Field-Theoretic Model of High-Energy Scattering. III. Inelastic Electron-Proton Scattering

H. M. FRIED*
Physics Department, Brown University, Providence, Rhode Island 02912

AND

T. K. GAISSER†
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, England

(Received 16 April 1970)

A field-theoretic model of soft, neutral-meson production is used to bracket recent deep-inelastic electron-proton scattering data.

I. INTRODUCTION

RECENT experiments[1] have tended to confirm Bjorken's scaling prediction[2] suggesting that the structure functions $W_{1,2}$ of inelastic electron-proton scattering[3] are dependent upon the combination $x=\omega^{-1}=q^2/2m\nu$ and appear to yield a very small ratio for the quantity $R=\sigma_L/\sigma_T$. In particular, as x decreases from the elastic region ($x=1$), νW_2 rises to a maximum at $x\sim0.2$, and thereafter apparently decreases only slightly down to the minimum measured $x\sim0.025$. Various theoretical explanations[4–6] have been proposed to obtain a νW_2 which exhibits scaling and is more or less constant for very small x. It is the purpose of these remarks to describe the predictions which follow from a simplified version of the soft, virtual, neutral vector meson (SVNVM) field-theoretic model previously applied to the nucleon electromagnetic form factors and elastic pp scattering,[7] and in a qualitative way, to πp elastic and charge-exchange scattering.[8]

The ideas underlying this treatment are that (i) neutral vector mesons (NVM's) are copiously produced in the inelastic e-p channels, and (ii) they can be described by a simple generalization of the "soft-photon" methods, conserving energy and momentum exactly in all processes, while approximating the nucleon emission operators by those applicable to the case of soft mesons. We do not attempt to compute the emission probabilities directly, but infer them via unitarity from the off-shell Compton amplitudes, the latter calculated by summing over SVNVM exchanges between the proton legs of the simplest "hard" (i.e., nonsoft) amplitude, as in Fig. 1. It is not yet known if (i) is true, nor if large numbers of neutral mesons other than ρ^0 or ω are produced; in the context of Refs. 7 and 8 it was useful to deal with vector mesons, and we continue to do so here, even though there is no compelling reason why multiple soft-π^0 production cannot be included. Assumption (ii) is a natural approximation in any eikonal-type calculation where the proton shares in a very large collision energy, and is not appreciably deflected by the emission of relatively soft quanta. It should be noted that the addition of SVNVM exchanges between the proton legs of the graph of Fig. 1 represents a generalization of recent eikonal models,[9] in the sense that all virtual particles are soft except the internal nucleon, which is very hard, as befits the essential constituent of a far-off-shell Compton amplitude.

The present calculation was suggested by those of Refs. 4 and 5, and a comparison with the content of these papers may be worthwhile. The computation of Ref. 4 sums the large ω dependence of the set of rainbow (ladder) graphs, each approximated in an infinite momentum frame by an upper cutoff to the virtual-meson transverse momentum. The forms which result are very suggestive of a soft, or eikonal, approach, producing for small x the behavior $\nu W_2\sim ax^b$, with a and b constants; the choice $b\sim0$ then yields a crude description of the data. The Regge model of Ref. 5 suggests that apart from an unknown background term, $\nu W_2\sim\beta(q^2)(m/\nu)^{1-\alpha}$, where α and β are $t=0$ trajectory and residue functions, respectively; in order to obtain dependence on the variable x only, $\beta(q^2)$ must have a similar power be-

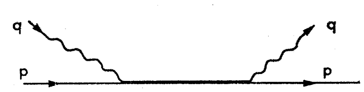

FIG. 1. s-channel Born approximation used as the "hard" part of the Compton amplitude.

