It is this asymptotic behavior, which is nicely decreasing for Ren positive, that one takes advantage of when defining the Lorentz Laplace transform of a function of a longitudinal boost angle θ. We always have in mind the connection of an invariant subenergy s with $\cosh\theta$. A behavior like s^α leads to $e^{\alpha\theta}$, so suppose $g(\theta)$ is bounded by $e^{\alpha\theta}$. We then define the partial amplitude which is analytic in Re$n>$ Re$\alpha-|m-j_0|$:

$$g_{l_1l_2}{}^{j_0n}=\sum_m\int_0^\infty(\sinh\theta)^2d\theta\,\frac{g_{l_1ml_2}(\theta)a_{ml_1l_2}{}^{j_0,n+1}(\theta)}{B(j_0,n;l_1,m,l_2)}\,,\qquad\text{(A29)}$$

where the usual measure $(\sinh\theta)^2d\theta$ on the $SO(1,3)$ hyperboloid has entered, and B is a combinatorial coefficient which is specified shortly. By using the asymptotic behavior of the $a^{j_0,n+1}$ functions, and the connection between them and the usual representation functions $d^{j_0,n}$ on $SL(2,C)$,[11] one may show that if we define $g_{l_1l_2}{}^{j_0n}$ via (A29), we can recover $g(\theta)$ for positive θ by

$$g_{l_1ml_2}(\theta)=\int_{c-i\infty}^{c+i\infty}\frac{dn}{2\pi i}[j_0^2-(n+1)^2]d_{l_1ml_2}{}^{j_0,n}(\theta)g_{l_1l_2}{}^{j_0n}\qquad\text{(A30)}$$

when we take for $m\geqslant j_0$:

$$B(j_0,n,l_1,m,l_2)=\frac{j_0^2-(n+1)^2}{4(m-j_0)!(m+j_0)!}(2l_1+1)(2l_2+1)\times\frac{(l_1+j_0)!(l_2+j_0)!}{(l_1-j_0)!(l_2-j_0)!}\times\frac{[n+2-j_0]_{l_2+j_0}[-n-2m+j_0]_{l_1-j_0}}{[-n-1-m]_{l_2+m+1}[n+1+m]_{l_1+m+1}}\,,\qquad\text{(A31)}$$

with

$$[a]_q=a(a+1),\ldots,(a+q-\tfrac{1}{2})\,.\qquad\text{(A32)}$$

In the text we defined $e_{l_1ml_2}{}^{j_0n}(\theta)$ as

$$e_{l_1ml_2}{}^{j_0n}(\theta)=a_{ml_1l_2}{}^{j_0,n+1}(\theta)/B(j_0,n,l_1,m,l_2)\,,\qquad\text{(A33)}$$

so our transform pair is (46) and (47).

PHYSICAL REVIEW D VOLUME 3, NUMBER 9 1 MAY 1971

Study of Chiral $SU(2)\times SU(2)$ Current-Algebra Models*

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(Received 10 December 1970)

The smoothness assumptions of the chiral $SU(2)\times SU(2)$ hard-pion current-algebra method are examined in detail. A new model, the current-smoothness model, emerges as a plausible alternative to the standard hard-pion model of Schnitzer and Weinberg, and others. The two models give satisfactory predictions for the decay $A_1\to\rho+\pi$, but quite different predictions for the decay $A_1\to\pi+\gamma$ and the colliding-beam reaction $e^+e^-\to A_1{}^\pm\pi^\mp$. Other possible models are also discussed.

I. INTRODUCTION

The hard-pion current-algebra method,[1,2] which consists of the chiral $SU(2)\times SU(2)$ current commutation relations proposed by Gell-Mann,[3] conservation of the vector current (CVC), partial conservation of the axial current (PCAC), together with certain "smoothness" assumptions, provides a useful phenomenological tool for the analysis and correlation of various strong, electromagnetic, and weak processes. In particular, it leads to relations between the pion electromagnetic form factor, A_1-meson decays,[1,2] pion-pion scattering,[4,5] and pion-nucleon scattering.[6] Extension to the $SU(3)\times SU(3)$ current algebra, with nonconservation of the strangeness-changing vector current appropriately taken into account, leads to further relations between these processes, K-π scattering,[7] and the form factors of the K_{l3} decay,[8–10] although these relations are not entirely in agreement with experiment. The method can also be applied, with rather more

* Supported in part by the National Science Foundation under Grant No. GP-12396.

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theoretical ambiguity, to the electromagnetic decays of ω, φ, π^0, and η mesons.[11,12]

The present work evolved from an attempt to extend the method to situations in which the zero-width (pole-dominance) approximation to the particle and current propagators is inappropriate[13] (such as in the colliding-beam reaction $e^+e^- \rightarrow \pi^+\pi^-$, discussed by several authors,[14] or the physical decay $A_1 \rightarrow 3\pi$). In the course of this attempt, it was found that the original smoothness assumptions are in fact not unique—that by a slight modification of the point of view of Schnitzer and Weinberg, a wide variety of models with equal *a priori* plausibility could be constructed. To select one or another of these models required an appeal to some higher principle, for example, (1) field-current identities,[15] (2) explicit Lagrangians,[2,16] (3) complete single-particle dominance,[1] (4) high-energy limiting behavior.[17,18] These principles are not necessarily wrong; indeed, they have intuitive appeal to many physicists. But it seemed worthwhile to give an account of the more general class of models, and to discuss experimental tests of the various models.

Section II contains a review of the current commutation relations, CVC, PCAC, and the Ward identities relevant to the exposition of the models, and introduces the propagators and the A-A-V vertex functions (the T products of two axial-vector currents and one vector current). Section III contains the decomposition of the vertex functions into invariant amplitudes, and the implications of the Ward identities for the invariant amplitudes. Section IV contains a decomposition of the matrix elements of the current operators between single-particle states; the large-momentum-transfer behavior of these matrix elements is of some interest. Section V contains the crucial discussion of the smoothness assumptions; it is here that the general models are introduced. Section VI contains predictions of the models for various processes related to the A-A-V vertex function—the decays $A_1 \rightarrow \rho\pi$, $A_1 \rightarrow \pi\gamma$, and the colliding-beam reaction $e^+e^- \rightarrow A_1^{\pm}\pi^{\mp}$, using analysis[19] of the $e^+e^- \rightarrow \pi^+\pi^-$ data as input.

From this emerges a second model, the current-smoothness model, which assumes maximum smoothness of the T product of three currents, *without* explicit decomposition of the axial-vector current into spin-0 and spin-1 parts. It does not satisfy any of the higher principles listed above (except in a certain limit which is not consistent with present experimental data), but it gives reasonable predictions for the decay $A_1 \rightarrow \rho\pi$ (as does the standard hard-pion model). The predictions of the model for the decay $A_1 \rightarrow \pi\gamma$ and the colliding-beam reaction are quite different from those of the standard hard-pion model, and can lead to an experimental choice between the models.

