$0^{\pm}$ Mesons, Equal-Time Commutators, and Symmetry Breaking

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$0^{\pm}$ mesons are studied in connection with equal-time commutators and soft-meson extrapolations whereby values of interesting quantities including $SU(3)\times SU(3)$-violating parameters are found.

I. INTRODUCTION

Much attention has been devoted recently to the study of the properties of the low-lying mesonic states with J^P equal to $0^{\pm}$, $1^{\pm}$, trying to predict, e.g., their masses, widths, and weak-coupling constants.

In this paper we shall be concerned with the properties exhibited by the π, K, and κ mesons. In our approach we shall use $SU(3)\times SU(3)$ charge algebra and the well-defined σ terms obtained through a symmetry-breaking Lagrangian transforming as a $(3,3^*)+(3^*,3)$ representation of $SU(3)\times SU(3)$.[1] The consequences of the use of higher commutators are found and related to the usual assumptions.

We shall discuss the saturation of commutators and the structure of matrix elements in the hard-meson spirit. Only one particle plus the corresponding Z-graph intermediate states will be considered; furthermore, the form factors will be "once subtracted,"[2] i.e., we give up the simplest pole-dominance hypothesis for matrix elements of current divergences.

The pole-dominance hypothesis has been used systematically in Refs. 3 and 4, for instance. However, using the experimentally measurable parameter $\beta=f_K/\sqrt{2}f_\pi f_+(0)=1.23\pm0.03$,[5] pole dominance for every divergence (PD) gives, for the κ mass (m_κ) and width (Γ), the values $m_\kappa=1.107_{+0.064}^{-0.053}$ BeV and $\Gamma=1.143_{+0.331}^{-0.233}$ BeV. These values can be compared with the experimental values found by Trippe *et al.*,[6] $m_\kappa=1.1$–1.2 BeV and $\Gamma\approx400$ MeV, and by Crennell *et al.*,[7] $m_\kappa=1.160$ BeV and $\Gamma=90\pm30$ MeV.

The disagreement between PD and experiment appears clearly and justifies a treatment in the more general framework mentioned above.

The work is divided into six sections as follows. In Sec. II we explain the method in detail, showing the role played by higher commutators. Section III is devoted to elaborate general formulas and discusses constraints and limiting cases. In Sec. IV our formulas are compared with experiment and the best fitting parameters selected. In Sec. V the connection with another work is drawn, and finally in Sec. VI we summarize the conclusions.

II. METHOD

We use the equal-time commutators[8]

$$[F_5{}^3,F^{4+i5}]=\tfrac{1}{2}F_5{}^{4+i5}, \tag{2.1}$$

$$[F_5{}^{4+i5},F^{4-i5}]=F_5{}^3, \tag{2.2}$$

$$[F_5{}^3,F_5{}^{4+i5}]=\tfrac{1}{2}F^{4+i5}, \tag{2.3}$$

and the σ terms given by Gell-Mann, Oakes, and Renner (GMOR),[1]

$$[F_5{}^3,D^{4+i5}]=-\tfrac{3}{4}C[1/(\sqrt{2}-\tfrac{1}{2}C)]D_5{}^{4+i5}, \tag{2.4}$$

$$[F_5{}^{4+i5},D^{4+i5}]=\tfrac{3}{2}[C/(\sqrt{2}+C)]D_5{}^3, \tag{2.5}$$

$$[F_5{}^3,D_5{}^{4+i5}]=-[(\sqrt{2}-\tfrac{1}{2}C)/3C]D^{4+i5}. \tag{2.6}$$

The whole procedure can be described in three steps.

(A) The equal-time commutation relations (ETCR) are sandwiched between the vacuum and the corresponding π, K, κ one-particle states taken at rest ($\mathbf{p}=0$). Thus, only $0^{\pm}$ objects play a role, and we can select and study them without worrying about $1^{\pm}$ parameters. The procedure is advantageous because we can verify better if a determinate set of hypotheses works or not.

No other commutators similar to the above are useful if we do not want to enter into the η-η' mixing problem.

(B) The next step consists in introducing a complete set of intermediate states in the commutator and then truncating the sum rule, retaining only one particle plus the corresponding Z graph.

From a technical point of view, what we are doing is to retain only those intermediate states which permit us to relate the appearing matrix elements to known parameters. This procedure may seem somewhat arbitrary. Similar saturations were done by the pioneers

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[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); S. L. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968); M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

[2] See, for instance, S. G. Brown and G. B. West, Phys. Rev. **168**, 1605 (1968); P. P. Srivastava, Nucl. Phys. **B15**, 461 (1970).