* Work supported in part by the U. S. Atomic Energy Commission (Report No. NYO-2262TA-217).
† NATO postdoctoral fellow, 1969–70.
[1] E. D. Bloom *et al.*, Phys. Rev. Letters **23**, 930 (1969); **23**, 935 (1969).
[2] J. D. Bjorken, Phys. Rev. **179**, 1547 (1969).
[3] S. Drell and J. Walecka, Ann. Phys. (N. Y.) **28**, 18 (1964).
[4] S. D. Drell, D. J. Levy, and T.-M. Yan, Phys. Letters **22**, 744 (1969); Phys. Rev. **187**, 2159 (1969); Phys. Rev. D **1**, 1035 (1970); **1**, 1617 (1970).
[5] H. D. I. Abarbanel, M. L. Goldberger, and S. B. Treiman, Phys. Rev. Letters **22**, 500 (1969).
[6] J. J. Sakurai, Phys. Rev. Letters **22**, 981 (1969); J. D. Bjorken and E. A. Paschos, Phys. Rev. **185**, 1975 (1969).
[7] H. M. Fried and T. K. Gaisser, Phys. Rev. **179**, 1491 (1969).
[8] H. M. Fried, Phys. Rev. D **1**, 596 (1970).
[9] H. D. I. Abarbanel and C. Itzykson, Phys. Rev. Letters **23**, 53 (1969); M. Lévy and J. Sucher, Phys. Rev. **186**, 1656 (1969); R. Blankenbecler and R. L. Sugar, *ibid.* **183**, 1387 (1969); S. J. Chang and S. Ma, Phys. Rev. Letters **22**, 1334 (1969). The relation between the present approach and the eikonal models is discussed in more detail by T. K. Gaisser, Phys. Rev. D **2**, 1337 (1970).

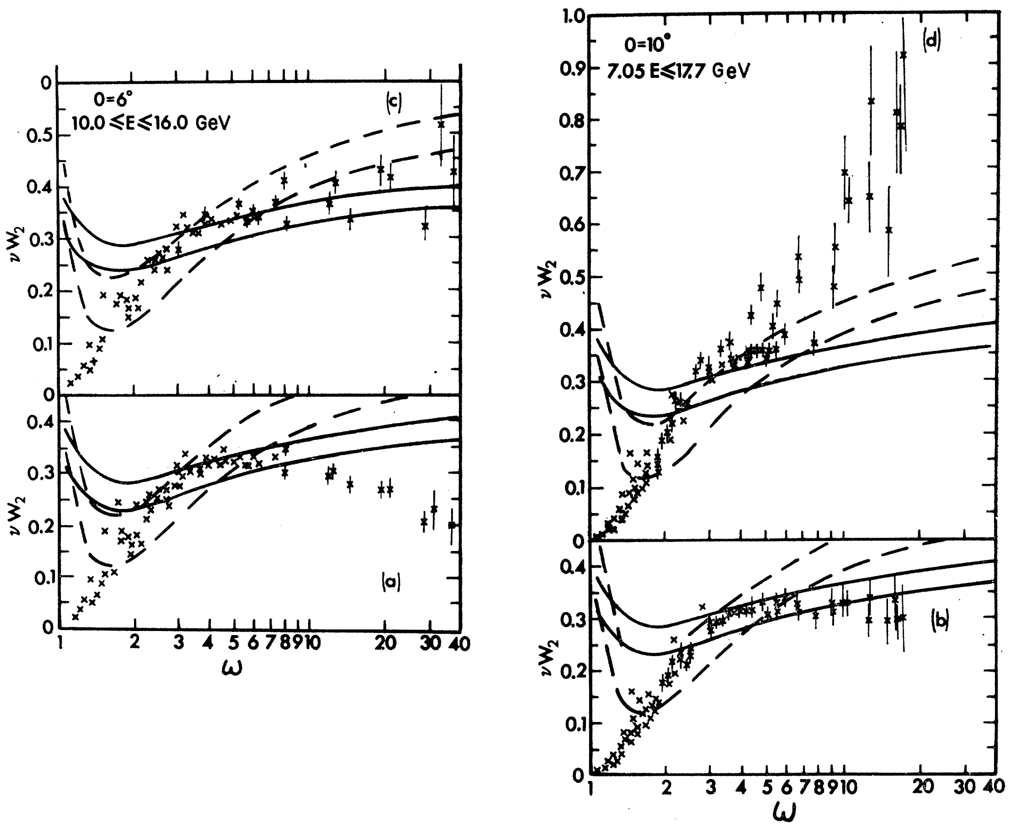

FIG. 2. Curves of Eq. (14) superimposed on the data [presented by R. Taylor in Proceedings of the 1969 Conference on Electron and Photon Interactions at High Energies, Daresbury, England (unpublished)], as described in the text. Plots (a) and (b) have been made assuming $R=0$; (c) and (d) use $R=\infty$.

havior, while a roughly constant dependence for small x can be achieved if $\alpha \sim 1$.