II. BASIC PRINCIPLES

A. Commutation Relations

The vector currents $V_\mu{}^a(x)$ and axial-vector currents $A_\mu{}^a(x)$ ($a=1, 2, 3$) are assumed to satisfy the standard $SU(2)\times SU(2)$ commutation relations[3]

$$\begin{aligned}\delta(x_0-y_0)[V_0{}^a(x),V_\mu{}^b(y)] &= i\epsilon^{abc}V_\mu{}^c(x)\delta(x-y)+\mathrm{ST}\\ &=\delta(x_0-y_0)[A_0{}^a(x),A_\mu{}^b(y)], \quad (2.1\mathrm{a})\end{aligned}$$

with the usual c-number Schwinger terms (ST), and

$$\begin{aligned}\delta(x_0-y_0)[V_0{}^a(x),A_\mu{}^b(y)] &= i\epsilon^{abc}A_\mu{}^c(x)\delta(x-y)\\ &=\delta(x_0-y_0)[A_0{}^a(x),V_\mu{}^b(y)] \quad (2.1\mathrm{b})\end{aligned}$$

without Schwinger terms. Furthermore,

$$\delta(x_0-y_0)[V_0{}^a(x),\partial_\mu A_\mu{}^b(y)] = i\epsilon^{abc}\partial_\mu A_\mu{}^c(x)\delta(x-y). \quad (2.2)$$

B. Two-Point Functions

The basic two-point functions are

$$\Delta_{\mu\nu}(q)\delta_{ab}=\int[i\langle 0|T(V_\mu{}^a(x)V_\nu{}^b(y))|0\rangle - C_V g_{\mu 0}g_{\nu 0}\delta(x-y)\delta_{ab}]e^{iq\cdot(x-y)}d^4x, \quad (2.3)$$

$$\Delta_{\mu\nu}{}^A(q)\delta_{ab}=\int[i\langle 0|T(A_\mu{}^a(x)A_\nu{}^b(y))|0\rangle - C_A g_{\mu 0}g_{\nu 0}\delta(x-y)\delta_{ab}]e^{iq\cdot(x-y)}d^4x. \quad (2.4)$$

We have

$$\Delta_{\mu\nu}(q)=F(q^2)g_{\mu\nu}-G(q^2)q_\mu q_\nu, \quad (2.5)$$

$$\Delta_{\mu\nu}{}^A(q)=F_A(q^2)g_{\mu\nu}-G_A(q^2)q_\mu q_\nu, \quad (2.6)$$

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[18] H. J. Schnitzer and M. Wise [Ann. Phys. (N. Y.) **59**, 129 (1970)] examine the Bjorken limit of the vertex functions.

[19] M. T. Vaughn, Ref. 14.

with the standard spectral representations

$$F(q^2)=\int \frac{\sigma(\kappa)}{q^2-\kappa}d\kappa\,, \tag{2.7}$$

$$G(q^2)=\int \frac{\sigma(\kappa)}{\kappa(q^2-\kappa)}d\kappa\,, \tag{2.8}$$

$$F_A(q^2)=\int \frac{\sigma_A(\kappa)}{q^2-\kappa}d\kappa\,, \tag{2.9}$$

$$G_A(q^2)=\int\left[\frac{\sigma_A(\kappa)}{\kappa}+\sigma_P(\kappa)\right]\frac{1}{q^2-\kappa}d\kappa\,, \tag{2.10}$$

with positive definite spectral functions $\sigma(\kappa)$, $\sigma_A(\kappa)$, and $\sigma_P(\kappa)$.

The Schwinger constants C_V, C_A are then given by

$$C_V=\int \frac{\sigma(\kappa)}{\kappa}d\kappa\,, \tag{2.11}$$

$$C_A=\int\left[\frac{\sigma_A(\kappa)}{\kappa}+\sigma_P(\kappa)\right]d\kappa \tag{2.12}$$

and, according to the first Weinberg sum rule,[20]

$$C_V=C_A\equiv C\,. \tag{2.13}$$

Note that

$$q_\mu\Delta_{\mu\nu}(q)=[F(q^2)-q^2G(q^2)]q_\nu = -C_Vq_\nu \tag{2.14}$$

and

$$q_\mu\Delta_{\mu\nu}{}^A(q)=[F_A(q^2)-q^2G_A(q^2)]q_\nu = -[C_A+\bar\Delta(q^2)]q_\nu\,, \tag{2.15}$$

where

$$\bar\Delta(q^2)=\int \frac{\kappa\sigma_P(\kappa)}{q^2-\kappa}d\kappa\,. \tag{2.16}$$

Also required are the two-point functions

$$i\langle 0|T(\partial_\mu A_\mu{}^a(x)A_\nu{}^b(y))|0\rangle =\left[\frac{i}{(2\pi)^4}\int q_\nu\bar\Delta(q^2)e^{-iq\cdot(x-y)}d^4q\right]\delta_{ab} \tag{2.17}$$

and

$$\langle 0|T(\partial_\mu A_\mu{}^a(x)\partial_\nu A_\nu{}^b(y))|0\rangle =\left[\frac{i}{(2\pi)^4}\int \Delta_0(q^2)e^{-iq\cdot(x-y)}d^4q\right]\delta_{ab}\,, \tag{2.18}$$

where

$$\Delta_0(q^2)=\int \frac{\kappa^2\sigma_P(\kappa)}{q^2-\kappa}d\kappa\equiv (F_\pi m_\pi{}^2)^2\Delta_\pi(q^2)\,. \tag{2.19}$$

Here the PCAC condition

$$\partial_\mu A_\mu{}^a(x)=F_\pi m_\pi{}^2\varphi_\pi{}^a(x) \tag{2.20}$$

defines the interpolating field $\varphi_\pi{}^a(x)$ for the pion; thus

$$\langle 0|T(\varphi_\pi{}^a(x)\varphi_\pi{}^b(y))|0\rangle =\left[\frac{i}{(2\pi)^4}\int \Delta_\pi(q^2)e^{-iq\cdot(x-y)}d^4q\right]\delta_{ab}\,. \tag{2.21}$$

The residues at the poles of the propagators (in the narrow-width approximation for ρ and A_1) are determined from the single-particle matrix elements

$$\langle 0|V_\mu{}^a(0)|\rho^b(k)\rangle=g_\rho N_\rho\rho_\mu{}^\rho(k)\delta_{ab}\,, \tag{2.22}$$

$$\langle 0|A_\mu{}^a(0)|A_1{}^b(q)\rangle=g_AN_A\rho_\mu{}^A(q)\delta_{ab}\,, \tag{2.23}$$

$$\langle 0|A_\mu{}^a(0)|\pi^b(p)\rangle=iF_\pi N_\pi p_\mu\delta_{ab}\,, \tag{2.24}$$

where $\rho^\rho(k)$ and $\rho^A(q)$ are the polarization vectors for the ρ and A_1, respectively, N_ρ, N_A, and N_π are the usual covariant normalization factors, and the constants g_ρ, g_A, and F_π are *defined* by these equations.

