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Finite-Energy Sum Rules and Inelastic Electron Scattering*

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We use finite-energy sum rules (FESR) to analyze the inelastic-electron-scattering data. We show that the sum of resonances build up a scaling function although each resonance contribution falls off according to a dipole formula. This is achieved by assuming that the dipole mass which enters into the form factor increases when we go to higher resonances. The assumption seems to be confirmed by the data. We also investigate the contribution of the Regge trajectories to the FESR, assuming that they all scale. We find that the equations can be satisfied only if there is another contribution of a $J=0$ fixed pole. The residue function of the fixed pole is calculated explicitly and compared with that of the fixed pole found at real Compton scattering. The two residue functions are found to have opposite signs.

I. INTRODUCTION

THE new results on electron scattering off deuterium, presented at the Kiev conference,[1] raised again the question of the consistency of duality and scaling behavior. Duality tells us that the low-energy part of a scattering amplitude builds the high-energy part via finite-energy sum rules (FESR). The low-energy part, however, is dominated by resonances, and in the case of electron scattering their contributions decrease very fast with[2] q^2 (probably like the elastic contribution which obeys the dipole formula[3]), while the high-energy cross section was found to decrease very slowly with q^2 (probably like $1/q^2$ for fixed c.m. energy[4]). We are thus led to an inconsistency, namely, the two sides of the FESR seem to have a completely different q^2 behavior.

So far, the only attempt to explain this inconsistency, is that of Harari,[5] who suggested, in accord with the commonly accepted picture of strong interactions,[6] that the resonances should be related only to the lower-lying trajectories [with $\alpha(0)<1$] and the Pomeranchukon trajectory should be related to the background. This picture therefore suggests that the residue functions of the lower trajectories decrease very rapidly with $Q^2(=-q^2)$ and that the Pomeranchukon (scaling) contribution starts to dominate at rather low Q^2 [around 1 $(\mathrm{GeV}/c)^2$]. This explanation seems to be ruled out by the new data,[1] which seem to indicate that the difference between the *e-p* and *e-n* cross sections (which is dominated by the A_2 trajectory at high energies) does scale. Since this difference is certainly dominated by resonances, at low energies, we are faced again with the above-mentioned difficulty: How can a sum of rapidly falling-off resonances build up a scaling behavior?

We attempt to show, in this paper, that *a dipole behavior for all the resonances' form factors is compatible with scaling* and we suggest that the compensating effect to the rapid decrease of the high power comes from the fact that the higher the resonance, the bigger is the mass of the dipole which dominates its form factor. This suggestion seems to be confirmed by the resonances' data and we also show that it leads to a scaling behavior in a "semilocal" sense; i.e., when integrating the resonances' contributions to $\nu W_2(Q^2,\nu)$ over any fixed interval of the scaling variable, we get a result which does not vanish when $Q^2\to\infty$, although every resonance contribution by itself vanishes like $1/Q^8$ (Sec. II). Apart from checking the consistency of this assumption with the present available data, we discuss some of its further implications for experiment (Sec. III).

Having shown that the resonance contribution to the FESR is compatible with scaling behavior, we go on to examine the Regge-pole side of the FESR. We assume scaling behavior for the $\alpha(0)\simeq0.5$ trajectories as well as for the Pomeranchukon and find that the FESR cannot be satisfied unless there is a $J=0$ fixed pole which couples to the real part of the forward virtual Compton amplitude. We calculate explicitly this fixed-pole contribution, and going to the $Q^2=0$ limit, we try to relate this fixed pole to the one found in the analysis of real forward Compton scattering.[7] We find that the two fixed poles are about equal in absolute magnitude but have opposite signs (Sec. IV).

II. SCALING RESONANCES

The first step toward establishing a relation between the resonances' contributions to $\nu W_2(Q^2,\nu)$ and the scaling curve was made recently by Bloom and Gilman.[8] They have managed to show that by choosing a new scaling variable,

$$\omega'=(2M\nu+M^2)/Q^2=\omega+M^2/Q^2\,, \qquad (1)$$

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[1] E. D. Bloom *et al.*, SLAC Report No. SLAC-PUB-796, 1970 (unpublished).