[3] L. Schülke, Nucl. Phys. **B14**, 619 (1969).

[4] Y. Y. Lee, Nuovo Cimento **64**, 474 (1969).

[5] N. Brene as quoted by L. K. Pande, Phys. Rev. Letters **23**, 353 (1969).

[6] T. G. Trippe *et al.*, Phys. Letters **28B**, 203 (1968).

[7] D. J. Crennell *et al.*, Phys. Letters **22**, 487 (1969).

[8] The notation used is the following: The charges F^i are normalized so that $[F^i,F^j]=if^{ijk}F^k$, $[F^i,F_5{}^j]=if_{ijk}F_5{}^k$, with f_{ijk} as defined by Gell-Mann (Ref. 1). Weak-coupling constants are defined through $\langle 0|A_\mu{}^a|\bar{a}(q)\rangle=iq_\mu f_a$, where $f_a=f_{\bar{a}}$, and $\langle 0|V_\mu{}^b|\bar{b}(q)\rangle=f_{\bar{b}}q_\mu$, with $f_b=-f_{\bar{b}}$. In particular, we have

$$\langle 0|V_\mu{}^{4+i5}|\kappa^-\rangle=f_\kappa,\quad \langle 0|A_\mu{}^{4+i5}|K^-\rangle=f_K,$$
$$\langle 0|A_\mu{}^{4+i5}|\pi^-\rangle=f_\pi,\quad \langle 0|A_\mu{}^3|\pi^0\rangle=f_{\pi^0},$$

with $f_\pi=\sqrt{2}f_{\pi^0}$. The notation for matrix elements of current divergences is $\langle a|D^{\bar{b}}|d\rangle=-id^{abd}(t)$, $\langle a|D_5{}^{\bar{b}}|d\rangle=d_5{}^{abd}(t)$, with $t=(p_a-p_d)^2$. We shall use $f_{\pi^0}=94$ MeV.

working on the charge-algebra method,[9] but afterwards the whole procedure was justified through the reformulation of the problem from a dispersive point of view[10] and the use of Regge asymptotic theory.

We think that a similar justification can be used here by employing the dispersive soft-pion-corrections method introduced in an elegant way by Fubini and Furlan[11] and afterwards applied to vertex functions.[12,13] By means of a straightforward extension of this formalism to the soft kaon, one obtains dispersive equations whose convergence is known by arguments given by Björken. Saturating these equations with the nearby poles, we have obtained the same results as those of the saturation procedure above.

(C) The third step consists in using the so-called "once-subtracted dispersion relations" (quadratic smoothness) for the divergences of the matrix elements appearing in the equations. We shall use

$$d^{\pi^0\kappa^-K^-}(t)=m_\kappa^2 f_\kappa \frac{G-H^\kappa(t-m_\kappa^2)}{t-m_\kappa^2}, \qquad (2.7)$$

$$d^{\kappa^-\pi^0K^-}(t)=-m_\pi^2 f_{\pi^0} \frac{G-H^\pi(t-m_\pi^2)}{t-m_\pi^2}, \qquad (2.8)$$

$$d^{\pi^0K^-\kappa^-}(t)=-m_K^2 f_K \frac{G-H^K(t-m_K^2)}{t-m_K^2}, \qquad (2.9)$$

where G is the coupling constant controlling the $\kappa\to K\pi$ decay and $H^{\pi,K,\kappa}$ are the parameters measuring the breaking of pole dominance. The κ width is given by

$$\Gamma=\tfrac{3}{2}(p/m_\kappa^2)G^2/4\pi, \qquad (2.10)$$

where p stands for the pion momentum in the κ rest frame. The above assumption (C) can be related to the characteristics of higher commutators. Let us consider the equal-time commutators $[D_\alpha,D_\beta]$, $[\dot{D}_\alpha,D_\beta]$, and $[\dot{D}_\alpha,\dot{D}_\beta]$.

In any field-theoretical model where the meson fields are proportional to current divergences, the above objects vanish for $\alpha\neq\beta$ (canonical commutation relations). If we repeat the *Ansätze* (A) and (B) for the corresponding once-integrated commutators, interesting results can be drawn about matrix elements of current divergences. From $[D^\alpha,D^\beta]$ we obtain no equations but identities whenever our once-subtracted assumptions are used. Reversing the argumentation, we have proved that the use of $[D^\alpha,D^\beta]=0$ commutators implies quadratic smoothness. Although this is a simple consequence of the usual manipulations, it seems never to have been written explicitly.

