TABLE VI. Results of a two-angle fit for $b/a=1.2$.

| Model | $\chi^2$ | C.L. (%) | $\theta_V$ | $\theta_A$ | $h_2$ |
|---|---|---|---|---|---|
| $(\alpha_0+\beta_0, \alpha_1, \beta_1, d_1)$ | 6.0 | 57 | $0.267\pm0.01$ | $0.205\pm0.03$ | $-1.22\pm0.09$ |
| $(\alpha_0+\beta_0, \alpha_1, \beta_1, e_2)$ | 7.3 | 45 | $0.255\pm0.01$ | $0.218\pm0.03$ | $-1.19\pm0.09$ |
| $(\alpha_0+\beta_0, \alpha_1, \beta_1, \gamma_2)$ | 8.3 | 34 | $0.258\pm0.01$ | $0.219\pm0.03$ | $-1.39\pm0.09$ |
| $(c_0+d_0, c_1, d_1, \alpha_1)$ | 7.2 | 43 | $0.272\pm0.007$ | $0.190\pm0.02$ | $-0.97\pm0.08$ |

expense of having very unreasonable values for $\theta_A$ and $\theta_V$. For $(\alpha_0+\beta_0, \alpha_1, \beta_1, d_1)$, we get the results displayed in Table III.

For each decay, an effective $\alpha=D/(D+F)$ may be defined. An independent determination of this number can be obtained for $\Lambda\rightarrow p$ and $\Sigma\rightarrow\Lambda$ decays,[16] giving as some sort of average for these two decays an $\alpha$ in the range

$$0.58\leq\alpha\leq0.60. \tag{18}$$

Our two-angle fits for $b/a\geq1.4$ are excluded if we impose the requirement (18). Acceptable fits are only obtained for $b/a\leq1.2$. From now on we fix $b/a$ to be

$$b/a=1.2$$

as the maximum value compatible with lbd.

We now list some results of a two-parameter fit. Table IV contains the $\chi^2$ and parameters for the models giving the lowest $\chi^2$; details are collected in Table V. For a two-angle fit we obtain the results of Table VI.

The case $(\alpha_0+\beta_0, \alpha_1, \beta_1, d_1)$ actually has a lower $\chi^2$ than the symmetric two-angle fit, again at the expense of a greater value for $\theta_V-\theta_A$. Since $Z_K\neq Z_\pi\neq Z_\kappa$, one may hope to get good fits for $\theta_A$ closer to $\theta_V$, since our symmetry breaking distinguishes between $A$ and $V$. Unfortunately, this is not the case.

We conclude by stressing that any model of $SU(3)$-symmetry breaking should be tested with lbd. For the type of symmetry breaking used in this paper, the breaking parameter $b/a$ cannot be larger than about 1.2. One may of course also take for granted the Gell-Mann–Okubo formula and pure mass mixing with $b/a\sim1$ and try to fit meson data and $K^+$-nucleon scattering into this picture.

[16] See, e.g., R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley, New York, 1969), p. 441.

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# Inclusive Reactions and Dual-Resonance Models*

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The theoretical and phenomenological advantages of looking at inclusive-type reactions in dual-resonance models are discussed. The case of single-particle distributions is considered in some detail, and part of the contributions explicitly evaluated. We find that such contributions exhibit "scaling" behavior of the type observed long ago by Amati, Fubini, and Stanghellini in the multiperipheral model and which was reproposed in more precise terms recently by Feynman and by Benecke, Chou, Yang, and Yen. The explicit form of the limiting distribution is given. A similar behavior is found in the case of the multiparticle distribution. The need of manageable techniques for computing high-energy limits of dual loop discontinuities is stressed. We finally comment on the possible relevance of our results on deriving bootstrap-type conditions from duality and in providing a way to understand the connection between the dual-resonance model and other models of current interest.

## I. INTRODUCTION

ALMOST ten years ago, prior to any substantial high-energy data, Amati, Stanghellini, and Fubini[1] noted a remarkable *scaling* property of their multiperipheral model,[1,2] namely, that in the laboratory system, fixed low-mass clusters of secondary particles from high-energy collisions have a transverse momentum distribution which is independent of the incident energy and of the energy of the cluster.

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[1] D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento **26**, 896 (1962). See also K. Wilson, Acta Phys. Austriaca **17**, 37 (1963).

[2] L. Bertocchi, S. Fubini, and M. Tonin, Nuovo Cimento **25**, 626 (1962).

During the past year there has been a renewed interest in this fascinating subject. Much of the recent discussion concerns inclusive reactions (on which there are now considerable data) such as $pp \to \pi+$anything. Feynman[3] as well as Benecke, Chou, Yang, and Yen[4] have conjectured[4] that the momentum distribution of those observed secondary particles whose lab momentum is of the order of 1 approaches a limiting distribution which is independent of the incident energy, and in fact, apart from normalization, independent of the identity of the incident particle.

This conjecture has spurred considerable theoretical work directed at verifying this striking property in a host of different models, such as the multi-Regge-pole model[5] and even quantum electrodynamics.[6] Here we shall analyze these scaling phenomena[7] in the framework of the dual-resonance model.[8]

Before proceeding to our analysis, we would like to make a few preliminary remarks concerning (1) the importance of inclusive reactions as a phenomenological testing ground for dual models, and (2) the plausibility of a purely multi-Regge-pole or multiperipheral model for the study of inclusive reactions.

Turning to the latter we note that, as it has been stressed by Chew, nature chooses to produce a substantial part of the multiparticle cross sections with low subenergies. This might appear to lower our confidence in a multi-Regge-pole dominance of the matrix elements. A similar statement applies to the use of multiperipheral models.

Thus it appears to us that the dual-resonance model which has reasonable high- and low-energy behavior, and in which the effects of resonance formation are explicitly included, might provide both a useful theoretical laboratory and a practical tool in which to investigate inclusive reactions and scaling behavior at high energy.