[2] We follow the notation of J. D. Bjorken, Phys. Rev. **179**, 1547 (1969).

[3] The resonance data can be found in W. K. H. Panofsky, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968); M. Breidenbach, thesis, MIT Report No. MIT-2098-635, 1970 (unpublished).

[4] M. Breidenbach *et al.*, Phys. Rev. Letters **23**, 935 (1969).

[5] H. Harari, Phys. Rev. Letters **22**, 1078 (1969).

[6] See, e.g., H. Harari, lecture notes of the Brookhaven Summer School, 1969 (unpublished).

[7] M. Damashek and F. J. Gilman, Phys. Rev. D **1**, 1319 (1970).

[8] E. D. Bloom and F. J. Gilman, Phys. Rev. Letters **25**, 1140 (1970).

the scaling function $F_2(\omega')$ averages the experimental curve and passes through the resonances's peaks at every Q^2, provided that $Q^2 \gtrsim 1(\text{GeV}/c)^2$. In other words, the following relation between the Q^2-dependent amplitude $\nu W_2(Q^2,\nu)$ and the Q^2-independent scaling function $F_2(\omega')$ should hold for any $Q^2[\gtrsim 1(\text{GeV}/c)^2]$:

$$\frac{2M}{Q^2}\int_{\nu_1}^{\nu_2} \nu W_2(Q^2,\nu)d\nu = \int_{\omega_1}^{\omega_2} F_2(\omega')d\omega', \tag{2}$$

where $\omega_{1,2}$ are related to $\nu_{1,2}$, respectively, via relation (1).

The importance of Eq. (2) [which is a triviality when we send Q^2 to infinity and assume that $\nu W_2(Q^2,\nu)$ does scale] comes from the fact that it should hold for finite values of Q^2, independent of Q^2. We can thus write $W_2(Q^2,\nu)$ as a sum of resonances, insert this form into the left-hand side of Eq. (2) and see under which conditions the result is Q^2 independent. Let us therefore integrate the right-hand side of Eq. (2) between the fixed limits $\omega_1=1$ and $\omega_2=a$, where a is some fixed number. Using Eq. (2), we get that S, defined as

$$S=\frac{2M}{Q^2}\int_{Q^2/2M}^{(aQ^2-M^2)/2M} \nu W_2(Q^2,\nu)d\nu, \tag{3}$$

should be a constant, independent of Q^2. Let us now express $W_2(Q^2,\nu)$ as a sum of resonances (we use the narrow-width approximation for the resonances contributions):

$$W_2(Q^2,\nu)=\sum_n W_n(Q^2)\delta(\nu-(Q^2+M_n^2-M^2)/2M), \tag{4}$$

where M_n is the nth resonance mass and $W_n(Q^2)$ defines its contribution to the amplitude and it is essentially a coupling constant times a form factor normalized to unity at $Q^2=0$:

$$\begin{aligned} W_n(Q^2)&=g_n^2[G_n(Q^2)]^2, \\ G_n(0)&=1. \end{aligned} \tag{5}$$

We are not going to take the background into account since the discussion, which will use expression (4) for the amplitude, remains completely unchanged under either of the following two possibilities.

(1) The background contribution should be added to the resonances, and it scales by itself.

(2) The complete amplitude is given by resonances which also build the background in some (as yet) unknown way, probably via unitarity.

