PHYSICAL REVIEW D VOLUME 3, NUMBER 9 1 MAY 1971

Uniqueness of the Interaction Involving Spin-$\frac{3}{2}$ Particles

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(Received 21 December 1970)

The free-field Lagrangian as well as the propagator for spin-$\frac{3}{2}$ particles contains an arbitrary parameter A. However, the S-matrix elements for the interaction of a spin-$\frac{3}{2}$ field with other fields are always independent of this parameter, provided the interaction Lagrangian is properly chosen. For a system of a spin-$\frac{3}{2}$ field coupled to a nucleon field and a pion field, a two-parameter (A,Z) interaction Lagrangian is introduced in such a way that it is invariant under a point transformation of the spin-$\frac{3}{2}$ field. The transition amplitude for such an interaction is independent of the parameter A. However, it does depend on the second parameter Z. By requiring the interaction to be consistent with the principles of second quantization, the value of the second parameter has been fixed. Therefore, the $\Delta(1238)$ contribution to the elastic πN scattering amplitudes can be uniquely determined. Using this model for the evaluation of the $\Delta(1238)$ contribution and the chiral-invariant Lagrangians for the calculation of the nucleon pole and ρ-pole contributions, the πN scattering lengths have been computed and compared with the experimental data. The calculated results are in good agreement with experiment.

I. INTRODUCTION

THE Lagrangian field theory of particles with spin $\geq\frac{3}{2}$ is beset with a difficulty that the Lagrangian and the propagator are not unique.[1–5] There are arbitrary parameters in the theory. However, it can be shown that the physically interesting quantities such as energy and momentum and the canonical commutation relations are not dependent on the parameters which occur in the Lagrangian.[2–4] Similarly, the S-matrix elements involving the interaction of particles with spin $\geq\frac{3}{2}$ can be defined uniquely, provided the interaction is chosen in such a way that it is consistent with the constraints in the theory. In this paper we discuss in detail the theory of a spin-$\frac{3}{2}$ field coupled to a nucleon field and a boson field. The propagator for spin-$\frac{3}{2}$ particles contains an arbitrary parameter A. Our aim here is to construct a model for this interacting system of a spin-$\frac{3}{2}$ particle, a nucleon, and a boson in such a way that the S-matrix elements can be defined uniquely and that the interaction is consistent with the well-known principles of quantum field theory. However, it is found that there exists not only one interaction Lagrangian, but a set of interaction Lagrangians for which the S-matrix elements are independent of the parameter A. The most general form of such an interaction Lagrangian containing only the first-order derivative of the boson field is given by[6]

$$\mathcal{L}_1=g^*\bar{\psi}_\mu\Theta_{\mu\nu}\psi\partial_\nu\phi+\text{H.c.}, \tag{1.1}$$

where

$$\Theta_{\mu\nu}=\{\delta_{\mu\nu}+[\tfrac{1}{2}(1+4Z)A+Z]\gamma_\mu\gamma_\nu\}. \tag{1.2}$$

The spinor-vector ψ_μ represents the spin-$\frac{3}{2}$ field, ψ and ϕ are, respectively, the nucleon field and the pseudoscalar boson field. The coupling constant g^* has the dimensions of MeV^{-1}. The new parameter Z is purely arbitrary. The S-matrix elements for the interaction Lagrangian (1.1) are indeed independent of A, but they depend on Z. It should be pointed out here that the parameter Z does not occur in the free-field Lagrangian or the propagator. After the explicit evaluation of the $\Delta(1238)$ contribution to the elastic πN scattering amplitudes, it is shown that the parameter Z measures the nonpole part of the amplitudes compared to the pole term. In order to fix the value of Z, the interacting fields are quantized on a spacelike surface. By demanding that the interaction be consistent with the principles of second quantization, the value of Z is found to be $\frac{1}{2}$. Having thus defined the interaction uniquely, the $\Delta(1238)$ contribution to the S- and P-wave pion-nucleon scattering lengths are calculated explicitly. Then the nucleon-pole contribution and the ρ-pole contribution to the πN scattering lengths evaluated from the chiral-invariant Lagrangian models[7–9] for the πN scattering are added to the $\Delta(1238)$ contribution, and the final results are compared with the experimental data. It may be mentioned here that the interaction Lagrangian (1.1), after the isospin indices have been put in, can be easily made chiral invariant by replacing the ordinary derivative of the pion field with a covariant one. The effective interaction Lagrangian for the calculation of the $\Delta(1238)$ contribution to elastic πN scat-

[1] P. A. Moldauer and K. M. Case, Phys. Rev. **102**, 279 (1956).
[2] L. M. Nath, Nucl. Phys. **68**, 660 (1965).
[3] S. C. Bhargava and H. Watanabe, Nucl. Phys. **87**, 273 (1966).
[4] A. Kawakami and S. Kamefuchi, Nuovo Cimento **48A**, 239 (1967); A. Kawakami, Nucl. Phys. **B2**, 33 (1967).
[5] Using a $(2S+1)$-dimensional wave function for the description of a particle with spin S, S. Weinberg [Phys. Rev. **133**, B1318 (1964)] has developed a theory of higher spin particles in which a knowledge of the Lagrangian or the field equations is not necessary. In this approach, there are no superfluous components in theory and the wave function is required only to satisfy the Klein-Gordon equation. Weinberg's theory, as it is not based on the principles and concepts of the Lagrangian field theory, does not have the troubles associated with the conventional field-theoretical description of particles with spin $\geq\frac{3}{2}$. However, in this paper we are concerned with the problems of the Lagrangian field theory of higher-spin particles. The Lagrangian approach is manifestly covariant and it is this formalism which is always used for practical calculations.
[6] The reason for choosing this particular form for the interaction has been fully explained in Sec. II.
[7] S. Weinberg, Phys. Rev. Letters **18**, 188 (1967); Phys. Rev. **166**, 1568 (1968).
[8] J. Schwinger, Phys. Letters **24B**, 473 (1967).
[9] R. D. Peccei, Phys. Rev. **176**, 1812 (1968).

tering is the same as (1.1), apart from the isospin factors. The theoretically predicted values for the πN scattering lengths from this model are in reasonably good agreement with experiment.

The theory of a spin-$\frac{3}{2}$ field in interaction with a nucleon field and pseudoscalar boson field is developed in Sec. II. In Sec. III the interacting fields are quantized on a spacelike surface and the value of Z is fixed. Section IV contains the details of the calculation of the πN scattering lengths and the results given in two tables. Finally, a brief discussion of our results is given in Sec. V. The subsidiary conditions in the presence of interaction, which are necessary to eliminate the redundant components of the field ψ_μ, are derived in the Appendix.