There is also a second good reason for looking at the inclusive-type predictions of dual amplitudes. The point which is well known, is that the dual amplitudes have poles on the real axis, which make them nonunitary. This problem has been recently investigated in Ref. 9 where it was shown that one gets typical behaviors for exclusive cross sections of the form $\sigma \sim 1/\Gamma_R$ ($\sigma \to \infty$ for $\Gamma_R \to 0$) if $\Gamma_R$ is the total width of the resonance $R$.

Thus, in order to get a finite, meaningful result, one has to take into account explicitly the total width of each resonance. This point is well known to everyone who has used dual amplitudes for phenomenological fits to *exclusive* type of experiments.[10] In such fits one simply puts an imaginary part in the trajectory function fitted to give correctly the width of some low-lying resonances. However, the arguments of Ref. 9 show that the dependence of $\sigma$ upon $\Gamma_R$ is quite critical and one therefore expects the results to be quite sensitive[11] to the explicit input form of $\text{Im}\alpha$.

The point made in Ref. 9 is that on the contrary, whenever one sums over all the unobserved final states (i.e., one is looking into the *inclusive* type of experiments), the above critical dependence on $\Gamma_R$ is washed out and one is entitled to compute experimental cross sections by looking at certain discontinuities of the *unmodified* (i.e., zero width) dual amplitudes and by smoothing them out by hand. The main assumption of course is that, once the resonances have acquired a finite width, the asymptotic behavior of the resulting amplitudes will still be given approximately by the (multi-) Regge limit of the unmodified dual amplitudes. Indeed one expects other effects, like absorption, to be already included at least in part, once resonances are given nonzero widths. The forward amplitudes considered here, however, should not be affected too much by these corrections.

In view of the two points mentioned above, we believe that dual amplitudes and the inclusive type of experiments do match nicely: (i) The theoretical predictions are less model dependent than in exclusive experiments; (ii) dual amplitudes take nicely into account both low- and high-energy effects, which seem to play comparable roles in the physical situation. The rest of the paper is organized as follows. In Sec. II we recall for the sake of completeness the results found in Ref. 9 on the total cross sections. In Sec. III we discuss the single-particle distribution in some detail and we evaluate some of the contributions to it. In Sec. IV we extend our considerations to the case of multiparticle distributions, and finally, in Sec. V, we summarize the results and draw some conclusions.

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[3] R. P. Feynman, Phys. Rev. Letters **23**, 1415 (1969).

[4] J. Benecke, T. T. Chou, C. N. Yang, and E. Yen, Phys. Rev. **188**, 2159 (1969). Their general definition of scaling is not restricted to particles whose lab momentum is of order one, but applies to any volume of phase space.

[5] D. Silverman and Chung-I Tan, Phys. Rev. D **2**, 233 (1970); **2**, 3089 (E) (1970); N. F. Bali, A. Pignotti, and D. Steele, *ibid.* **3**, 1167 (1971); Carleton E. DeTar, *ibid.* **3**, 128 (1971); A. H. Mueller, *ibid.* **2**, 2963 (1970); K. Wilson, Cornell University report, 1970 (unpublished). See also N. Bali, L. S. Brown, R. D. Peccei, and A. Pignotti, Phys. Letters **33B**, 175 (1970), for a discussion of the experimental situation.

[6] See H. Cheng and T. T. Wu, Phys. Rev. Letters **24**, 1456 (1970), and references therein.

[7] We are not considering the deep-inelastic type of scaling in dual models as done by V. Rittenberg and H. R. Rubinstein, Weizmann Institute report, 1970 (unpublished).

[8] For a review see, e.g., Chan Hong-Mo, CERN Report No. TH1057, 1969 (unpublished); V. Alessandrini, D. Amati, M. Le-Bellac, and D. Olive, CERN Report No. TH1160, 1970 (unpublished); G. Veneziano, MIT Report No. 151, 1970 (unpublished).

[9] A. DiGiacomo, S. Fubini, L. Sertorio, and G. Veneziano, Phys. Letters **33B**, 171 (1970).

[10] See, for instance, Chan Hong-Mo, R. O. Raitio, G. H. Thomas, and N. A. Törnquist, Nucl. Phys. **B19**, 173 (1970); and J. Bartch *et al.*, Aachen-Berlin-CERN-London-Vienna collaboration, *ibid.* **B20**, 63 (1970), where references to earlier works can be found.

[11] We should, however, remark that at high energy the presence of so many overlapping resonances will presumably simply produce a smoothing out of the behavior on the real axis. It is then reasonable to assume, as we shall also do, that such smoothing can be obtained there by replacing the dual amplitudes by their (multi-) Regge-pole limits.

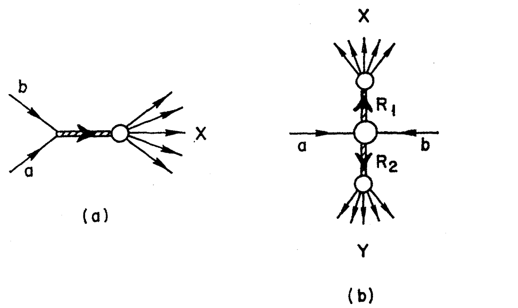


FIG. 1. Dual contributions to the amplitude for $a+b \to$ everything: (a) resonant contributions; (b) nonresonant contributions.

## II. TOTAL CROSS SECTIONS

The most inclusive experiment of all, the total cross section, is also the simplest one to study in the duality framework. As noticed in Ref. 9, the scattering amplitude for $a+b \to$ everything can be conveniently decomposed in the dual model into the sum of two classes of terms: those which contain resonances in the direct channel and those which do not. The two types of diagrams are indicated in Figs. 1(a) and 1(b), respectively. In drawing Fig. 1(b), use has been made of the general property of the planar dual model according to which whenever $a+b$ do not form a single resonance, they have to produce two resonances (which then decay into the final state).

