Photon Low-Energy Theorem and the Radiative Decays of the Ω^- Particle*†

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The photon low-energy theorem due to Low, and to Adler and Dothan is applied to the two-body radiative decays of the Ω^- particle. The decay amplitudes are evaluated in the resulting pole model. With the usual assumptions of octet dominance and CP invariance for the weak Hamiltonian, it is shown that the parity-violating amplitude for $\Omega^- \to \Xi^{*-}\gamma$ should be zero for a current×current weak interaction. The weak vertex parameters obtained from a fit to the experimental data on the nonleptonic decays of the hyperons are used to estimate the decay amplitudes and the decay rates. The implications of the results obtained earlier by Raszillier and Burlacu on the three-body radiative decays $\Omega^- \to B_8+P+\gamma$ are discussed. Theoretical estimates for the decay rates of weak decay modes of the Ω^- particle have been compiled.

I. INTRODUCTION

THE Ω^- particle decays only via its weak interactions, and it is expected to have decay modes analogous to the weak decays of the strange baryons. In particular, the radiative decays $\Omega^-\to\Xi^-\gamma$ and $\Omega^-\to\Xi^{*-}\gamma$ are expected to compete favorably with the semileptonic decay modes. A study of these radiative decay modes is therefore of interest. The necessary formalism for a phenomenological analysis of these decay modes is presented.

It is shown in Sec. II that the parity-violating (p.v.) amplitude for $\Omega^-\to\Xi^{*-}\gamma$ should be zero for a current ×current weak interaction with the usual assumptions of octet dominance and CP invariance for the weak Hamiltonian. This result parallels the results for the weak radiative decays of the octet hyperons derived by Hara,[1] using the $TL(1)$ symmetry[2] of the current ×current model. The decay amplitudes are evaluated in the pole model. It is estimated that the $\Xi^{*-}\gamma$ mode has a negligible decay rate. On the other hand, the $\Xi^-\gamma$ decay mode has the branching ratios

$$\Gamma(\Omega^-\to\Xi^-\gamma)/\Gamma(\Omega^-\to\Xi^-\pi^0)=1.38\%$$

and

$$\Gamma(\Omega^-\to\Xi^-\gamma)/\Gamma(\Omega^-\to\Xi^0e^-\bar{\nu})=19\%.$$

Theoretical estimates for the decay rates of the semileptonic, the nonleptonic, and the weak radiative decays of the Ω^- particle have been compiled in Table I.

The following analysis assumes that the spin of the Ω^- is $J_\Omega=\frac{3}{2}$. In Sec. III the possibility of measuring the spin of the Ω^- particle by a measurement of the decay distribution function in the nonleptonic decays $\Omega^-\to\Xi\pi$ is discussed. For such an analysis, a study of the $\Xi\pi$ decay mode would be preferable since two completely independent theoretical considerations, one using current algebra and the current×current model[3] and the other from a study[4] of the radiative decays $\Omega^-\to B+P+\gamma$, predict large asymmetries in the decay distribution for these decays. If such large asymmetries are observed experimentally, and if the decay distribution function is measured, then the Lee-Yang inequalities[5] satisfied by the averages, over the distribution function, of certain test functions would allow an unambiguous measurement of the spin of the Ω^- particle.

II. TWO-BODY RADIATIVE DECAYS OF Ω^- PARTICLE

A. Symmetry Considerations

The electromagnetic interactions conserve U-spin[6] and hence the effective Hamiltonian for $\Omega^-\to\Xi^-\gamma, \Xi^{*-}\gamma$ will have the same U-spin properties as the current ×current interaction. Thus the $TL(1)$ symmetry[2] of the current×current Hamiltonian will be preserved. If it is assumed that the weak Hamiltonian transforms as the $T_3{}^2$ member of an octet, then the effective Hamiltonian transforms as the irreducible $SU(3)$ tensors contained in the direct product $T_3{}^2\times T_1{}^1$. The requirement of $TL(1)$ invariance, or invariance under the interchange $2\leftrightarrow3$ of the $SU(3)$ indices, does not lead to any restrictions on the decay amplitudes for $\Omega^-\to\Xi^-\gamma$. However, for the decay $\Omega^-\to\Xi^{*-}\gamma$, $TL(1)$ invariance requires the p.v. decay amplitude to be zero. This follows from the fact that the Ω^- and the Ξ^{*-} particles belong to the same U-spin multiplet.