II. LAGRANGIAN FIELD THEORY OF SPIN-$\frac{3}{2}$ PARTICLES IN PRESENCE OF INTERACTIONS

The Lagrangian and the equations of motion for the free spin-$\frac{3}{2}$ field can be written, respectively, as[1]

$$\mathcal{L}=\bar{\psi}_\mu\Lambda_{\mu\nu}\psi_\nu\,, \tag{2.1}$$

$$\Lambda_{\mu\nu}\psi_\nu=0\,, \tag{2.2}$$

where

$$\Lambda_{\mu\nu}=-[(\gamma_\lambda\partial_\lambda+M)\delta_{\mu\nu}+A(\gamma_\mu\partial_\nu+\gamma_\nu\partial_\mu)+B\gamma_\mu\gamma_\lambda\partial_\lambda\gamma_\nu-CM\gamma_\mu\gamma_\nu]\,, \tag{2.3}$$

$$B=(\tfrac{3}{2}A^2+A+\tfrac{1}{2})\,,\quad C=(3A^2+3A+1)\,. \tag{2.4}$$

The parameter A is arbitrary except that $A\neq-\frac{1}{2}$. From Eq. (2.2) it is possible to derive the following subsidiary conditions:

$$\gamma_\mu\psi_\mu=0\,, \tag{2.5}$$

$$\partial_\mu\psi_\mu=0\,. \tag{2.6}$$

The constraints (2.5) and (2.6) are necessary to eliminate the redundant components of the field ψ_μ and they are valid for all values of A. Expression (2.1) shows that for the spin-$\frac{3}{2}$ field there exists not only one Lagrangian, but a set of Lagrangians all of which are equally suitable for the description of the spin-$\frac{3}{2}$ field. This fact was first pointed out by Moldauer and Case[1] and later discussed further by Fronsdal.[10] In the presence of interactions, Eqs. (2.5) and (2.6) do not in general hold. However, it is possible to derive the necessary number of subsidiary conditions for at least a selected class of interactions.[11,12] The propagator is given by[13]

$$\langle 0|\,T(\psi_\mu(x)\psi_\nu^\dagger(y))\,|0\rangle=d_{\mu\nu}(\partial)\Delta_F(x-y)\,, \tag{2.7}$$

where

$$d_{\mu\nu}\Lambda_{\nu\lambda}=(\Box-M^2)\delta_{\mu\lambda}\,,$$
$$\Delta_F(x-y)=-i(2\pi)^{-4}\int e^{ik(x-y)}(k^2+M^2-i\epsilon)^{-1}d^4k\,, \tag{2.8}$$
$$d^4k=d^3k(-idk_4)\,.$$

Using Eq. (2.8), the operator $d_{\mu\nu}$ can be evaluated explicitly and written as

$$\begin{aligned}d_{\mu\nu}(\partial)=&-(\gamma_\lambda\partial_\lambda-M)\left[\delta_{\mu\nu}-\tfrac{1}{3}\gamma_\mu\gamma_\nu+\frac{1}{3M}(\gamma_\mu\partial_\nu-\gamma_\nu\partial_\mu)-\frac{2}{3M^2}\partial_\mu\partial_\nu\right]-\frac{1}{3M^2}\left(\frac{A+1}{2A+1}\right)\\&\times\left\{\left[\frac{1}{2}\left(\frac{A+1}{2A+1}\right)\gamma_\lambda\partial_\lambda+\left(\frac{A}{2A+1}\right)M\right]\gamma_\mu\gamma_\nu+\gamma_\mu\partial_\nu+\left(\frac{A}{2A+1}\right)\gamma_\nu\partial_\mu\right\}(\Box-M^2)\,.\end{aligned} \tag{2.9}$$

The reason for writing the operator $d_{\mu\nu}$ in the form (2.9) is to show that the A-dependent terms will disappear from (2.9) if $P^2=M^2$ and that the propagator is in general arbitrary. For this reason, the S-matrix elements involving the propagator of spin-$\frac{3}{2}$ particles cannot be defined uniquely unless the interactions of the spin-$\frac{3}{2}$ field are properly chosen. It should be pointed out here that the free-field Lagrangian (2.1) is invariant under the point transformation,

$$\psi_\mu\to\psi_\mu'=\psi_\mu+a\gamma_\mu\gamma_\lambda\psi_\lambda\,,\qquad A\to A'=\frac{A-2a}{1+4a}\,, \tag{2.10}$$

where a is an arbitrary parameter except that $a\neq-\frac{1}{4}$. The meaning of the invariance of the free-field Lagrangian under the transformation (2.10) is that the physically interesting quantities such as energy and momentum and the canonical commutation relations should not depend on A. The free-field Lagrangian (2.1) shows that the parameter A simply determines the proportion in which the spin-$\frac{1}{2}$ field $\gamma_\lambda\psi_\lambda$ is mixed with the other fields and the invariance of the Lagrangian under the transformation (2.10) implies that the physical content of the theory does not depend on the value of A. Now, we demand that the interaction Lagrangian for a spin-$\frac{3}{2}$ field coupled to a nucleon field and a pseudoscalar boson field should be constructed in such a way that the total Lagrangian is invariant under the point transformation,

$$\psi_\mu\to\psi_\mu'=\psi_\mu+a\gamma_\mu\gamma_\lambda\psi_\lambda\,,\qquad A\to A'=\frac{A-2a}{1+4a}\,,\qquad \psi\to\psi'=\psi\,,\quad \phi\to\phi'=\phi\,. \tag{2.11}$$

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[10] C. Fronsdal, Nuovo Cimento Suppl. 9, 416 (1958).

[11] See the Appendix.

[12] M. Fierz and W. Pauli, Proc. Roy. Soc. (London) A173, 211 (1939).

[13] Y. Takahashi and H. Umezawa, Progr. Theoret. Phys. (Kyoto) 9, 1 (1953).

The most general form of such an interaction Lagrangian containing only the first-order derivative of the pion field is given by

$$\mathcal{L}_1 = g^* \bar{\psi}_\mu \Theta_{\mu\nu} \psi \partial_\nu \phi + \text{H.c.}, \qquad (2.12)$$

where

$$\Theta_{\mu\nu} = \{\delta_{\mu\nu} + [\tfrac{1}{2}(1+4Z)A + Z]\gamma_\mu\gamma_\nu\},$$

and the parameter Z is purely arbitrary. The reason for coupling the N^*N current to the derivative of the pion field, not to the pion field itself, is that the nonlinear realization of the chiral symmetry[7,9] predicts a derivative coupling for this interaction. Interaction Lagrangians with higher-order derivatives of the pion field have been found to be unnecessary. Now, the S-matrix elements for the interaction Lagrangian (2.12) should be independent of A according to an equivalence theorem proved by Kamefuchi *et al.*[14] This theorem states that the physical content of a theory does not change if the field variables undergo a point transformation in such a way that the transformed field can be expressed as a power series in terms of the original field variables and vice versa. The $\Delta(1238)$ contribution to the elastic πN scattering is calculated explicitly in the second-order perturbation theory and it is found that the transition amplitudes are indeed independent of A. This result is true in all orders of perturbation theory. However, this is a nonrenormalizable theory and we are interested only in the second-order transition amplitudes for elastic πN scattering. The N^* contribution to the amplitudes $A^{(\pm)}$ and $B^{(\pm)}$ can be written explicitly as[15]