The total cross section will come out, to first order in unitarity corrections, by squaring the amplitude, integrating over phase space, and dividing by the flux. When we square the amplitudes of Fig. 1(a) the result is simply[9] the averaged imaginary part of the elastic $a+b \to a+b$ amplitude times the usual kinematical factor. We normalize our four-point function in such a way that

$$\langle a',b'|S|a,b\rangle = \delta_{aa'}\delta_{bb'} + \frac{i(2\pi)^4\delta^{(4)}(p_a+p_b-p_{a'}-p_{b'})}{(2p_{a,0}\times 2p_{b,0}\times 2p_{a',0}\times 2p_{b',0})^{1/2}} T_{ab\to a'b'}, \quad (2.1)$$

$$T_{ab\to a'b'} = -2\gamma^2 \frac{\Gamma(-\alpha_s)\Gamma(-\alpha_t)}{\Gamma(-\alpha_s-\alpha_t)} + \text{permutations}, \quad (2.2)$$

where $\alpha_s = \alpha(s) = \alpha_0 + \alpha' s$. Then we find

$$\sigma_{\text{tot}}^{(1a)} \underset{s\text{ large}}{\sim} \frac{2\gamma^2\pi\alpha'}{\Gamma(1+\alpha_0)} (\alpha_s)^{\alpha_0-1}. \quad (2.3)$$

It is perhaps worth stressing that, in order to get Eq. (2.3), one has only to assume that interference between different dual multiplets can be neglected ($\Gamma_R \ll \alpha'^{-1}$). On the other hand, interferences inside the same multiplet *have* to be neglected in the case of total cross section since this is just what defines the states with a definite lifetime.

The second contribution to $\sigma_{\text{tot}}$ comes from squaring the amplitudes of Fig. 1(b). As discussed in Ref. 9, this will hopefully bring in diffraction (together with nonresonating backgrounds). Unfortunately, no convincing evaluation of this contribution (which involves calculation of the discontinuity of a nonplanar loop) is available at present. This will also limit the generality of our discussion in the subsequent sections. Although such calculation is certainly nontrivial, it is also a well-defined and solvable mathematical problem. We hope that the answer will be known soon.

Finally the last contribution to $\sigma_{\text{tot}}$ comes from interferences between Figs. 1(a) and 1(b). These can be proven not to be small because of a well-defined phase between some diagrams of Fig. 1(a) and some of Fig. 1(b). Again evaluation of these terms is beyond the scope of our present investigation.

## III. SINGLE-PARTICLE DISTRIBUTIONS

### A. Kinematics

We consider the inclusive reaction of Fig. 2:

$$a+b \to 1+X,$$

where $X$ = everything represents for us any number of scalar particles. The differential cross section at a fixed value $M_X^2$ of the missing mass is a function of three independent variables (see Sec. IV for the discussion of the multiparticle distribution). The natural, Mandelstam-type, invariant variables can be taken as

$$s=(p_a+p_b)^2, \quad t=(p_b-p_1)^2, \quad M_X^2 = p_X^2 = (p_a+p_b-p_1)^2. \quad (3.1)$$

A different and particularly physical choice of variables for the scaling limit is the one of Feynman[3]:

$$s=(p_a+p_b)^2, \quad |p_1^{\perp}|, \quad x = 2p_1^{\parallel}/\sqrt{s}, \quad (3.2)$$

where $p_1^{\perp}$ and $p_1^{\parallel}$ are the center-of-mass transverse and parallel momenta of the detected particle.

The relation between the variables (3.1) and (3.2) is, in the high-energy limit,

$$t = -(1/x)[\mu^2(1-x)^2 + |p_1^{\perp}|^2] + O(1/s) \text{ for } x>0, \quad \rho \equiv M_X^2/s = 1-\bar{x}+O(1/s), \quad (3.3)$$

where $\mu$ is the mass of the scalar particles and

$$\bar{x} = \left(x^2 + \frac{|p_1^{\perp}|^2+\mu^2}{\frac{1}{4}s}\right)^{1/2} = \frac{2E_1}{\sqrt{s}}.$$

Scaling (or limiting fragmentation[4]) is a statement about the limit of the above-mentioned differential

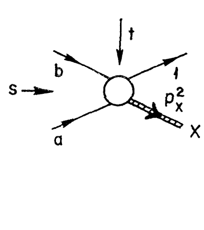


FIG. 2. Reaction $a+b \to 1+$everything and its kinematical variables.

cross section $d\sigma$ when $s \to \infty$ and both $|p_1^{\perp}|$ and $x$ are kept fixed.

In the most general case in which constant asymptotic total cross sections are not assumed, one defines scaling as[12]

$$\lim_{\substack{s\to\infty;\, x \text{ and} \\ |p^{\perp}| \text{ fixed}}} \frac{d\sigma}{\sigma_{\text{tot}}} = \frac{d^3p_1}{E_1} F(x, |p_1^{\perp}|), \tag{3.4}$$

where $F$ is neither zero nor infinity.

It is quite obvious that, in terms of the variable (3.1), scaling will be defined as

$$\lim_{\substack{s\to\infty;\, \rho = Mx^2/s \\ \text{and } t \text{ fixed}}} \frac{d\sigma}{\sigma_{\text{tot}}} = \frac{d^3p_1}{E_1} G(\rho,t). \tag{3.5}$$

The condition $t$ fixed will be replaced by $u$ fixed if $x<0$.

### B. Discussion of Dual Contributions

As in the case of total cross sections, we can conveniently divide the dual contribution to the scattering amplitude of Fig. 2 into several classes. It appears useful to introduce for each dual graph a "resonance number" $r$ defined as the least number of resonances out of which all the unobserved ("anything") final states come out. For instance, the terms of Fig. 1(a) have $r=1$, and those of Fig. 1(b) have $r=2$.

In the case of the single-particle distribution of Fig. 2, we can have $r=1$, 2, or 3. In general, for two initial and $n$ final detected particles, we can have $r=1, 2, \ldots, n+2$, and also $r=0$ if $n\geq 2$.

Going back to Fig. 2, for each $r$ we have

$$\binom{3}{r}$$

topologically different terms. The total number is then 7, of which there are 3 with $r=1$, 3 with $r=2$, and 1 with $r=3$. The whole set of diagrams has been depicted in Fig. 3.