The $SU(3)$ structure of the effective Hamiltonian for $B_{10}\to B_{10}+\gamma$, where B_{10} are the spin-$\frac{3}{2}$ decuplet particles with $SU(3)$ wave functions ψ_{ijk}, is given in terms of the irreducible tensors **8** and **27** in the product $T_3{}^2\otimes T_1{}^1$, and we have

$$H_w(B_{10}\to B_{10}\gamma)=a(\bar{\psi}^{ij2}\psi_{ij3}\pm\bar{\psi}^{ij3}\psi_{ij2}) +b(\bar{\psi}^{j21}\psi_{j31}\pm\bar{\psi}^{j31}\psi_{j21}), \quad (1)$$

where the upper sign holds for the parity-conserving (p.c.) amplitude and the lower sign holds for the p.v. amplitude. Here the coefficients a and b are real, as required by time-reversal invariance. Thus $TL(1)$ invariance would require the vanishing of the p.v.

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TABLE I. Decay rates of the semileptonic, nonleptonic, and weak radiative decays of the Ω^- particle.

Decay mode	Rate (10^8 sec^{-1}) Theory	Expt
1. Semileptonic (Ref. 14)		
$\Omega^- \to \Xi^0 e^- \bar{\nu}$	0.47	...
$\Omega^- \to \Xi^0 \mu^- \bar{\nu}$	0.32	...
$\Omega^- \to \Xi^{*0} e^- \bar{\nu}$	0.17	...
$\Omega^- \to \Xi^{*0} \mu^- \bar{\nu}$	0.01	...
2. Nonleptonic (Ref. 3)		
$\Omega^- \to \Xi^- \pi^0$	6.5	9.6
$\Omega^- \to \Xi^0 \pi^-$	13.0	26.0
$\Omega^- \to \Lambda^0 K^-$	59.0	42.0
$\Omega^- \to \Xi^{*-} \pi^0$	0.37	...
$\Omega^- \to \Xi^{*0} \pi^-$	0.74	...
3. Weak electromagnetic		
$\Omega^- \to \Xi^- \gamma$	0.09	...
$\Omega^- \to \Xi^{*-} \gamma$	2.4×10^{-6}	...
Total rate (10^8 sec^{-1})	80.67	~77.6

amplitudes. For the decay $\Omega^- \to \Xi^{*-}\gamma$ the second term does not contribute. The result that the p.v. amplitude for $\Omega^- \to \Xi^{*-}\gamma$ is zero is analogous to the results for the weak radiative decays of the octet hyperons derived by Hara,[1] who was able to show that under the same conditions the p.v. amplitudes in the decays $\Sigma^+ \to p\gamma$ and $\Xi^- \to \Sigma^-\gamma$ vanish.

B. Weak Electromagnetic Decay $\Omega^- \to \Xi^-\gamma$

The most general decay-matrix element consistent with Lorentz invariance is

$$\begin{aligned} M(\Omega^-(p) \to \Xi(p'); \gamma(k)) &= \bar{u}(p')\{[a_1\delta_{\mu\nu} + a_2 i\gamma_\mu k_\nu + a_3 k_\nu(p'+p)_\mu + a_4 k_\mu k_\nu] \\ &\quad + [b_1\delta_{\mu\nu} + b_2 i\gamma_\mu k_\nu + b_3 k_\nu(p'+p)_\mu + b_4 k_\mu k_\nu]\gamma_5\}\psi_\nu(p)\epsilon_\mu \\ &= M_\mu \epsilon_\mu , \end{aligned} \tag{2}$$

where ϵ_μ is the photon polarization.