$$A^{(-)} = \frac{g^{*2}}{48}\left\{\frac{8}{3}\left[(3t-4\mu^2)(M+m)+2M^3+3M^2m-2Mm^2-4m^3-\frac{2\mu^2}{M}(m^2-\mu^2)+\frac{m}{M^2}(m^2-\mu^2)^2\right]\right.$$
$$\left.\times\left(\frac{1}{s-M^2}-\frac{1}{u-M^2}\right)+\frac{8}{3M^2}[(1-\xi)m-2\eta M](s-u)\right\}, \qquad (2.13)$$

$$A^{(+)} = -\frac{g^{*2}}{24}\left\{\frac{8}{3}\left[(3t-4\mu^2)(M+m)+2M^3+3M^2m-2m^2M-4m^3-\frac{2\mu^2}{M}(m^2-\mu^2)+\frac{m}{M^2}(m^2-\mu^2)^2\right]\right.$$
$$\times\left(\frac{1}{s-M^2}+\frac{1}{u-M^2}\right)+\frac{8}{3M^2}[(1-\xi)m-2\eta M](s+u)$$
$$\left.+\frac{16}{3M^2}[3mM^2+2M^3+2\eta m^2M+2\mu^2(M+m)+(\xi-2)m^3]\right\}, \qquad (2.14)$$

$$B^{(-)} = \frac{g^{*2}}{48}\left\{\frac{8}{3}\left[(-M^2-3t+2Mm+6m^2+2\mu^2)+\frac{2m}{M}(m^2-\mu^2)-\frac{1}{M^2}(m^2-\mu^2)^2\right]\left(\frac{1}{s-M^2}+\frac{1}{u-M^2}\right)\right.$$
$$\left.-\frac{8}{3M^2}(1+\eta-\sqrt{\xi})(s+u)+\frac{16}{3M^2}[-M^2-(2+4\eta)Mm+(2+\eta-\sqrt{\xi}-2\xi)m^2-\mu^2(2+\eta-\sqrt{\xi}-\xi)]\right\}, \qquad (2.15)$$

$$B^{(+)} = -\frac{g^{*2}}{24}\left\{\frac{8}{3}\left[(-M^2-3t+2mM+6m^2+2\mu^2)+\frac{2m}{M}(m^2-\mu^2)-\frac{1}{M^2}(m^2-\mu^2)^2\right]\right.$$
$$\left.\times\left(\frac{1}{s-M^2}-\frac{1}{u-M^2}\right)-\frac{8}{3M^2}(1+\eta-\sqrt{\xi})(s-u)\right\}, \qquad (2.16)$$

where

$$\eta = 2Z(2Z+1), \quad \sqrt{\xi} = (2Z+1)$$

and M, m, and μ are, respectively, the masses of the N^* resonance, the nucleon, and the pion. Following convention, the T matrix can be written in terms of the invariant amplitudes $A(s,t)$ and $B(s,t)$:

$$T^{\alpha\beta}(s,t) = \bar{u}(p_2)[A^{\alpha\beta}(s,t) + \tfrac{1}{2}i\gamma\cdot(q_1+q_2)B^{\alpha\beta}(s,t)]u(p_1), \qquad (2.17)$$

where q_1 and q_2 are, respectively, the four-momenta of the incoming pion and the outgoing pion in the s-channel c.m. frame. Similarly, p_1 and p_2 represent the four-momenta of the incoming nucleon and the outgoing nucleon, respectively.[16] In isospin space, the amplitudes

[14] S. Kamefuchi, L. O'Raifeartaigh, and A. Salam, Nucl. Phys. 28, 529 (1961).

[15] For the calculation of the isospin coefficients, see P. Carruthers and M. M. Nieto, Ann. Phys. (N.Y.) 51, 359 (1969).

[16] In the metric we are using, $x_\mu^2 = \mathbf{x}^2 + x_4^2 = \mathbf{x}^2 - x_0^2$; $s = -(p_1+q_1)^2$, $t = -(p_1-p_2)^2$, $u = -(p_1-q_2)^2$; γ_1–γ_5 are all Hermitian.

$A^{\alpha\beta}$ and $B^{\alpha\beta}$ can be further decomposed as

$$A^{\alpha\beta}=\delta_{\alpha\beta}A^{(+)}+\tfrac{1}{2}[\tau_\alpha,\tau_\beta]A^{(-)}, \qquad (2.18)$$

and a similar expression for the amplitude $B^{\alpha\beta}$.

In the expressions (2.13)–(2.16), the amplitudes can be separated into two parts: a pole term and a nonpole term. As expected, the nonpole terms depend on Z. It should be emphasized here that the constant Z is in no way related to the parameter A and it occurs only in the interaction Lagrangian. In Sec. III we fix the value of Z from theoretical considerations. For the calculation of the N^* contribution to elastic πN scattering, Peccei[9] has developed a model which is similar to this model in some respects. However, he claims that the interaction Lagrangian cannot be invariant under the point transformation (2.11) unless $\Theta_{\mu\nu}$ satisfies a subsidiary condition, namely, $\gamma_\mu\Theta_{\mu\nu}=0$. We have found that this condition is very stringent and really unnecessary. The most general form of the interaction Lagrangian, where the N^*N current is coupled to the first-order derivative of the pion field, is given in (2.12). Further, this subsidiary condition, $\gamma_\nu\Theta_{\mu\nu}=0$, if it is imposed on the interaction, will require that Z be equal to $-\frac{1}{4}$. In Sec. III, it is shown that the interaction for $Z=-\frac{1}{4}$ is inconsistent with the principles of second quantization.

III. QUANTIZATION OF SPIN-$\frac{3}{2}$ FIELD COUPLED TO A NUCLEON FIELD AND A PION FIELD

In order to quantize the spin-$\frac{3}{2}$ field in the presence of interaction, let us define a field $\psi_i^{3/2}$ by the relation

$$\psi_i^{3/2}(x)=A_{il}\psi_l(x), \qquad (3.1)$$

where

$$A_{il}=(\delta_{il}-\tfrac{1}{3}\gamma_i\gamma_l), \quad i,l=1,2,3.$$

The spinor-vector $\psi_i^{3/2}$ has eight independent components and is indeed characterized by spin $\frac{3}{2}$ for rotations in three-dimensional space. In the canonical formalism the field $\gamma_l\psi_l$ and $\gamma_4\psi_4$ are treated as dependent "coordinates" and these redundant components can always be expressed in terms of the canonically independent field variables. Using the formalism of Yang and Feldman,[17,18] the field variables on a spacelike surface σ in the Heisenberg representation can be expressed in terms of associated quantities which satisfy the free-field commutation relations on the surface σ. More explicitly, a field variable $\phi_\alpha(x)$ can be written as

