Conspiracy versus Regge Cuts in $\pi^- p$ Charge Exchange*

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(Received 29 June 1970)

A phenomenological study of $\pi^- p$ charge-exchange differential cross section and polarization is made using the ρ plus a conspiring ρ' and using the ρ plus an absorptive cut. In both cases the Regge residues are taken to have the Veneziano form const/$\Gamma(\alpha)$. The conspiracy model is found to agree with the data considerably better than the cut model.

I. INTRODUCTION

HIGH-ENERGY phenomenological calculations have been made using two different models: (1) the conspiracy model which consists of leading plus conspiring Regge trajectories[1-3]; (2) the Regge-pole-cut model with leading Regge trajectories plus absorptive cuts.[4,5] Since the reaction $\pi^- p \to \pi^0 n$ only involves the quantum numbers of the ρ in the t channel, this reaction should give a useful phenomenological comparison of the two models.

In the conspiracy model, using Veneziano-type residues, it has been found[2,3] that the model gives a good agreement with the experimental data.[6,7] A detailed calculation involving the cut model has also been obtained in Refs. 4 and 5. However, in order to calculate the cut integral analytically, the authors use Regge formulas of nonconventional type and obtain a least-χ^2 value which is considerably larger than the conspiracy model. Therefore it would be interesting to use Veneziano-type residues also in the cut model in the hope of improving agreement with the data.

The purpose of this paper is to present a comparison of the conspiracy and the cut models using the same form for the Regge-pole expressions. The cut integral is calculated numerically in order to incorporate Veneziano-type residues. It is found here that the Veneziano-type residues improve the cut model significantly, but still the experimental data strongly favor the conspiracy model.

* Research supported in part by the U. S. Air Force Office of Scientific Research, Office of Aerospace Research, under Grant No. AF-AFOSR-1294-67.

† National Defense Education Act Title IV Graduate Fellow.

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II. DESCRIPTION OF MODELS

In the conspiracy model, the amplitudes are given by[2,3]

$$A'=A_\rho'+A_{\rho'}', \quad B=B_\rho+B_{\rho'}.$$

The ρ amplitudes are

$$A_\rho'=\frac{\beta_\rho{}^n\xi_\rho}{\Gamma(\alpha_\rho)}(as)^{\alpha_\rho}, \tag{1}$$

$$B_\rho=\frac{\beta_\rho{}^f\xi_\rho}{\Gamma(\alpha_\rho)}(as)^{\alpha_\rho-1}, \tag{2}$$

where $\xi_\rho=(1-e^{-i\pi\alpha_\rho})/\sin\pi\alpha_\rho$ and where the ρ trajectory is given by

$$\alpha_\rho=0.5+at, \quad a=0.9\ \mathrm{GeV}^{-2}.$$

The ρ' amplitudes are given by

$$A_{\rho'}'=\frac{t\beta_{\rho'}{}^n\xi_{\rho'}}{\Gamma(\alpha_{\rho'})}(as)^{\alpha_{\rho'}}, \tag{3}$$

$$B_{\rho'}=\frac{\beta_{\rho'}{}^f\xi_{\rho'}}{\alpha_{\rho'}\Gamma(\alpha_{\rho'})}(as)^{\alpha_{\rho'}-1}. \tag{4}$$

The factors t and $\alpha_{\rho'}$ in Eqs. (3) and (4) have already been suggested by another paper.[3] The ρ' trajectory is given by

$$\alpha_{\rho'}=-0.02+at, \quad a=0.9\ \mathrm{GeV}^{-2}.$$

There are four residue parameters $\beta_\rho{}^n$, $\beta_\rho{}^f$, $\beta_{\rho'}{}^n$, and $\beta_{\rho'}{}^f$, which are varied.

In the absorptive Regge-pole-cut model the amplitude is the sum of the ρ Regge pole plus an absorptive cut; that is,[4,5]

$$M_{\mu'\mu}=M_{\mu'\mu}{}^\rho+\lambda_{\mu'\mu}M_{\mu'\mu}{}^{\mathrm{cut}}, \tag{5}$$

where

$$M^{\mathrm{cut}}=\frac{-i}{32\pi^2}\int d\Omega\, M_{\mu'\mu}{}^\rho M_{\mu'\mu}{}^{\mathrm{el}}.$$

The differential cross section and polarization are given by

$$\frac{d\sigma}{dt}=\frac{|M_{++}|^2+|M_{+-}|^2}{64\pi q^2 s}$$

and

$$P=\frac{2\,\mathrm{Im}(M_{++}M_{+-}{}^*)}{|M_{++}|^2+|M_{+-}|^2}.$$

The subscripts μ' and μ are s-channel helicities.

TABLE I. Least χ^2 for conspiracy and Regge pole-cut models.