At this point, one should in principle square all these contributions and, by use of the arguments of Ref. 9, one can reduce everything to uniquely defined discontinuities of dual amplitudes. With the exception of Figs. 3(a$_1$)–3(a$_3$), one will have to compute loop discontinuities. This is by no means trivial, although the recent developments in constructing multiloop amplitudes in a general way[13] seem to offer ground for some optimism. Of course interference terms must also be kept in general. Some of them are clearly negligible [such as the one between 3(a$_1$) and 3(a$_2$)]; some others [such as the one between 3(a$_1$) and 3(a$_3$)] are clearly important as we shall see.

[12] See DeTar, Ref. 5.

[13] C. Lovelace, Phys. Letters **32B**, 703 (1970); V. A. Alessandrini, CERN Report No. 1215, 1970 (unpublished); M. Kaku and L. Yu, Phys. Letters **33B**, 166 (1970).

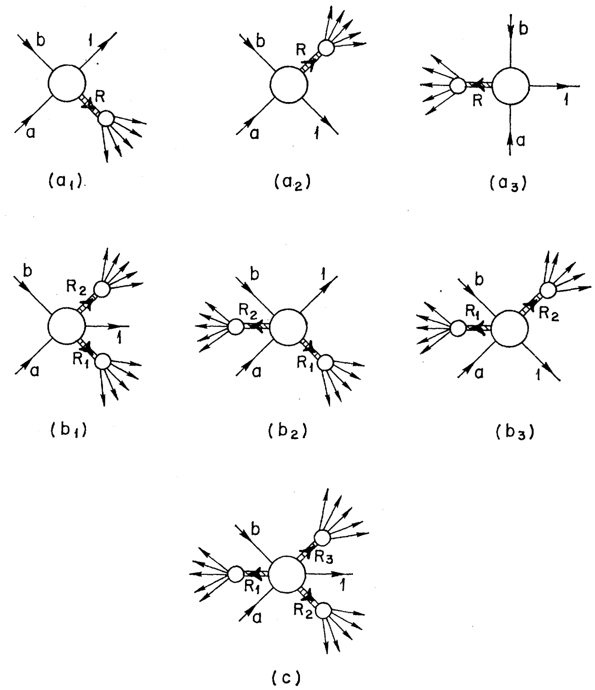


FIG. 3. Classification of the dual contributions to the single-particle distribution.

At this point we note, however, an important simplification which occurs if we restrict ourselves to the distribution of the fastest particle with c.m. longitudinal momentum parallel to particle $b(\rho\gtrsim 0, x\sim 1)$. We will be left then with only three important types of graphs, namely, in Figs. 3(a$_1$), 3(a$_3$), and 3(b$_2$). This is because in all other graphs one *cannot* exchange a Regge trajectory in the $t$ channel [remember that $t=(p_b-p_1)^2$]. Therefore they are depressed for $s\to\infty$, $t$ fixed as the $B_{su}$ "third term" in the four-point function.[8] With similar arguments one becomes convinced of the fact that Figs. 3(a$_2$), 3(a$_3$), and 3(b$_3$) should give the distribution of the particle moving fastest and parallel to $p_a(x\sim -1)$, while figures of the type shown in 3(b$_1$) and 3(c) will give the distribution for $x\sim 0$ and will be therefore relevant for determining multiplicities and pionization.[14]

Because of the aforementioned technical difficulties in studying loops, we shall limit ourselves to the $x\simeq 1$ region and therefore to the Figs. 3(a$_1$), 3(a$_3$), and 3(b$_2$).

Before proceeding to the actual calculation, we wish to remark about two of the consequences of limiting ourselves to $d\sigma^{\text{fast}}$.

[14] This separation is similar to that of Bali *et al.*, second paper in Ref. 5. In this respect we should also mention that we are not including graphs in which the detected particle comes out of the decay of an intermediate resonance. When such a resonance is formed by the two initial particles, this effect is clearly taken already into account, because of duality. Although we feel that this is the case in general, we are unable to prove such a statement as yet. We hope that, in any event, the corrections to our approximation will be small. Again, such an assumption is consistent with Bali *et al.*, Ref. 5.

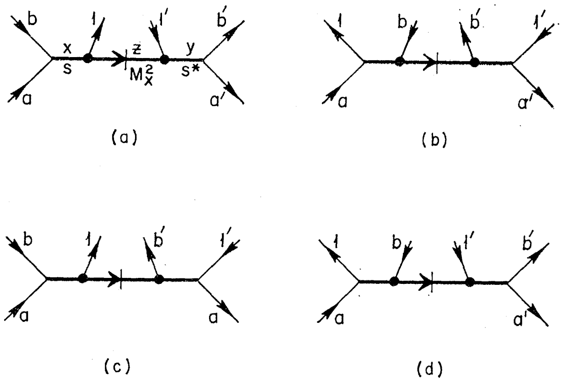


FIG. 4. Elastic six-point functions needed to compute the contribution to the cross section coming from the diagrams of Figs. 3($a_1$) and 3($a_3$).

(i) If we integrate $d\sigma^{\text{fast}}$ over the whole phase space, we get, within the usual multiperipheral approximations, $\sigma_{\text{tot}}$ without any multiplicity factor, which usually comes in if we integrate the full distribution.

(ii) By comparing the value of $\sigma_{\text{tot}}$ (or better its resonance formation component) as in (i) and as in Sec. II, one gets[15] a condition resulting in a nonlinear constraint upon $\gamma^2$ of Eq. (2.2). The fact that such a constraint can be satisfied comes indeed from the validity of Eq. (3.5).

### C. Computation of Simplest Contributions

Here we compute the contributions to $d\sigma^{\text{fast}}$ coming from Figs. 3($a_1$) and 3($a_3$). It is quite unfortunate that we have to omit, for the moment, Fig. 3($b_2$) from the discussion. Indeed, if the graph of Fig. 1(b) would prove to be able to produce diffraction with constant (or almost constant) cross sections, then Fig. 3($b_2$) would be expected to dominate $d\sigma^{\text{fast}}$. We hope to be able to fill this gap in a future publication.