From $[\dot{D}_\alpha,D_\beta]$ commutators we obtain, respectively, the equations $H^\kappa+H^\pi=0$, $H^\kappa+H^K=0$, and $H^\pi+H^K=0$ for the same index choice in Eqs. (2.1), (2.2), and (2.3). Therefore the values $H^\pi=H^K=H^\kappa=0$ are obtained when the three equations are used simultaneously.

From the $[\dot{D}_\alpha,\dot{D}_\beta]$ commutators we obtain, respectively, $H^\kappa=0$, $H^\pi=0$, and $H^K=0$, again for the same index choice as in Eqs. (2.1), (2.2), and (2.3).

We have shown how higher canonical commutators impose constraints on the kind of smoothness to be chosen. Fourth-order smoothness arises naturally in the general approach of Ward identities considered by Gerstein, Schnitzer, and Weinberg,[14] but in fact quadratic smoothness has been used in applications. We have proved in our context that quadratic smoothness is equivalent to the use of canonical commutators $[D^\alpha,D^\beta]=0$. Higher commutators imply PD and therefore must be rejected, at least as far as our simple saturation procedure is concerned.

III. APPLICATIONS

Applying (A), (B), and (C) assumptions to the ETCR (2.1), (2.2), and (2.3), we find, respectively,

$$f_K=2f_{\pi^0}f_+(0)+2f_\kappa h_+(0), \qquad (3.1)$$

$$2f_{\pi^0}=2f_Kf_+(0)+2f_\kappa g_+(0), \qquad (3.2)$$

$$f_\kappa=2f_{\pi^0}g_+(0)+2f_Kh+(0), \qquad (3.3)$$

where $f_\pm(t)$, $g_\pm(t)$, and $h_\pm(t)$ are form factors defined by

$$\langle\pi^0(p)|V_\mu^{4+i5}(0)|K^-(k)\rangle =f_+(t)(k+p)_\mu+f_-(t)(k-p)_\mu, \qquad (3.4)$$

$$\langle\pi^0(p)|A_\mu^{4+i5}(0)|\kappa^-(k)\rangle =i[g_+(t)(k+p)_\mu+g_-(t)(k-p)_\mu], \qquad (3.5)$$

$$\langle\kappa^-(p)|A_\mu^{3}(0)|K^-(k)\rangle =i[h_+(t)(k+p)_\mu+h_-(t)(k-p)_\mu]. \qquad (3.6)$$

From (3.1), (3.2), and (3.3) we get the $f_+(0)$, $g_+(0)$, and $h_+(0)$ values

$$f_+(0)=\frac{f_K^2+2f_{\pi^0}^2-f_\kappa^2}{4f_Kf_{\pi^0}}, \quad h_+(0)=\frac{f_\kappa^2+f_K^2-2f_{\pi^0}^2}{4f_Kf_\kappa},$$
$$g_+(0)=\frac{f_\kappa^2+2f_{\pi^0}^2-f_K^2}{4f_\kappa f_{\pi^0}}. \qquad (3.7)$$

The $f_+(0)$ expression is the well-known Glashow-Weinberg formula[1] obtained independently by several people.[15] Our procedure is similar to that of Dahmen, Rothe, and Schülke.

[9] S. Fubini and G. Furlan, Physics **1**, 229 (1967).

[10] See, for instance, G. Furlan, Brandeis University reports, 1967 (unpublished).

[11] S. Fubini and G. Furlan, Ann. Phys. (N. Y.) **48**, 322 (1968).

[12] M. Ademollo, G. Denardo, and G. Furlan, Nuovo Cimento **47A**, 1 (1968).

[13] S. P. de Alwis and S. A. Nutbrown, Nuovo Cimento **58B**, 876 (1968).

[14] I. S. Gerstein, H. J. Schnitzer, and S. Weinberg, Phys. Rev. **175**, 1873 (1968); I. S. Gerstein and H. J. Schnitzer, *ibid.* **185**, 1876 (1968).

[15] H. D. Dahmen, K. D. Rothe, and L. Schülke, Nucl. Phys. **B7**, 472 (1968); Riazuddin, A. Q. Sarker, and Fayyazuddin, *ibid.* **B6**, 515 (1968); P. P. Srivastava, *ibid.* **B7**, 224 (1968).

