)^2-\mu^2][(k_2-k)^2-\mu^2]}\\ &\left.-\int d^4k \frac{2\epsilon_1\cdot\epsilon_2}{(k^2-\mu^2)[(Q-k)^2-\mu^2]}\right\}\,,\end{aligned} \tag{4}$$

where Q is the four-momentum of K_1, M_K is the mass of K_1, μ is the mass of pion, k_i is the four-momentum of the ith photon, and ϵ_i is the polarization of the ith photon. The decay rate is then

$$d\Gamma_\pi = \frac{1}{2}\frac{1}{2M_K}|\mathfrak{M}|^2 \frac{d^3k_1}{2\omega_1(2\pi)^3}\frac{d^3k_2}{2\omega_2(2\pi)^3} \times\delta(Q-k_1-k_2)(2\pi)^4\,. \tag{5}$$

Now by using the Cutkosky rule, we can get the discontinuity[6] of $\mathfrak{M}$, which is just $2i\,\text{Im}\mathfrak{M}_\pi$. The result is

$$\text{Im}\mathfrak{M}_\pi(s) = 2\alpha(\lambda_{K\pi^+\pi^-})\frac{\mu^2}{s^2}\ln\left[\frac{1+\beta(s)}{1-\beta(s)}\right] \times[(\epsilon_1\cdot\epsilon_2)(k_1\cdot k_2)-(\epsilon_1\cdot k_2)(\epsilon_2\cdot k_1)]\,, \tag{6}$$

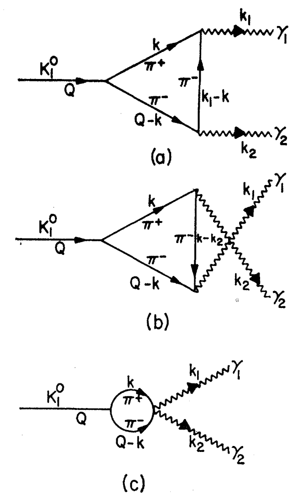

FIG. 2. Three Feynman diagrams corresponding to perturbation calculation of $K_1 \to \gamma\gamma$ decay. The seagull term is needed for gauge invariance.

where

$$\beta(s) = (1-4\mu^2/s)^{1/2}\,,\quad s = M_K^2\,,\quad \alpha = \text{the electromagnetic constant.} \tag{7}$$

Hence

$$\text{Im}\mathcal{H}_\pi(s) = 2\alpha(\lambda_{K\pi^+\pi^-})\frac{\mu^2}{s^2}\ln\left[\frac{1+\beta(s)}{1-\beta(s)}\right]\,, \tag{8}$$

where $\mathcal{H}_\pi(s)$ is just the coefficient of $[(\epsilon_1\cdot\epsilon_2)(k_1\cdot k_2) - (\epsilon_1\cdot k_2)(\epsilon_2\cdot k_1)]$ in Eq. (1).

Now to get the real part of $\mathfrak{M}_\pi$, $\text{Re}\mathfrak{M}_\pi$, we write a dispersion relation for $\text{Re}\mathcal{H}_\pi(s)$, since it is $\mathcal{H}_\pi(s)$ that we regard as form factor[7]:

$$\begin{aligned}\text{Re}\mathcal{H}_\pi(s) &= \frac{P}{\pi}\int_{4\mu^2}^\infty ds' \frac{\text{Im}\mathcal{H}_\pi(s')}{s'-s}\\ &= 2\alpha(\lambda_{K\pi^+\pi^-})\frac{\mu^2}{2\pi s}\times\left\{-\frac{1}{\mu^2}+\frac{1}{s}\left[\pi^2-\ln^2\frac{(1+\beta)}{(1-\beta)}\right]\right\}\,.\end{aligned} \tag{9}$$

The decay rate[8,9] is

$$\Gamma_\pi = (M_K^3/64\pi)|\mathcal{H}_\pi|^2\,, \tag{10}$$

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[3] A. Barbaro-Galtieri, S. Derenzo, L. R. Price, A. Rittenberg, A. Rosenfeld, N. Barash-Schmidt, C. Bricman, M. Roos, P. Söding, and C. G. Wohl, Rev. Mod. Phys. **42**, 87 (1970).