In contrast, the SVNVM model of this paper does not require an *ad hoc* upper cutoff to the virtual momenta, does not single out an infinite-momentum frame, and does include properly weighted contributions from all ladder and crossed graphs. It takes advantage of the other possibility associated with Regge behavior, producing a function with $\beta \simeq$constant, $\alpha \sim 1$, and an explicit background term which scales. One finds an expression for νW_2 which separates into two distinct parts: a Regge-like term proportional to $(m/\nu)^p$, where p is a small positive paramрter, together with a background term of form

$$f\left(x+\frac{1}{2}\frac{m_R{}^2}{m^2}\frac{m}{\nu}\right),$$

where m is the proton mass and m_R denotes the internal nucleon state, as pictured in Fig. 1. It is the first term which produces a constant (or more precisely, weakly dependent upon m/ν) limiting behavior as x vanishes, while for moderate values of $\omega < 2\nu/m$, the second term effectively scales and provides the approach to that limit. For $\omega \gg 2\nu/m$ we do not obtain scaling, which property depends crucially upon the sequence in which the limits $\nu/m \to \infty$ and $\omega \to \infty$ are taken. Our final formula, Eq. (14), which owes its simplicity in part to several technical approximations, is a function of three parameters which may be partially inferred from the work of Refs. 7 and 8, and provides a moderately good fit to the existing data in the deep inelastic region, Fig. 2. It will be interesting to see, experimentally, if and how νW_2 decreases for very small values of m/ν and/or x in the deep inelastic region; as indicated in the text, corrections to (14) are dependent upon possibly different masses and coupling constants of the SVNVM exchanged, and on the resonant structure of the γp channel.

II. DERIVATION

The simplest Born approximation to the Compton amplitude is given by[10]

$$M_{\mu\nu} = -(2\pi)^{-4}\int dx\, e^{ip\cdot x}\int du\, e^{iq\cdot u} \times\int dy\, e^{-ip'\cdot y}\int dw\, e^{-iq'\cdot w} \times\bar{u}(p')\delta(y-w)\gamma_\nu S_c(w-u)\gamma_\mu\delta(u-x)u(p)\,, \quad (1)$$

[10] We neglect the graph with a π^0 pole in the t channel, since it vanishes for $t=0$, and omit the crossed graph of Fig. 1, which cannot contribute to $\mathrm{Im}T_{\mu\nu}$. Equation (1) and subsequent expressions may be understood to be averaged over nucleon spins.

where q, q' and p, p' represent the initial and final photon and proton momenta, respectively. The model in which all possible SVNVM's are exchanged between proton legs of these graphs is obtained by replacing the factors $\delta(x-u)\cdots\delta(w-y)$ of (1) by the combination

$$\exp\left(-i\frac{\delta}{\delta A_1}\Delta_c\frac{\delta}{\delta A_2}\right)\times G_{\rm BN}{}^{(p')}(\bar{y},w|A_1)\cdots G_{\rm BN}{}^{(p)}(u,\bar{x}|A_2)_{A_1=A_2=0}\,, \quad (2)$$

where

$$\frac{\delta}{\delta A_1}\Delta_c\frac{\delta}{\delta A_2}\equiv\sum_\mu\int\int\frac{\delta}{\delta A_\mu{}^{(1)}(z_1)}\Delta_c(z_1-z_2)\frac{\delta}{\delta A_\mu{}^{(2)}(z_2)}d^yz_1d^yz_2\,,$$

and only the $\delta_{\mu\nu}\Delta_c$ part of the NVM propagator has been retained. The $G_{\rm BN}$ denote once-amputated, mass shell, Bloch-Nordsieck Green's functions, corresponding to the propagation of a very heavy nucleon in a fictitious, external c-number potential $A_\mu(z)$. These Green's functions may be given by Eqs. (9) of Ref. 7,

$$\int dx\, e^{ip\cdot x}G_{\rm BN}{}^{(p)}(u,\bar{x}|A)\bigg|_{\rm m.s.}=e^{ip\cdot u}\exp\left[ig\int_0^\infty d\xi\, p_\mu A_\mu(u-\xi p)\right], \quad (3a)$$

$$\int dy\, e^{-ip'\cdot y}G_{\rm BN}{}^{(p')}(\bar{y},w|A)\bigg|_{\rm m.s.}=e^{-ip'\cdot w}\exp\left[ig\int_0^\infty d\xi\, p_\mu' A_\mu(w+p'\xi)\right], \quad (3b)$$

where m.s. means "mass shell"; and since they are not more complicated than the exponential of a linear form in A_μ, the functional differentiation operation of (2) may be carried through immediately, with the result

$$M_{\mu\nu}=-\delta(q+p-q'-p')\int d^4z\,[\exp\mathfrak{F}(z)]\times\bar{u}(p')\gamma_\mu S_c(-z)\gamma_\nu u(p)e^{i(p+q)\cdot z}\,, \quad (4)$$

where

$$\mathfrak{F}(z)=-i\frac{g^2}{(2\pi)^4}(p\cdot p')\times\int\frac{d^yk}{k^2+\mu^2-i\epsilon}\,\frac{e^{ik\cdot z}}{(k\cdot p-i\epsilon)(k\cdot p'-i\epsilon)}\,. \quad (5)$$

