Decay Widths of the Boson Resonances*

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Using the method due to Carruthers to express the decay rates of baryon resonances, formulas are derived to express the decay rates of the boson resonances with arbitrary spins which decay into a pion and a boson with spin 0, 1, or 2.

A DECOMPOSITION of the high-spin spinor into a summation over the products of the Clebsch-Gordan coefficients, the polarization vector for spin 1, and the lower-spin spinor have been proposed by Carruthers.[1] By analogy to this decomposition, we can express the polarization tensor for spin J as a summation over the products of the Clebsch-Gordan coefficients, the polarization vector for spin 1, and the polarization tensor for spin J-1. The expression is

$$\epsilon_{\mu_1\mu_2\cdots\mu_J}(p,\lambda_J) = \sum_{\lambda_1\lambda_{J-1}} C(1, J-1, J; \lambda_1, \lambda_{J-1}, \lambda_J)\epsilon_{\mu_1}(p,\lambda_1) \times\epsilon_{\mu_2\cdots\mu_J}(p,\lambda_{J-1}), \quad (1)$$

where the λ's are the helicities. We can prove by using mathematical induction that the polarization tensor given by (1) is totally symmetric in the μ's and traceless with respect to the pair of the μ's. It is easy to show that it is transverse to the momentum p in the sense that $p_{\mu_1}\epsilon_{\mu_1\cdots\mu_J}(p,\lambda_J)=0$. The normalization condition is

$$\sum_{\text{all } \mu\text{'s}} \bar{\epsilon}_{\mu_1\cdots\mu_J}(p,\lambda_J)\epsilon_{\mu_1\cdots\mu_J}(p,\lambda'_J)=\delta_{\lambda_J\lambda_J'} \quad (2)$$

with $\bar{\epsilon}_{\mu_1\cdots\mu_J}(p,\lambda_J)=\pm\epsilon_{\mu_1\cdots\mu_J}{}^*(p,\lambda_J)$, where the double sign assumes + or − depending upon whether

$$\sum_{i=1}^{J} \delta_{\mu_i 4}$$

is even or odd, respectively. By repeating the above decomposition (1), we can express the polarization tensor for spin J as the summation over the products of the Clebsch-Gordan coefficients, and the polarization vectors all with spin 1. It is then straightforward to derive the expressions for the decay rates of the boson resonances.

The above expressions for the decay widths assume simple formulas depending upon whether the normality of the boson resonances changes or not: The normality is +1 or −1, according as the spin and parity of the bosons are either $J^P=0^+,1^-,2^+,\ldots$ or $J^P=0^-,1^+,2^-,\ldots$.

For the decays into the bosons with spin 0, the decays are forbidden when the normality does not change because of the parity conservation. When the normality changes, the decay matrix element can be expressed as

$$(4q_0p_0)^{1/2}\langle 0(q)|j_\pi(0)|J(p)\rangle = q_{\mu_1}\cdots q_{\mu_J}\epsilon_{\mu_1\cdots\mu_J}(p,\lambda_J)F \quad (3)$$

and the decay rate is given, in terms of the c.m. momentum q, by

$$\Gamma(J\to\pi+0)=\frac{J!}{(2J+1)!!}\frac{q^{2J+1}}{8\pi m_J{}^2}|F|^2. \quad (4)$$

For the decays into the bosons with spin 1, the decay matrix element and the decay rate are given, when the normality changes, by

$$(4q_0p_0)^{1/2}\langle 1(q)|j_\pi(0)|J(p)\rangle = \bar{\epsilon}_\mu(q,\lambda_1)(\delta_{\mu\nu_1}F+p_\mu q_{\nu_1}G)q_{\nu_2}\cdots q_{\nu_J}\epsilon_{\nu_1\nu_2\cdots\nu_J}(p,\lambda_J), \quad (5)$$

$$\Gamma(J\to\pi+1) = \frac{q^{2J-1}}{8\pi m_J{}^2}\frac{(J-1)!}{(2J+1)!!}\left((J+1)|F|^2 + J\left|\frac{q_0}{m_1}F-\frac{q^2m_J}{m_1}G\right|^2\right), \quad (6)$$

where m_1 is the mass of the boson with spin 1. When the normality does not change, the corresponding expressions are

$$(4q_0p_0)^{1/2}\langle 1(q)|j_\pi(0)|J(p)\rangle = \bar{\epsilon}_\mu(q,\lambda_1)e_{\mu\nu\tau\mu_1}q_\nu p_\tau q_{\mu_2}\cdots q_{\mu_J}\epsilon_{\mu_1\mu_2\cdots\mu_J}(p,\lambda_J)F, \quad (7)$$

$$\Gamma(J\to\pi+1)=\frac{q^{2J+1}}{8\pi}\frac{(J-1)!(J+1)}{(2J+1)!!}|F|^2. \quad (8)$$

Both expressions (6) and (8) refer to the c.m. system.