It is easy to check that a more complicated structure for matrix elements of current divergences would not lead to the Glashow-Weinberg formula (3.7), which is therefore a result of quadratic smoothness. PD approximations are equivalent to $H^\pi = H^K = H^\kappa = 0$ (in fact, two of them give the third one) and then we get the following results:

$$\sqrt{2} f_+(0) = 1 \quad [\text{exact } SU(3) \text{ value}], \tag{3.8}$$

$$\lambda = \xi + 1, \tag{3.9}$$

$$\beta = \lambda, \tag{3.10}$$

$$m_\kappa^2 = (m_K^2 \lambda - m_\pi^2)/(\lambda - 1), \tag{3.11}$$

$$-2 f_{\pi^0} G = (m_K^2 - m_\pi^2)/(\lambda - 1), \tag{3.12}$$

where

$$\lambda = f_K/f_\pi, \quad \xi = f_\kappa/f_\pi, \quad \beta = f_K/\sqrt{2} f_\pi f_+(0).$$

We see that everything can be expressed as a function of the measurable quantity β. Therefore, PD gives very definite predictions to be checked by experiments. Using the β value[5] $\beta = 1.23 \pm 0.03$, we obtain

$$\begin{gathered} \lambda = 1.23 \pm 0.03, \quad \xi = 0.23 \pm 0.03, \\ m_\kappa = 1.107_{+0.064}^{-0.053} \text{ BeV}, \\ \Gamma = 1.143_{+0.331}^{-0.233} \text{ BeV}. \end{gathered} \tag{3.13}$$

As pointed out in the Introduction, PD gives an incorrect value for Γ as compared with the experimental results.[6,7]

Let us now apply our assumptions to the σ commutators (2.4) and (2.5) [the equation arising from (2.6) is not independent]. The resulting equations are

$$-m_\kappa^2 f_\kappa \frac{G}{m_\kappa^2 - m_K^2} - m_\kappa^2 f_\kappa f_{\pi^0} \left(H^\kappa + \frac{m_\pi^2}{m_\kappa^2 - m_K^2} H^\pi \right) = -\tfrac{3}{4} C \frac{1}{\sqrt{2} - \tfrac{1}{2} C} m_K^2 f_K, \tag{3.14}$$

$$m_\kappa^2 f_\kappa f_K \frac{G}{m_\kappa^2 - m_\pi^2} + m_\kappa^2 f_K f_\kappa \left(H^\kappa + \frac{m_K^2}{m_\kappa^2 - m_\pi^2} H^K \right) = \tfrac{3}{2} C \frac{1}{\sqrt{2} + C} m_\pi^2 f_{\pi^0}. \tag{3.15}$$

Subtracting (3.14) and (3.15), we obtain

$$\frac{m_\kappa^2 f_\kappa^2}{2} = -\tfrac{3}{4} C \frac{1}{\sqrt{2} - \tfrac{1}{2} C} m_K^2 f_K + \tfrac{3}{2} C \frac{1}{\sqrt{2} + C} m_\pi^2 f_{\pi^0}. \tag{3.16}$$

Equation (3.14) can be written in a more convenient way:

$$G = (m_\pi^2 - m_K^2) \frac{f_+(0)}{f_\kappa} - m_\kappa^2 \frac{h_+(0)}{f_{\pi^0}} - \tfrac{3}{4} C \frac{1}{\sqrt{2} - \tfrac{1}{2} C} \frac{m_\kappa^2 f_K}{f_\kappa f_{\pi^0}}. \tag{3.17}$$

We have obtained Eqs. (3.1), (3.2), and (3.3) [or, equivalently, (3.7)] and (3.16) and (3.17) as the basic relations. Therefore, we have only two theoretical relations plus an experimental value involving the unknowns G, λ, ξ, C, and m_κ^2.

By searching the experimental m, Γ bounds, one can find zones for λ, ξ, and C. This point will be discussed in Sec. IV.

Solving our equations for two unknowns as a function of the rest, we obtain

$$2 f_{\pi^0} G = -\frac{-Y^{1/2} + m_\pi^2(\xi^2 - \lambda^2) + m_K^2(1 - \xi^2) + m_\kappa^2(\lambda^2 - 1)}{2\xi\lambda}, \tag{3.18}$$

$$C = -\sqrt{2} \frac{3Y^{1/2} + m_\kappa^2 \xi^2 + 3 m_K^2 \lambda^2 - 3 m_\pi^2}{-2 m_\kappa^2 \xi^2 + 6 m_K^2 \lambda^2 + 3 m_\pi^2}, \tag{3.19}$$

where $Y = \lambda(m_\pi^2, m_K^2 \lambda^2, m_\kappa^2 \xi^2)$ and $\xi^2 = 1 + \lambda^2(1 - 2/\beta)$. The symbol $\lambda(a,b,c)$ means $a^2 + b^2 + c^2 - 2ab - 2bc - 2ac$. The sign in front of $Y^{1/2}$ has been chosen to obtain the correct expression in the PD limit.