[4] See Ref. 2, p. 142, for comment on $\pi^0\pi^0 \to \gamma\gamma$ scattering.

[5] We use the conventions adopted by J. Bjorken and S. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1965), Appendix B, pp. 381–386.

[6] K. Nishijima, *Fields and Particles* (Benjamin, New York, 1969), pp. 351–359.

[7] For further comments on what part of the invariant matrix element $\mathfrak{M}$ should be considered as the form factor for which a dispersion relation for the real part can be written, see B. R. Martin, E. de Rafael, and J. Smith, Phys. Rev. D **2**, 179 (1970). Their H is related to $\mathcal{H}$ here by $H = \frac{1}{8}M_K\mathcal{H}$.

[8] For the decay rate of boson → boson+boson, the decay-rate expression can be obtained from J. D. Jackson, in *Brandeis University Summer Institute Lectures in Theoretical Physics* (Benjamin, New York, 1963), pp. 269–272.

[9] The values of the masses and constants used are obtained from Ref. 3, pp. 121–122.

and using values obtained from Ref. 3, we get the results

$$\left.\begin{aligned} \operatorname{Im}\mathcal{H}_\pi &= 22.1\times10^{-2} \\ \operatorname{Re}\mathcal{H}_\pi &= -12.5\times10^{-2} \end{aligned}\right\} \text{ in natural units} \tag{11a}$$

$$\Gamma_\pi = 2.3\times10^4/\text{sec}. \tag{11b}$$

The decay rate obtained can be compared with previous calculations; e.g., Martin, de Rafael, and Smith[10] got 2.2×10^4/sec.

III. CALCULATIONS FOR CONTRIBUTIONS OF ω, ρ, φ EXCHANGES

Using the gauge-invariant Lagrangian[11] for the decay $V\to\pi\gamma$,

$$\mathcal{L}(V\to\pi\gamma) = \frac{if_V}{m_V}\left[V^\mu \frac{\partial\phi^\dagger}{\partial X^\nu}F^{\alpha\beta}\right]\epsilon_{\mu\nu\alpha\beta} + \text{H.c.}$$

and the Lagrangian $\mathcal{L}=\lambda\phi^\dagger\phi$ for the $K\pi\pi$ vertex, the invariant matrix $\mathfrak{M}_V$ for the graphs in Fig. 3 is obtained:

$$\begin{aligned} \mathfrak{M}_V = \int \frac{d^3k}{(2\pi)^4}\, \frac{i}{(k^2-\mu^2)}\,\lambda\, \frac{i}{(Q-k)^2-\mu^2}\, \frac{-if_V}{m_V} \\ \times \epsilon_{\alpha\beta\gamma\delta}\epsilon^\alpha(V)(k-k_1)^\beta k_1{}^\gamma \epsilon_1{}^\delta \\ \times \frac{-ig_{\alpha\alpha'}}{(k_1-k)^2-m_V{}^2}\, \frac{-if_V}{m_V}\epsilon_{\alpha'\beta'\gamma'\delta'} \\ \times \epsilon^{\alpha'}(V)(k-k_1)^{\beta'}k_2{}^{\gamma'}\epsilon_2{}^{\delta'} + 1\leftrightarrow 2. \end{aligned} \tag{12}$$

Here V labels variables for the vector meson, and 1 and 2 refer to the two photons.

Whereas the logarithmic divergences of the first two graphs (a) and (b) of Fig. 2 were canceled by the seagull term (c) which has to be added for gauge invariance; here (a) and (b) of Fig. 3 are gauge-invariant already and the logarithmic divergences are not remedied. However, as we shall see later, $\operatorname{Im}\mathfrak{M}_V$ is finite and the divergence manifests itself in the $\operatorname{Re}\mathfrak{M}_V$. We will then introduce reasonable cutoff values to get a feeling of how the variation of $\operatorname{Re}\mathfrak{M}_V$ affects the decay rate.