Time-reversal invariance requires the coefficients a_i and b_i to be real. For photons the gauge $k_\mu\epsilon_\mu = 0$ is used, and hence $a_4 = 0$ and $b_4 = 0$. In the rest frame of the Ω^- particle the kinematical restrictions for this two-body decay require the vanishing of a_3 and b_3. Finally, gauge invariance relates a_1 to a_2 and b_1 to b_2, leaving only two arbitrary parameters corresponding to the p.c. and the p.v. decay amplitudes. Gauge invariance is expressed by

$$M_\mu k_\mu = 0 . \tag{3}$$

This leads to the relations

$$a_1 = (M_\Omega - M_\Xi) a_2 \tag{4}$$

and

$$b_1 = -(M_\Omega + M_\Xi) b_2 . \tag{5}$$

The gauge-invariant matrix element can also be expressed in the more compact form

$$M(\Omega^- \to \Xi^-\gamma) = \bar{u}(p')\gamma_\mu(a_2 + \gamma_5 b_2)\psi_\nu(p) F_{\mu\nu} . \tag{6}$$

The decay rate, evaluated using the matrix element of Eq. (2) with the above restrictions, is given by

$$\begin{aligned} \Gamma(\Omega^- \to \Xi^-\gamma) = \frac{(M_\Omega^2 - M'^2)}{12\pi M_\Omega^2} \{ & (E'+M')[|a_1|^2 \\ & + (E'-M')^2|a_2|^2 - (E'-M')\,\mathrm{Re}(a_1^* a_2)] \\ & + (E'-M')[|b_1|^2 + (E'+M')^2|b_2|^2 \\ & + (E'+M')\,\mathrm{Re}(b_1^* b_2)]\} . \end{aligned} \tag{7}$$

The decay amplitudes are evaluated using the perturbation theory approach of Low[7] and of Adler and Dothan.[8] It has been shown by Low[7] that the amplitude for any process describing the emission of a photon of momentum k_μ has terms of order k_μ^{-1} (infrared-divergent term) and of order zero in k_μ, which depend only on the corresponding nonradiative amplitude and on the electromagnetic constants of the initial and final particles. Adler and Dothan[8] note that the result arises from the fact that the divergence of the electric current is measurable. The radiative amplitude may be expressed in terms of two terms, one in which the photon is radiated from an external charged-particle line and the other in which the photon is emitted from an internal line. Insertion of the photon lines on external charged lines give rise to the terms M^{ext} of $O(k^{-1})$ and $O(k^0)$ from the propagators of the charged hadrons. The term M^{int}, obtained from photon emission from internal lines, is finite at $k_\mu = 0$ and contains terms independent of k and of $O(k)$. The lemmas proved by Adler and Dothan[8] lead to a simple recipe for the construction of the radiative matrix element which are of order k^0 and of order k^{-1}. One writes M^{ext} as a sum of all terms in which the photon is radiated from external charged lines. Terms in M^{ext} which are explicitly independent of k_μ are dropped, and terms independent of k_μ are added so as to make the truncated matrix element gauge invariant.

For the special case of the two-body decays of hadrons, $A \to B + \gamma$, where A and B are single particles, it has been shown by Pestieau[9] that in the soft-photon limit the above procedure reduces to the usual pole model.[10–12] In the following it will be assumed that the pole model gives adequately accurate results even for physical photons. Similar assumptions are usually made in evaluating the pion photoproduction amplitudes in the isobaric models.[13]

The decay amplitudes for $\Omega^- \to \Xi^-\gamma$ are evaluated in the pole model with the pole diagrams of Fig. 1. The Ξ^{*-} pole vanishes because the electromagnetic vertex

