Scattering and Pair Production from the Deuteron at Large Momentum Transfers*

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Expressions are derived for Glauber double scattering and pair production via twin scattering from the deuteron at large momentum transfers. In the double-scattering region, it is shown that twin scattering is comparable to the Drell-Söding contribution and can even dominate the process.

I. INTRODUCTION

The use of the deuteron as a target in high-energy scattering experiments has been of considerable help in understanding the scattering process. The simple yet amazingly accurate picture afforded by the Glauber approximation[1] has been extended to many different reactions.[2] The small size of the deuteron and the fact that the double-scattering region can be quite clearly separated and studied experimentally has allowed one to study the scattering amplitudes of unstable particles from nucleons[3] and possible off-mass-shell and/or dispersion effects.

Our purpose here is to discuss corrections to the Glauber double-scattering formula and the production of a pair of particles, for example pions, from the deuteron at large momentum transfers. In addition to the familiar Drell-Söding process,[4] it will be shown that twin scattering, in which each member of the pair scatters from a nucleon, is very important at large momentum transfers. In certain kinematic regimes it can in fact dominate the process. The theoretical treatment will follow the eikonal Green's-function approach developed in Ref. 5. Since we are interested in large momentum transfers, the standard Glauber treatment is not applicable.[1,6] As an introduction to our approach and to derive an interesting correction factor to the usual Glauber double-scattering term, we first consider the scattering of a particle by the deuteron. Nonrelativistic kinematics is used since it is not clear how to distinguish relativistic effects from those resulting from different deuteron models.

II. DOUBLE SCATTERING

Our purpose in this section is to discuss the scattering of a single projectile from a two-particle bound state at large momentum transfer. This is a regime where one expects the standard Glauber approximation to fail. The evaluation of the contribution of the double-scattering diagram shown in Fig. 1 requires a knowledge of the three-particle Green's function. The eikonal Green's-function formulation is used together with the analysis of the important diagrams for the process of interest, which can be found in Ref. 5.

To the zeroth approximation, we neglect the relative motion in the initial and final bound states. This means obviously that

$$\mathbf{p}\sim\mathbf{n}\sim\tfrac{1}{2}\mathbf{P}\equiv\mathbf{n}_0$$

and

$$\mathbf{p}\sim\mathbf{n}'\sim\tfrac{1}{2}\mathbf{P}'\equiv\mathbf{p}_0\,,$$

which in turn imply

$$\mathbf{l}\sim\mathbf{l}_0\equiv\tfrac{1}{2}(\mathbf{k}'+\mathbf{k})\,.$$

The next step in constructing the Green's function is to expand the energy of the intermediate state about the above "zeroth"-momentum values and to keep only the linear correction terms. The energy of the intermediate state is written as

$$E_{\text{int}}=(E_0-\mathbf{V}\cdot\mathbf{l}_0-\mathbf{v}_p\cdot\mathbf{p}_0-\mathbf{v}_n\cdot\mathbf{n}_0)+\mathbf{V}\cdot\mathbf{l}+\mathbf{v}_p\cdot\mathbf{p}'+\mathbf{v}_n\cdot\mathbf{n}\,,\quad(1)$$

where E_0 and the velocities are all evaluated at the zeroth-momentum values given above. The free three-particle Green's function in coordinate space then becomes

$$G=-i\int_0^\infty d\tau\; e^{i\tau A}\delta(\mathbf{r}_p'-\mathbf{r}_p-\mathbf{v}_p\tau)\times\delta(\mathbf{r}_n'-\mathbf{r}_n-\mathbf{v}_n\tau)\delta(\mathbf{x}'-\mathbf{x}-\mathbf{V}\tau)\,,\quad(2)$$

where

$$A=E-E_0+\mathbf{V}\cdot\mathbf{l}_0+\mathbf{v}_p\cdot\mathbf{p}_0+\mathbf{v}_n\cdot\mathbf{n}_0\,,$$

and E is the initial or final-state energy.

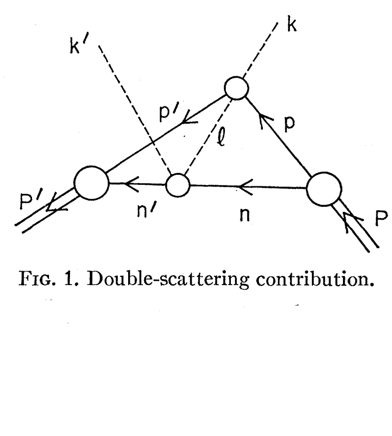

Fig. 1. Double-scattering contribution.