$$\phi_\alpha(x)=\phi_\alpha(x/\sigma)+i\int_{-\infty}^{+\infty}d^4x'[\theta(x_0-x_0'),d_{\alpha\beta}(\partial)]\times\Delta(x-x')j_\beta(x'), \qquad (3.2)$$

where $\phi_\alpha(x)$ is the Heisenberg operator and $\phi_\alpha(x/\sigma)$ is the associated field variable which obeys the free-field commutation relation. The point x lies on the spacelike surface σ. The current j_α and the operator $d_{\alpha\beta}(\partial)$ are defined by

$$\Lambda_{\alpha\beta}(\partial)\phi_\beta(x)=j_\alpha(x), \quad \Lambda_{\alpha\beta}(\partial)\phi_\beta{}^{\text{free}}(x)=0, \qquad (3.3)$$

$$d_{\alpha\beta}(\partial)\Lambda_{\beta\gamma}(\partial)=\delta_{\alpha\gamma}(\Box-M^2), \qquad (3.4)$$

where M is the mass associated with the field ϕ_α. The function $\Delta(x-y)$ is given by

$$\Delta(x-y)=(8\pi^3)^{-1}\int e^{ik(x-y)}\delta(k^2+M^2)\epsilon(k^0)d^4k.$$

For a system of a spin-$\frac{3}{2}$ field coupled to a nucleon field and a pion field, using the formulas (3.2) and the interaction Lagrangian (2.12), all Heisenberg operators can be expressed in terms of the associated field variables. Thus we find

$$\psi_l^{3/2}(x)=\psi_l^{3/2}(x/\sigma)+\tfrac{2}{3}(g^*/M^2)A_{li}\partial_i[\tfrac{1}{2}(1+2Z)\gamma_k\psi(x)\partial_k\phi(x)-\tfrac{1}{2}(1-2Z)\gamma_4\psi(x)\partial_4\phi(x)], \qquad (3.5)$$

$$\psi(x)=\psi(x/\sigma), \qquad (3.6)$$

$$\phi(x)=\phi(x/\sigma), \qquad (3.7)$$

$$\partial_i\phi(x)=\partial_i\phi(x/\sigma), \qquad (3.8a)$$

$$\partial_4\phi(x)=\partial_4\phi(x/\sigma)+\tfrac{1}{2}g^*\{\psi^\dagger(x)[3(A+1)\gamma_4\psi_4(x)+(3A+1)\gamma_i\psi_i(x)]-\text{H.c.}\}-\tfrac{1}{2}g^*(1-2Z)(1+2A)\times[\bar\psi(x)\gamma_4\gamma_\mu\psi_\mu(x)-\text{H.c.}], \qquad (3.8b)$$

$$\partial_\mu\phi(x/\sigma)\equiv[\partial_\mu\phi(x,\sigma)]_{x/\sigma}, \quad \partial_\mu\equiv(\partial/\partial x_\mu).$$

The operators $\psi_l^{3/2}(x/\sigma)$, $\psi(x/\sigma)$, and $\phi(x/\sigma)$ satisfy the free-field commutation relations on the spacelike surface σ. The field variables such as $\psi_i^{3/2}(x)$, $\psi(x)$, and $\phi(x)$ are the Heisenberg operators on the surface σ. Further, using the equation of motion for the spin-$\frac{3}{2}$ field, the redundant components $\gamma_l\psi_l$ and $\gamma_4\psi_4$ can be expressed in terms of the canonically independent coordinates[11]

$$-\tfrac{1}{2}(\tfrac{2}{3}\gamma_l\partial_l-M)[3(A+1)\gamma_4\psi_4+(3A+1)\gamma_k\psi_k]=\partial_l\psi_l^{3/2}+\tfrac{1}{2}(1-2Z)g^*\gamma_4\psi\partial_4\phi-\tfrac{1}{2}(1+2Z)g^*\gamma_l\psi\partial_l\phi \qquad (3.9)$$

and

$$3(2A+1)M\gamma_\nu\psi_\nu=g^*\gamma_\mu\Theta_{\mu\nu}'\psi\partial_\nu\phi+(2g^*/M)\partial_\mu(\Theta_{\mu\nu}'\psi\partial_\nu\phi), \qquad (3.10)$$

where

$$\Theta_{\mu\nu}'=\delta_{\mu\nu}-\tfrac{1}{2}(1+2Z)\gamma_\mu\gamma_\nu.$$

Now we postulate the following commutation relations on a spacelike surface σ:

$$\{\psi_i^{3/2}(x),\psi_l^{3/2}(y)\}\delta(x^0-y^0)=0, \qquad (3.11a)$$

$$\{\psi(x),\psi(y)\}\delta(x^0-y^0)=0, \qquad (3.11b)$$

$$[\phi(x),\phi(y)]\delta(x^0-y^0)=0, \qquad (3.11c)$$

$$\{\psi_i^{3/2}(x),\psi(y)\}\delta(x^0-y^0)=0, \qquad (3.12a)$$

$$[\psi_i^{3/2}(x),\phi(y)]\delta(x^0-y^0)=0, \qquad (3.12b)$$

[17] C. N. Yang and D. Feldman, Phys. Rev. **79**, 972 (1950).

[18] Y. Takahashi, *An Introduction to Field Quantization* (Pergamon, New York, 1969), p. 201. We are using the notations of this book for the Heisenberg fields and the associated field variables.

$$[\psi(x),\phi(y)]\delta(x^0-y^0)=0. \qquad (3.12c)$$

It should be noted here that the spin-statistics theorem does not necessarily require the commutation relations (3.12). However, *PCT* symmetry implies at least the weak form of the local commutation relations given in (3.12).[19] By demanding that the interaction be consistent with the commutation relations (3.11) and (3.12), we find from Eqs. (3.5)–(3.10) that the parameter Z should be equal to $\frac{1}{2}$. This is also true if we assume only the weak local commutativity instead of the relations (3.12). Further, if we take abnormal commutation relations such as

$$[\psi_i^{3/2}(x),\psi(y)]\delta(x^0-y^0)=0, \qquad (3.13a)$$

$$\{\psi_i^{3/2}(x),\phi(y)\}\delta(x^0-y^0)=0, \qquad (3.13b)$$

$$\{\psi(x),\phi(y)\}\delta(x^0-y^0)=0 \qquad (3.13c)$$

and require that the interaction should be consistent with relations (3.11) and (3.13), we reach the same conclusion as before, that is, Z should equal $\frac{1}{2}$. It is not surprising that the physical content of this theory does not depend on whether the normal set of commutation relations (3.12) or the abnormal set (3.13) is used. Using the Klein transformation, it is possible to show that a theory with the abnormal commutation relations is a special case of a theory with the normal local commutativity.[19,20]