Then, in the pole-dominance limit, Eqs. (2.5) and (2.6) are, respectively,

$$\Delta_{\mu\nu}(q)=\frac{g_\rho{}^2}{q^2-m_\rho{}^2}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{m_\rho{}^2}\right), \tag{2.5$'$}$$

$$\Delta_{\mu\nu}{}^A(q)=\frac{g_A{}^2}{q^2-m_A{}^2}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{m_A{}^2}\right)-\frac{F_\pi{}^2}{q^2-m_\pi{}^2}q_\mu q_\nu\,. \tag{2.6$'$}$$

C. Three-Point Functions

The basic three-point functions are defined by

$$\begin{aligned}M_{\mu\nu\lambda}{}^{abc}(p,q) &\equiv -i\int\langle 0|T(A_\mu{}^a(x)A_\nu{}^b(y)V_\lambda{}^c(0))|0\rangle \times e^{ip\cdot x}e^{iq\cdot y}d^4xd^4y\\ &\equiv \epsilon^{abc}\Delta_{\mu\rho}{}^A(p)\Delta_{\nu\sigma}{}^A(q)\Delta_{\lambda\tau}(p+q)\Gamma_{\rho\sigma\tau}(p,q)\,,\end{aligned} \tag{2.25}$$

$$\begin{aligned}M_{\pi\nu\lambda}{}^{abc}(p,q) &\equiv \int\langle 0|T(\varphi_\pi{}^a(x)A_\nu{}^b(y)V_\lambda{}^c(0))|0\rangle e^{ip\cdot x}e^{iq\cdot y}d^4xd^4y\\ &\equiv \epsilon^{abc}\Delta_\pi(p^2)\Delta_{\nu\sigma}{}^A(q)\Delta_{\lambda\tau}(p+q)\Gamma_{\pi\sigma\tau}(p,q)\,,\end{aligned} \tag{2.26}$$

$$\begin{aligned}M_{\pi\pi\lambda}{}^{abc}(p,q) &\equiv i\int\langle 0|T(\varphi_\pi{}^a(x)\varphi_\pi{}^b(y)V_\lambda{}^c(0))|0\rangle e^{ip\cdot x}e^{iq\cdot y}d^4xd^4y\\ &\equiv \epsilon^{abc}\Delta_\pi(p^2)\Delta_\pi(q^2)\Delta_{\lambda\tau}(p+q)\Gamma_{\pi\pi\tau}(p,q)\,,\end{aligned} \tag{2.27}$$

with the symmetry properties

$$\Gamma_{\rho\sigma\tau}(p,q)=-\Gamma_{\sigma\rho\tau}(q,p)\,, \tag{2.28}$$

$$\Gamma_{\pi\pi\tau}(p,q)=-\Gamma_{\pi\pi\tau}(q,p) \tag{2.29}$$

required by Bose statistics.

[20] S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

D. Vector Ward Identities

As a consequence of the CVC condition

$$\partial_\lambda V_\lambda^c(z)=0, \tag{2.30}$$

the three-point functions satisfy the vector Ward identities

$$(p+q)_\lambda M_{\mu\nu\lambda}{}^{abc}(p,q)=\epsilon^{abc}[\Delta_{\mu\nu}{}^A(p)-\Delta_{\mu\nu}{}^A(q)], \tag{2.31}$$

$$(F_\pi m_\pi^2)(p+q)_\lambda M_{\pi\nu\lambda}{}^{abc}(p,q) =-\epsilon^{abc}[\bar\Delta(p^2)p_\nu+\bar\Delta(q^2)q_\nu], \tag{2.32}$$

$$(p+q)_\lambda M_{\pi\pi\lambda}{}^{abc}(p,q)=\epsilon^{abc}[\Delta_\pi(p^2)-\Delta_\pi(q^2)]. \tag{2.33}$$

E. Axial-Vector Ward Identities

As a consequence of the PCAC condition (2.20), the three-point functions are related by the axial-vector identities

$$p_\mu M_{\mu\nu\lambda}{}^{abc}(p,q)=\epsilon^{abc}[\Delta_{\nu\lambda}(p+q)-\Delta_{\nu\lambda}{}^A(q)] +F_\pi m_\pi^2 M_{\pi\nu\lambda}{}^{abc}(p,q) \tag{2.34}$$

and

$$(F_\pi m_\pi^2)q_\nu M_{\pi\nu\lambda}{}^{abc}(p,q)=(F_\pi m_\pi^2)^2 M_{\pi\pi\lambda}{}^{abc}(p,q) -\epsilon^{abc}\bar\Delta(p^2)p_\lambda, \tag{2.35}$$

so that $M_{\pi\nu\lambda}{}^{abc}(p,q)$ and $M_{\pi\pi\lambda}{}^{abc}(p,q)$ are determined from $M_{\mu\nu\lambda}{}^{abc}(p,q)$ and the two-point functions. In the derivation of Eq. (2.34), the first Weinberg sum rule, Eq. (2.13), has been used to cancel the Schwinger terms from the vector and axial-vector current propagators.

III. DECOMPOSITION OF VERTEX FUNCTIONS

The vertex function $\Gamma_{\pi\pi\lambda}(p,q)$ is decomposed according to

$$\Gamma_{\pi\pi\lambda}(p,q)=\Gamma(p^2,q^2,s)K_\lambda +C_V^{-1}[\Delta_\pi^{-1}(p^2)-\Delta_\pi^{-1}(q^2)](p+q)_\lambda/s, \tag{3.1}$$

where $\Gamma(p^2,q^2,s)$ is a function of the scalar variables, symmetric under interchange of p^2 and q^2, $s\equiv(p+q)^2$, and

$$K_\lambda\equiv(p-q)_\lambda-(p^2-q^2)(p+q)_\lambda/s. \tag{3.2}$$

The vector Ward identity (2.33) is evidently satisfied; furthermore, if $\Gamma_{\pi\pi\lambda}(p,q)$ is to have no pole at $s=0$, then it is necessary that

$$\Gamma(p^2,q^2,0)=C_V^{-1}\left[\frac{\Delta_\pi^{-1}(p^2)-\Delta_\pi^{-1}(q^2)}{p^2-q^2}\right], \tag{3.3}$$

and thus, independently of any assumptions of single-particle dominance,

$$\Gamma(m_\pi^2,m_\pi^2,0)=C_V^{-1}. \tag{3.4}$$

$\Gamma_{\pi\nu\lambda}(p,q)$ is decomposed according to

$$\Gamma_{\pi\nu\lambda}(p,q)=A[g_{\nu\lambda}-(p+q)_\nu(p+q)_\lambda/s]+[Bp_\nu+Cq_\nu]K_\lambda +[C_VF_\pi m_\pi^2\Delta_\pi(p^2)]^{-1}\left[F_A^{-1}(q^2)\bar\Delta(p^2)\left(p_\nu-\frac{p\cdot q}{q^2}q_\nu\right) -\frac{\bar\Delta(q^2)+\bar\Delta(p^2)p\cdot q/q^2}{C_A+\bar\Delta(q^2)}q_\nu\right]\frac{(p+q)_\lambda}{s}, \tag{3.5}$$

where A, B, and C depend on the scalar variables p^2, q^2, and s. The vector Ward identity (2.32) is satisfied; if $\Gamma_{\pi\nu\lambda}(p,q)$ is to have no pole at $s=0$, then it is necessary that

$$A(p^2,q^2,0)+(p^2-q^2)B(p^2,q^2,0) =[C_VF_\pi m_\pi^2\Delta_\pi(p^2)F_A(q^2)]^{-1}\bar\Delta(p^2), \tag{3.6}$$

$$A(p^2,q^2,0)+(p^2-q^2)C(p^2,q^2,0) =-\{C_VF_\pi m_\pi^2\Delta_\pi(p^2)[C_A+\bar\Delta(q^2)]\}^{-1} \times[p\cdot qF_A^{-1}(q^2)G_A(q^2)\bar\Delta(p^2)+\bar\Delta(q^2)]. \tag{3.7}$$