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TABLE I. Double-scattering form factor.

$\eta/\langle 1/r^2\rangle^{1/2}$	0	0.30	0.77	2.3
$\lvert G(\eta)\rvert^2$	1	0.96	0.75	0.31

It is now straightforward to evaluate the contribution of Fig. 1 to the scattering amplitude using standard perturbation theory. The result is

$$i\frac{2\pi}{\mu}\int_0^\infty d\tau\, e^{i\tau(E-E_0)}\int \frac{d^3y d^3\delta}{(2\pi)^3} f_n(\tfrac{1}{2}\mathbf{\Delta}+\boldsymbol{\delta})f_p(\tfrac{1}{2}\mathbf{\Delta}+\boldsymbol{\delta})$$
$$\times e^{i\delta\cdot y}\psi^*(\mathbf{y}+(\mathbf{v}_p-\mathbf{V})\tau)\psi(\mathbf{y}+(\mathbf{v}_n-\mathbf{V})\tau),$$

where f_n and f_p are the projectile-nucleon scattering amplitudes with the intermediate projectile possibly far from the mass shell. The main difference between this formula and the Glauber formula is the τ integration, which takes into account the motion of the constituents between scatterings. A more exact formula can also be derived.[7]

To compare in more detail with the Glauber formula, it is convenient to consider the situation in which the ranges of the forces that give rise to f_n and f_p are small compared to the separation of the bound particles. In this situation, the δ dependence of f_n and f_p can be neglected and both the $\mathbf{y}$ and $\boldsymbol{\delta}$ integrations can be performed. The final answer for the scattering amplitude including the diagram with the nucleon roles reversed from Fig. 1 can be written in the form

$$f(t,\eta,l_0)=F(t)[f_p(t)+f_n(t)]+i\frac{f_n(\frac{1}{4}t)f_p(\frac{1}{4}t)}{l_0}\langle 1/r^2\rangle G(\eta), \quad (3)$$

where $t=-\Delta^2$,

$$\eta=(E-E_0)/|\mathbf{V}-\mathbf{v}_p|\simeq -t/8k,$$

and

$$G(\eta)\langle 1/r^2\rangle=\langle e^{ir\eta}/r^2\rangle. \quad (4)$$

The first two terms are the single- or impulse-scattering contribution with the bound-state form factor $F(t)$. The last term is the double-scattering contribution and differs from the Glauber formula by the factor $G(\eta)$, which can be considered as the double-scattering form factor. This factor has been evaluated for a number of deuteron models and the results, which are presented in Table I, are quite insensitive to the models chosen.

[7] If one does not linearize the energy of the intermediate projectile, then a more exact formula is easily derived:

$$\frac{i2\pi}{\mu}\int d\tau\, e^{i\tau(E-E_0)}\int\frac{d^3yd^3\delta}{(2\pi)^3}\left(\frac{\mu}{2\pi i\tau}\right)^{3/2}dz\, f_n(\tfrac{1}{2}\Delta-\delta)f_p(\tfrac{1}{2}\Delta+\delta)$$
$$\times e^{i\delta\cdot(y-z)}\psi^*[y+(v_p-V_0)\tau]\psi[y+(v_n-V)\tau]$$

Integration by parts in the variable z yields the result in the text. plus correction terms.

There has not yet been a systematic comparison with experimental data of the effect of the factor $G(\eta)$. The effect of this factor should, however, be substantial for large momentum transfer and small energy.

III. TWIN SCATTERING

We now turn to the second process of interest, which is the breakup of a projectile into a pair of particles by scattering from a two-particle bound state. Again we are interested in the regime in which a large momentum is transferred to the bound state. This is a nonrelativistic model which should give insight into processes such as pair production by photons at large momentum transfers. The projectile is treated as a tightly bound scalar object described by the wave function $\phi(x)$.