We shall require these quantities in the forward direction only, with $q=q'$, $p=p'$. The function of (5) has been discussed in some detail in Ref. 8, where an argument is made to the effect that, at very high scattering energies, the μ^2z^2 dependence of $\mathfrak{F}((\mu/m)z\cdot p,\mu^2z^2)$ may be neglected. A similar situation exists here, in the limit of very large ν/m and away from the elastic region: If one introduces parametric representations for the $(\mu/m)z\cdot p$ and μ^2z^2 dependence of $\mathfrak{F}$, integrates exactly over $\int d^4z$, and then passes to the limit $m/\nu\rightarrow 0$, for $x<1$, *under* the remaining parametric integrals, one finds the same result that follows by simply neglecting the μ^2z^2 dependence of $\mathfrak{F}$. This "restriction to the light cone" comes about because $-(p+q)^2$ is very large in the deep inelastic region, and $p+q$ is the 4-momentum conjugate to z. While not introducing any infrared divergence, this approximation does shrink to zero the gap which must exist[11] between the elastic peak and the onset of the inelastic continuum in the functions $W_{1,2}$, and is the first of two main reasons why the calculation cannot be trusted in the immediate vicinity of the elastic peak. The only reason for introducing the approximation of neglecting the μ^2z^2 dependence of $\mathfrak{F}$ at this stage is that the calculation is then simplified considerably.

The structure functions are themselves defined by $W_{1,2}=(1/\pi)\,{\rm Im}T_{1,2}$, where $T_{1,2}$ denote the $q^2>0$ continuation of the Compton amplitudes,

$$M_{\mu\nu}=\delta(q+p-q'-p')T_{\mu\nu}\,,\quad T_{\mu\nu}=\delta_{\mu\nu}T_1+(p_\mu p_\nu/m^2)T_2$$

plus terms proportional to q_μ and/or q_ν. Current conservation may be preserved while continuing to spacelike values of q^2 by simply dropping the q_μ and/or q_ν terms, replacing $\delta_{\mu\nu}$ by $\delta_{\mu\nu}{}^T=\delta_{\mu\nu}-q_\mu q_\nu/q^2$ and p_μ by $p_\mu{}^T=p_\mu-q_\mu(q\cdot p)/q^2$; this is equivalent to adding into the original form terms proportional to q_μ and/or q_ν so that the properties $q_\mu T_{\mu\nu}=q_\nu T_{\mu\nu}=0$ are maintained. The function $S_c(-z)$ should denote the complete, renormalized proton propagator, which we later approximate in terms of a sum over the proton pole and all (narrow-width) resonances of the γp channel. No structure has been introduced to represent the proton's electromagnetic vertices, which are of course more complicated than those of a simple γ_μ. Hence when we calculate the structure functions there will always be present a "point charge" contribution incorrectly overestimating the complete elastic form factors, as well as an overestimate of the functions in the continuum near $x=1$. The model can be extended to correct this imbalance, but it is a somewhat ticklish matter, which will be deferred to a separate attempt[12]; here, we follow the simplest path of dropping such purely elastic terms and expecting the model to be relevant only in the deep inelastic region.

Introducing a convenient representation for

$$\mathfrak{F}((\mu/m)z\cdot p,0)\equiv\mathfrak{F}((\mu/m)z\cdot p)\,,$$

$$\exp\mathfrak{F}((\mu/m)z\cdot p)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}d\lambda\,\exp\mathfrak{F}(\lambda)\int_{-\infty}^{+\infty}da\,e^{-ia\lambda}e^{ia(\mu/m)z\cdot p}\,,$$

[11] We are indebted to D. R. Yennie for this remark.