From the axial-vector Ward identity (2.35), it then follows that

$$(F_\pi m_\pi^2)^2\Delta_\pi(p^2)\Delta_\pi(q^2)F(s)\Gamma(p^2,q^2,s) =(F_\pi m_\pi^2)\Delta_\pi(p^2)[C_A+\bar\Delta(q^2)]F(s)[\tfrac{1}{2}A(p^2,q^2,s) -p\cdot qB(p^2,q^2,s)-q^2C(p^2,q^2,s)]+\tfrac{1}{2}\bar\Delta(p^2), \tag{3.8}$$

which determines $\Gamma(p^2,q^2,s)$ in terms of A, B, and C.

Finally, $\Gamma_{\mu\nu\lambda}(p,q)$ is decomposed according to

$$\begin{aligned}\Gamma_{\mu\nu\lambda}(p,q) =&[A_+g_{\mu\nu}+(D_++D_-)p_\mu p_\nu+(D_+-D_-)q_\mu q_\nu +F_+p_\mu q_\nu+G_+q_\mu p_\nu]K_\lambda\\ &+[(B_++B_-)(p-q)_\nu+(C_++C_-)(p+q)_\nu]g_{\mu\lambda}\\ &+[(B_+-B_-)(p-q)_\mu-(C_+-C_-)(p+q)_\mu]g_{\nu\lambda}\\ &-2[B_+(p_\mu p_\nu-q_\mu q_\nu)-B_-(p_\mu q_\nu-q_\mu p_\nu) +C_-(p+q)_\mu(p+q)_\nu](p+q)_\lambda/s\\ &+C_V^{-1}\{[F_A^{-1}(p^2)-F_A^{-1}(q^2)]g_{\mu\nu} -\varphi(p^2)p_\mu p_\nu+\varphi(q^2)q_\mu q_\nu\}(p+q)_\lambda/s,\end{aligned} \tag{3.9}$$

where

$$\varphi(x)\equiv F_A^{-1}(x)G_A(x)/[C_A+\bar\Delta(x)] \tag{3.10}$$

and the $X_\pm$ ($X=A, \ldots, G$) depend on the scalar variables p^2, q^2, and s, with

$$X_\pm(q^2,p^2,s)=\pm X_\pm(p^2,q^2,s). \tag{3.11}$$

The vector Ward identity (2.31) is satisfied; if $\Gamma_{\mu\nu\lambda}(p,q)$ is to have no pole at $s=0$, then it is necessary that

$$A_+(p^2,q^2,0)=C_V^{-1}\left[\frac{F_A^{-1}(p^2)-F_A^{-1}(q^2)}{p^2-q^2}\right], \tag{3.12}$$

$$B_-(p^2,q^2,0)-C_-(p^2,q^2,0)=\tfrac{1}{2}(p^2-q^2)F_+(p^2,q^2,0), \tag{3.13}$$

$$B_-(p^2,q^2,0)+C_-(p^2,q^2,0)=-\tfrac{1}{2}(p^2-q^2)G_+(p^2,q^2,0), \tag{3.14}$$

$$2C_-(p^2,q^2,0)+(p^2-q^2)D_+(p^2,q^2,0) = -\tfrac{1}{2}C_V^{-1}[\varphi(p^2)-\varphi(q^2)], \quad (3.15)$$

$$2B_+(p^2,q^2,0)+(p^2-q^2)D_-(p^2,q^2,0) = -\tfrac{1}{2}C_V^{-1}[\varphi(p^2)+\varphi(q^2)]. \quad (3.16)$$

From the axial-vector Ward identity (2.34), it then follows that

$$(F_\pi m_\pi^2)\Delta_\pi(p^2)A(p^2,q^2,s)=F^{-1}(s)-F_A^{-1}(q^2) -[C_A+\bar{\Delta}(p^2)][p\cdot(p-q)(B_+-B_-) -p\cdot(p+q)(C_+-C_-)], \quad (3.17)$$

$$(F_\pi m_\pi^2)\Delta_\pi(p^2)B(p^2,q^2,s) =-[C_A+\bar{\Delta}(p^2)][A_++p^2(D_++D_-)+p\cdot qG_+ +\tfrac{1}{2}(B_++B_-+C_++C_-)], \quad (3.18)$$

$$(F_\pi m_\pi^2)\Delta_\pi(p^2)C(p^2,q^2,s) =-[C_A+\bar{\Delta}(p^2)][p\cdot q(D_+-D_-)+p^2F_+ -\tfrac{1}{2}(B_++B_--C_+-C_-)]-\tfrac{1}{2}\varphi(q^2). \quad (3.19)$$

Then also

$$(F_\pi m_\pi^2)^2\Delta_\pi(p^2)\Delta_\pi(q^2)\Gamma(p^2,q^2,s) =[C_A+\bar{\Delta}(p^2)][C_A+\bar{\Delta}(q^2)] \times\{p\cdot q[A_++p^2(D_++D_-)+q^2(D_+-D_-)+q\cdot pG_+] +\tfrac{1}{2}[(s-2p^2-2q^2)B_++sC_+ +(p^2-q^2)(B_--C_-)]+p^2q^2F_+\} +\tfrac{1}{2}[C_A+\bar{\Delta}(p^2)+\bar{\Delta}(q^2)+F(s)]F^{-1}(s). \quad (3.20)$$

IV. SINGLE-PARTICLE MATRIX ELEMENTS; FORM FACTORS

Consider the matrix elements

$$\langle\pi^a(p)|V_\lambda{}^c(0)|\pi^b(q)\rangle=\epsilon^{abc}N_\pi{}^aN_\pi{}^bF_\pi(t)(p+q)_\lambda, \quad (4.1)$$

where $t=(p-q)^2$, and

$$\langle\pi^a(p)|\varphi_\pi{}^\rho(0)|\rho^c(k)\rangle=2\epsilon^{abc}N_\pi{}^aN_\rho\gamma(q^2)p_\lambda\rho_\lambda{}^\rho(k), \quad (4.2)$$

where $q^2=(k-p)^2$.