By the arguments of Ref. 9 the contributions to $d\sigma$ coming from Figs. 3($a_1$) and 3($a_3$) can be reduced to the calculation of the discontinuities of elastic six-point functions[16] in the tree approximation. To be more precise one gets the diagrams of Fig. 4, where 4(a) comes from squaring 3($a_1$), 4(b) comes from 3($a_3$), and 4(c) and 4(d) come from the interference.

In order to get the desired cross section, one has to take each of these graphs and compute the $\delta$-function-type discontinuity in the variable $M_X{}^2=(p_a+p_b-p_1)^2$ while keeping the other variables $(p_a+p_b)^2=s$, $(p_b-p_1)^2=t$, and $(p_{a'}+p_{b'})^2=\bar{s}$ independent of each other. The physical result is then obtained by letting $s=\text{Re}s+i\epsilon$, $\bar{s}=\text{Re}s-i\epsilon$. Finally one smooths out the $M_X{}^2$ discontinuity by the obvious replacement $dM^2\delta(M^2-M_N{}^2)\to \alpha' dM^2$. For the graph in Fig. 4(a), for instance, we have, apart from trivial factors,

$$B_6{}^{(4a)}=\int\int_0^1\int dxdydz\, x^{-\alpha_s-1}y^{-\alpha_{\bar{s}}-1}z^{-\alpha_M-1}(1-x)^{-\alpha_t-1}$$
$$\times(1-y)^{-\alpha_t-1}(1-z)^{-\alpha_0-1}(1-xz)^{t+2\alpha_0}$$
$$\times(1-yz)^{t+2\alpha_0}(1-xyz)^{-2\alpha_0},\quad(3.6)$$

where we have defined $\alpha(t)=\alpha_t$, $\alpha(s)=\alpha_s$, $\alpha(\bar{s})=\alpha_{\bar{s}}$, and $\alpha(M_X{}^2)=\alpha_M$. We have also set the slope $\alpha'$ of the trajectory equal to 1. We are interested in the limit in which $t$ is fixed and $\alpha_s$, $\alpha_{\bar{s}}$, and $\alpha_M\to\infty$. We first consider the limit when $\text{Re}\alpha_s$, $\text{Re}\alpha_{\bar{s}}$, and $\text{Re}\alpha_M\to-\infty$. By the change of variables

$$x=\exp[-u/(-\alpha_s-1)],$$
$$y=\exp[-v/(-\alpha_{\bar{s}}-1)],\quad(3.7)$$
$$z=\exp[-w/(-\alpha_M-1)],$$

one gets

$$B_6{}^{(4a)}=(-\alpha_s-1)^{\alpha(t)}(-\alpha_M-1)^{-2\alpha(t)+\alpha_0}(-\alpha_s-1)^{\alpha(t)}$$
$$\times\int\int_0^\infty\int dudvdw\, e^{-(u''+v''+M'')}(uv)^{-\alpha(t)-1}w^{-\alpha_0-1}$$
$$\times[(w+u\rho)(w+v\bar{\rho})]^{t+2\alpha_0}[w+\rho u+\bar{\rho}v]^{-2\alpha_0}$$
$$\times[F(u')F(v')]^{-\alpha(t)-1}F(w')^{-\alpha_0-1}$$
$$\times[F(u'+w')F(v'+w')]^{t+2\alpha_0}F(u'+v'+w')^{-2\alpha_0},\quad(3.8)$$

where

$$u'=u/(-\alpha_s-1),$$
$$v'=v/(-\alpha_{\bar{s}}-1),$$
$$w'=w/(-\alpha_M-1),$$
$$\rho=(-\alpha_M-1)/(-\alpha_s-1),$$
$$\bar{\rho}=(-\alpha_M-1)/(-\alpha_{\bar{s}}-1),\quad(3.9)$$
$$u''=u'+u,$$
$$v''=v'+v,$$
$$w''=w'+w,$$

and finally

$$F(x)=(1/x)(1-e^{-x}),\quad 0\leq F(x)\leq\min(1,1/x)$$
$$(x\geq 0).\quad(3.10)$$

By using the arguments of Bardakci and Ruegg,[17] we can prove at this stage that the limit $\alpha_s$, $\alpha_{\bar{s}}\to\infty$, $\alpha_M\to\infty$ can be taken inside the integral and thus we get

$$B_6{}^{(4a)}\underset{\substack{\text{Re}\alpha(s)\to-\infty;\\ \text{Re}\alpha_M\to-\infty;\\ t\text{ fixed}}}{\sim}(-\alpha_s)^{\alpha(t)}(-\alpha_s)^{\alpha(t)}$$
$$\times(-\alpha_M)^{-2\alpha(t)+\alpha_0}\Phi(t,\rho,\bar{\rho}),\quad(3.11)$$

[15] G. Veneziano, Phys. Letters **34B**, 59 (1971).

[16] In the context of the multiperipheral model, the convenience of reducing the calculation to that of an elastic six-point function has been noted by Mueller (Ref. 5). We are grateful to G. F. Chew for pointing out this paper to us.

[17] K. Bardakci and H. Ruegg, Phys. Letters **28B**, 242 (1968).

where

$$\Phi(t,\rho,\bar\rho)=\int\int_0^\infty\int dudvdw\, e^{-(u+v+w)}(uv)^{-\alpha(t)-1}w^{-\alpha_0-1}$$
$$\times[(w+u\rho)(w+v\bar\rho)]^{t+2\alpha_0}[w+\rho u+\bar\rho v]^{-2\alpha_0}. \quad (3.12)$$

The arguments of Ref. 17 can be brought further to prove Eqs. (3.11) and (3.12) in any direction of the complex planes of $\alpha_M$ and $\alpha_s$, $\alpha_{\bar s}$, with the exception of the real axis (as usual). Therefore one obtains

$$B_6^{(4a)}=(-\alpha_s)^{\alpha_0/2}(-\alpha_{\bar s})^{\alpha_0/2}\left(\frac{-\alpha_M}{-\alpha_s}\right)^{-\alpha(t)+\alpha_0/2}$$
$$\left(\frac{-\alpha_M}{-\alpha_{\bar s}}\right)^{-\alpha(t)+\alpha_0/2}\Phi\left(t,\frac{-\alpha_M}{-\alpha_s},\frac{-\alpha_M}{-\alpha_{\bar s}}\right). \quad (3.13)$$