Let us therefore insert the resonances contribution (4) into the integral (3), obtaining

$$S=\sum_{n=0}^{N} [1+(M_n^2-M^2)/Q^2]W_n(Q^2), \tag{6}$$

where N is the number of resonances which can contribute to the sum S. N is determined by the fact that all the arguments of the δ functions appearing in Eq. (4) should be within the integration range. It is therefore the index which corresponds to the highest resonance which can obey the relation

$$M_N^2 \leqslant Q^2(a-1). \tag{7}$$

This is perhaps the point to mention that we clearly must have an infinite number of resonances with increasing masses which can couple to the γN system if we are going to be able to describe the entire amplitude as a sum of s-channel exchanges alone (in accord with the usual concepts of duality[6]). We are not, however, going to assume anything about any specific mass relation which the resonances should obey. The number N of resonances which can contribute to the sum S at any Q^2 will be determined at any particular model by inserting the appropriate mass formula into Eq. (7). It is important to note, however, that Eq. (7) is a kinematical constraint which is model independent. The mass of the highest resonance which can contribute to the sum S is therefore model independent and increases as $\sqrt{Q^2}$.

Let us now look at the nth-resonance form factor $G_n(Q^2)$. As we have mentioned earlier, the form factors decrease very rapidly with Q^2 and the data[3] seem to indicate that they all follow a dipole behavior. Let us then write

$$G_n(Q^2)=(1+Q^2/d_n^2)^{-2}. \tag{8}$$

Bloom and Gilman[8] have speculated, using Eq. (2), that all form factors should indeed have the same Q^2 dependence. The essence of their argument is that the nth-resonance contribution is peaked around

$$\omega_n'=1+M_n^2/Q^2, \tag{9}$$

and by increasing Q^2 it will move toward $\omega'=1$. But since $F_2(\omega')$ is a good average for the physical amplitude at all Q^2, this means that all the resonances should "slide" along the scaling curve and the Q^2 behavior of every form factor should be related to the behavior near threshold of $F_2(\omega')$ in the same way. That is, we get the behavior

$$G_n(Q^2) \xrightarrow[Q^2\to\infty]{} 1/Q^4$$

for every n. But this argument tells us that the higher the resonance, the bigger is Q^2 where the falloff should start because the resonance has a longer way to go before reaching the $\omega'\simeq 1$ region. In other words, if the resonances are to follow the scaling curve, it is clear that the higher resonances should start falling off rapidly in Q^2 only at higher values of Q^2. This means that d_n, the mass of the nth-resonance dipole formula [Eq. (8)], should increase with n.

The data seem to support this idea. The best test for it would be of course to fit every resonance form factor with a dipole expression and see whether d_n really follows our assumption. This has not yet been done, however, and the most transparent way the experimental resonance data are published[3] is in the form of

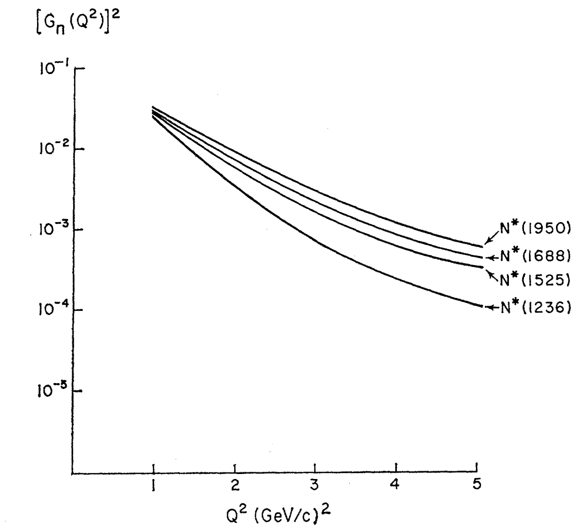

FIG. 1. First four resonances' form factors as functions of Q^2. Curves were extracted from the data (Ref. 3).

curves of $(d\sigma/d\Omega)_{\text{res}}/(d\sigma/d\Omega)_{\text{el}}$ as a function of Q^2. To obtain $G_n(Q^2)$ from these curves, we first multiplied them by the elastic dipole formula [with $d_0^2=0.71$ $(\text{GeV}/c)^2$]. We then extrapolated the $Q^2>1$ $(\text{GeV}/c)^2$ data all the way down to $Q^2=0$ to get rid of the threshold behavior. We then divided out the effective coupling constant which one gets in this way. The results are shown in Fig. 1 and they show that the form factor of any resonance lies above the form factor of the former, at least for the first four prominent resonances.