$F_\pi(t)$ is the pion electromagnetic form factor; evidently

$$F_\pi(t)=-F(t)\Gamma(m_\pi^2,m_\pi^2,t). \quad (4.3)$$

Also,

$$\gamma(q^2)=g_\rho\Delta_\pi(q^2)\Gamma(m_\pi^2,q^2,m_\rho^2). \quad (4.4)$$

Moreover, consider the matrix elements

$$\langle\pi^a(p)|V_\lambda{}^c(0)|A_1{}^b(q)\rangle=i\epsilon^{abc}N_\pi N_AG_{\nu\lambda}(p,q)\rho_\nu{}^A(q), \quad (4.5)$$

with

$$G_{\nu\lambda}(p,q)\equiv G_1(t)\left[g_{\nu\lambda}-\frac{(p-q)_\nu(p-q)_\lambda}{t}\right] +G_2(t)p_\nu\left[(p+q)_\lambda+\frac{m_A{}^2-m_\pi{}^2}{t}(p-q)_\lambda\right], \quad (4.6)$$

where $t\equiv(p-q)^2$, and

$$G_1(t)=g_AF(t)A(m_\pi^2,m_A^2,t), \quad (4.7a)$$

$$G_2(t)=g_AF(t)B(m_\pi^2,m_A^2,t), \quad (4.7b)$$

$$\langle A_1{}^b(q)|\varphi_\pi{}^a(0)|\rho^c(k)\rangle =i\epsilon^{abc}N_AN_\rho\rho_\nu{}^{A*}(q)B_{\nu\lambda}(q,k)\rho_\lambda{}^\rho(k), \quad (4.8)$$

with

$$B_{\nu\lambda}(q,k)=\beta_1(p^2)g_{\nu\lambda}-\beta_2(p^2)k_\nu q_\lambda, \quad (4.9)$$

where $p^2\equiv(q-k)^2$, and

$$\beta_1(p^2)=g_Ag_\rho\Delta_\pi(p^2)A(p^2,m_A^2,m_\rho^2), \quad (4.10a)$$

$$\beta_2(p^2)=2g_Ag_\rho\Delta_\pi(p^2)B(p^2,m_A^2,m_\rho^2), \quad (4.10b)$$

$$\langle\pi^a(p)|A_\nu{}^b(0)|\rho^c(k)\rangle=i\epsilon^{abc}N_\pi N_\rho H_{\nu\lambda}(p,k)\rho_\lambda{}^\rho(k), \quad (4.11)$$

with

$$H_{\nu\lambda}(p,k)=H_1(q^2)g_{\nu\lambda}+[H_2(q^2)p_\nu+H_3(q^2)q_\nu]p_\lambda, \quad (4.12)$$

where $q\equiv k-p$, and

$$H_1(q^2)=g_\rho F_A(q^2)A(m_\pi^2,q^2,m_\rho^2), \quad (4.13a)$$

$$H_2(q^2)=2g_\rho F_A(q^2)B(m_\pi^2,q^2,m_\rho^2), \quad (4.13b)$$

$$q^2H_3(q^2)=H_1(q^2)-p\cdot qH_2(q^2)-2(F_\pi m_\pi^2)^{-1}\gamma(q^2). \quad (4.13c)$$

Finally, consider the matrix elements

$$\langle A_1{}^a(p)|V_\lambda{}^c(0)|A_1{}^\rho(q)\rangle =\epsilon^{abc}N_A{}^aN_A{}^b\rho_\mu{}^A(p)M_{\mu\nu\lambda}(p,q)\rho_\nu{}^A(q), \quad (4.14)$$

with

$$M_{\mu\nu\lambda}(p,q)=M_1(t)g_{\mu\nu}(p+q)_\lambda+M_2(t)(g_{\mu\lambda}p_\nu+g_{\nu\lambda}q_\mu) +M_3(t)q_\mu p_\nu(p+q)_\lambda, \quad (4.15)$$

where $t\equiv(p-q)^2$, and

$$M_1(t)=-g_A{}^2F(t)A_+(m_A^2,m_A^2,t), \quad (4.16a)$$

$$M_2(t)=-g_A{}^2F(t)[B_+(m_A^2,m_A^2,t) +C_+(m_A^2,m_A^2,t)], \quad (4.16b)$$

$$M_3(t)=g_A{}^2F(t)G_+(m_A^2,m_A^2,t). \quad (4.16c)$$

$$\langle A_1{}^b(q)|A_\mu{}^a(0)|\rho^c(k)\rangle =\epsilon^{abc}N_AN_\rho\rho_\nu{}^{A*}(q)N_{\mu\nu\lambda}(q,k)\rho_\lambda{}^\rho(k), \quad (4.17)$$

with

$$N_{\mu\nu\lambda}(p,q)=N_1(p^2)(q_\mu g_{\nu\lambda}-k_\nu g_{\mu\lambda})+N_2(p^2)(q_\mu g_{\nu\lambda}+k_\nu g_{\mu\lambda}) -N_3(p^2)g_{\mu\nu}q_\lambda-N_4(p^2)q_\mu k_\nu q_\lambda +[N_5(p^2)g_{\nu\lambda}-N_6(p^2)k_\nu q_\lambda](k-q)_\mu, \quad (4.18)$$

where $p^2\equiv(q-k)^2$,

$$N_1(p^2)=-g_Ag_\rho F_A(p^2)[B_+(p^2,m_A^2,m_\rho^2) +C_+(p^2,m_A^2,m_\rho^2)], \quad (4.19a)$$

$$N_2(p^2)=g_Ag_\rho F_A(p^2)[B_-(p^2,m_A^2,m_\rho^2) +C_-(p^2,m_A^2,m_\rho^2)], \quad (4.19b)$$

$$N_3(p^2)=2g_Ag_\rho F_A(p^2)A_+(p^2,m_A^2,m_\rho^2), \quad (4.19c)$$

$$N_4(p^2)=2g_Ag_\rho F_A(p^2)G_+(p^2,m_A{}^2m,_\rho{}^2), \quad (4.19d)$$

and $N_5(p^2)$ and $N_6(p^2)$ are expressed in terms of matrix elements of the pion field with the aid of the PCAC condition.

V. SMOOTHNESS ASSUMPTIONS

In order to make predictions, it is necessary to make assumptions about the amplitudes $X_\pm(p^2,q^2,s)$ which appear in the vertex function $\Gamma_{\mu\nu\lambda}(p,q)$. These assumptions involve the approximation of the amplitudes by rational functions of p^2, q^2, and s (in the pole-dominance approximation for the axial-vector current propagator), invoking the constraints at $s=0$ expressed in Eqs. (3.12)–(3.16), and restrictions on the number of subtractions required in dispersion relations for the single-particle matrix elements introduced in Sec. IV (these correspond to restrictions on derivative couplings in an effective-Lagrangian approach).

It must be noted that the constraints at $s=0$ involve the function $\varphi(x)$ introduced in Eq. (3.10), which has a pole when

$$C_A+\bar{\Delta}(x)=0. \tag{5.1}$$

This equation is certain to be satisfied in the interval $0<x<m_\pi^2$, since

$$C_A+\bar{\Delta}(0)=\int\frac{\sigma_A(\kappa)}{\kappa}d\kappa>0 \tag{5.2}$$

and

$$C_A+\bar{\Delta}(x)\cong F_\pi^2m_\pi^2/(x-m_\pi^2)+\text{const} \tag{5.3}$$

for $x\approx m_\pi^2$.