We can be confident that taking the discontinuity across the $\alpha_M$ cut in the form (3.13) will immediately give us the average over the true $\delta$-type discontinuity. Therefore the physical quantity will be

$$\tilde B_6^{(4a)}=(\alpha_s)^{\alpha_0}\,\mathrm{Disc}_{\alpha_M}\left[\left(\frac{-\alpha_M}{+\alpha_s}\right)^{-2\alpha(t)+\alpha_0}\Phi(t,\rho,\bar\rho)\right]_{\bar s=s^*}, \quad (3.14)$$

which is therefore of the form $\alpha_s^{\alpha_0}\psi(t,\alpha_M/\alpha_s)$. In order to get the real cross section $d\sigma$, we have to multiply by the appropriate powers of the coupling constant and divide by the flux. The result is

$$\frac{d\sigma^{(4a)}}{d^3p_1/p_{10}}\underset{s\to\infty}{\sim}\frac{\alpha'^2\gamma^4}{4\pi^2}(\alpha_s)^{\alpha_0-1}f(t,\rho=\alpha_M/\alpha_s), \quad (3.15)$$

where

$$f(t,\rho)=\left[\frac{\mathrm{Disc}_{\alpha_M}}{2i\pi}\left(\frac{-\alpha_M}{\alpha_s}\right)^{-2\alpha(t)+\alpha_0}\Phi(t,\rho,\bar\rho)\right]_{\bar s=s^*}. \quad (3.16)$$

Comparing this result with Eq. (2.3), it is then immediately possible to verify that scaling, in the sense of Eq. (3.5), does indeed hold.

There is still, however, a tricky point and this is that the integral (3.12) as such is simply divergent. The reason for this is that the process of Fig. 4(a) has a pole in the channel $b+1+1'\to 1'+a'+a$ and, since $p_1=p_{1'}$, we are just sitting on top of the pole. Nevertheless, the $M_X^2$ discontinuity is finite [poles in $M_X^2$ are incompatible with poles in $(p_b+p_1+p_{1'})^2$] and it is just enough to regularize the integral by subtracting a real counterterm. This can be done in many equivalent ways. A particularly elegant one is to let the trajectory of channels $b+1+1'$ and $b'+1+1'$ have intercept $\alpha_0-\delta$ instead of $\alpha_0$. For $\delta>0$, everything is then defined and scaling is seen to hold for any $\delta$. The amplitude, however, will have a behavior for $\delta\to 0^+$ of the form $(1/\delta)$ times a function with no discontinuity in $\alpha_M$. It will then be enough to subtract a real counterterm with the same behavior as $\delta\to 0$. We just state that the final outcome is to have again Eqs. (3.15) and (3.16) hold, but with $\Phi(t,\rho,\bar\rho)$ now given by

$$\Phi(t,\rho,\bar\rho)^{\text{regularized}}=\int\int_0^\infty\int dudvdw(uv)^{-\alpha(t)-1}w^{-\alpha_0-1}$$
$$\times[e^{-u-v-w}(w+u\rho)^{\alpha_0+\alpha_t}(w+v\bar\rho)^{\alpha_0+\alpha_t}$$
$$\times(w+u\rho+v\bar\rho)^{-2\alpha_0}-e^{-u\rho-v}(w+u\rho)^{\alpha(t)+\alpha_0}(v\bar\rho)^{\alpha_t-\alpha_0}$$
$$-e^{-u-v\bar\rho}(w+v\bar\rho)^{\alpha_t+\alpha_0}(\rho u)^{\alpha_t-\alpha_s}], \quad (3.17)$$

which is now well defined.

The evaluation of the other tree contributions [Figs. 4(b)–4(d)] is similar to the one given here. In fact, it is identical provided we reinterpret $s$ and $\bar s$. Since $-\alpha_M/-\alpha_u\simeq-\alpha_M/\alpha_s-\alpha_M\simeq\rho/\rho-1$, we obtain for instance

$$\frac{d\sigma^{(4b)}}{d^3p_1/p_{10}}\underset{\substack{s\to\infty;\\ \rho,\,t\text{ fixed}}}{\sim}\frac{\alpha'^2\gamma^4}{4\pi^2}(\alpha_s)^{\alpha_0-1}$$
$$\times(1-\rho)^{\alpha_0}\left[\frac{\mathrm{Disc}_{\alpha_M}}{2i\pi}\left(\frac{-\alpha_M}{-\alpha_u}\right)^{\alpha_0-2\alpha_b}\right.$$
$$\left.\times\Phi\left(t,\frac{-\alpha_M}{-\alpha_u},\frac{-\alpha_M}{-\alpha_u}\right)\right]_{-\alpha_M/-\alpha_\mu=\rho/(\rho-1)}. \quad (3.18)$$

Similar results hold for $d\sigma^{(4c)}$ and $d\sigma^{(4d)}$. Equations (3.16) and (3.17) are indeed quite complicated and the properties of the function $\Phi$ are presently under study. It is possible, for instance, to perform explicitly one of the three integrations in (3.17). The remaining two-dimensional integral is of the kind

$$\int_0^\infty\int dxdy(xy)^a[(1+x)(1+y)]^b$$
$$\times(1+x+y)^c(1+\rho x+\bar\rho y)^d, \quad (3.19)$$

which we hope can be reduced to some hypergeometric function.