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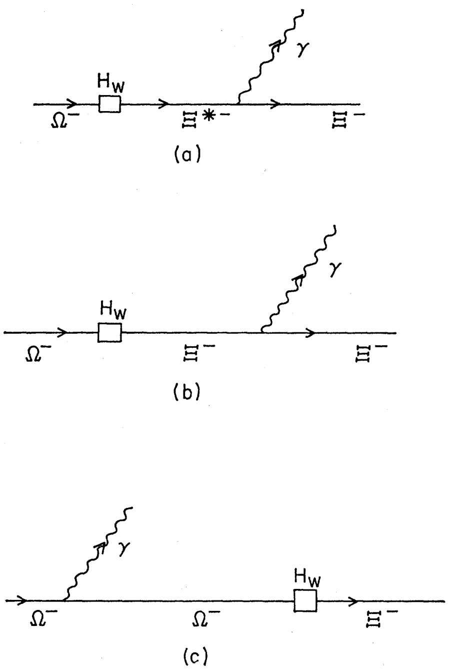

FIG. 1. Pole diagrams for the weak radiative decay $\Omega^- \to \Xi^- \gamma$.

must conserve U spin whereas the Ξ^-—Ξ^{*-} transition involves a change of U spin with the Ξ^{*-} having U spin $\frac{3}{2}$ and the Ξ^- having U spin $\frac{1}{2}$. The Ξ^- pole does not contribute because of the kinematic subsidiary conditions satisfied by the spin-$\frac{3}{2}$ initial state. Thus only pole diagram (c) of Fig. 1 with the Ω^- pole contributes to the decay amplitudes.

The weak vertex parameters for the Ξ^--Ω^- vertex are obtained from the phenomenological analysis of Ram Mohan,[12] and the two-body weak vertex is given by

$$\langle \Xi^- | H_w | \Omega^- \rangle = -i\partial_\mu \bar{u}(p')(G_{\text{p.v.}} + \gamma_5 G_{\text{p.c.}})\psi_\mu(p)\,, \quad (8)$$

with

$$G_{\text{p.v.}} = -4.37\times 10^{-7}\ \text{MeV}^{-1}, \qquad G_{\text{p.c.}} = -2.32\times 10^{-7}\ \text{MeV}^{-1}. \quad (9)$$

The gauge-invariant weak Hamiltonian is therefore given by the minimal substitution

$$H_w = -i(\partial_\mu - ieA_\mu)\bar{u}(p')(G_{\text{p.v.}} + \gamma_5 G_{\text{p.c.}})\psi_\mu(p)\,, \quad (10)$$

where it assumed that there is no momentum transfer across the weak vertex.

The electromagnetic interaction of the Ω^- is expressed by

$$H_{\text{e.m.}} = -e[\bar{\psi}_\mu i\gamma_\lambda \psi_\mu A_\lambda + (\kappa_\Omega/4M_\Omega)\bar{\psi}_\mu \sigma_{\lambda\rho}\psi_\mu F_{\lambda\rho}]\,, \quad (11)$$

where terms of higher order in the momentum transfer are neglected, and κ_Ω is the anomalous magnetic moment of the Ω^- particle.

The contribution to the matrix element for the decay from the pole diagram of Fig. 1 is given by

$$M(\Omega^- \to \Xi^- \gamma) = M_1 + M_c\,, \quad (12)$$

where

$$M_1 = e\bar{u}(p')(G_{\text{p.v.}} + \gamma_5 G_{\text{p.c.}}) p_\mu' [\delta_{\mu\nu} - \tfrac{1}{3}\gamma_\mu\gamma_\nu - (i/3M_\Omega)(\gamma_\mu p_\nu' - \gamma_\nu p_\mu') + (2p_\mu' p_\nu'/3M_\Omega^2)] \times \frac{-i\gamma_\alpha\cdot(p_\alpha - k_\alpha) + M_\Omega}{(p_\alpha - k_\alpha)^2 + M_\Omega^2} \times \left[i\gamma_\rho \epsilon_\rho + i\left(\frac{\kappa_\Omega}{2M_\Omega}\right)\sigma_{\rho\xi}\epsilon_\rho k_\xi \right]\psi_\nu(p)\,, \quad (13)$$