Another way to present the same data, which is perhaps less impressive but at the same time free of any speculations, is to take the experimental curves for consecutive resonances and divide them by each other. By doing this we directly obtain the experimental curves for $[G_n(Q^2)]^2/[G_{n-1}(Q^2)]^2$ as a function of Q^2 without any speculation.[9] If our suggestion is correct, then these curves should show increasing functions of Q^2, and Fig. 2 shows that they really do.

Further support for our assumption comes from the way in which the experimentalists analyze the scaling curve.[1] They claim that they can get a good fit to νW_2 near threshold of the form

$$\nu W_2=\sum_{k=3}^{5}\frac{a_k}{[1+Q^2/(W^2-M^2+\mu^2)]^k}, \tag{10}$$

where a_k are some constants, W is the γN system c.m. energy, and $\mu^2\simeq 0.9$ GeV^2. Projecting the leading term in Eq. (10) on the nth resonance, we get a dipole fit to $G_n(Q^2)$ with

$$d_n^2=M_n^2-M^2+\mu^2, \tag{11}$$

in agreement with our suggestion.

We will take the above-mentioned facts as evidence in favor of our assumption and look at its implications for scaling behavior.

By combining the suggested form (11) for d_n^2 with the mass relation [Eq. (7)] for the highest resonance which can contribute to the sum S, we see that d_N^2, the squared dipole mass of the highest contributing resonance, grows like Q^2 and the form factor of the highest resonance in the sum S is always a finite number, even when we send Q^2 to infinity. That is,

$$G_N(Q^2) \underset{Q^2\to\infty}{\longrightarrow} \text{const}, \tag{12}$$

and although every resonance contribution by itself decreases with the same power behavior of the elastic contribution, the highest resonances which contribute to the sum S are always finite and enable S to remain constant in Q^2, in agreement with scaling behavior.

Having shown that the integral over resonances from threshold to any finite value of the scaling variable is Q^2 independent, we immediately get that the integral over any finite interval of the scaling axis has this same property. We have thus shown that the sum of resonances builds an amplitude which scales in a "semi-local" sense. That is, for any finite interval in the scaling variable, the area under the sum of the resonances is equal to the area under the scaling curve, for any $Q^2[\gtrsim 1\ (\text{GeV}/c)^2]$.

The specific form we have taken for the dipole masses [Eq. (11)] is of course just for illustration purposes and we by no means suggest that it should be the ultimate form which the data must obey. What we do, however, suggest, in view of the above calculation, is that the

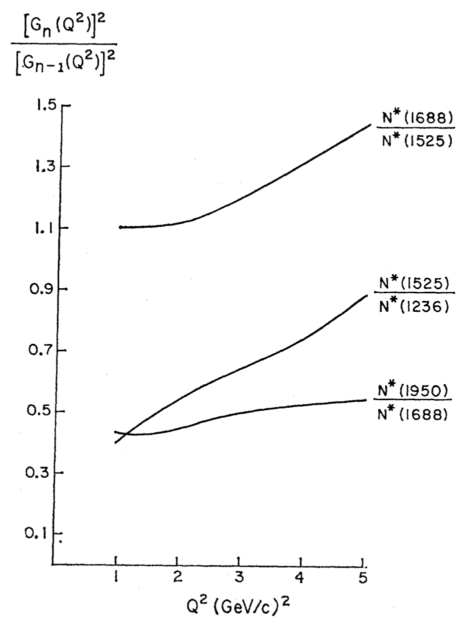

FIG. 2. Ratio of form factors of consecutive resonances as functions of Q^2 (experimental curves). Curves are not normalized and contain the ratio of the coupling constants (see Ref. 9).

[9] In order to make this presentation as free of speculation as possible, the curves were not normalized as demanded by Eq. (5). If that were done, all the curves of Fig. 2 would just be shifted to have a common origin at $Q^2=0$.

following two features of resonances, which seem to be obeyed by the data, build up scaling behavior.