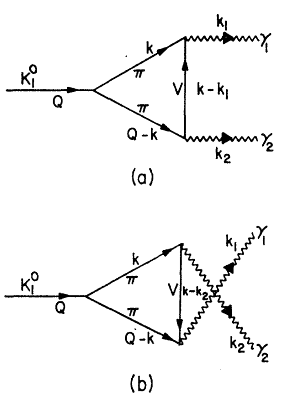

FIG. 3. Feynman diagrams for $K_1\to\gamma\gamma$ with vector-meson exchange in $\pi\pi\to\gamma\gamma$ scattering part. No seagull term is included, because the two graphs are already gauge invariant in the model assumed.

Again, using the Cutkosky rule to get the discontinuity of $\mathfrak{M}_V$, we obtain

$$\begin{aligned} \operatorname{Im}\mathfrak{M}_V = \frac{1}{16\pi}\lambda\left(\frac{f_V}{m_V}\right)^2\left[\frac{m_V{}^2}{s}\hat{V}-\beta\right] \\ \times[(\epsilon_1\cdot\epsilon_2)(k_1\cdot k_2)-(\epsilon_1\cdot k_2)(\epsilon_2\cdot k_1)], \end{aligned} \tag{13}$$

where

$$\beta = (1-4\mu^2/s)^{1/2}, \tag{13'}$$

$$\delta_V = 1+2\mu^2/(m_V{}^2-\mu^2), \tag{13''}$$

$$\hat{V} = \ln\left(\frac{1+\beta}{1-\beta}\,\frac{\delta_V-\beta}{\delta_V+\beta}\right), \tag{13'''}$$

and hence

$$\operatorname{Im}\mathcal{H}_V = \frac{1}{16\pi}\lambda\left(\frac{f_V}{m_V}\right)^2\left(\frac{m_V{}^2}{s}\hat{V}-\beta\right). \tag{14}$$

We use the dispersion relation again to get the $\operatorname{Re}\mathcal{H}_V$:

$$\begin{aligned} \operatorname{Re}\mathcal{H}_V(s) &= \frac{P}{\pi}\int_{4\mu^2}^\infty ds' \frac{\operatorname{Im}\mathcal{H}_V(s')}{s'-s} \\ &= \frac{1}{16\pi}\lambda\left(\frac{f_V}{m_V}\right)^2\frac{1}{\pi}[m_V{}^2P(\Omega')-P(\Omega'')], \end{aligned} \tag{15}$$

where P indicates principal value and the Ω's are integrals, i.e.,

$$\begin{aligned} \Omega'(s) &= \int_{4\mu^2}^\infty ds' \frac{\hat{V}(s')}{s'(s'-s)} \\ &= \frac{1-\beta^2}{4\mu^2}\left[-\tfrac{1}{2}\pi^2-\tfrac{1}{2}\ln^2\left(\frac{1+\beta}{1-\beta}\right)+\operatorname{Li}\left(\frac{\delta_V-\beta}{1-\beta}\right)\right. \\ &\quad -\operatorname{Li}\left(-\frac{\delta_V-\beta}{1+\beta}\right)+\operatorname{Li}\left(\frac{\delta_V+\beta}{1+\beta}\right) \\ &\quad \left.-\operatorname{Li}\left(-\frac{\delta_V+\beta}{1-\beta}\right)\right], \end{aligned} \tag{16}$$

$$\begin{aligned} \Omega''(s,\Lambda^2) &= \int_{4\mu^2}^{\Lambda^2} ds' \frac{\beta(s')}{(s'-s)} \\ &= \frac{1}{2}\left[(1-\beta)\ln\left(\frac{1+\beta''}{1-\beta''}\,\frac{\beta''+\beta}{\beta''-\beta}\right)\right. \\ &\quad \left.+(1+\beta)\ln\left(\frac{1+\beta''}{1-\beta''}\,\frac{\beta''-\beta}{\beta''+\beta}\right)\right], \end{aligned} \tag{17}$$

$$\beta(s') = (1-4\mu^2/s')^{1/2},\quad \beta(s) = (1-4\mu^2/s)^{1/2},\quad \beta''(\Lambda^2) = (1-4\mu^2/\Lambda^2)^{1/2}. \tag{18}$$