We now proceed to compare our formula with the pioneering Glashow-Weinberg work.[1] It is easy to find the values of their "renormalization constants" Z_π, Z_K, and Z_κ as a function of the C parameter used by GMOR by realizing that they are essentially u or v expectation values between vacuum and one-particle states. Thus we obtain[16]

$$\begin{gathered} Z_\pi^{1/2} = m_\pi^2 f_\pi \frac{\sqrt{3}}{\sqrt{2} + C}, \quad Z_K^{1/2} = m_K^2 f_K \frac{-\sqrt{3}}{-\sqrt{2} + \tfrac{1}{2} C}, \\ Z_\kappa^{1/2} = m_\kappa^2 f_\kappa \frac{-2}{\sqrt{3} C}, \end{gathered} \tag{3.20}$$

and Eq. (19) of Glashow-Weinberg,

$$m_\pi^2 f_\pi Z_\pi^{-1/2} = m_K^2 f_K Z_K^{-1/2} - m_\kappa^2 f_\kappa Z_\kappa^{-1/2}, \tag{3.21}$$

turns out to be an identity.

Their Eq. (20),

$$f_\pi Z_\pi^{1/2} = f_K Z_K^{1/2} - f_\kappa Z_\kappa^{1/2}, \tag{3.22}$$

is the same as Eq. (3.16) above.[17]

So we see that our procedure includes the original Glashow-Weinberg formulas. In this context PD means $Z_\pi = Z_K = Z_\kappa$, giving, for the C parameter, the value

$$C = -\sqrt{2} \frac{2 m_K^2 \lambda - 2 m_\pi^2}{2 m_K^2 \lambda + m_\pi^2}. \tag{3.23}$$

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[16] We are using the contrary sign convention for f_κ as used by Glashow and Weinberg (Ref. 1).

[17] Glashow and Weinberg obtain the numerical results $\lambda^2 = 1.17$, $\xi^2 = 0.34$, $\sqrt{2} f_+(0) = 0.85$, $m_\kappa \leq 0.670$ BeV, using both Weinberg sum rules. We think that the second Weinberg sum rule must be avoided and in such a case the difficulties arising with the m_κ bound disappear.

As for the original GMOR expression for C,

$$C\approx -\sqrt{2}\frac{2m_K^2-2m_\pi^2}{2m_K^2+m_\pi^2}, \qquad (3.24)$$

we can check, as will be described more clearly later, that our general formula (3.19) agrees with (3.24) in the sense that the predicted C parameters are of the same order, $C\approx -1.2$, but we must remark that formula (3.24) cannot be incorporated as another equation. The approximations giving (3.24) are $Z_\pi=Z_K$ and $f_\pi=f_K$ (see GMOR[1]), i.e., $f_\kappa Z_\kappa^{1/2}=0$ and therefore $f_\kappa=0$. It is obvious that we are disregarding this eventuality.[18]

Returning to the general case, we shall now exhibit some bounds that naturally arise in the model.

If we assume C to be negative and smaller in modulus than $\sqrt{2}$, we immediately obtain the conditions $Z_\pi^{1/2}f_\pi\geq 0$ and $Z_K^{1/2}f_K\geq 0$. From the relation $f_\pi f_K>0$ suggested by $SU(3)$ we obtain the bound first written by Glashow-Weinberg[1,19]:

$$m_\kappa\leq |m_\pi f_\pi - m_K f_K|/|f_\kappa|, \qquad (3.25)$$

where the equal sign arises when $Z_\pi^{1/2}/Z_K^{1/2}=m_\pi/m_K$. From the condition $\xi^2>0$ and the experimental relation $\beta>1$, we obtain the λ bound $\lambda^2<1/(2/\beta-1)$. If the Quinn-Björken[20] condition $\sqrt{2}f_+(0)\leq 1$ holds, we obtain $\lambda\leq\beta$, which is a better bound than the former one. An obvious condition must be, of course, $m_\kappa>m_K+m_\pi$.

We shall now discuss particular conditions, less restrictive than PD, in order to check their implications with experiment. It is noteworthy that whenever a condition of the set written down is considered, the ambiguities arising in the general C and G formulas disappear.

Some of the following particular cases have already been used by other authors in a related framework.