Now, using the expressions (3.5)–(3.12) and taking $Z=\frac{1}{2}$, the commutator or the anticommutator between any two field variables on a spacelike surface can be written down. Some of these commutation relations are

$$\{\psi_i^{3/2}(x),\psi_l^{3/2\dagger}(y)\}_{x^0=y^0}=\left[\delta_{il}-\tfrac{1}{3}\gamma_i\gamma_l-\frac{2}{9M^2}\partial_i\partial_l+\frac{2}{9M^2}(\boldsymbol{\gamma}\cdot\boldsymbol{\partial})\partial_i\gamma_l-\frac{2}{9M^2}(\boldsymbol{\gamma}\cdot\boldsymbol{\partial})\gamma_i\partial_l-\frac{2}{27M^2}\boldsymbol{\partial}^2\gamma_i\gamma_l\right]\delta^{(3)}(\mathbf{x}-\mathbf{y})$$
$$+\frac{4}{9M^4}g^{*2}A_{ij}\frac{\partial^2}{\partial x_j\partial y_k}[\delta^{(3)}(\mathbf{x}-\mathbf{y})(\boldsymbol{\gamma}\cdot\boldsymbol{\partial})\phi(x)(\boldsymbol{\gamma}\cdot\boldsymbol{\partial})\phi(y)]_{x^0=y^0}A_{kl}, \qquad (3.14)$$

$$\{\psi(x),\psi_l^{3/2\dagger}(y)\}_{x^0=y^0}=\frac{2}{3M^2}g^*\frac{\partial}{\partial y_i}[\delta^{(3)}(\mathbf{x}-\mathbf{y})(\boldsymbol{\gamma}\cdot\boldsymbol{\partial})\phi(y)]_{y^0=x^0}A_{il}, \qquad (3.15)$$

$$-\{[3(A+1)\gamma_4\psi_4(x)+(3A+1)\gamma_l\psi_l(x)],[3(A+1)\psi_4^\dagger(y)\gamma_4-(3A+1)\psi_k^\dagger(y)\gamma_k]\}_{x^0=y^0}$$
$$=-\frac{8}{3M^2}\boldsymbol{\partial}^2\delta^{(3)}(\mathbf{x}-\mathbf{y})+\frac{4}{M^4}g^{*2}\overrightarrow{B}(x)[\delta^{(3)}(\mathbf{x}-\mathbf{y})(\boldsymbol{\gamma}\cdot\boldsymbol{\partial})\phi(x)(\boldsymbol{\gamma}\cdot\boldsymbol{\partial})\phi(y)]_{x^0=y^0}\overleftarrow{B}(y), \qquad (3.16)$$

$$\{\psi(x),\psi^\dagger(y)\}_{x^0=y^0}=\delta^{(3)}(\mathbf{x}-\mathbf{y}), \qquad (3.17)$$

where

$$(\boldsymbol{\gamma}\cdot\boldsymbol{\partial})\phi(x)=\gamma_l\frac{\partial\phi(x)}{\partial x_l},\quad(\boldsymbol{\gamma}\cdot\boldsymbol{\partial})\phi(y)=\gamma_l\frac{\partial\phi(y)}{\partial y_l},\quad l=1,2,3$$

and

$$A_{il}=(\delta_{il}-\tfrac{1}{3}\gamma_i\gamma_l),\quad B(x)=\left(\tfrac{2}{3}\gamma_l\frac{\partial}{\partial x_l}+M\right).$$

In the derivation of (3.14)–(3.17), a knowledge of the free-field commutators and anticommutators is necessary. However, the equal-time commutation relations for the free fields can be easily obtained from the well-known covariant commutators and anticommutators. Expression (3.14) shows that in the presence of interaction the anticommutator of a spin-$\frac{3}{2}$ field with its Hermitian conjugate contains the pion field to which it has been coupled. A similar result was derived by Johnson and Sudarshan,[21] who quantized the spin-$\frac{3}{2}$ field in the presence of minimal electromagnetic interaction and found that the anticommutator between $\psi_i^{3/2}(x)$ and $\psi_l^{3/2\dagger}(x)$ depends on the electromagnetic field tensor. However, it was found in Ref. 21 that the quantization of the spin-$\frac{3}{2}$ field in the presence of minimal electromagnetic interaction leads to some inconsistency, such as the anticommutator of a field with its Hermitian conjugate is not always positive definite. It should be emphasized here that no such inconsistency exists for the system we are dealing with, provided the parameter Z equals $\frac{1}{2}$. The spin-$\frac{3}{2}$ field in the presence of interaction can be quantized also by using an alternative formalism developed by Schwinger.[22] We have found that the results are identical whether the Yang-Feldman technique or Schwinger's action principle is used for the quantization of the spin-$\frac{3}{2}$ field coupled to a nucleon field and a pion field.

[19] R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics, and All That* (Benjamin, New York, 1964), pp. 146 and 147.

[20] T. Kinoshita, Phys. Rev. **110**, 978 (1958).

[21] K. Johnson and E. C. G. Sudarshan, Ann. Phys. (N.Y.) **13**, 126 (1961).

[22] J. Schwinger, Phys. Rev. **82**, 919 (1951).

TABLE I. Isospin-even scattering lengths.

Scattering lengths	N exchange	N* exchange	Total	Experiment
$a_S{}^+$	−0.011	0.000	−0.011	−0.012±0.004[a]; −0.001±0.003[b]
$a_{P_{3/2}}{}^+$	+0.055	+0.053	+0.108	+0.135[c]; +0.134±0.005[d]
$a_{P_{1/2}}{}^+$	−0.107	+0.018	−0.089	−0.036[c]; −0.059±0.005[d]

[a] Ref. 24. [b] Ref. 25. [c] Ref. 26. [d] Ref. 27.

IV. CALCULATION OF SCATTERING LENGTHS

The S- and P-wave pion-nucleon scattering lengths are defined as

$$a_S=[\mu f_{0^+}]_{q^2=0},\quad a_{P_{1/2}}=[\mu^3 f_{1^-}/q^2]_{q^2=0},\quad a_{P_{3/2}}=[\mu^3 f_{1^+}/q^2]_{q^2=0},\tag{4.1}$$

where μ and q are, respectively, the mass and the c.m. momentum of the pion. The partial waves $f_{l^\pm}$ can be obtained from the invariant amplitudes $A(s,t)$ and $B(s,t)$. More explicitly,

$$f_{l^\pm}=\tfrac{1}{2}\int_{-1}^{+1} d(\cos\theta)[F_1P_l(\cos\theta)+F_2P_{l\pm1}(\cos\theta)],\tag{4.2}$$

where

$$F_1=\frac{E+m}{8\pi W}[A-(W-m)B],\qquad F_2=-\frac{E-m}{8\pi W}[A+(W+m)B],\tag{4.3}$$

$W=\sqrt{s}$, and m and E are, respectively, the mass and the c.m. energy of the nucleon. In our notation, the relationship between the conventional helicity amplitudes and the amplitudes F_1 and F_2 is given by

$$F_{++}=(F_1+F_2)\cos\tfrac{1}{2}\theta,\quad F_{+-}=(F_1-F_2)\sin\tfrac{1}{2}\theta.\tag{4.4}$$