[10] In Ref. 6 the decay rate for $K_1\to2\gamma$ contains an error. See Ref. 7.

[11] C. H. Chan, Proc. Phys. Soc. (London) **80**, 39 (1962); M. Gourdin, *Unitary Symmetries* (North-Holland, Amsterdam, 1967), pp. 99–100.

The Li in Eq. (16) denotes the dilogarithmic function.[12]

The logarithmic divergence we mentioned before appears in $\Omega''(s,\Lambda^2)$ as $\Lambda\to\infty$. Two cutoff values $\Lambda=5\mu$ and $\Lambda=10\mu$ are then introduced to get finite $\mathrm{Re}\mathcal{H}_V(s)$. The reason why higher cutoff values were not used is that doing so would mean other exchange particles of higher and higher masses have to be considered. Intuitively, we need only values of s that are commensurate with energies of the particles involved.

Before we can determine numerical rates, we have to determine values for λ and f_V for each case of ω, ρ, and ϕ exchange.

We need two λ's, λ_0 and $\lambda_\pm$, corresponding to $K_1\to\pi^0\pi^0$ and $K_1\to\pi^+\pi^-$. These can easily be determined with data from Ref. 3. With regard to the f_V's, things are less definite. We have data on direct measurement of the rate $\Gamma(\omega\to\pi^0\gamma)$,[13] but for $\rho^\pm$ and ϕ, we only have data on upper limits of $\Gamma(\rho^\pm\to\pi^\pm\gamma)$[13] and $\Gamma(\phi\to\pi^0\gamma)$[14]:

$$\begin{aligned}\Gamma(\omega^0\to\pi^0\gamma)&=9.4\%\ \Gamma(\omega^0\to\text{all})\\&=1.19\ \text{MeV}/\hbar,\\ \Gamma(\rho^\pm\to\pi^\pm\gamma)&<0.2\%\ \Gamma(\rho^\pm\to\text{all})\\&<0.250\ \text{MeV}/\hbar,\\ \Gamma(\phi\to\pi^0\gamma)&<0.35\%\ \Gamma(\phi\to\text{all})\\&<0.0136\ \text{MeV}/\hbar.\end{aligned}\tag{19}$$

Using the formula for the decay rate of a vector boson of mass m_V into two spinless bosons of mass μ

$$\Gamma=\frac{f_V^2}{24\pi}m_V\left(1-\frac{\mu^2}{m_V^2}\right)^3,\tag{20}$$

we obtain the results in Table I.[15]

The results of the calculations for the $\mathrm{Im}\mathcal{H}_V$, $\mathrm{Re}\mathcal{H}_V(1)$, and $\mathrm{Re}\mathcal{H}_V(2)$, where 1, 2 refer to cutoff values $\Lambda=5\mu$ and $\Lambda=10\mu$, respectively, are tabulated in Table II.