For the decays into the bosons with spin 2, the decay matrix element and the decay rate are given, when the normality changes, by

$$(4q_0p_0)^{1/2}\langle 2(q)|j_\pi(0)|J(p)\rangle = \bar{\epsilon}_{\mu_1\mu_2}(q,\lambda_2)(\delta_{\mu_1\nu_1}\delta_{\mu_2\nu_2}F+\delta_{\mu_1\nu_1}p_{\mu_2}q_{\nu_2}G+p_{\mu_1}p_{\mu_2}q_{\nu_1}q_{\nu_2}H) \times q_{\nu_3}\cdots q_{\nu_J}\epsilon_{\nu_1\nu_2\nu_3\cdots\nu_J}(p,\lambda_J), \quad (9)$$

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[1] P. Carruthers, Phys. Rev. **152**, 1345 (1966); **169**, 1398(E) (1968).

$$\Gamma(J\to\pi+2)=\frac{q^{2J-3}}{8\pi m_J^2}\frac{(J-2)!}{(2J+1)!!}\left[\frac{(J+2)(J+1)}{2}|F|^2+2(J^2-1)\left|\frac{q_0}{m_2}F-\frac{q^2m_J}{2m_2}G\right|^2+\frac{2J(J-1)}{3}\left|\left(\frac{1}{2}+\frac{q_0^2}{m_2^2}\right)F-\frac{q_0q^2m_J}{m_2^2}G+\frac{q^4m_J^2}{m_2^2}H\right|^2\right],\quad(10)$$

in terms of the c.m. variables and m_2, the mass of the boson with spin 2.

The above derivations can easily be extended to the decays into the bosons with spin higher than 2, as long as the normality of the boson resonances changes. If the normality does not change, however, no analytic expressions can be derived for the decay rates even when the final boson has spin 2. This is because of the appearance of the totally antisymmetric tensor in the decay matrix element. We remark that such a mathematical difficulty does not arise in the case of the baryon resonances since we do not have to use the totally antisymmetric tensor.

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Errata

On-Shell Current Algebra and the Radiative Leptonic Decay of K^+, N. J. Carron and R. L. Schult [Phys. Rev. D **1**, 3171 (1970)]. Following Eq. (7.1), the values quoted for $|F_4|$ from Refs. 29 and 30 should be divided by $\sin\theta\approx0.22$. Thus, from Ref. 29, $|F_4|$ should be 5.64, not 1.24. From Ref. 30, $4.36\lesssim|F_4|\lesssim6.23$, not as given. All calculations of $|F_4|$ are therefore in rough agreement and lie between 4 and 7.

K^-p Amplitudes near the $\Lambda(1520)$, D. Berley, P. Yamin, R. Kofler, A. Mann, G. Meisner, S. Yamamoto, J. Thompson, and W. Willis [Phys. Rev. D **1**, 1996 (1970)]. Several significant numerical errors occur; correct values are given below.

Table II. Neutral-channel Legendre coefficients, $K^+ + p$ interactions near 400 MeV/c.

P_{K^-} (MeV/c)	$\sigma(\Lambda\pi^0)$ (mb)	$\sigma(\Sigma^0\pi^0)$ (mb)	$\sigma(\bar{K}^0n)$ (mb)
350	4.5	5.0	6.4
370	4.6	6.5	7.7
390	3.5	7.1	7.3
390	4.5	9.1	11.0
410	3.7	6.8	8.2
430	3.2	5.6	6.3

Table VI. Parameters of constant K-matrix fit.

				$P_{3/2}$
Isospin 0	$K_{\Sigma\Sigma}{}^0$			0.01
		S	$P_{1/2}$	$P_{3/2}$
Isospin 1	$K_{KK}{}^1$		−0.092	0.041
	$K_{K\Sigma}{}^1$	−0.66	−0.087	−0.027
	$K_{K\Lambda}{}^1$	−0.39	−0.045	−0.044
	$K_{\Sigma\Sigma}{}^1$		−1.02	−0.094
	$K_{\Lambda\Sigma}{}^1$		0.39	−0.089
	$K_{\Lambda\Lambda}{}^1$		0.04	0.045