(a) There is an infinite number of resonances, with masses which increase indefinitely, which can couple to the γN system.

(b) The resonance form factors obey a dipole formula, and the higher the resonance, the bigger is the dipole mass.

As a result of these two properties, the sum S remains constant when we increase Q^2, although each resonance vanishes very fast. The reason is that when we increase Q^2, the contributions to S come from higher resonances, whose form factors have not yet vanished since their behavior is characterized by a bigger dipole mass.

Although point (a) is quite conventional and accepted by most people, point (b) has not yet been investigated and we suggest that it deserves more study.

A word should be added here about the effective coupling constants g_n^2. In the above explanation we did not take them into account because we just wanted to get a qualitative description of the way in which the resonances build up a scaling curve. It is clear, however, that in order to make the sum S constant we must also demand some specific behavior for the coupling constants. This behavior will depend, of course, on the specific model we take for the resonances. If we take, for instance, a model with a quadratic mass formula and a single resonance at every mass, we find that the coupling constants should obey $g_n^2 \sim 1/n$ in order to make the sum S constant. In the case of a model with multiplicity of resonances, our analysis can be applied as it is in the case that all resonances with the same mass have the same form factor. In that case we find that it is the sum of the squares of the coupling constants of resonances belonging to the same excited state which should decrease like $1/n$. It is worth mentioning that an explicit resonance model (with multiplicity) which obeys scaling was constructed recently by Domokos *et al.*[10] and the form factors in this model follow our conjecture.

III. EXPERIMENTAL CONSEQUENCES

The expression we wrote down for the resonances' form factors [Eq. (8)] and the assumption we made about the increase of the dipole mass when we go to higher resonances, suggest that the form factors become more flat when we go to higher resonances. This means that the peak of any resonance will become more pronounced, compared to the former resonances, when we increase Q^2. This conclusion (which is verified by the data[3]) suggests that by increasing Q^2 we enhance the contribution of a certain resonance (compared to the former ones) and create more favorable kinematical conditions for its study. On the other hand, by increasing Q^2 we decrease the absolute magnitude of the resonance contribution because of the rapid falloff of the form factor. We want, therefore, to find the Q^2 region where the contribution of a certain resonance is both most enhanced compared to the former one, and not too small in absolute magnitude.

To do that, let us first write down the contribution of the resonance relative to the former one as

$$f_n(Q^2) \equiv \frac{[G_n(Q^2)]^2}{[G_{n-1}(Q^2)]^2} = \left(\frac{1+Q^2/d_{n-1}^2}{1+Q^2/d_n^2}\right)^4 = \begin{cases} 1, & Q^2=0 \\ (d_n^2/d_{n-1}^2)^4, & Q^2\to\infty . \end{cases} \tag{13}$$

Since $d_n^2 > d_{n-1}^2$, we see that $f_n(Q^2)$ rises monotonically from 1 to some finite value when we increase Q^2. If we assume that $d_n^2 \sim M_n^2$ [Eq. (11)] and that the resonance masses obey some kind of linear or quadratic mass relation, then the ratio between consecutive masses will approach unity when we go to higher resonances and we get

$$\frac{d_n^2}{d_{n-1}^2} \xrightarrow[n \text{ large}]{} 1 . \tag{14}$$

We find that the higher resonances remain "equally important"[11] when we increase Q^2 and we cannot enhance the contribution of a certain resonance by increasing Q^2. For the lower resonances we can obtain from Eq. (13) the Q^2 region where we have

$$f_n(Q^2) \geqslant C_1,$$

where C_1 is some constant which obeys

$$1 \leqslant C_1 \leqslant (d_n^2/d_{n-1}^2)^4.$$

This region is given by

$$Q^2 \geqslant \frac{d_n^2(C_1^{1/4}-1)}{d_n^2/d_{n-1}^2 - C_1^{1/4}}. \tag{15}$$