one obtains the expressions for $T_{1,2}$:

$$T_{1,2}(q,p)=\frac{1}{2\pi}\int d\lambda\,\exp\mathfrak{F}(\lambda)\int da\,e^{-ia\lambda}T_{1,2}{}^{(H)}(q,p\xi)\,,\quad (6)$$

where $T_{1,2}{}^{(H)}$ denotes the "hard" part of each amplitude,

$$T_1{}^{(H)}(q,p\xi)=(\nu+m\xi-m_R)D(s,\xi)^{-1}\,,\quad (7)$$

$$T_2{}^{(H)}(q,p\xi)=2\xi mD(s,\xi)^{-1}\,,\quad (8)$$

with $\xi=1+a\mu/m$ and $D(s,\xi)=m_R^2+(q+\xi p)^2$. Equations (7) and (8) should properly be summed over all resonance contributions $m_R\geqslant m$, with appropriate weighting factors; we return to this point below.

With the aid of the integrals

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}d\lambda\,\exp\mathfrak{F}(\lambda)\int_{-\infty}^{+\infty}da\,e^{-ia\lambda}\{1;\xi\}D(s,\xi)^{-1}$$
$$=\frac{ir}{2m^2\eta_R}\int_0^\infty d\lambda\Bigg\{\{1;\eta_R-\nu/m\}\times\exp\left[\mathfrak{F}(\lambda)-i\lambda r\left(\eta_R-\frac{\nu}{m}-1\right)\right]$$
$$+\{1;-\eta_R-\nu/m\}\times\exp\left[\mathfrak{F}^*(\lambda)-i\lambda r\left(\eta_R+\frac{\nu}{m}+1\right)\right]\Bigg\}\,,\quad (9)$$

expressions for $T_{1,2}$ may be obtained from (6)–(8). Here, $r=m/\mu$ and the quantity η_R is given by $m^{-1}[\nu^2+q^2+m_R^2]^{1/2}$ and enters in the combinations $X_R{}^{(\pm)}=\eta_R\pm\nu/m$. {The variable $X_R{}^{(-)}$ ranges between the deep inelastic limiting value of $(\nu/m)\times[(1+m_R^2/\nu^2)^{1/2}-1]$, and the quasi-elastic value of 1, where the missing mass equals the resonance mass.} In principle it is necessary to sum over any arbitrarily large m_R in this channel; but if the relative coupling, or weighting, of such dependence decreases as the resonance mass increases, the terms with $m_R\gtrsim\nu$ may give little contribution. Without any justification other than simplicity, we here neglect all such dependence, taking $m_R=m$ inside η; thus we write $X_R{}^{(\pm)}=X^{(\pm)}$ with $X^{(+)}\sim 2\nu/m$, $X^{(-)}\sim x+m/2\nu$ in the large ν/m, ν^2/q^2 limit, and replace the summation over all resonant states by a multiplicative factor Σ, which may be expected to be of the order of three (by a comparison of the strength of the experimental elastic and quasi-elastic peaks), and whose numerical value is later adjusted to fit the data. Since $T_1{}^{(H)}$ contains a numerator factor of m_R, one might expect this approximation to be rather drastic for W_1.

The function $\mathfrak{F}(\lambda)$ may be represented by $\mathrm{Im}\mathfrak{F}(\lambda)=\frac{1}{2}\pi pe^{-\lambda}$ and the parametric statement

$$\mathrm{Re}\mathfrak{F}(\lambda)=-p\int_0^\infty\frac{zdz}{z^2+1}\cos(z\lambda)\,,$$

where $p=g^2/4\pi^2$. One then finds

$$\nu W_2=\frac{\Sigma\nu}{\pi\mu\eta}\int_0^\infty d\lambda\,\exp[\mathrm{Re}\mathfrak{F}(\lambda)]\times\{X^{(-)}\cos[r\lambda(1-X^{(-)})+\tfrac{1}{2}\pi p\,e^{-\lambda}]-X^{(+)}\cos[r\lambda(1+X^{(+)})+\tfrac{1}{2}\pi p\,e^{-\lambda}]\}\quad (10)$$

and

$$2mW_1=\frac{\Sigma}{\pi\mu\eta}\int_0^\infty d\lambda\,\exp[\mathrm{Re}\mathfrak{F}(\lambda)]\times\{(m\eta-m_R)\cos[r\lambda(1-X^{(-)})+\tfrac{1}{2}\pi p\,e^{-\lambda}]-(m\eta+m_R)\cos[r\lambda(1+X^{(+)})+\tfrac{1}{2}\pi p\,e^{-\lambda}]\}\,.\quad (11)$$