Assumptions[a]		Number of parameters	χ^2(total) 109 points	$\chi^2(d\sigma/dt)$ 84 points	χ^2(pol.) 25 points
$\rho+\rho'+\text{cuts}^V$	(a) λ's$\gtrless 0$	8	124	90	34
	(b) λ's>0	8	136	102	34
$\rho+\rho'^V$		4	143	111	32
$\rho+\text{cuts}^V$	(a) λ's$\gtrless 0$	4	257	122	135
	(b) λ's>0	4	257	122	135
ρ (alone)^V		2	364	199	165
$\rho+\text{cuts}^n$	(a) λ's$\gtrless 0$	4	363	311	52
	(b) λ's>0	4	367	317	50
ρ (alone)^n		2	1042	877	165

[a] Veneziano-type residues of form $1/\Gamma(\alpha)$ are denoted by the superscript V, and nonconventional residues by the superscript n. See Ref. 4.

Following Ref. 4, we reduce the double integral in Eq. (5) to a single integral by using the following approximation for the elastic scattering amplitude:

$$M_{\mu'\mu}^{\text{el}}=-\delta_{\mu'\mu}(i+\rho)s\sigma_T e^{At/2},$$

where ρ is the ratio of the real to the imaginary part of the forward πN peak, and σ_T is the πN total cross section. The experimental values used (see Ref. 5) are $\rho=0$, $\sigma_T=25$ mb, and $A/2=3.75$ GeV^{-2}.

The integral then reduces to

$$M_{\mu'\mu}^{\text{cut}}=-(\sigma_T/4\pi)(1-i\rho)e^{At/2}\times\int_{-\infty}^{0}\frac{dt'}{2}e^{At'/2}I_n(A(tt')^{1/2})M_{\mu'\mu}^{\rho}(t'),\quad(6)$$

where

$$n=|-\mu'+\mu|$$

and

$$I_n(Z)=(-i)^n J_n(iZ).$$

The amplitudes A' and B are related to M_{++} and M_{+-} by

$$M_{++}^{\rho}=2m(1+t/4q^2)^{1/2}\left[A_\rho'+\left(\omega-\frac{\omega+t/4m}{1-t/4m^2}\right)B_\rho\right]\quad(7)$$

and

$$M_{+-}^{\rho}=\frac{(-t)^{1/2}}{q}\left\{EA_\rho'+\left[m(s^{1/2}-E)-E\frac{\omega+t/4m}{1-t/4m^2}\right]B_\rho\right\},\quad(8)$$

where

$$E=\frac{s+m^2-\mu^2}{2s^{1/2}},\quad\omega=\frac{s-m^2-\mu^2}{2m},$$

and m = mass of nucleon, μ = mass of π.

III. RESULTS AND DISCUSSION

Equations (7) and (8), in which A' and B are given by (1) and (2), were substituted into Eq. (6) and numerically integrated. This gives four parameters for the cut model. They are two residue constants β_ρ^n and β_ρ^f and two cut parameters λ_{++} and λ_{+-}. We have considered two cases: (a) where the λ's were unrestricted; (b) where they were restricted to positive values. The experimental data used here are identical to those in Ref. 3. The data consisted of 84 $d\sigma/dt$ points at laboratory momentum of 4.83–18.2 GeV/c and t values out to -3 GeV^2. Also included were 25 polarization points at laboratory momenta 5.9 and 11.2 GeV/c. The fit to the data of the conspiracy model is quite similar to those given in Ref. 3. The χ^2 results for the two models are given in Table I. There we also give the χ^2 values where Regge cuts (for both ρ and ρ') were added to the conspiracy model. The resulting χ^2 in this case did not differ significantly from the conspiracy model. For completeness the χ^2 values are given for the nonconventional amplitudes (see Ref. 4) and when only the ρ term is considered. The parameter values obtained from the least-χ^2 fit are given in Table II.

It should be noted that the values of the λ obtained here differ from $\lambda=1$ considerably. Furthermore if we make the restriction $\lambda=1$, the $\rho+\text{cuts}^V$ model (see Table I) would be essentially the same as the cut model

TABLE II. Parameter values obtained in least-χ^2 fit to data. Energies are in GeV.