For any Q^2 which obeys relation (15), the contribution of the nth resonance, relative to that of the $(n-1)$ resonance, will be C_1 times stronger than its contribution at $Q^2=0$. On the other hand, we want to restrict the falloff of the resonance form factor and we will require

$$[G_n(Q^2)]^2 \geqslant C_2 , \tag{16}$$

where C_2 is another constant between 0 and 1. Relation (16) can be satisfied only by

$$Q^2 \leqslant d_n^2(C_2^{-1/4}-1) . \tag{17}$$

Since we want relations (15) and (17) to be obeyed simultaneously, we see that C_1 and C_2 cannot be chosen

[10] G. Domokos, S. Kovesi-Domokos, and E. Schonberg, Phys. Rev. D **3**, 1184 (1971).

[11] Figure 2 represents the functions $f_n(Q^2)$ up to the ratio of the coupling constants (see Ref. 9) and it clearly indicates that they become flatter for the higher resonances.

arbitrarily but must obey the relation

$$C_1^{1/4}+C_2^{1/4}\left(\frac{d_n^2}{d_{n-1}^2}-1\right)\leqslant\frac{d_n^2}{d_{n-1}^2}. \tag{18}$$

Equation (18) merely tells us that we cannot improve indefinitely the relative contribution of a resonance if we want it to have a finite absolute contribution. By deciding what is the lowest C_2 which still gives a detectable counting rate, the experimentalists can see from Eq. (18) how much the contribution of a certain resonance, relative to the former one, can be enhanced. For example, if we look at the $N^*(1920)$ and demand that its form factor does not fall off below 0.5, we get (assuming $d_n^2\sim M_n^2$) that its contribution relative to that of the $N^*(1680)$ can be enhanced only by 25%, at most, compared with the corresponding value at $Q^2=0$. The corresponding Q^2 happens to be around 1 $(\mathrm{GeV}/c)^2$.

IV. SCALING REGGE POLES

Having shown how the resonances build a scaling curve in the sense of FESR, let us look now at the Regge-pole contributions to the equation. The high-energy behavior of $\nu W_2(Q^2,\nu)$ is given by

$$\nu W_2(Q^2,\nu)\xrightarrow[\nu\to\infty]{}\sum_i\beta_i(Q^2)\nu^{\alpha_i-1}, \tag{19}$$

where α_i are the intercepts of the contributing Regge trajectories. As we have mentioned earlier, the data[1] seem to indicate that the A_2 trajectory scales, as well as the Pomeranchukon. We will therefore assume scaling for all Regge trajectories and write

$$\beta_i(Q^2)=C_i(2M/Q^2)^{\alpha_i-1}, \tag{20}$$

where C_i are constants. We see now that in the scaling region $\nu W_2(Q^2,\nu)$ [or equivalently in this region, $F_2(\omega)$] has a Regge expansion in the variable ω.[12] Assuming that Regge behavior starts from some $\omega=\omega_0$, we can write the following FESR:

$$\int_1^{\omega_0}F_2(\omega)d\omega=\sum_i c_i\frac{\omega_0^{\alpha_i}}{\alpha_i}. \tag{21}$$

The derivation of this equation took into account only $\alpha>0$ poles.

Equation (21) gives us the usual result of FESR theory: The area under the small ω region of the scaling curve is equal to the area under the extrapolated Regge tail. Using the experimental curve for the left-hand side of the FESR and the Regge-fit parameters for the right-hand side, we can see whether there is any ω_0 for which Eq. (21) can be satisfied.

We performed this calculation using a recent Regge fit for $F_2(\omega)$.[13] This fit gives us

$$C_1=0.28,\quad C_2=0.18\quad\text{if}\quad\sigma_s/\sigma_t=0.18,$$

and

$$C_1=0.11,\quad C_2=0.68\quad\text{if}\quad\sigma_s/\sigma_t=0,$$

where C_1 corresponds to the Pomeranchukon contribution and C_2 to the $\alpha=0.5$ trajectories.