Summing the imaginary parts and real parts in Table II, adding them to the $\mathrm{Im}\mathcal{H}_\pi$ and $\mathrm{Re}\mathcal{H}_\pi$ and indicating the respective quantities as $\mathrm{Im}\mathcal{H}_T$, $\mathrm{Re}\mathcal{H}_T(1)$, and $\mathrm{Re}\mathcal{H}_T(2)$, we get the following results (in natural units):

$$\mathrm{Im}\mathcal{H}_T=22.5\times10^{-1},$$
$$\mathrm{Re}\mathcal{H}_T(1)=-8.4\times10^{-2},$$
$$\mathrm{Re}\mathcal{H}_T(2)=-12.95\times10^{-2},$$

with corresponding decay rates

$$\Gamma_T(1)=2.10\times10^4/\text{sec}=10.3\%\ \Gamma_\pi,$$
$$\Gamma_T(2)=2.45\times10^4/\text{sec}=\ 4.75\%\ \Gamma_\pi.$$

TABLE I. Values of f_V^2 and α_V for the ω, ρ, and ϕ mesons (in natural units). $\lambda_0=1.35\times10^7$; $\lambda_\pm=2.02\times10^7$.

	f_V^2	$\alpha_V=(1/32\pi)f_V^2$
ω^0	1.26×10^{-1}	$1.25\times10^{-3}=1.7\times10^{-1}\alpha_{\text{e.m.}}$
ρ	2.72×10^{-2}	$2.71\times10^{-4}=3.7\times10^{-2}\alpha_{\text{e.m.}}$
ϕ	1.06×10^{-3}	$1.06\times10^{-5}=1.4\times10^{-3}\alpha_{\text{e.m.}}$

TABLE II. Values of $\mathrm{Im}\mathcal{H}_V$, $\mathrm{Re}\mathcal{H}_V(1)$, and $\mathrm{Re}\mathcal{H}_V(2)$ for the ω, ρ, and ϕ exchanges (in natural units).

	$\mathrm{Im}\mathcal{H}_V$	$\mathrm{Re}\mathcal{H}_V(1)$	$\mathrm{Re}\mathcal{H}_V(2)$
ω^0	-6.51×10^{-3}	3.13×10^{-2}	-2.52×10^{-3}
ρ	1.10×10^{-2}	9.53×10^{-3}	-1.57×10^{-3}
ϕ	-2.08×10^{-5}	2.05×10^{-4}	3.59×10^{-5}

IV. ANALYSIS AND CONCLUSION

From the above results, we note the following:

The ρ contributes most in increasing the imaginary part of $\mathcal{H}_\pi$, the ω contributes most in decreasing the real part of $\mathcal{H}_\pi$, and the ϕ contributes the least in both the real and imaginary parts of $\mathcal{H}_\pi$, as expected even in the $SU(3)$ limit. All contributions are small compared to the pion-exchange term.

However, we bear in mind that for ρ and ϕ we used upper limits for f_ρ^2 and f_ϕ^2. It may so happen that the contributions of ρ and ω to the imaginary part of $\mathcal{H}_\pi$ cancel each other but still the ω would contribute most to the real part and hence be dominant in altering the decay rate due to π above.

We note also that in the $SU(3)$ limit (without mixing), the amplitude with ω exchange is expected to be three times that of the ρ exchange amplitude. This result does not follow from our calculations because we use the experimental upper limit for f_ρ^2. Inclusion of mixing angle will not alter the order of magnitude of our results.

Hence the three vector-meson exchanges do not alter very much the decay rate calculated using π exchange alone as long as we use reasonable cutoff mass values for the real parts of the invariant amplitudes.

ACKNOWLEDGMENT

The author would like to thank Dr. Jack Smith for most helpful discussions.

[12] The dilogarithm function used here is defined by L. Lewin, *Dilogarithms and Associated Functions* (McDonald, London, 1958), i.e.,

$$\mathrm{Li}(x)=-\int_0^x\frac{\ln(1-t)}{t}dt.$$

[13] See Ref. 3, p. 122.

[14] C. Bemporad, P. O. Braccini, R. Castaloi, L. Foa, K. Ludels-Meyer, and D. Schmitz, Phys. Letters **29B**, 383 (1969); see also Ref. 3.

[15] $\alpha_V=(1/32\pi)f_V^2$ can be considered as the structure constant corresponding to $\alpha_{\text{e.m.}}=e^2/4\pi$. This can be seen from Eqs. (8) and (15).