Assumptions[a]		β_ρ^n	β_ρ^f	λ_{++}^ρ	λ_{+-}^ρ	$\beta_{\rho'}^n$	$\beta_{\rho'}^f$	$\lambda_{++}^{\rho'}$	$\lambda_{+-}^{\rho'}$
$\rho+\rho'+\text{cuts}^V$	(a) λ's$\gtrless 0$	18.5	212	0.39	-0.47	-23	21.7	-1.8	-5.2
	(b) λ's>0	17.8	202	0.25	≈ 0	-49	81	0.33	≈ 0
$\rho+\rho'^V$		16.5	191	...	...	-46.9	78.1	...	...
$\rho+\text{cuts}^V$	(a) λ's$\gtrless 0$	-22.4	219	1.28	0.06	...	...	...	...
	(b) λ's>0	-22.4	219	1.28	0.06	...	...	...	...
ρ (alone)^V		15.6	228	...	...	...	...	...	...
		γ_{++}	γ_{+-}	λ_{++}	λ_{+-}	Note (see Ref. 4):			
$\rho+\text{cuts}^n$	(a) λ's$\gtrless 0$	14.1	73.7	0.16	1.77	These parameters are fixed	$\rho=-0.2$, $\alpha_0=0.5$, $\alpha_1=0.9$, $E_0=0.165$		
	(b) λ's>0	-21.2	76.1	1.33	1.49				
ρ (alone)^n		-17.2	28.9	...	...				

[a] Veneziano-type residues of form $1/\Gamma(\alpha)$ are denoted by the superscript V, and nonconventional residues by the superscript n. See Ref. 4.

of Ref. 8. Obviously the additional restriction of setting $\lambda = 1$ would make the χ^2 values still larger. In particular the $\rho+\text{cuts}^V$ model would give a χ^2 value even larger than 257. Thus the conspiracy model is also shown to be favored by the data over the weak-cut model of Ref. 8.

In conclusion we find that Veneziano-type residues do improve the agreement of the cut model considerably. However, the data still favor the $\rho+\rho'$ conspiracy model over the various cut models. We would also like to point out that integrating numerically we are able to use any kind of amplitudes. In particular we could have used a suitable Veneziano formula instead of Eqs. (1)–(4). However, in the region where we are using these formulas it would make very little difference whether we used Eqs. (1)–(4) or a full Veneziano amplitude.

Note added in proof. Recently an article[9] has considered the same reaction with a different pole-cut model using a pair of complex-conjugate poles. Although no χ^2 values are given, the fits appear to be as good as the best fit with the absorptive-cut model. However, 11 free parameters were needed as opposed to four free parameters in the present calculations.

ACKNOWLEDGMENT

The author is indebted to Professor A. Ahmadzadeh for suggesting this problem, and for discussion and encouragement.

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PHYSICAL REVIEW D VOLUME 3, NUMBER 9 1 MAY 1971

Asymptotic Behavior of Particle Distributions in Hadron Collisions*

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(Received 22 December 1970)

The distribution functions for the "inclusive" production of N specified particles plus anything else are treated from a J-plane point of view. The variables relevant to the exhibition of the asymptotic behavior of these distributions are chosen during a group-theoretic discussion of the matrix elements involved. After the variables are located in this fashion, a crossed-channel partial-wave analysis is carried out to exploit the $SO(1,3)$ symmetry of the production cross sections, and in the context of this partial-wave structure the multi-Regge asymptotics are presented. Such features as pionization and limiting fragmentation are treated, as are certain phenomena involving the approach to limiting distributions, including the rate of approach and specific dependences on certain variables related to longitudinal momenta. Single- and double-particle production is treated in detail, and then a set of numerical estimates is made for proton-proton collisions with incident lab momenta of about 200–500 GeV/c to give an indication where many of the phenomenological results might be tested. A mathematical appendix is provided for those interested in group theory.

I. INTRODUCTION

THE study of the momentum distribution of selected secondary particles in hadron collisions characterized by $a+b \to (N \text{ detected objects})+(\text{anything else})$ offers the opportunity to probe the detailed structure of hadronic wave functions and provides the hadronic model builder with a source for determining various parameters of the model as well as a direct challenge to the fundamental features of the model itself. The multiplicity of models is easily as great as that of produced particles, and one would like to establish at least a common kinematical framework in which we might examine the individual candidates.

The first task of the present paper is to analyze the differential cross sections for the "inclusive" production[1] of N particles from a group-theoretical point of view, in order to identify variables which may prove useful in the consideration of various dynamical constructs. Essentially we take advantage of the observation that when the undetected particles are summed over in the process $a+b \to 1+2+\cdots+N+\text{anything}$, the differential cross section is related to a piece (but only a piece) of the forward absorptive part of a $(2+N)$-to-$(2+N)$ amplitude. The appropriate symmetry to be exploited in a group-theoretical analysis is then that of the little group of the respective null momentum transfers, namely, $SO(1,3)$, between particles with the same label.

The variables we choose for parametrizing the various momenta, and (by construction) the various little-group elements on which the transition matrix element depends, are not the usual boost in the z direction followed by a three-dimensional rotation; for, although they would be adequate, they do not bring out very clearly many of the interesting features of the secondary distribution. Instead we use a set of parameters strongly suggested by and intimately related to those introduced

* Research sponsored by the U. S. Atomic Energy Commission under Contract No. AT(30-1)-4159.
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[1] This name was introduced by R. P. Feynman in his lecture contained in *High Energy Collisions*, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1969), p. 237.