The poles of $\varphi(x)$, which appear in some of the amplitudes $X_\pm$, must not appear in the amplitudes $M_{\mu\nu\lambda}$, $M_{\pi\nu\lambda}$, and $M_{\pi\pi\lambda}$. Thus it is necessary that there be no poles in the combinations

(i) B_++C_+, (ii) B_-+C_-, (iii) C_+

as functions of either p^2 or q^2, in

(iv) D_++D_-,

(v) $(B_+-B_-)-(C_+-C_-)$

as functions of p^2, and thus in

(iv′) D_+-D_-,

(v′) $(B_++B_-)-(C_++C_-)$

as functions of q^2.

The amplitudes $X_\pm$ fall naturally into two classes—the amplitudes $(A_+,B_\pm,C_\pm)$, which multiply a linear function of the momenta, and the amplitudes $(D_\pm,F_+G_+)$, which multiply a cubic function of the momenta. The general procedure is to approximate the first class by rational functions of degree n in the scalar variables, and the second class by rational functions of degree $n-1$, in the limit of pole dominance of the axial-vector current propagator (although the form of the approximation does not require that limit to be taken). For $n=0$ and 1, this leads to the specific models described below.

A. Current-Smoothness Model

The first-class amplitudes are assumed to be of order zero in the scalar variables. Then

$$A_+=C_V^{-1}\left[\frac{F_A^{-1}(p^2)-F_A^{-1}(q^2)}{p^2-q^2}\right], \tag{5.4}$$

$$B_++C_+=C_V^{-1}g_A^{-2}\xi, \tag{5.5}$$

with ξ an arbitrary parameters, and

$$B_-+C_-=0 \tag{5.6}$$

(owing to the antisymmetry in p^2 and q^2). It is also assumed that

$$D_\pm=0. \tag{5.7}$$

(This is a nontrivial assumption as explained below.) Then the $s=0$ constraints require

$$B_+=-\tfrac{1}{4}C_V^{-1}[\varphi(p^2)+\varphi(q^2)], \tag{5.8}$$

$$B_-=\tfrac{1}{4}C_V^{-1}[\varphi(p^2)-\varphi(q^2)], \tag{5.9}$$

$$F_+=C_V^{-1}\left[\frac{\varphi(p^2)-\varphi(q^2)}{p^2-q^2}\right], \tag{5.10}$$

$$G_+=0. \tag{5.11}$$

This model contains a single parameter ξ, in terms of which the pion electromagnetic form factor is given by

$$F_\pi(t)=[F(t)/F(0)](1-\lambda t/4m_\rho^2), \tag{5.12}$$

with

$$\lambda=-(2F_\pi^2m_\rho^2/g_A^2)(1+\xi), \tag{5.13}$$

and the π-A-V vertex functions are given (on the pion mass shell) by

$$A(m_\pi^2,q^2,s)=C_V^{-1}F_\pi g_A^{-2}[m_\pi^2-m_A^2+\tfrac{1}{2}\xi(s+m_\pi^2-q^2)], \tag{5.14}$$

$$B(m_\pi^2,q^2,s)=-C_V^{-1}F_\pi g_A^{-2}(1+\tfrac{1}{2}\xi). \tag{5.15}$$

A model with a second arbitrary parameter is obtained by replacing Eq. (5.8) with

$$D_++D_-=C_V^{-1}d_0\Delta_\pi(p^2)/[C_A+\bar{\Delta}(p^2)], \tag{5.16}$$

$$D_+-D_-=C_V^{-1}d_0\Delta_\pi(q^2)/[C_A+\bar{\Delta}(q^2)] \tag{5.17}$$

(which is consistent with the general procedure, since these are rational functions of degree -1) and appropriately modifying Eqs. (5.9)–(5.11). However, the behavior of the amplitudes for $p^2\to\infty$ at fixed q^2 is altered, and the model is more appropriately considered as a special form of the linear model described below.

B. Linear Models

The first-class amplitudes are allowed to contain terms linear in the scalar variables; explicitly, assume

$$A_+=C_V^{-1}\left[\left(\frac{F_A^{-1}(p^2)-F_A^{-1}(q^2)}{p^2-q^2}\right)+g_A^{-2}\alpha s\right], \quad (5.18)$$

$$B_++C_+=C_V^{-1}g_A^{-2}[\xi+\eta s+\mu(p^2+q^2)], \quad (5.19)$$

$$G_+=C_V^{-1}g_A^{-2}\gamma, \quad (5.20)$$

and then the constraint at $s=0$ requires

$$B_-+C_-=-\tfrac{1}{2}C_A^{-1}g_A^{-2}(p^2-q^2)\gamma. \quad (5.21)$$

Also assume

$$D_++D_-=C_V^{-1}\left[d_1+(d_0+\delta s)\frac{\Delta_\pi(p^2)}{C_A+\bar{\Delta}(p^2)}\right], \quad (5.22)$$

$$D_+-D_-=C_V^{-1}\left[d_1+(d_0+\delta s)\frac{\Delta_\pi(q^2)}{C_A+\bar{\Delta}(q^2)}\right], \quad (5.23)$$

$$B_+=-\tfrac{1}{4}C_V^{-1}[\varphi(p^2)+\varphi(q^2)](1+\beta s) -\tfrac{1}{2}(p^2-q^2)D_-+C_V^{-1}g_A^{-2}\zeta s, \quad (5.24)$$

$$C_-=-\tfrac{1}{4}C_V^{-1}[\varphi(p^2)-\varphi(q^2)](1+\beta s) -\tfrac{1}{2}(p^2-q^2)D_+, \quad (5.25)$$

$$F_+=C_V^{-1}\left[\frac{\varphi(p^2)-\varphi(q^2)}{p^2-q^2}\right](1+\chi s) -C_V^{-1}g_A^{-2}\gamma+2D_+. \quad (5.26)$$

The eleven parameters $(\alpha,\xi,\eta,\mu,\gamma,d_0,d_1,\delta,\beta,\zeta,\chi)$ of the model are reduced to three by requiring the single-particle matrix elements of Sec. IV to have "reasonable" behavior for large momentum. This means requiring[21]

(i) $\gamma(q^2)\to 0$ for $q^2\to\infty$,

(ii) $F_\pi(t)\to$ const and for $t\to\infty$,

(iii) $G_1(t),\beta_1(p^2),H_1(q^2)\to$ const,

(iv) $G_2(t),\beta_2(p^2),H_2(q^2)\to 0$

for t, p^2, $q^2\to\infty$, respectively.

These conditions reduce to three the number of free parameters of the model; these parameters are most conveniently taken to be α (or γ), ξ, and β (or χ). The parameter α is related to the quadrupole moment of the A_1 meson, and does enter into the A_1-ρ-π or ρ-π-π vertex functions; also

$$\alpha+\tfrac{1}{2}\gamma=0=\alpha+\mu, \quad (5.27)$$

$$g_A^2d_1=\mu. \quad (5.28)$$

[21] These conditions correspond to those imposed by Brown and West, Ref. 17.