Consider first the twin-scattering process of Fig. 2. We return later to the Drell-Söding type of diagram. As before, the momenta of the particles in the intermediate state are expanded about their zeroth values:

$$\mathbf{n}\sim\mathbf{p}\sim\tfrac{1}{2}\mathbf{P}=\mathbf{n},$$
$$\mathbf{n}'\sim\mathbf{p}'\sim\tfrac{1}{2}\mathbf{P}'=\mathbf{p},$$
$$\mathbf{q}_2\sim\mathbf{k}_2+\tfrac{1}{2}\mathbf{\Delta}=\mathbf{q}_2^0.$$

The energy of the intermediate state is then written as before and expanded as in Eq. (1). The energy of particle k_1 is fixed and is included in E_{int}. The three-particle Green's function then takes a form essentially the same as that found previously.

The contribution of Fig. 2 to the scattering amplitude is

$$i\frac{2\pi}{\mu}\int_0^\infty d\tau\, e^{i\tau(E-E_0)}\int\frac{d^3yd^3\delta}{(2\pi)^3}f_n(\tfrac{1}{2}\mathbf{\Delta}-\boldsymbol{\delta})f_p(\tfrac{1}{2}\mathbf{\Delta}+\boldsymbol{\delta})$$
$$\times\tilde{\phi}(\tfrac{1}{2}(\mathbf{k}_1-\mathbf{k}_2)+\boldsymbol{\delta})e^{i\delta\cdot y}\psi^*(\mathbf{y}+(\mathbf{v}_p-\mathbf{V})\tau)\psi(\mathbf{y}+(\mathbf{v}_n-\mathbf{V})\tau),$$

where $\tilde{\phi}$ is the Fourier transform of $\phi(x)$. It is again convenient to assume that f_n and f_p are short-ranged and, in addition, that $\phi(x)$ has a small size in comparison with $\psi(r)$. These approximations together with the inclusion of the diagram in which the roles of n and p are interchanged then lead to a term in the scattering

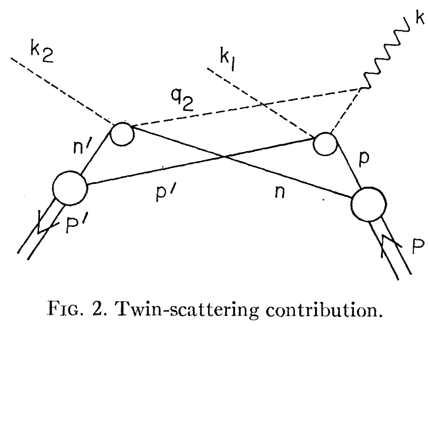

FIG. 2. Twin-scattering contribution.

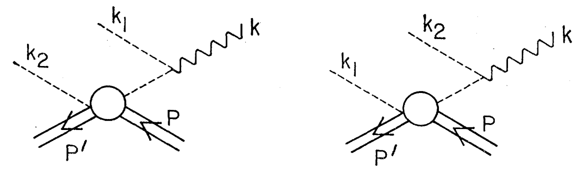

FIG. 3. Drell-Söding contribution.

amplitude of the form

$$i\frac{f_n(\frac{1}{4}t)f_p(\frac{1}{4}t)}{q_2^0}\left\langle\frac{e^{ir\beta_2}}{r^2}\right\rangle\tilde{\phi}(\tfrac{1}{2}(\mathbf{k}_1-\mathbf{k}_2)),\qquad(5)$$

where

$$\beta_2\cong\mathbf{\Delta}\cdot(\mathbf{k}_1-\mathbf{k}_2)/4q_2^0.$$

The extra phase factor $e^{ir\beta}$ in the bound-state average reflects as before a time-of-flight–off-energy-shell phase effect. The f's are again off-shell amplitudes.

In addition to the twin-scattering process of Fig. 2 and the three others which are obtained by permuting the particles, the Drell-Söding process also contributes as illustrated in Fig. 3. This process contributes terms of the form

$$\tilde{\phi}(\tfrac{1}{2}\mathbf{k}-\mathbf{k}_2)f(t,\eta_+,l_+)+\tilde{\phi}(\tfrac{1}{2}\mathbf{k}-\mathbf{k}_1)f(t,\eta_-,l_-),\qquad(6)$$

where f was defined in Eq. (3) and

$$l_\pm=\tfrac{1}{2}k\pm\tfrac{1}{2}(k_1-k_2),$$
$$\eta_\pm=-t/l_\pm.$$

At large t values, only the double-scattering part of the f's is important and it is convenient to write the total scattering amplitude in a form which separates the single from the double- and twin-scattering terms. The total scattering amplitude H then becomes