(*a*) *Pole dominance for D^{4+i5} matrix elements, i.e.,* $H^\kappa=0$. This condition has been used first by Chang and Leung[21] in the equivalent form

$$f_K^2+f_\kappa^2-2f_K f_\kappa(Z_K^{1/2}/Z_\kappa^{1/2})=f_\pi^2. \qquad (3.26)$$

We have found that this equation can be expressed in a convenient way by writing the corresponding m_κ formula

$$m_\kappa^2=\frac{m_K^2(\lambda^2-\beta)-\beta m_\pi^2(1-1/\beta)}{(\lambda^2-\beta)(1-1/\beta)}. \qquad (3.27)$$

For the sake of completeness, we also write down the formula for C:

$$C=-\sqrt{2}\frac{2m_K^2(\lambda^2-\beta)-2m_\pi^2(\beta-1)}{2m_K^2(\lambda^2-\beta)+m_\pi^2(\beta-1)}. \qquad (3.28)$$

(*b*) *Pole dominance for matrix elements of D_5^3, i.e.,* $H^\pi=0$. The m_κ and C formulas are

$$m_\kappa^2=\frac{2m_K^2\lambda^2(\lambda^2-\xi^2-1)+m_\pi^2[1-(\lambda^2-\xi^2)^2]}{2\xi^2(\lambda^2-\xi^2-1)}, \qquad (3.29)$$

$$C=-\sqrt{2}\frac{2m_K^2\beta-2m_\pi^2}{2m_K^2\beta+m_\pi^2}; \qquad (3.30)$$

the C formula has been obtained by Fuchs and Kuo.[22]

(*c*) *Pole dominance for D_5^{4+i5} matrix elements, i.e.,* $H^K=0$. The corresponding formulas are

$$m_\kappa^2=\frac{m_K^2[(\xi^2-1)^2-\lambda^4]+2m_\pi^2(\xi^2+\lambda^2-1)}{2\xi^2(\xi^2-\lambda^2-1)}, \qquad (3.31)$$

$$C=-\sqrt{2}\frac{m_K^2(\lambda^2+1-\xi^2)-2m_\pi^2}{m_K^2(\lambda^2+1-\xi^2)+4m_\pi^2}. \qquad (3.32)$$

Any two of the above conditions (a)–(c) are equivalent to PD.

The rest of the conditions considered correspond to the equality of renormalization constants. We have

(*d*) $Z_\pi=Z_K$. The m_κ and C formulas are

$$m_\kappa^2=\frac{m_K^2\lambda(\lambda-1)-m_\pi^2(\lambda-1)}{\xi^2}, \qquad (3.33)$$

$$C=-\sqrt{2}\frac{2m_K^2(\lambda^2-\beta)-2m_\pi^2(\beta-1)}{2m_K^2(\lambda^2-\beta)+m_\pi^2(\beta-1)}. \qquad (3.34)$$

(*e*) $Z_\pi=Z_\kappa$. The formulas are

$$m_\kappa^2=\frac{m_K^2\lambda^2-m_\pi^2(\xi+1)}{\xi(\xi+1)}, \qquad (3.35)$$

$$C=-\sqrt{2}\frac{2m_K^2\lambda^2-2m_\pi^2(\xi+1)}{2m_K^2\lambda^2+m_\pi^2(\xi+1)}. \qquad (3.36)$$

(*f*) $Z_K=Z_\kappa$. The formulas are

$$m_\kappa^2=\frac{m_\pi^2+m_K^2\lambda(\xi-\lambda)}{\xi(\xi-\lambda)}, \qquad (3.37)$$

$$C=-\sqrt{2}\frac{2m_K^2\lambda(\xi-\lambda)+2m_\pi^2}{2m_K^2\lambda(\xi-\lambda)-m_\pi^2}. \qquad (3.38)$$

Of course any two of the above conditions (d)–(f) are equivalent to PD.

[18] Such a procedure has been used by Fayyazuddin and Riazuddin, Phys. Rev. D **1**, 317 (1970).

[19] The same condition arises from the necessary constraint $Y\geq 0$.

[20] H. R. Quinn and J. D. Björken, Phys. Rev. **171**, 1660 (1968).

[21] L. N. Chang and Y. C. Leung, Phys. Rev. Letters **21**, 122 (1968).

[22] N. H. Fuchs and T. K. Kuo, Nuovo Cimento **64A**, 382 (1969).

TABLE I. Scalar-meson parameters and symmetry-breaking parameters are displayed, with β fixed at the value 1.23 and λ running from 1.16 to 1.23.