and

$$M_c = -e\bar{u}(p')(G_{\text{p.v.}} + \gamma_5 G_{\text{p.c.}}) \times \left(1 + \frac{2p_\alpha'^2}{3M_\Omega^2} - \frac{i\gamma_\alpha p_\alpha'}{3M_\Omega}\right)\psi_\nu(p)\epsilon_\nu\,. \quad (14)$$

Gauge invariance of the matrix element obtained from the pole diagram of Fig. 1 requires the introduction of an additional "contact" term in which the photon is emitted from the weak vertex. It ensures that the gauge condition of Eq. (3) is explicitly satisfied. The first term in the contact term [Eq. (14)] arises from the minimal substitution of Eq. (10). The terms proportional to the momenta are expected to originate from minimal substitutions in the nonradiative two-body weak Hamiltonian of Eq. (8) when momentum dependence of the weak vertex is taken into account. The need for the contact term arises because the propagator for the spin-$\frac{3}{2}$ intermediate state has terms of order k. The soft-photon calculation represented by the pole model cannot determine amplitudes beyond order k^{-1} and order k^0, as mentioned earlier.

The decay amplitudes evaluated from Eqs. (12) and (9) using physical masses for the initial and final baryons are given by

$$M(\Omega^- \to \Xi^- \gamma)_{\text{p.c.}} = -e\bar{u}(p')\left(0.81\times 10^{-9}\delta_{\mu\nu} + \frac{0.36\times 10^{-7}}{M_\Omega + M_\Xi} i\gamma_\mu k_\nu\right)\gamma_5 \psi_\nu \epsilon_\mu$$

and

$$M(\Omega^- \to \Xi^- \gamma)_{\text{p.v.}} = e\bar{u}\left(1.83\times 10^{-7}\delta_{\mu\nu} + \frac{1.98\times 10^{-7}}{M_\Omega - M_\Xi} i\gamma_\mu k_\nu\right)\psi_\nu \epsilon_\mu\,.$$

The contributions from the p.c. and p.v. amplitudes to the decay rates are given by

$$\Gamma_{\text{p.c.}}(\Omega^- \to \Xi^- \gamma) = 2.8\times 10^{-17}\ \text{MeV}$$

and

$$\Gamma_{\text{p.v.}}(\Omega^- \to \Xi^- \gamma) = 5.89\times 10^{-15}\ \text{MeV}. \quad (15)$$

The total decay rate $\Gamma(\Omega^- \to \Xi^-\gamma) = 5.92 \times 10^{-15}$ MeV can be compared with the decay rate for the $\Xi^-\pi^0$ mode evaluated in Ref. 3; the value predicted for this branching ratio is

$$\Gamma(\Omega^- \to \Xi^-\gamma)/\Gamma(\Omega^- \to \Xi^-\pi^0) = 1.38 \times 10^{-2} = 1.38\%. \quad (16)$$

The radiative decay mode $\Omega^- \to \Xi^-\gamma$ competes more favorably with the semileptonic decay modes, and the branching ratio for the $\Xi^0\gamma$ to the $\Xi^0 e^- \bar{\nu}$ decay mode is predicted to be

$$\Gamma(\Omega^- \to \Xi^-\gamma)/\Gamma(\Omega^- \to \Xi^0 e^- \bar{\nu}) = 0.19 = 19\%. \quad (17)$$

This estimate is based on the semileptonic decay rate evaluated using $SU(6)$ symmetry by Muzinich and by Pakvasa and Rosen.[14]