$$H=H_1+H_2,\qquad(7)$$

where

$$H_1=F(t)[f_n(t)+f_p(t)][\tilde{\phi}(\tfrac{1}{2}\mathbf{k}-\mathbf{k}_2)+\tilde{\phi}(\tfrac{1}{2}\mathbf{k}-\mathbf{k}_1)]\qquad(8)$$

and

$$H_2=if_n(\tfrac{1}{4}t)f_p(\tfrac{1}{4}t)\left\langle\frac{1}{r^2}\right\rangle\left[\tilde{\phi}(\tfrac{1}{2}\mathbf{k}-\mathbf{k}_2)\frac{G(\eta_+)}{l_+}+\tilde{\phi}(\tfrac{1}{2}\mathbf{k}-\mathbf{k}_1)\frac{G(\eta_-)}{l_-}+\tilde{\phi}(\mathbf{k}_2-\mathbf{k}_1)\frac{G(\beta_1)}{q_1^0}+\tilde{\phi}(\mathbf{k}_1-\mathbf{k}_2)\frac{G(\beta_2)}{q_2^0}\right],\qquad(9)$$

where β_1 and q_1^0 are obtained from β_2 and q_2^0 by the change $k_1\leftrightarrow k_2$.

One immediately sees that for a sufficiently large momentum transfer, where H_1 can be neglected, the twin-scattering terms are certainly of comparable magnitude to the Drell-Söding double-scattering terms. In fact, at large t, it is possible to choose events which have $k_1-k_2\ll\frac{1}{2}k-k_{1,2}\simeq\frac{1}{2}\Delta$ and thus the $\tilde{\phi}$ factor suppresses the double-scattering terms relative to the twin-scattering terms. The twin scattering is further enhanced in this region by virtue of the phase factors since the β's are much smaller than $\eta_\pm$. In any case, the differing k_1-k_2 dependence of the two contributions should make it possible to separate them experimentally.

Finally we note the changes that result when the members of the produced pair have isospin 1. Referring to Eq. (8), we find that instead of H_1 involving the factor $[f_n(t)+f_p(t)]$, one actually measures $\frac{2}{3}[2f_3(t)+f_1(t)]$, where f_3 and f_1 are, respectively, the isospin-$\frac{3}{2}$ and isospin-$\frac{1}{2}$ scattering amplitudes. Similarly in Eq. (9) one finds that the double-scattering terms involving η_+ and η_- are multiplied not by $f_n(\frac{1}{4}t)f_p(\frac{1}{4}t)$ but rather by $\frac{1}{9}[2f_3^2(\frac{1}{4}t)+8f_3(\frac{1}{4}t)f_1(\frac{1}{4}t)-f_1^2(\frac{1}{4}t)]$. This is the combination normally encountered in pion-deuteron scattering.[8] These results are independent of whether the bound state being broken up is isoscalar or isovector. The scattering-amplitude factor multiplying the twin-scattering terms in β_1 and β_2 is, however, different in the two cases. For an isoscalar projectile, one finds a factor of

$$\tfrac{1}{9}[6f_3^2(\tfrac{1}{4}t)+3f_1^2(\tfrac{1}{4}t)],$$

whereas for an isovector projectile one finds

$$\tfrac{1}{9}[5f_3^2(\tfrac{1}{4}t)+2f_3(\tfrac{1}{4}t)f_1(\tfrac{1}{4}t)+2f_1^2(\tfrac{1}{4}t)].$$

It thus becomes possible to measure somewhat different combinations of the scattering amplitudes from those normally encountered in pion-deuteron scattering.

IV. CONCLUSIONS

The main results of this paper have been the derivation of expressions for double scattering and for the production of a pair of particles from the deuteron which should be accurate at large momentum transfer and small relative momentum of the pair. Resonant production of a pair, through a ρ meson in the case of pion pairs, has been neglected but could be easily included if necessary. Further inelastic mechanisms have also been neglected. The presence of the double-scattering form factor G and its dependence on η for double scattering and β for twin scattering may provide an experimental way of distinguishing these two mechanisms at low energies. The two scattering amplitudes in twin scattering need not be the same distance from the mass shell, whereas they must be the same in double scattering. This may allow a more accurate determination, especially of the phase, of off-mass-shell effects. If so, it will provide a further test of the picture of high-energy scattering from bound systems.

[8] C. Wilkin, Phys. Rev. Letters **17**, 561 (1966).