λ	ξ	m_κ maxima	$\sqrt{2}f_+(0)$	m_κ	C	Γ	$\left(\frac{Z^K}{Z^\pi}\right)^{1/2}$	$\left(\frac{Z^\kappa}{Z^\pi}\right)^{1/2}$	$\frac{\langle u_8\rangle}{\langle u_0\rangle}$
1.16	0.397	1.103	0.943	1.100	−1.010	0.479	3.629	7.032	−0.920
1.17	0.378	1.171	0.951	1.150	−1.075	0.587	2.721	5.733	−0.838
				1.100	−1.143	0.452	2.140	3.975	−0.708
1.18	0.358	1.250	0.959	1.200	−1.120	0.746	2.350	4.950	−0.766
				1.150	−1.164	0.599	1.977	3.721	−0.665
				1.100	−1.192	0.485	1.744	2.953	−0.585
1.19	0.337	1.344	0.967	1.200	−1.187	0.798	1.802	3.397	−0.612
				1.150	−1.208	0.654	1.629	2.786	−0.544
				1.100	−1.224	0.529	1.489	2.322	−0.485
1.20	0.314	1.458	0.975	1.200	−1.225	0.897	1.504	2.563	−0.494
				1.150	−1.236	0.740	1.404	2.183	−0.444
				1.100	−1.246	0.602	1.323	1.871	−0.398
1.21	0.289	1.601	0.983	1.200	−1.250	1.048	1.304	1.999	−0.393
				1.150	−1.258	0.866	1.2402	1.742	−0.355
				1.100	−1.264	0.705	1.190	1.521	−0.320
1.22	0.261	1.790	0.991	1.200	−1.269	1.276	1.157	1.574	−0.304
				1.150	−1.274	1.056	1.118	1.394	−0.276
				1.100	−1.278	0.861	1.084	1.233	−0.250
1.23	0.230	2.054	1	1.200	−1.284	1.646	1.043	1.229	−0.224
				1.150	−1.287	1.363	1.019	1.102	−0.204
				1.100	−1.290	1.111	0.967	0.985	−0.186

IV. NUMERICAL VALUES

Our formulas have been explored maintaining a fixed β value and moving λ in the interval allowed by the above-mentioned bounds. For some given values of β and λ, everything is determined whenever m_κ is known. In Table I the numerical values of the parameters are shown using the mean value $\beta=1.23$ and three different m_x values, i.e., 1.1, 1.15, and 1.2 BeV. As an output, the values of C, Γ, $(Z_K/Z_\pi)^{1/2}$, $(Z_\kappa/Z_\pi)^{1/2}$, $\langle u_8\rangle/\langle u_0\rangle$, and $f_+(0)$ have been considered. $\langle u_{8,0}\rangle$ represents the vacuum expectation value of $u_{8,0}$. Several features can be extracted from an analysis including β from 1.20 to 1.29.

(a) The κ width is a very sensitive quantity as a function of λ in the $\lambda_{\max}$ zone [in general, for $\lambda\gtrsim\lambda_{\min}$, there is a minimum that is not displayed in the table; i.e., $\Gamma=\Gamma(\lambda)$ is a function decreasing as λ increases in a reduced zone near λ threshold and afterwards steadily increasing up to $\lambda_{\max}$].

(b) Γ changes by 30–50% when m_κ is varied from 1.1 to 1.2, the most important effect being in the $\lambda_{\max}$ zone.

(c) C values are approximately -1.2, similar to the GMOR value.

(d) $\sqrt{2}f_+(0)$ values lie very near to 1 ($\simeq$5% error) as compared with 0.85 obtained using both Weinberg sum rules.

(e) No narrow peak can be reached.

(f) When the sign in front of $Y^{1/2}$ in (3.18) and (3.19) is reversed, it is impossible to get Γ values within the bounds allowed by both experiments. Taking the central figure $\beta=1.23$, and $m_\kappa=1.1$ BeV, we see that the best Γ value, $\Gamma=452$ MeV, is obtained for $\lambda=1.17$, For $\beta=1.26$, Γ values $\approx$400 MeV are not yet accessible, but with $\beta=1.20$ we again obtain that region if we select $\lambda\approx1.15$. If m_κ increases, β must decrease in order to obtain the width $\approx$400 MeV.

We see that the model is compatible with the enhancement of Trippe *et al.*,[6] although more precise experimental data are needed in order to choose a well-defined set of theoretical parameters.

Concerning the conditions listed at the end of Sec. II, it turns out that they select solutions $\Gamma=\Gamma(\lambda)$ which are very steep (with an exception to be discussed below). In fact, when the experimental m_κ bounds are taken into account, only λ lying closely to $\lambda_{\max}=\beta$ are acceptable (note $\lambda=\beta$ plus whatever condition considered in Sec. II give PD) and we cannot find a κ width $\approx$400 MeV.