Using the formulas (4.1)–(4.3) and the expressions (2.13)–(2.16) for the invariant amplitudes A and B with $Z=\frac{1}{2}$, the $\Delta(1238)$ contributions to the πN scattering lengths have been calculated explicitly. The N^* contributions to the S- and P-wave scattering lengths are

$$a_S{}^+=\tfrac{1}{3}(a_1+2a_3)=0,\tag{4.5}$$

$$a_S{}^-=\tfrac{1}{3}(a_1-a_3)=0,\tag{4.6}$$

$$a_{P_{3/2}}{}^+=\tfrac{1}{3}(a_{13}+2a_{33})=\frac{1}{18\pi}g^{*2}\frac{\mu^3 m}{m+\mu}\left\{\frac{1}{M-(m+\mu)}-\frac{1}{M^2-(m-\mu)^2}\left[M+m+\mu-\frac{1}{3M}(4m^2-6m\mu+6\mu^2)-\frac{4m^2}{3M^2}(m-\mu)\right]\right\},\tag{4.7}$$

$$a_{P_{3/2}}{}^-=\tfrac{1}{3}(a_{13}-a_{33})=\frac{1}{36\pi}g^{*2}\frac{\mu^3 m}{m+\mu}\left\{\frac{1}{m+\mu-M}+\frac{1}{(m-\mu)^2-M^2}\left[M+m+\mu-\frac{1}{3M}(4m^2-6m\mu+6\mu^2)-\frac{4m^2}{3M^2}(m-\mu)\right]\right\},\tag{4.8}$$

$$a_{P_{1/2}}{}^+=\tfrac{1}{3}(a_{11}+2a_{31})=a_{P_{3/2}}{}^+-\frac{\mu^2}{4m^2}a_S{}^+-\frac{\mu}{16\pi m}R^{(+)},\tag{4.9a}$$

$$R^{(+)}=\frac{16}{9}g^{*2}\mu^3 m\times\frac{2m^2+mM+(m/M)(m^2-\mu^2)}{[(m+M)^2-\mu^2][(m-M)^2-\mu^2]},\tag{4.9b}$$

$$a_{P_{1/2}}{}^-=\tfrac{1}{3}(a_{11}-a_{31})=a_{P_{3/2}}{}^--\frac{\mu^2}{4m^2}a_S{}^--\frac{\mu}{16\pi m}R^{(-)},\tag{4.10a}$$

$$R^{(-)}=\frac{2}{9}\frac{g^{*2}\mu^2}{[(m+M)^2-\mu^2][(m-M)^2-\mu^2]}\times\left\{(m^2+\mu^2-M^2)\left[12m(m+M)-\frac{4m}{M}(2m^2+3\mu^2)-\frac{4m^2}{M^2}(2m^2+\mu^2)\right]+4m^2\mu^2\left[1+\frac{10m}{M}+\frac{1}{M^2}(7m^2-\mu^2)\right]\right\},\tag{4.10b}$$

The coupling constant g^* can be determined from the relation

$$\Gamma_{\Delta\to\pi N}=\frac{1}{12\pi}g^{*2}\frac{p^3(E+m)}{M},\tag{4.11a}$$

where Γ and M are, respectively, the decay width and the mass of the resonance $\Delta(1238)$. The momentum and the energy of the nucleon in the pion-nucleon c.m. frame are denoted by p and E, respectively. Putting in the experimentally known values[23] of Γ and M in (4.11a), we find

$$g^{*2}/4\pi=18.90\ \text{GeV}^{-2}.\tag{4.11b}$$

The N^* contributions to the pion-nucleon scattering lengths given on the right-hand side of (4.5)–(4.10) have been evaluated numerically and are shown in

[23] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **42**, 87 (1970).

Tables I and II.[25–28] However, before the predictions from this model on the πN scattering lengths can be compared with experiment, the contributions due to nucleon exchange and direct pion-nucleon scattering or the ρ-mediated interaction will have to be obtained and added to the N^* contribution. This can be accomplished by using the chiral-invariant Lagrangian models for pion-nucleon scattering.[7–9] Using Weinberg's[7] model, the contributions of the nucleon exchange term and the direct pion-nucleon scattering term to the invariant amplitudes can be written as

$$\begin{aligned} A^- &= 0, \\ B^- &= \frac{2f^2}{\mu^2}+\frac{4m^2f^2}{\mu^2}\left(\frac{1}{s-m^2}+\frac{1}{u-m^2}\right)-\frac{2f_0^2}{\mu^2}, \\ A^+ &= \frac{4mf^2}{\mu^2}, \\ B^+ &= \frac{4m^2f^2}{\mu^2}\left(\frac{1}{s-m^2}-\frac{1}{u-m^2}\right), \end{aligned} \tag{4.12}$$

where

$$f=1.01\pm0.01\,,\quad f/f_0=1.18\pm0.02\,.$$

It may be mentioned here that in Weinberg's chiral-invariant Lagrangian for pion-nucleon scattering, the nucleon current is coupled to the derivative of the pion field. In addition to this, in the effective interaction Lagrangian for pion-nucleon scattering there is a direct pion-nucleon scattering term which, of course, does not contribute to any of the invariant amplitudes except B^-. The numerical values of the contributions due to the nucleon exchange and the direct pion-nucleon scattering are given in Tables I and II. However, it is also possible to construct a chiral-invariant Lagrangian for pion-nucleon scattering, where the direct scattering term is replaced effectively by the ρ-mediated interaction terms.[8,9] As far as the S-wave scattering lengths are concerned, these two models give identical results. For the P-wave scattering lengths, the contributions of the ρ-mediated interaction terms are appreciably different from that of the direct scattering term. This is shown explicitly in the tables.[28] Experimental results on the pion-nucleon scattering lengths are displayed in the tables beside the predictions from this model, and our comments and conclusions are given in Sec. V.

V. DISCUSSION

We have presented here a theory of the spin-$\frac{3}{2}$ field coupled to a spin-$\frac{1}{2}$ fermion field and a pseudoscalar boson field, and this theory is free from many troubles associated with the interactions of particles with spin $\geq\frac{3}{2}$. In particular, it has been possible to define uniquely the S-matrix elements for this system, though the propagator for the spin-$\frac{3}{2}$ particles contains an arbitrary parameter A. However, it has been found that the most general form of the interaction Lagrangian containing only the first-order derivative of the boson field contains a second parameter Z which can have any value. The S matrix for these interaction Lagrangians is independent of the parameter A, but it depends on the parameter Z which occurs only in the interaction Lagrangian. In order to determine the value of Z, the spin-$\frac{3}{2}$ field in the presence of interaction has been quantized on a spacelike surface. Then, by demanding that the interaction should be consistent with the principles of second quantization, the value of Z has been fixed. It is shown in the Appendix that it is possible to derive for this interaction the required number of subsidiary conditions from the equation of motion for the spin-$\frac{3}{2}$ field. This demonstrates that the theory developed here is also consistent at the classical level with the constraints on the components of the spin-$\frac{3}{2}$ field.