C. Weak Electromagnetic Decay $\Omega^- \to \Xi^{*-}\gamma$

This decay is characterized by the gauge-invariant phenomenological Hamiltonian

$$H_w(\Omega^- \to \Xi^{*-}\gamma) = \tfrac{1}{2}\bar{\psi}_\mu(p')\{[(a_1^* + a_2^* k_\mu k_\nu / 2M_\Omega^2) + (b_1^* + b_2^* k_\mu k_\nu / 2M_\Omega^2)\gamma_5]\sigma_{\rho\xi}\}\psi_\nu(p) F_{\rho\xi}. \quad (18)$$

It will be assumed that the amplitudes a_2^* and b_2^* can be neglected in comparison with the amplitudes a_1^* and b_1^*, respectively. This assumption is a reasonable one because the contribution to the decay rate from these amplitudes will be suppressed by kinematical factors proportional to the photon momentum. Secondly, the amplitudes will be evaluated in the soft-photon limit using the pole model, and in this limit the amplitudes a_2^* and b_2^* are zero. The pole model is expected to give good results even when the photon momentum is extrapolated to its physical value for the decay. The decay rate, in this approximation, is given by

$$\Gamma(\Omega^- \to \Xi^{*-}\gamma) = [(M_\Omega^2 - M_{\Xi^*}^2)^3 / 8\pi M_\Omega^3] \times \{|a_1^*|^2[1 + 2(E_{\Xi^*} + M_{\Xi^*})/9M_{\Xi^*}^2] + |b_1^*|^2[1 + 2(E_{\Xi^*} - M_{\Xi^*})/9M_{\Xi^*}^2]\}. \quad (19)$$

The amplitudes a_1^* and b_1^* are evaluated in the pole approximation using the pole diagrams of Fig. 2. The electromagnetic vertex given by Eq. (11) is used in the calculations. The weak vertex is given by

$$\langle \Xi^{*-} | H_w | \Omega^- \rangle = \bar{\psi}_\mu(p')[G^*_{\text{p.c.}} + \gamma_5 G^*_{\text{p.v.}}]\psi_\mu(p). \quad (20)$$

The parameter $G^*_{\text{p.c.}}$ is the matrix element of the p.c. decuplet spurion between Ω^- and Ξ^{*-} states and has been evaluated in Ref. 3 by inserting a complete set of states between the currents in a current×current model for the weak Hamiltonian. By assuming that the octet and decuplet baryon intermediate states alone saturate the matrix element, it was shown that the matrix element exhibits octet dominance quite apart from the detailed assumptions regarding the form factors used in defining the currents. Thus the strength of the p.c. vertex is known, and the parameter $G^*_{\text{p.c.}}$ is given by

$$G^*_{\text{p.c.}} = 1.075 \times 10^{-5}\ \text{MeV}. \quad (21)$$

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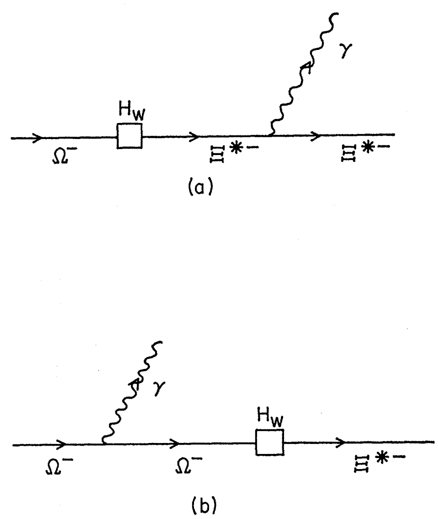

FIG. 2. Pole diagrams for the weak radiative decay $\Omega^- \to \Xi^{*-}\gamma$.