These integrals are not quite trivial. They both contain the badly overestimated elastic peak contribution, proportional to $\delta(X^{(-)}-1)$, which can be seen arising from the oscillations of the $X^{(-)}$ dependence as λ becomes very large. We remove these terms by subtracting from the $X^{(-)}$ dependence of (10) and (11) their values at $p=0$:

$$\nu W_2\to\frac{\nu r}{\pi m\eta}\Sigma\int_0^\infty d\lambda\,\exp[\mathrm{Re}\mathfrak{F}(\lambda)]\times X^{(+)}[\sin(\lambda r+\tfrac{1}{2}\pi p\,e^{-\lambda})\sin(\lambda rX^{(+)})-\cos(\lambda r+\tfrac{1}{2}\pi p\,e^{-\lambda})\cos(\lambda rX^{(+)})]$$
$$+\frac{\nu r}{\pi m\eta}\Sigma\int_0^\infty d\lambda\,X^{(-)}\{\exp[\mathrm{Re}\mathfrak{F}(\lambda)]\times\cos[\lambda r(1-X^{(-)})+\tfrac{1}{2}\pi p\,e^{-\lambda}]-\cos[\lambda r(1-X^{(-)})]\}\,,\quad (12)$$

with a similar expression for mW_1. In the limit of $m/\nu\ll 1$, the two terms of (12) exhibit qualitatively different behavior. If $X^{(+)}\sim 2\nu/m$, only small values of that term's integration variable λ can be important, and it is permissible to make the replacements $\mathrm{Re}\mathfrak{F}(\lambda)\sim p\ln\lambda$, $\sin\frac{1}{2}\pi p\sim\sin(\frac{1}{2}\pi pe^{-\lambda}+\lambda r)$, and $\cos\frac{1}{2}\pi p\sim\cos(\frac{1}{2}\pi pe^{-\lambda}+\lambda r)$; the resulting integral can then be evaluated and yields

$$\Sigma\frac{\sin\pi p}{\pi}\Gamma(1+p)\left(\frac{m}{2r\nu}\right)^p.\quad (13)$$

The second term of (12), on the other hand, must vanish linearly as $X^{(-)}\sim x+m/2\nu$ becomes small. In the small-x portion of the deep-inelastic limit, this term is of order m/ν, so that the entire contribution to νW_2 is due to (13). Experimentally, the rough constancy of the very deep inelastic measurements requires a small p, and it is reassuring to observe that treating the SVNVM exchanges as ρ^0 exchange suggests a value of $p\lesssim 1/\pi$. The corresponding contribution due to soft-pion exchange, with a much larger value of p, would not even be seen at large ν/m. If $0.2<p<0.3$, a doubling of ν will just result in a diminution of (13) by a factor ~ 0.8, a numerical variation not incompatible with the accuracy of the present data. For such small p, we will replace (13) by $p(m/2r\nu)^p$. Note that it would be wrong (by at least a factor of 2) to replace this quantity by just p, its leading term in a perturbation expansion in powers of p, since the validity of such an expansion would require $p \ll [\ln(2r\nu/m)]^{-1}$.

Because the parameter p is small, the contribution of (13) is effectively (at present energies) a constant, and hence the fact that it does not scale is irrelevant. The property of scaling is exhibited by the $X^{(-)}$ term of (12), which is essentially a function of x only for $1>x>m/2\nu$; this term describes the approach to the deep inelastic limit. Numerical evaluation of the integral is a rather delicate affair because great care is necessary in combining the oscillatory factors of the integrand, while simultaneously performing a numerical integration to obtain $\mathrm{Re}\mathfrak{F}(\lambda)$. Rather, we have here employed a perturbation expansion, in powers of p, to carry out the integrals of the $X^{(-)}$ dependence in closed form. There is no reason to suspect the perturbation expansion of this term, and, in fact, the corrections of order p^2 have been computed and appear to change the answer below only slightly. In this way one finds for the entire expression (12) the approximate form

$$\nu W_2 \simeq p\Sigma\left\{\left(\frac{m}{2r\nu}\right)^p + f(X^{(-)})\right\},$$

$$f(z) = -z(1-z)[1/r^2+(1-z)^2]^{-1}. \qquad (14)$$

In Fig. 2 we have superimposed on the existing data our curves of (14) for $\nu/m=5$ (the upper curve of each pair) and for $\nu/m=10$ (the lower member of each pair) using the parameters $r=1.3$, $p=0.26$, $\Sigma=4.3$ (dashed curves), and $r=1$, $p=0.2$, $\Sigma=3.5$ (solid curves). The first set of γ, p values is what one would expect from a model of soft ρ^0 mesons only, while the second set of γ, p values has been chosen to give a slightly better fit at large ω. In both cases, the accurately known data (plotted assuming $R=0$) is bracketed by the curves of (14), as in Figs. (2a) and (2b).