We found that with either of these fits one cannot satisfy the FESR (21) for any value of ω_0 where measurements exist (that is, up to $\omega\sim20$). If one wants to extrapolate roughly the experimental curves to large values of ω, one finds that, using the first fit, the FESR can be satisfied only with $\omega_0\sim65$, and using the second fit, one needs $\omega_0\sim45$. This is of course highly undesirable since the data clearly indicate that Regge behavior starts around $\omega\sim5$, so that we expect ω_0 to be of this order of magnitude. One can argue that there is some inconsistency in our numerical calculation since, to compare Eq. (21) with experiment, one should in principle use only Regge fits which were done in the $\omega>\omega_0$ region, for every ω_0 which we choose. In any case, this is certainly not a big effect and even with this remark in mind it is very unlikely that the $\alpha>0$ trajectories alone will satisfy the FESR (21) with $\omega_0\sim5$. The point is that when we extrapolate the $\omega>5$ tail to threshold, we get too much area between the extrapolated curve and the scaling curve, owing to the rapid decrease toward threshold of $F_2(\omega)$. This area cannot be compensated for by the area left in the broad peak of $F_2(\omega)$ above the extrapolated Regge tail. We conclude that we need another contribution to the FESR [Eq. (21)]. Such a contribution could come from the presence of a $J=0$ fixed pole.[14]

Let us be a little bit more specific. The forward virtual Compton amplitude for the scattering of a photon with four-momentum q_μ and polarization vector ϵ_μ off a hadron with four-momentum P_μ is given by $\epsilon_\mu{}^*T_{\mu\nu}(P,q)\epsilon_\nu$ and $T_{\mu\nu}$ can be decomposed[2]

$$T_{\mu\nu}(P,q)=(1/M^2)P_\mu P_\nu T_2(q^2,\nu)+\cdots \tag{22}$$

and

$$W_2(q^2,\nu)=(1/\pi)\,\mathrm{Im}T_2(q^2,\nu).$$

$W_2(q^2,\nu)$ is related to the cross section for photoabsorption of the virtual photon by the hadron:

$$W_2(Q^2,\nu)=\frac{1}{4\pi^2\alpha}\frac{Q^2}{(\nu^2+Q^2)^{1/2}}[\sigma_t(Q^2,\nu)+\sigma_s(Q^2,\nu)]. \tag{23}$$

Our conclusion about the $J=0$ fixed pole can be

[12] We will be working in the $Q^2\to\infty$ limit so that the scaling variables ω and ω' are essentially the same.

[13] H. Pagels, Phys. Letters **34B**, 299 (1971).

[14] A $J=0$ fixed pole in the electroproduction amplitudes has already been discussed in the literature. In particular, J. M. Cornwall, D. Corrigan, and R. E. Norton [Phys. Rev. Letters **24**, 1141 (1970)] derived equations similar to ours from quite a different point of view. See also a discussion of the Q^2 dependence of the residue function by T. P. Cheng and W. K. Tung, *ibid.* **24**, 851 (1970), and Ref. 17.

written as

$$\nu^2[T_2(Q^2,\nu)-T_R(Q^2,\nu)] \xrightarrow[\nu\to\infty]{} \beta_0(Q^2)\,, \qquad (24)$$

where $T_R(Q^2,\nu)$ is the contribution to the amplitude from the $\alpha(0)>0$ Regge poles and $\beta_0(Q^2)$ is the residue of the fixed pole.[15] The FESR (21) will now be modified and we simply have to add to its right-hand side the function $-(2M/Q^2)\beta_0(Q^2)$. But since any other term in the equation is Q^2 independent, we clearly must have

$$\beta_0(Q^2)=C_0(Q^2/2M) \qquad (25)$$

and the $J=0$-pole residue function obeys the scaling relation (20), although the pole cannot couple to $W_2(Q^2,\nu)$. The constant C_0 can be determined from the FESR (21), and we get for it the equation

$$C_0=\sum_{\alpha_i>0} C_i\frac{{\omega_0}^{\alpha_i}}{\alpha_i}-\int_1^{\omega_0} F_2(\omega)d\omega\,. \qquad (26)$$

Equation (26) makes it explicitly clear that the conclusion about the presence of the fixed pole is not just an accident due to the choice of the Regge parameters. The data really have to change drastically in order to make $C_0=0$.