The parameter $G^*_{\text{p.v.}}$ is zero in the limit of $SU(3)$ symmetry for the initial and final states in the current ×current model, as has been shown in Sec. II A. However the medium-strong interactions would break the $TL(1)$ symmetry of the weak Hamiltonian and this would lead to a nonzero value for $G^*_{\text{p.v.}}$, and the p.v. amplitude will no longer vanish. No reliable estimate exists for this p.v. decuplet spurion. In the absence of such information let us assume that its strength is of the same order of magnitude as the matrix elements of the p.v. spurion for octet states[12]; then $G^*_{\text{p.v.}}$ is given by

$$G^*_{\text{p.v.}} \cong (1\text{–}5) \times 10^{-6}\ \text{MeV}. \quad (22)$$

The p.c. and p.v. amplitudes a_1^* and b_1^*, respectively, are given in the pole model by

$$a_1^* = -e\frac{G_{\text{p.c.}}^*}{M_\Omega - M_{\Xi^*}}\left(\frac{\kappa_\Omega}{2M_\Omega} - \frac{\kappa_{\Xi^{*-}}}{2M_{\Xi^*}}\right) \quad (23)$$

and

$$b_1^* = -e\frac{G_{\text{p.v.}}^*}{M_\Omega + M_{\Xi^*}}\left(\frac{\kappa_\Omega}{2M_\Omega} + \frac{\kappa_{\Xi^{*-}}}{2M_{\Xi^*}}\right). \quad (24)$$

Here κ_Ω and $\kappa_{\Xi^{*-}}$ are the anomalous magnetic moments of the Ω and the Ξ^{*-} particles. $SU(6)$ symmetry can be used to predict these magnetic moments in the absence of experimental data, and we have $\kappa_\Omega = \kappa_{\Xi^{*-}} = -1.79$. With these values as input, the decay amplitudes are estimated to be

$$a_1^* = -e(3.74 \times 10^{-12})\ \text{MeV}^{-1} \quad (25)$$

and

$$b_1^* = -e(3.48\text{–}17.4) \times 10^{-13}\ \text{MeV}^{-1}. \quad (26)$$

The contributions to the decay rate from the p.c. and the p.v. amplitudes are evaluated using Eq. (19) and are given by

$$\Gamma(\Omega^- \to \Xi^{*-}\gamma)_{\text{p.c.}} = 1.44\times10^{-19}\text{ MeV}$$
and
$$\Gamma(\Omega^- \to \Xi^{*-}\gamma)_{\text{p.v.}} = 0.16\times10^{-19}\text{ MeV}, \tag{27}$$

where the *upper* limit of the strength of the p.v. decuplet spurion has been used.

The total decay rate is $\Gamma(\Omega^- \to \Xi^{*-}\gamma) = 1.60\times10^{-19}$ MeV. The branching ratio for the $\Xi^{*-}\gamma$ decay mode to the $\Xi^{*-}\pi^0$ decay mode can be evaluated using the estimated decay rate of the $\Xi^{*-}\pi^0$ mode given in Ref. 3. The predicted branching ratio is

$$\Gamma(\Omega^- \to \Xi^{*-}\gamma)/\Gamma(\Omega^- \to \Xi^{*-}\pi^0) = 6.6\times10^{-6}. \tag{28}$$

In this section the two-body radiative decays of the Ω^- have been evaluated with the weak vertex parameters determined from pole-model calculations made earlier.[12] The estimates for the decay rates show that the branching ratio $\Gamma(\Omega^- \to \Xi^-\gamma)/\Gamma(\Omega^- \to \Xi^-\pi^0) = 1.38\%$ is of the same order of magnitude as the ratio $\Gamma(\Sigma^+ \to p\gamma)/\Gamma(\Sigma^+ \to p\pi^0)$ and the decay mode $\Omega^- \to \Xi^-\gamma$ is the analog, in Ω^- decays, of $\Sigma^+ \to p\gamma$ in the weak decays of the Σ^+ particle.