It is very interesting to consider instead the limit in which $\alpha(s)\gg\alpha(M_X^2)\gg 1$, say, for instance, of $\alpha(M_X^2)\gg 1$, but fixed. One finds it possible to pass to this limit by setting $\rho=\bar\rho=0$ in the integral defining $\Phi$. On the other hand,

$$\Phi(t,0,0)=\Gamma^2(-\alpha_t)\Gamma(2\alpha_t-\alpha_0). \quad (3.20)$$

Therefore from Eqs. (3.15) and (3.16) we obtain

$$\frac{d\sigma^{(4a)}}{d^3p_1/p_{10}}\underset{\substack{s\to\infty;\\ 1\ll\alpha_M\ll\alpha_s;\\ t\text{ fixed}}}{\sim}\frac{\alpha'^2\gamma^4}{4\pi^2}\alpha_s^{\alpha_0-1}$$
$$\times\frac{\Gamma^2(-\alpha_t)}{\Gamma(1+\alpha_0-2\alpha_t)}\left(\frac{\alpha_s}{\alpha_M}\right)^{2\alpha_t-\alpha_0}. \quad (3.21)$$

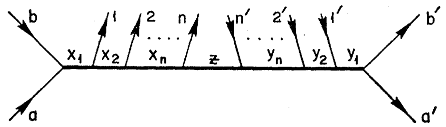


FIG. 5. Elastic $(2n+4)$-point function needed to compute a particular contribution to the $n$-particle distribution.

Combining all the four contributions of Fig. 4, we finally obtain

$$\frac{d\sigma}{d^3p_1/p_{10}} \underset{\substack{s\to\infty;\\ 1\ll\alpha_M\ll\alpha_s;\\ t\text{ fixed}}}{\sim} \alpha_s^{\alpha_0-1}\frac{\alpha'^2\gamma^4}{\pi^2}\frac{\Gamma^2(-\alpha_t)}{\Gamma(1+\alpha_0-2\alpha_t)} \times\left(\frac{\alpha_s}{\alpha_M}\right)^{2\alpha_t-\alpha_0}\cos^2[\pi\alpha(t)/2]. \quad (3.22)$$

Equation (3.22) agrees with the result of Chang, Freund, and Nambu,[18] which is based on a "statistical approach" to dual amplitudes. We hope also that our derivation will eventually clarify the connections between dual and statistical models.[19] As remarked by the authors of Ref. 18, a behavior of $d\sigma/dtdM^2$ similar to that of Eq. (3.22) agrees well with experiments.

We stress, however, that the simple result (3.22) only holds for $\rho\to 0$. To extrapolate such a form outside $\rho=0$ does not seem *a priori* justified here. Our result thus differs from the corresponding one of Bali, Pignotti, and Steele[5] in Sec. II of their paper. We agree, however, with them in the $\rho\to 0$ limit. Their calculation is based on a multi-Regge-pole model in which one drops the Toller-angle dependence of the Regge residue. How good this approximation is in the case of dual amplitudes can be explicitly checked by studying the small-$\rho$ behavior of the function $\Phi$.

Finally we remark that we have to restrict ourselves to the case of $\alpha_0<0$. In the physically interesting case of $\alpha_0>0$, factorized dual amplitudes exhibit poles in the physical negative-$t$ region. These (tachyon) poles cause the integral over $\cos\theta$ of the differential cross section to diverge. For phenomenological purposes one might try to use for $\alpha_0>0$ the type of amplitudes recently proposed by Rittenberg and Rubinstein and by Olive and Zakrzewski,[20] although they violate one of the important properties we used, i.e., factorization.[21]

[18] L. N. Chang, P. G. O. Freund, and Y. Nambu, Phys. Rev. Letters **24**, 628 (1970).

[19] R. Hagedorn, Nuovo Cimento **56A**, 1027 (1968). Actually, before comparing with his results one should also compute the loop-type contributions. It is amusing to speculate that there could be a connection between the density of states which explicitly enters into the loop calculations and the transverse momenta distribution which, in Hagedorn, depends crucially on the "ultimate" temperature $KT_0\simeq 160$ MeV.

[20] V. Rittenberg and H. R. Rubinstein, Phys. Rev. Letters **25**, 191 (1970); D. Olive and W. J. Zakrzewski, Nucl. Phys. **B21**, 303 (1970).

[21] A. P. Balachandran, L. N. Chang, and P. H. Frampton, Nuovo Cimento **1A**, 545 (1971).

## IV. MULTIPARTICLE DISTRIBUTIONS

The discussion of multiparticle distributions follows conceptually the same patterns as that of the single-particle distribution. However, the resonance number $r$ of Sec. III can now assume values as high as $n+2$ for an $n$-particle distribution. When, for instance, a diagonal contribution is considered, one gets immediately into the technical problem of computing the discontinuity of a nonplanar $(r-1)$-loop amplitude. Thus a thorough discussion of the multiparticle distribution leads one quickly into the realm of quite involved multiloop amplitudes. In order to get some answer to compare with experiments, a systematic technique for dealing with such loops is felt to be needed.

Here we shall content ourselves to consider only the simplest contributions to the multiparticle distribution. If again we consider only the distribution of the $n$-fastest moving particles in the direction of $p_b$, considerable simplifications are expected, namely, the momenta $p_b$ and $p_1$ through $p_n$ should be adjacent in the dual graph.

Before going into the calculation we briefly discuss the kinematics of the process.

### A. Kinematics

We consider the inclusive process

$$a+b\to 1+2+3\cdots+n+X.$$

The differential cross section

$$\frac{d\sigma}{(d^3p_1/E_1)(d^3p_2/E_2)\cdots(d^3p_n/E_n)}$$

will be a function of $3n$ independent variables [it is an $(n+3)$-point function with one external leg of variable mass]. For our purposes a natural choice of variables is (see also Fig. 5)

$$\begin{aligned} &s_0=s=(p_a+p_b)^2, \quad s_i(i=1,2,\ldots,n),\\ &t_i(i=1,\ldots,n), \quad p_1p_j(j=2,\ldots,n), \end{aligned} \quad (4.1)$$

where

$$\begin{aligned} s_i&=(p_a+p_b-p_1-p_2\cdots-p_i)^2,\\ t_i&=(p_b-p_1-p_2\cdots-p_i)^2. \end{aligned} \quad (4.2)$$