The conditions also require

$$\eta=0=\delta, \quad (5.29)$$

$$\beta+\mu-2\zeta=0. \quad (5.30)$$

The parameter ξ is related to the parameter δ of Schnitzer and Weinberg[1] by

$$\xi=-2-\delta \quad (5.31)$$

and to the anomalous magnetic moment λ_A of Arnowitt *et al.*[2] by

$$\xi=-1-\lambda_A. \quad (5.32)$$

d_0 is given in terms of ξ by

$$d_0=-C_Vg_A^{-2}(1+\tfrac{1}{2}\xi). \quad (5.33)$$

The pion electromagnetic form factor has the form of Eq. (5.12), with

$$(g_A^2/2m_\rho^2F_\pi^2)\lambda=2C_VF_\pi^{-2}(1+\tfrac{1}{2}\xi-\chi m_A^2) +2\beta m_A^2-1-\xi, \quad (5.34)$$

with the additional relation

$$C_VF_\pi^{-2}\chi m_A^2-C_VF_\pi^{-2}(1+\tfrac{1}{2}\xi-\chi m_A^2) -\beta m_A^2=\frac{m_A^2}{m_\rho^2}\left(1-\frac{C_V}{2F_\pi^2}\lambda\right) \quad (5.35)$$

connecting β and χ.

The π-A-V vertex functions are now given (on the pion mass shell) by

$$A(m_\pi^2,q^2,s)=C_V^{-1}F_\pi g_A^{-2}[(\tfrac{1}{2}\xi-\beta m_A^2)s+q^2-m_A^2 -(C_VF_\pi^{-2}-1)(m_\pi^2-q^2)(1+\tfrac{1}{2}\xi)], \quad (5.36)$$

$$B(m_\pi^2,q^2,s)=C_V^{-1}F_\pi g_A^{-2}(C_VF_\pi^{-2}-1)(1+\tfrac{1}{2}\xi). \quad (5.37)$$

There are several possible assumptions which can be made to eliminate the parameter β (or χ). None of these assumptions is very compelling, but we give three examples below.

Model 2A. The second-class amplitudes are required to be bounded by constants when one variable is large with the remaining two fixed. This requires $\delta=0$, which is already ensured by Eq. (5.29), and

$$\chi=0. \quad (5.38)$$

This model leads to a large width for $A_1\to\rho\pi$ (2–5 BeV) and will not be discussed further.

Model 2B. The constraints (3.12)–(3.14) are required to be satisfied for all s. This requires $\alpha=0$ (which affects only the A_1-A_1-ρ vertex), and

$$\beta=\chi. \quad (5.39)$$

Then

$$\lambda=-(2m_\rho^2/m_A^2)(C_VF_\pi^{-2}-1)(1+\xi) +4[1-\tfrac{1}{2}C_VF_\pi^{-2}(C_Vm_\rho^2g_A^{-2})] \quad (5.40)$$

and

$$\tfrac{1}{2}\xi-\chi m_A{}^2=\frac{m_A{}^2}{m_\rho{}^2}-\frac{(C_VF_\pi{}^{-2})^2}{C_VF_\pi{}^{-2}-1}-\tfrac{1}{2}(C_VF_\pi{}^{-2}-1)\xi. \quad (5.41)$$

Model 2C. (Standard hard-pion model). Although there seems to be no elegant justification for the model in the present context, the standard hard-pion model[1,2] is reproduced if

$$\beta m_A{}^2=\tfrac{1}{2}C_VF_\pi{}^{-2}\xi+(C_VF_\pi{}^{-2}-1)m_A{}^2/m_\rho{}^2, \quad (5.42)$$

$$\chi m_A{}^2=\tfrac{1}{2}C_VF_\pi{}^{-2}(1+\xi)+\tfrac{1}{2}(C_VF_\pi{}^{-2}-1)m_A{}^2/m_\rho{}^2, \quad (5.43)$$

when

$$\lambda=-(2m_\rho{}^2/m_A{}^2)(C_VF_\pi{}^{-2}-1)(1+\xi)+4(1-\tfrac{1}{2}C_VF_\pi{}^{-2}), \quad (5.44)$$

$$\tfrac{1}{2}\xi-\beta m_A{}^2=(1-C_VF_\pi{}^{-2})\left(\tfrac{1}{2}\xi+\frac{m_A{}^2}{m_\rho{}^2}\right). \quad (5.45)$$

In spite of the lack of justification[22] for Eqs. (5.42) and (5.43), it is reassuring that Eqs. (5.44) and (5.45), which are the standard hard-pion results, can be reproduced by adjusting a single parameter.

Model 2B coincides with the standard hard-pion model in the limit

$$g_A{}^2=C_Vm_\rho{}^2, \quad (5.46)$$

which is approximately true experimentally. However, the models are sufficiently sensitive to deviations from this relation that they lead to distinct predictions.

The current-smoothness model is an independent model; it is equivalent to the other two only if Eq. (5.46) is satisfied and, in addition,

$$C_V=2F_\pi{}^2 \quad (5.47)$$

(generalized Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation),

$$m_A{}^2=2m_\rho{}^2 \quad (5.48)$$

and

$$\xi=-2 \quad (5.49)$$

(corresponding to $\lambda_A=1$ or $\delta=0$). These relations are not well satisfied experimentally.

VI. PREDICTIONS OF MODELS AND CONCLUSIONS

With the ρ-π-π vertex determined from an analysis[19] of the data on the colliding-beam reaction

$$e^++e^-\to\pi^++\pi^-, \quad (6.1)$$

the parameters can be fixed in each of the three models (current-smoothness, 2B, standard hard-pion), and predictions made about processes related to the A_1-ρ-π vertex.

1. *Decay* $A_1\to\rho+\pi$

The width for the decay $A_1\to\rho+\pi$ (including both charge states) is given by

$$\Gamma(A_1\to\rho\pi)=\frac{g_A{}^2g_\rho{}^2}{4\pi m_A{}^2}\left[R_1(m_\rho{}^2)+\frac{q_{\pi\rho}{}^2}{3m_\rho{}^2}R_2(m_\rho{}^2)\right]q_{\pi\rho}, \quad (6.2)$$

where $q_{\pi\rho}$ is the pion momentum in the A_1 rest system, and

$$R_1(t)=|A(m_\pi{}^2,m_A{}^2,t)|^2, \quad (6.3)$$

$$R_2(t)=|A(m_\pi{}^2,m_A{}^2,t)-(t+m_A{}^2-m_\pi{}^2)B(m_\pi{}^2,m_A{}^2,t)|^2-4m_A{}^2t|B(m_\pi{}^2,m_A{}^2,t)|^2. \quad (6.4)$$

The relation of this width to the experimental width for the decay $A_1\to3\pi$ is obscured by several effects.

(i) The width for $A_1\to\rho+\pi\to3\pi$ computed using only tree diagrams is affected by the finite width of the ρ and interference between the crossed ρ bands on the Dalitz plot. The over-all effect is to decrease the width from the result of Eq. (6.2), unless that width is predicted to be small because of cancellations exactly at $t=m_\rho{}^2$ (see the discussion of Model 2B below).

(ii) The decay mode $A_1\to\sigma+\pi$ is present; it may be small, but it cannot be predicted by standard hard-pion methods.

(iii) Contact terms in the four-point function involved in the decay $A_1\to3\pi$ may be present; their effect on the width is unknown at present.