It may be noted that, as ν/m increases, our expression for νW_2 tends to go negative; this tendency is enhanced for small μ, and in the limit $\mu \to 0$ and/or $m/\nu \to 0$ (14) is negative for all $\omega>1$. Since νW_2 must be positive, the approximations which have been made in passing to (14) are at fault. In the elastic region, the replacement of $\mathfrak{F}((\mu/m)z,p,\mu^2z^2)$ by $\mathfrak{F}((\mu/m)z,p,0)$ removes the gap between $x=1$ and the onset of the inelastic continuum [which should occur at $x=1-\mu(\mu+2m)/2m\nu$] and does not, for finite ν, treat the small-μ limit properly. In the large-ω deep-inelastic region, however, the value of μ should be irrelevant, and a calculational procedure which treats both terms of (12) in the same way should yield a positive answer. If we suppose, for example, that p is extremely small, so that a perturbation expansion of both terms of (12) is permissible, the curly bracket of (14) will be replaced by the combination $f(X^{(-)})+f(-X^{(+)})$, which, for large ν/m and in the worst case of $\mu=0$, is positive for all $\omega>2$. Further, a more refined calculation should include the higher-resonance quasi-elastic peaks, neglected above, which move out of the deep-inelastic continuum as ν increases, and tend to maintain the positivity of the cross section. In the light of these remarks it is perhaps fortunate to find that the curves corresponding to the crude approximations of (14) do actually resemble and bracket the deep-inelastic data.

An analogous calculation can be carried through for mW_1,

$$2mW_1 \simeq p\Sigma\left\{r(rX^{(+)})^{-1-p}\left(1+\frac{m_R}{m\eta}\right) + \left(1-\frac{m_R}{m\eta}\right)(X^{(-)})^{-1}f(X^{(-)})\right\}, \quad (15)$$

which may then be used to compute

$$R=(W_2/W_1)(1+\nu^2/q^2)-1.$$

In the limit $m/\nu \to 0$, $x\neq 0$, a comparison of (14) and (15) yields $\nu W_2 \sim 2mxW_1$, or $R\sim q^2/\nu^2$, the same value found in Ref. 4, and in agreement with experiment. However, the derivation here is open to question because of the tendency for both νW_2 and mW_1 to become negative for extremely large ν. At smaller values of ν, (15) is simply not adequate since, for $m_R=m$, it produces negative values for mW_1 over most of the inelastic region, and a more accurate evaluation is required.

III. SUMMARY

Of the various simplifying assumptions employed in this calculation, probably the most severe is the overestimate of the elastic region, which is unavoidable if no attempt is made to use the correct form factors; this difficulty is closely related to questions of normalization (low-energy theorems) and will be treated separately.[12] If, in the deep-inelastic region, the tendency of the data points to drop as x decreases persists, the model will need further refinement, probably in the direction of taking the resonance sum into account in a more accurate way. A good numerical evaluation of the relatively simple expression (12) would be welcome. Nevertheless, in spite of the crudeness of the computation, it is gratifying to be able to employ SVNVM exchange to produce a model of νW_2 which qualitatively agrees with the deep-inelastic data.

ACKNOWLEDGMENTS

We would like to thank numerous colleagues for useful discussions, and especially to thank H. Kendall

[12] H. M. Fried and Hector Moreno, Phys. Rev. Letters **25**, 625 (1970).

for kindly sending reams of data, with error bars. B. Kirby was of great assistance in all matters of computation. This calculation was originally begun when one of us (H. M. F.) was a guest at Queen Mary College, London, and at C.N.R.S., Marseille, and it is a pleasure to thank these Institutions for their hospitality and support. One of us (T. K. G.) would like to thank J. C. Polkinghorne for extending the hospitality of D.A.M.T.P., Cambridge, and to express appreciation for the award of a NATO postdoctoral fellowship.