Taking $\omega_0=5$, we get $C_0\simeq 1.2$ using the $\sigma_s/\sigma_t=0.18$ fit and $C_0\simeq 2.5$ using the $\sigma_s/\sigma_t=0$ fit.

The form which we find for the fixed pole enables us to connect it to the one found in real forward Compton scattering by Damashek and Gilman.[7] The point is that when $Q^2\to 0$ the longitudinal photoabsorption cross section vanishes and we obtain from (23)

$$W_2(Q^2,\nu) \xrightarrow[Q^2\to 0]{} \frac{1}{4\pi^2\alpha}\frac{Q^2}{\nu}\sigma(\nu)\,, \qquad (27)$$

where $\sigma(\nu)$ is the real photoabsorption cross section at lab energy ν. But this is related to the forward Compton nonflip amplitude $f_1(\nu)$, by the optical theorem:

$$\mathrm{Im} f_1(\nu)=(\nu/4\pi)\sigma(\nu)\,. \qquad (28)$$

Combining Eqs. (22), (27), and (28), we see that the imaginary parts of $T_2(Q^2,\nu)$ and $f_1(\nu)$ are related via

$$\mathrm{Im} T_2(Q^2,\nu) \xrightarrow[Q^2\to 0]{} \frac{1}{\alpha}\frac{Q^2}{\nu^2}\mathrm{Im} f_1(\nu)\,. \qquad (29)$$

[15] Note that an $\alpha=0$ pole cannot couple to W_2, the imaginary part of T_2.

What can we say about the real parts? Are they also related in a similar way? The analysis of Damashek and Gilman[7] tells us that $f_1(\nu)$ obeys

$$f_1(\nu)-f_R(\nu) \xrightarrow[\nu\to\infty]{} -\alpha/M\,, \qquad (30)$$

where $f_R(\nu)$ stands for the contribution of the $\alpha(0)>0$ Regge trajectories to $f_1(\nu)$. Assuming that the real parts of $T_2(Q^2,\nu)$ and $f_1(\nu)$ obey a similar relation to Eq. (29), we get

$$\nu^2[T_2(Q^2,\nu)-T_R(Q^2,\nu)] \xrightarrow[Q^2\to 0;\ \nu\to\infty]{} -2Q^2/2M\,. \qquad (31)$$

We see that the fixed pole we get in this way has an opposite sign to the one found in the analysis of the scaling curve.

One can of course argue that one should not make a continuation in Q^2 from the scaling region[16] [that is, $Q^2\geqslant 1$ $(\mathrm{GeV}/c)^2$] to $Q^2=0$ and also that there is no reason why the real parts of T_2 and f_1 should be related to each other in the same way that the imaginary parts are [Eq. (29)]. If one accepts the above analysis, however, then one is led to the conclusion that the fixed-pole residue function changes sign when going from $Q^2=1$ $(\mathrm{GeV}/c)^2$ to $Q^2=0$.[17] A mechanism which can explain this somewhat unexpected behavior is not yet known, and if found, could have very interesting implications.

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[16] Note that the $J=0$ fixed pole is the only pole that we can continue to $Q^2=0$ using Eq. (29). The $\alpha>0$ poles will blow up in this limit and the $\alpha<0$ ones will vanish.

[17] An analysis identical to ours was done independently by S. R. Choudhury and R. Rajaraman, Phys. Rev. D 2, 2728 (1970). Using the 6° and 10° data, they get the same result that we get for C_0. Using preliminary data for 18° (which at that time were available only as a free-hand curve through the preliminary data points), they claim to obtain the opposite sign for C_0. The 18° data have meanwhile been published and they agree with the data on 6° and 10°. Our analysis is based on fits to the over-all data.