2. *Decay* $A_1\to\pi+\gamma$

The partial width for the decay $A_1\to\pi+\gamma$ is given by

$$\Gamma(A_1\to\pi+\gamma)=\frac{1}{6}\left(\frac{g_{A\pi\gamma}{}^2}{4\pi}\right)\left(\frac{m_A{}^2-m_\pi{}^2}{m_A{}^3}\right), \quad (6.5)$$

where

$$g_{A\pi\gamma}=-eC_Vg_AA(m_\pi{}^2,m_A{}^2,0). \quad (6.6)$$

This is the most definite prediction of the model; it is also the most difficult to verify experimentally.

3. *Colliding-Beam Reaction* $e^++e^-\to A_1{}^\pm+\pi^\mp$

The colliding-beam reaction

$$e^++e^-\to A_1{}^\pm+\pi^\mp\to2\pi^+2\pi^- \quad (6.7)$$

is a promising method of determining properties of the A_1, since the four-charged-pion final state, which is relatively easy to detect, should be dominated by A_1-π states (ω-π and ρ-ρ states are forbidden, while σ-ρ is

[22] Of course, appeal to an effective Lagrangian, as in Ref. 2, will lead to those results, as will separation of the axial-vector current into spin-0 and spin-1 structures, followed by insistence that each spin part separately be single-particle dominated, as in Ref. 1. However, these arguments seem to treat the particles as more fundamental than the currents, a point of view which is not altogether convincing.

allowed, but presumably not too important at c.m. energies of 1.2–1.4 BeV). Assuming ρ dominance into the 1–2 BeV region, the two-body reaction cross section at c.m. energy $E=\sqrt{t}$ is given by

$$\sigma(e^+ + e^- \to A_1^+ + \pi^-)$$

$$=2\pi\left(\frac{e^2}{4\pi}\right)^2 g_A{}^2 \frac{|F(t)|^2}{t^2}\left[R_1(t)+\frac{q_{\pi A}{}^2}{3m_A{}^2}R_2(t)\right]\frac{q_{\pi A}}{E}, \quad (6.8)$$

where $q_{\pi A}$ is the pion momentum in the c.m. system, $F(t)$ is the coefficient of $g_{\mu\nu}$ in the vector current propagator, and $R_1(t)$ and $R_2(t)$ are given by Eqs. (6.3) and (6.4).

While there are again complications owing to finite width and interference effects, contact terms, and other channels $(\sigma-\rho)$, the two-body cross section calculated here should be a reasonable approximation to the total cross section for the four-body reaction in the c.m. energy range 1.2–1.4 BeV (and perhaps somewhat higher) so long as no ρ' meson, which couples strongly to both the photon and the A_1-π system, exists in this energy range.

Numerical results are shown in Table I for each of the three models under consideration, for ρ properties corresponding to best fits to each set of colliding-beam data taken from Ref. (14). The colliding-beam cross sections for $\pi^+ + \pi^-$ and $A_1^+ + \pi^-$ final states are evaluated at c.m. energy 1.4 BeV using the Vaughn-Wali propagator[14] for the vector current, with cutoff parameter $\alpha^2 = 50m_\pi{}^2$.

If the fit to the Novosibirsk data is ignored, then the following conclusions can be drawn.

(i) The strong decay of the A_1 is in general not a good test of the models unless $A_1 \to 3\pi$ is treated more fully. However, the small width for $A_1 \to \rho\pi$ in Model 2B, which is due to almost complete cancellation between various terms in the amplitude $A(m_\pi{}^2, m_A{}^2, m_\rho{}^2)$, and which persists in the calculation of the width for $A_1 \to \rho\pi \to 3\pi$ via the tree diagram, is sufficient to rule out Model 2B unless the ρ data changes drastically.

(ii) A clear distinction between the current-smoothness model and the standard hard-pion model can be made by a measurement either of the decay rate for $A_1 \to \pi + \gamma$ or of the cross section for the colliding-beam reaction (6.7). The latter measurement is feasible at ADONE, and will certainly be carried out at DESY.[23]

TABLE I. Predictions of three models of A_1 properties and colliding-beam cross sections using ρ-meson parameters obtained from fits to the low-energy colliding-beam data (Ref. 19). The first set of values are based on Orsay data, the second and third on Novosibirsk data, the fourth on combined data. The three values shown for $\Gamma(A_1 \to \rho\pi)$ are for A_1 mass $m_A = 1.05$, 1.07, and 1.09 BeV; elsewhere, $m_A = 1.07$ BeV is used. The widths quoted as small are explained in the text.

	Current smoothness	Model 2B	Standard hard-pion
$m_\rho = 0.7678$ BeV, $\Gamma_\rho = 0.1106$ BeV, $\lambda = 0$			
$\Gamma(A_1 \to \rho\pi)$ (BeV)	0.1541		0.1295
	0.1623	small	0.1579
	0.1696		0.1894
$\Gamma(A_1 \to \pi\gamma)$ (MeV)	0.19	0.31	0.028
$\sigma(\pi^+\pi^-)$ (nb)	2.46	2.46	2.46
$\sigma(A_1{}^+\pi^-)$ (nb)	1.86	2.85	8.41
$m_\rho = 0.765$ BeV, $\Gamma_\rho = 0.1404$ BeV, $\lambda = 0$			
$\Gamma(A_1 \to \rho\pi)$ (BeV)	0.299	0.0764	0.0573
	0.315	0.0467	0.0688
	0.329	0.0229	0.0814
$\Gamma(A_1 \to \pi\gamma)$ (MeV)	0.28	0.28	0.24
$\sigma(\pi^+\pi^-)$ (nb)	2.55	2.55	2.55
$\sigma(A_1{}^+\pi^-)$ (nb)	2.92	6.92	7.65
$m_\rho = 0.763$ BeV, $\Gamma_\rho = 0.1015$ BeV, $\lambda = 0.84$			
$\Gamma(A_1 \to \rho\pi)$ (BeV)	0.348	0.634	0.0672
	0.352	0.550	0.0811
	0.352	0.463	0.0964
$\Gamma(A_1 \to \pi\gamma)$ (MeV)	0.058	0.019	0.069
$\sigma(\pi^+\pi^-)$ (nb)	0.20	0.20	0.20
$\sigma(A_1{}^+\pi^-)$ (nb)	5.87	11.1	4.40
$m_\rho = 0.7666$ BeV, $\Gamma_\rho = 0.1238$ BeV, $\lambda = 0$			
$\Gamma(A_1 \to \rho\pi)$ (BeV)	0.209		0.0958
	0.220	small	0.1162
	0.230		0.1378
$\Gamma(A_1 \to \pi\gamma)$ (MeV)	0.23	0.31	0.095
$\sigma(\pi^+\pi^-)$ (nb)	2.50	2.50	2.50
$\sigma(A_1{}^+\pi^-)$ (nb)	2.28	4.20	8.01

The methods and models of this paper can be generalized to higher-point functions and to a $U(3)\times U(3)$ current algebra. We shall report on these problems in a future publication.

ACKNOWLEDGMENTS

It is a pleasure to thank R. Arnowitt, M. Friedman, P. Nath, P. Polito, R. Suitor, and K. C. Wali for helpful discussions related to this work.

[23] It is a pleasure to thank T. Walsh for illuminating conversations on the experimental possibilities.