PHYSICAL REVIEW D VOLUME 3, NUMBER 1 1 JANUARY 1971

Fifth Interaction and Baryon Masses

S. Nussinov and Amir Schorr
Department of Physics and Astronomy, Tel-Aviv University, Tel-Aviv, Israel

AND

Y. Ne'eman*
Department of Physics and Astronomy, Tel-Aviv University, Tel-Aviv, Israel
and
Center for Particle Theory, University of Texas, Austin, Texas

(Received 20 August 1970)

We determine the fifth-interaction coupling from large-angle high-energy nucleon-nucleon scattering, and extract the D/F value for baryon matrix elements of a current-current product from hyperon decays. Assuming symmetric-nonet conserved vector current, we predict the baryon octet D/F value. The result fits observations to within 30%. The actual baryon masses fix $\sin\phi \approx \frac{1}{9} - \frac{1}{12}$ for the eighth-component admixture in the fifth-interaction current.

INTRODUCTION

SEVERAL authors[1–4] have suggested that at least part of $SU(3)$ symmetry breaking may be due to the "fifth interaction" described by the Hamiltonian[5]

$$H^V(x) = g_V^2 j_\mu{}^V(x) j_\mu{}^{V\dagger}(x), \qquad (1)$$

where

$$j_\mu{}^V(x) = \cos\phi\; j_\mu{}^0(x) + \sin\phi\; j_\mu{}^8(x). \qquad (2)$$

Since no external weak probe (such as the photon or a lepton pair) coupling to $j_\mu{}^V(x)$ is known, it is quite difficult to verify the existence of this interaction.

However, a particular feature of the interaction—the fact that in the nomenclature of the complex J plane it corresponds to a fixed pole—suggests a possible approach to its isolation. In the elastic scattering of particles with which j_μ couples, the Born term generated by H_V should be dominant for sufficiently large s and t. Here the usual Regge contributions, decaying exponentially in t, are quite negligible. In this region one expects for p-p scattering

$$\lim_{s\to\infty;\; -t\to\infty} \frac{dG}{dt}(s,t) \sim G_{Mp}{}^4(t), \quad \left(\frac{s}{t}\ \text{large}\right) \qquad (3)$$

where G_{Mp} is the proton's measured magnetic form factor.[6]

The indication that experimental data may approach the limit of Eq. (3) caused Abarbanel, Drell, and Gilman[7] to suggest that a current×current interaction might be responsible. It has been conjectured by one of us[3,4] that this interaction is identical with the "fifth interaction" H^V of Eq. (1). In this paper we base ourselves on this hypothesis, as well as on the value of g_V^2 as determined by Abarbanel *et al.*, and on Suzuki and Sugawara's[8] analysis of nonleptonic decays, to compute the mass splittings of baryons.

PROCEDURE

The Born term, derived from an effective $j\times j$ Hamiltonian, was written by Abarbanel *et al.* as follows:

$$T_{NN}{}^V = \bar{g}^2 G^2(t) \bar{u}(p_2{}')\gamma_\mu u(p_2) \bar{u}(p_1{}')\gamma^\mu u(p_1). \qquad (4)$$

* Research sponsored by the U. S. Air Force Office of Scientific Research, Office of Aerospace Research, under AFOSR Grant No. EOOAR-68-0010, through the European Office of Aerospace Research.

[1] M. Gell-Mann and Y. Ne'eman, *The Eightfold Way* (Benjamin, New York, 1969), p. 297.

[2] Y. Ne'eman, Physics 1, 203 (1965).

[3] Reference 1, p. 282.

[4] Y. Ne'eman, *Algebraic Theory of Particle Physics* (Benjamin, New York, 1967), pp. 103–105; Phys. Rev. **172**, 1818 (1968).

[5] Originally a $j_\mu{}^V(x)B^\mu(x)$ interaction was suggested. It is very likely, however, that m_B, the mass of the boson B, is large, so that we can use the effective local form of Eq. (1). [Some lower bounds on m_B have been established by D. Beder, R. Dashen, and S. Frautschi, Phys. Rev. **136B**, 1777 (1964)]. In principle, a large but finite m_B would be indicated if at very large t an additional factor $[m_B{}^2/(t-m_B{}^2)]^2$ would be needed on the right-hand side of Eq. (3).

[6] T. T. Wu and C. N. Yang, Phys. Rev. **137B**, 708 (1965).

[7] H. D. Abarbanel, S. D. Drell, and F. J. Gilman, Phys. Rev. **177**, 2458 (1969).

[8] M. Suzuki, Phys. Rev. Letters **15**, 986 (1965); H. Sugawara, *ibid.* **15**, 870 (1965).