In order to study the low-energy pion-nucleon interaction and the kaon-nucleon interaction, it is very often necessary to evaluate the $N^*(J^P=\frac{3}{2}^\pm)$ contribution and the $Y^*(J^P=\frac{3}{2}^\pm)$ contribution to elastic πN scattering and KN scattering, respectively, by using second-order perturbation theory. The common practice, as we see in the literature, is to assume the following:

$$\mathcal{L}_1=g^*\bar{\psi}_\mu\psi\partial_\nu\phi+(\text{H.c. for an even-parity resonance}), \tag{5.1a}$$

$$\mathcal{L}_1=g^*\bar{\psi}_\mu\gamma_5\psi\partial_\nu\phi+(\text{H.c. for an odd-parity resonance}), \tag{5.1b}$$

and

$$A=-1\,, \tag{5.2}$$

where A is the parameter which occurs in the propagator given by (2.7) and (2.9). It must be emphasized here that these assumptions do not have any physical basis except that the calculations are simplified to a large extent. However, such a choice as (5.1) for the inter-

TABLE II. Isospin-odd scattering lengths.

Scattering lengths	N exchange	N* exchange	Direct-scattering term	Total	Experiment
a_S^-	+0.001	0.000	+0.101	+0.102 +0.102[b]	+0.097±0.006[a] +0.090±0.002[c]
$a_{P_{3/2}}^-$	−0.055	−0.037	0.000	−0.092 −0.099[b]	−0.082[d] −0.081±0.005[e]
$a_{P_{1/2}}^-$	−0.054	+0.042	+0.008	−0.004 +0.021[b]	−0.003[d] −0.021±0.005[e]

[a] Ref. 24.
[b] The total value of the predicted scattering lengths if the ρ-exchange model is used instead of the direct scattering model is given.
[c] Ref. 25.
[d] Ref. 26.
[e] Ref. 27.

[24] V. Samaranayake and W. Woolcock, Phys. Rev. Letters **15**, 936 (1965).
[25] J. Hamilton, Phys. Letters **20**, 687 (1966).
[26] L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. **138**, B190 (1965). a_{33} has been taken as 0.217.
[27] J. Hamilton and W. Woolcock, Rev. Mod. Phys. **35**, 737 (1963).
[28] See Ref. 9 for the details of the calculation on the ρ-exchange contribution.

action Lagrangian is inconsistent with the other requirements of the theory, namely, (1) that the quantization of the spin-$\frac{3}{2}$ field in the presence of interaction should be possible without violating the well-known principles of physics[21] and (2) that the eight subsidiary conditions should follow from the equation of motion for the spin-$\frac{3}{2}$ field.[12] The model presented here for the interaction of a spin-$\frac{3}{2}$ field with a nucleon field and a pion field or a kaon field is compatible with these requirements.

The theoretical predictions on the basis of this model for the S-wave scattering lengths are in good agreement with the experimental results of Samaranayake and Woolcock.[24] However, the experimental value of a_S^+ given by Hamilton[25] is 12 times smaller in magnitude than the number cited in Ref. 24. Further experimental information is necessary in order to have a clearer idea as to the real value of a_S^+. It should be noted here that the $\Delta(1238)$ contributions to both a_S^+ and a_S^- are exactly zero in our scheme. In Peccei's model,[9] which corresponds to the interaction Lagrangian (2.12) with $Z=-\frac{1}{4}$, the magnitude of the N^* contribution to a_S^+ is very large. Consequently, the theoretical prediction for a_S^+ in Ref. 9 is five times larger in magnitude than the experimental estimate given by Samaranayake and Woolcock.[24] On the P-wave scattering lengths, the theoretical results obtained from this model are in reasonable agreement with the experiment except for $a_{P_{1/2}}{}^+$. However, there is considerable confusion on the experimental value of $a_{P_{1/2}}{}^+$. Further, we have not taken account of the higher resonances such as $P_{11}(1470)$, $S_{11}(1550)$, and $D_{13}(1525)$ which also can contribute to the P-wave scattering lengths. Finally, this model can be used to study the shape of the $\Delta(1238)$ resonance or to calculate the partial wave P_{33} near the resonance. This can be accomplished by replacing the mass M of the resonance with $M-\frac{1}{2}i\Gamma$ in the formulas (2.13)–(2.16), where Γ is the width of the resonance. Further work on the application of these ideas is in progress. The question of the surface terms has been investigated. Our calculations indicate that the contributions of the surface terms in the interaction Hamiltonian will be completely canceled by the contributions coming from the noncovariant terms of the propagator provided the parameter Z equals $\frac{1}{2}$. In other words, the S-matrix elements in this theory can be evaluated by using the covariant propagator and the invariant interaction Hamiltonian. This result can also be interpreted as that the Lorentz invariance of the S-matrix requires that the parameter Z should be equal to $\frac{1}{2}$. The details of these investigations will be published in a future paper.

ACKNOWLEDGMENTS

The authors would like to thank Professor Martin Kretzschmar, Donald Robson, M. H. Ross, P. K. Williams, and Dr. S. L. Cohen for some helpful discussions.

APPENDIX

The field equation for the spin-$\frac{3}{2}$ particles in the presence of the interaction (2.12) can be written as

$$(\gamma_\lambda\partial_\lambda+M)\psi_\mu+A(\gamma_\mu\partial_\nu+\gamma_\nu\partial_\mu)\psi_\nu+B\gamma_\mu(\gamma_\lambda\partial_\lambda)\gamma_\nu\psi_\nu -CM\gamma_\mu\gamma_\nu\psi_\nu-g^*\Theta_{\mu\nu}\psi\partial_\nu\phi=0\,, \quad \text{(A1)}$$

where B and C have been defined in (2.4) in terms of the parameter A which can have any value except that $A\neq-\frac{1}{2}$. The explicit expression for $\Theta_{\mu\nu}$ for an arbitrary Z has been given in (2.12). For the interaction (2.12), the equations of motion for the nucleon field and the pion field are, respectively,

$$(\gamma_\lambda\partial_\lambda+m)\psi-g^*(\partial_\nu\phi)\Theta_{\nu\mu}\psi_\mu=0\,, \quad \text{(A2)}$$

$$(\Box-\mu^2)\phi-g^*[\partial_\nu(\bar\psi\Theta_{\nu\mu}\psi_\mu)+\text{H.c.}]=0\,. \quad \text{(A3)}$$