In the limit of $SU(3)$ symmetry, $TL(1)$ invariance of the current$\times$current model requires the p.v. decay amplitude in $\Omega^- \to \Xi^{*-}\gamma$ to be zero. This result parallels the results obtained by Hara[1] that in the same symmetry limit, the p.v. amplitudes for $\Sigma^+ \to p\gamma$ and $\Xi^- \to \Sigma^-\gamma$ be zero. The decay rate for $\Omega^- \to \Xi^{*-}\gamma$ is estimated to be very small.

The theoretical estimates[3,14] for various decay modes of the Ω^- particle have been compiled in Table I. The experimental numbers[15] for the nonleptonic decay rates are based on 24 events of which 13 events are in the ΛK^- mode and 11 in the $\Xi\pi$ modes. The theoretical estimate for the total rate agrees with the experimental total rate based only on the nonleptonic decays.

III. DISCUSSION

It has been shown in Sec. II that an application of the photon low-energy theorem to the weak radiative decays leads to the pole model for the decay amplitudes. An experimental detection of the existence of the radiative decays would become possible with improved statistics on the production of Ω^- particles. The decay rates for the radiative decays given in Table I provide the theoretical estimates for these decays.

It has been assumed in the calculations that the spin of the Ω^- particle is $\frac{3}{2}$. While the success of the Gell-Mann–Okubo mass relation in predicting the existence of the Ω^- with a mass of 1672 MeV strongly suggests that the Ω^- particle belongs to the decuplet representation of $SU(3)$ and hence has positive parity and spin $=\frac{3}{2}$, there is no direct measurement of the spin of the Ω^-. A measurement of the decay distribution function in the nonleptonic decays $\Omega^- \to \Xi\pi$ would allow a test of the Lee-Yang inequalities[5]

$$\frac{-1}{2J+2} \le \langle\cos\theta\rangle \le \frac{1}{2J+2}, \tag{29}$$

leading to an unambiguous measurement of the spin of the Ω^-.

An experimental study of the decay distributions in the $\Xi\pi$ decay modes should lead to clean tests of the Lee-Yang inequalities leading to unambiguous measurements of the spin of the Ω^-, since calculations based on current algebra and the current$\times$current model of weak interactions predict large asymmetries in these decays.[3] An independent check on the result that the asymmetries in the $\Xi\pi$ modes should be large is provided by the work on the radiative decays $\Omega^- \to B+P+\gamma$ by Raszillier and Burlacu.[4] It has been shown by Barshay, Nauenberg, and Schultz[16] that by studying the radiative pionic decays of hyperons one can determine the partial waves in which the pion occurs in the nonradiative decay. Such a prediction is based on the photon low-energy theorem which is used in conjunction with information on the electromagnetic properties of the initial and final hadrons. This technique was used successfully to determine in which partial waves the pions occur in the decays $\Sigma^\pm \to n\pi^\pm$ and it was shown that the s-wave amplitude for $\Sigma^+ \to n\pi^+$ is zero, whereas the p-wave amplitude for $\Sigma^- \to n\pi^-$ is zero. Raszillier and Burlacu have studied the radiative decays of the Ω^- and have evaluated the branching ratios $\Gamma(\Omega^- \to BP\gamma)/\Gamma(\Omega^- \to BP)$ for the parity-conserving and the parity-violating parts of the decay rates. With the $SU(6)$ value $\kappa_\Omega = -1.79$ for the anomalous magnetic moment of the Ω^-, they show that the p-wave decay rate in the Λ^0K^- mode is nearly one order of magnitude higher than that for the K meson in a d-wave state. In the $\Xi\pi$ modes, the parity-conserving and the parity-violating parts of the amplitudes are shown to contribute comparable amounts to the nonleptonic decay rate, and therefore one may expect a large asymmetry in these modes.

A test of the predictions for the asymmetry parameters in the nonleptonic decays and an experimental detection of the existence of the radiative decays should become possible with improved statistics on the production of Ω^- particles in the near future.

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