The exception quoted above is the condition (a) first used by Chang and Leung.[21] When we choose $\beta=1.2$, there is a zone where the values are compatible with experiment. For instance, for $\lambda=1.14$, m_κ is 1.115 BeV and $\Gamma\approx400$ MeV. It is the zone discussed above in the general case.

V. COMMENTS

We shall briefly comment in this section on some related work, relating it to the general discussion above.

Schülke[3] (our work was motivated in part from reading his paper) works with the charge-charge commutators picking the same intermediate states as ours and using pole dominance everywhere, thus obtaining PD results. Lee[4] also works in the context of GMOR commutators and obtains the PD model results invoking asymptotic symmetry for the Fourier transforms of $\langle 0|T(u_j(x)u_i(0))|0\rangle$ and $\langle 0|T(v_j(x)v_i(0))|0\rangle$.

Asymptotic symmetry now implies that $Z_\pi = Z_K = Z_\kappa$. Thus Schülke arrives at PD through the condition $H^\kappa = H^K = H^\pi = 0$, while Lee does so from $Z^\pi = Z^K = Z^\kappa$.

Cleymans[23] uses the formulas (19) and (20) of Glashow and Weinberg and finds the κ width in the compatible zone with the experiment of Trippe *et al.*[6] using, instead of Weinberg's sum rules, a pair of relations from a model introduced by Sugawara. We think that the procedure is not meaningful because at least the first Weinberg sum rule is included in the formalism and so must be taken into account.

Chang and Leung[21] introduce the pole-dominance hypothesis of the divergence of the strangeness-changing vector current. We have shown that the experimental data support this conjecture. However, we do not agree with their numerical manipulations because it turns out that their values do not obey Glashow-Weinberg bounds: This discrepancy is caused by the numerical approximations carried out in the formula for m_κ.

Auvil and Deshpande[24] have worked using the approximations $Z^\pi = Z^K = Z^\kappa$. However, in a later work[25] they give up these assumptions and find numerical results analogous to ours.

In the application of the Fubini-Furlan method to vertex functions, pole-dominance approximations have always been used when numerical results have been obtained. So do Ademollo, Denardo, and Furlan[12] and de Alwis and Nutbrown,[12] who use pole dominance for D^{4+i5}. McKay, McKisic, and Wada[26] use in fact, what we have called PD. Denardo, Napolitano, and Soliani[27] employ the higher commutators $[\dot{D}^\alpha, \dot{D}^\beta] = 0$, obtaining a formula for λ as a function of m_κ, which corresponds to our (3.11).

Finally, we shall comment on several reports. Pande[28] has obtained a formula for C similar to ours working with phenomenological Lagrangians. Rodenberg and Zerwas[29] study two-, three-, and four-point functions in the context of the Fubini-Furlan dispersive methods, in fact, they include PD in their formulas. McKisic[30] has also employed the Fubini-Furlan method assuming $Z_\pi = Z_K = Z_\kappa$.

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VI. CONCLUSIONS

In this article we have discussed the properties of scalar and pseudoscalar mesons, emphasizing the role played by the κ meson. Our starting point has been that the hypothesis of pole dominance of matrix elements of any current divergences (PD), when combined with the usual saturation hypothesis of matrix elements of commutators, is not supported by experiment. We have shown that the most convenient framework to describe the actual situation are the hard-meson techniques, specifically the quadratic smoothness (the so-called once-subtracted dispersion relations). This particular form of smoothness arises naturally when our saturation *Ansatz* is applied to the canonical equal-time commutators $[D^\alpha, D^\beta] = 0$.

We have discussed the connections with the Fubini-Furlan soft-pion methods, checking that our direct procedure, when applied to charge-charge and σ commutators, is equivalent to the corresponding results derived from the application of the Fubini-Furlan method extended to soft kaons. This finding supports our procedure and also shows the connection between Fubini-Furlan extrapolations and hard-meson techniques.

As for the experimental situation, we have checked that our procedure is consistent with the broad resonance of Trippe *et al.*[6] rather than with the peak of Crennell *et al.*[7] Furthermore, we have shown that the pole-dominance model for matrix elements of the divergence of strangeness-changing vector current is also consistent with experiment. Values for relevant parameters including the $SU(3)\times SU(3)$-violating parameter C and the K_{l3} form factor $f_+(0)$ are obtained. This shows a system with a Lagrangian quasi-invariant under $SU(2)\times SU(2)$, with a vacuum state breaking both $SU(2)\times SU(2)$ and $SU(3)$ and giving quasi-$SU(3)$ limit values for the charge associated with strangeness-changing vector current.

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