It is no longer possible to derive the subsidiary conditions (2.5) and (2.6) from the field equation (A1). However, it does not necessarily mean that there are more degrees of freedom in the theory than are required to describe the spin-$\frac{3}{2}$ particles. If we can deduce from (A1) eight equations containing only the space derivatives of the field ψ_μ, but no derivative with respect to the time, the eight redundant components of ψ_μ can be successfully eliminated. Then the theory will have only the eight independent components of ψ_μ which are necessary for a relativistic description of a fermion with four states of polarization.[12] Multiplying (A1) with γ_μ from the left-hand side, we get

$$(3A+1)\gamma_\lambda\partial_\lambda\gamma_\nu\psi_\nu+2\partial_\lambda\psi_\lambda-3(2A+1)M\gamma_\nu\psi_\nu -(2A+1)^{-1}g^*\gamma_\mu\Theta_{\mu\nu}\psi\partial_\nu\phi=0\,. \quad \text{(A4)}$$

Subtracting the fourth component ($\mu=4$) of (A1) from (A4), a set of four subsidiary conditions can be obtained. More explicitly, these are

$$-\tfrac{1}{2}(\tfrac{2}{3}\boldsymbol{\gamma}\cdot\boldsymbol{\partial}-M)[3(A+1)\gamma_4\psi_4+(3A+1)\gamma_l\psi_l] =\partial_l\psi_l{}^{3/2}+\tfrac{1}{2}(1-2Z)g^*\gamma_4\psi\partial_4\phi -\tfrac{1}{2}(1+2Z)g^*\gamma_l\psi\partial_l\phi\,. \quad \text{(A5)}$$

Differentiation of Eq. (A1) with respect to x_μ yields

$$\tfrac{1}{2}(A+1)(\gamma_\lambda\partial_\lambda)[2\partial_\mu\psi_\mu+(3A+1)(\gamma_\mu\partial_\mu)\gamma_\nu\psi_\nu] -CM(\gamma_\lambda\partial_\lambda)\gamma_\nu\psi_\nu+M\partial_\lambda\psi_\lambda -g^*\partial_\mu(\Theta_{\mu\nu}\psi\partial_\nu\phi)=0\,. \quad \text{(A6)}$$

Now, comparing (A6) with (A4), we obtain

$$(3A+1)(\gamma_\lambda\partial_\lambda)\gamma_\nu\psi_\nu+2\partial_\lambda\psi_\lambda +(A+1)(2A+1)^{-1}(g^*/M)(\gamma_\lambda\partial_\lambda)(\gamma_\mu\Theta_{\mu\nu}\psi\partial_\nu\phi) -(2g^*/M)\partial_\mu(\Theta_{\mu\nu}\psi\partial_\nu\phi)=0\,. \quad \text{(A7)}$$

The second set of four subsidiary conditions can be derived by subtracting (A4) from (A7). These constraints can be written as

$$3(2A+1)M\gamma_\nu\psi_\nu=g^*\gamma_\mu\Theta_{\mu\nu}'\psi\partial_\nu\phi +(2g^*/M)\partial_\mu(\Theta_{\mu\nu}'\psi\partial_\nu\phi)\,, \quad \text{(A8)}$$

where

$$\Theta_{\mu\nu}'=[\delta_{\mu\nu}-\tfrac{1}{2}(1+2Z)\gamma_\mu\gamma_\nu]. \qquad (A9)$$

For $Z=\frac{1}{2}$,

$$\Theta_{44}'=0. \qquad (A10)$$

Equations (A2), (A3), and (A10) show that the right-hand side of (A8) is indeed free from the time derivative of ψ_μ and Eqs. (A8) can be considered as subsidiary conditions. This technique for the derivation of subsidiary conditions is essentially due to Fierz and Pauli.[12]

PHYSICAL REVIEW D VOLUME 3, NUMBER 9 1 MAY 1971

K^+-Nucleon Scattering in Broken $SU(3)\times SU(3)$*

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(Received 5 October 1970)

S-wave K^+-nucleon scattering is discussed in the framework of chiral $SU(3)\times SU(3)$, broken by a term transforming according to the $(3^*,3)+(3,3^*)$ representation, giving the usual partial conservation laws (partial conservation of vector current and partial conservation of axial-vector current). The scattering lengths are calculated for models giving the correct baryon spectrum and assuming that φ decouples from nucleons. Good agreement with experiment is obtained, supporting the breaking mechanism assumed. The numbers are $a(K^+p)=-0.27\ \mu_\pi^{-1}$ and $a(K^+n)=-0.14\ \mu_\pi^{-1}$.

I. INTRODUCTION

In this paper we discuss S-wave K^+-nucleon scattering in the framework of chiral $SU(3)\times SU(3)$. An effective Lagrangian for $\frac{1}{2}^+$ baryons, $0^\pm$ mesons, and $1^\pm$ mesons is constructed according to well-known rules,[1] which in the tree approximation generate scattering amplitudes satisfying the assumed symmetry properties.

Whereas soft-pion $SU(2)\times SU(2)$ current-algebra predictions are very good for π-N scattering,[2] the same is not true for K^+-N scattering. The kaon mass being so much larger than the pion's, hard-kaon methods are required. Since in this case we have to use $SU(3)\times SU(3)$, the symmetry-breaking mechanism has to be specified and K^+-nucleon scattering may be used to discriminate between the different models.

In this paper we take a conservative stand, trying to see whether the process in question can be understood via the usual symmetry-breaking scheme suggested by the quark model, where the symmetry-breaking term transforms as a $(3,3^*)+(3^*,3)$ representation, shifting the $0^\pm$ meson masses from zero to a finite value. We are thus assuming the usual partial conservation laws for the strangeness-changing weak vector currents (PCVC) and the weak axial-vector currents (PCAC). We furthermore assume the validity of the field algebra,[3] our $SU(3)\times SU(3)$-symmetric Lagrangian being thus of the Yang-Mills type with a mass term for the $1^\pm$ gauge fields.

As will be seen, the rather scarce experimental numbers can perfectly be understood within this frame. In this respect our conclusions differ from Schechter and Ueda,[4] who recently treated the same problem assuming $SU(3)\times SU(3)$ without scalar mesons but introducing symmetry breaking in a somewhat arbitrary fashion. In our case the terms of the effective Lagrangian giving the baryon and meson mass spectra are introduced in such a way as to destroy neither our exact and partial conservation laws, nor vector-meson dominance resulting from the algebra of fields. We thus do not agree with their tentative conclusion, that all $SU(3)$-symmetry breaking should come from the physical mass terms allowing the coupling constants to retain their $SU(3)$-symmetric values. Precisely these values forced them to use a somewhat unorthodox symmetry-breaking scheme.

In Sec. II we write down our Lagrangian, on which we also impose the requirement that the physical φ decouples from nucleons.[5] The meson part of this Lagrangian has been discussed extensively in the literature and we refer the reader to Ref. 1 for any more details. Section III contains the discussion of K^+-nucleon scattering.

II. LAGRANGIAN FOR K^+-NUCLEON SCATTERING

Our Lagrangian will contain an octet of $\frac{1}{2}^+$ baryons transforming according to the $[(8,1),(1,8)]$ repre-

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