In particular, $s_n=p_x^2=M_X^2$. We also define

$$\rho_i=s_i/s, \quad i=i,\ldots,n. \quad (4.3)$$

The Feynman-type variables[3] for the case would

$$s,\ p_i'',\ \mathbf{p}_{i\perp} \quad (i=1,\ldots,n), \quad (4.4)$$

which are also $3n$ in number since there are $2n-1$ independent $\mathbf{p}_{i\perp}$. Scaling will be defined in Feynman's variables as

$$\frac{d\sigma}{\sigma_{\text{tot}}} \underset{\substack{s\to\infty;\\ x_i \text{ fixed};\\ p_{i\perp} \text{ fixed}}}{\sim} \frac{d^3p_1}{E_1}\frac{d^3p_2}{E_2}\cdots\frac{d^3p_n}{E_n}F(x_i,\mathbf{p}_{i\perp}), \tag{4.5}$$

where $x_i=2p_i''/\sqrt{s}$. In other words, $F$, instead of being a function of $3n$ variables, becomes a function of $3n-1$ variables. The connection between variables (4.4) and (4.1) in the scaling limit is the following:

$$\rho_1=1-\bar{x}_1,$$

$$\rho_2=1-\bar{x}_1-\bar{x}_2,\ \ldots,\ \rho_n=\frac{M_X{}^2}{s}=1-\sum_{l=1}^{n}\bar{x}_l, \tag{4.6a}$$

$$t_i=-\Big(\sum_{l=1}^{i}\mathbf{p}_{l\perp}\Big)^2+\mu^2+\left(\sum_{n=1}^{i}\frac{p_{n\perp}{}^2+\mu^2}{x_n}\right)\Big(\sum_{m=1}^{i}x_m\Big)$$
$$-\sum_{r=1}^{i}x_r\left(\mu^2+\frac{p_{r\perp}{}^2+\mu^2}{x_r{}^2}\right), \tag{4.6b}$$

$$p_j\cdot p_i=-p_{j\perp}\cdot p_{i\perp}+\frac{1}{2}\left[\frac{x_i}{x_j}(p_{j\perp}{}^2+\mu^2)+\frac{x_j}{x_i}(p_{i\perp}{}^2+\mu^2)\right], \tag{4.6c}$$

where $\bar{x}_i=2E_i/\sqrt{s}$ and all $x_i$ are taken to be positive. One can get easily convinced that the set of variables (4.1) is equivalent to that in (4.4). In the variables (4.1), the statement of scaling takes the form

$$\frac{d\sigma}{\sigma_{\text{tot}}} \underset{\substack{s\to\infty;\\ \rho_i,\ p_1\cdot p_i,\\ t_i \text{ fixed}}}{\sim} \frac{d^3p_1}{E_1}\cdots\frac{d^3p_n}{E_n}A(\rho_i,t_i,p_1\cdot p_i). \tag{4.7}$$

Although the set of variables (4.1) is the most natural for our purposes, other variables might be more appropriate for studying the multiparticle distribution in some kinematical region.

### B. Explicit Computation of One Dual Contribution

We now prove that scaling in the sense of Eq. (4.7) does indeed hold for the analogous diagram Fig. 4(a) which, for the $n$-particle distribution, is given in Fig. 5. We have then to look at an elastic $(2n+4)$-point function, which apart from trivial powers of the coupling constant is given by

$$B_{2n+4}=\int_0^1\frac{1}{J}\prod_{i=1}^{n}dx_i\prod_{j=1}^{n}dy_j dz x_{i+1}{}^{-\alpha(s_i)-1}y_{i+1}{}^{-\alpha(\bar{s}_0)-1}\times z^{-\alpha_M-1}\prod_\alpha U_\alpha{}^{-\alpha-1}, \tag{4.8}$$

where $J$ is the usual Jacobian[8] giving cyclic symmetry and we have symbolically indicated by $\prod_\alpha U_\alpha{}^{-\alpha-1}$ all the remaining factors in the integrand in Chan's form.[8]

Using Eqs. (4.6), it is easy to see that all the variables $\alpha$ appearing in the above term are to be kept fixed in the scaling limit. In order to study such limits, we make the change of variables

$$\begin{aligned} x_i&=\exp[-u_i/(-\alpha(s_i)-1)],\\ y_i&=\exp[-v_i/(-\alpha(\bar{s}_i)-1)],\\ z&=\exp[-w/(-\alpha_M-1)]. \end{aligned} \tag{4.9}$$

We then obtain

$$B_{2n+4}=\prod_i[-\alpha(s_i)-1]^{-1}[-\alpha(\bar{s}_i)-1]^{-1}(-\alpha_M-1)^{-1}\times\int_0^\infty\frac{dudvdw}{J}\exp\Big(-\sum u_i-\sum w_i-w\Big)\times\prod_\alpha U_\alpha{}^{-\alpha-1}(u_i',v_j',w'), \tag{4.10}$$

where $u_i'=u_i/[-\alpha(s_i)]$, etc. Except for the case of $\alpha$ being the trajectory exchanged between $a$ and $a'$, all $U_\alpha$ have a limit for $u'$, $v'$, $w'\to 0$ which is a function of $u$, $v$, $w$ and the $p_i$ only. The $aa'$ trajectory gives a term

$$\Big\{1-\exp\Big[-\sum u_i'-\sum v_j'-w'\Big]\Big\}^{-\alpha(0)-1}\underset{u',v',w'\to 0}{\cong}\Big(\sum u_i'+v_i'+w'\Big)^{-\alpha(0)-1}=(-\alpha_M-1)^{\alpha(0)+1}f(u,v,w,\rho_i). \tag{4.11}$$

The Jacobian $J$ also has a simple limit. In conclusion, following the procedure of Sec. III and collecting all the factors, one gets

$$B_{2n+4}\underset{s\to\infty}{\sim}(-\alpha_s)^{\alpha_0/2}(-\alpha_{\bar{s}})^{\alpha_0/2}F(\rho_i,\bar{\rho}_i,p_i\cdot p_j,t_i). \tag{4.12}$$

Since all the $p_i\cdot p_j$ are functions of $\rho_i$ and $\mathbf{p}_{i\perp}$, Eq. (4.12) verifies the scaling limit for the distribution of the $n$ fastest particles. Of course, in order to get $d\sigma$ one has to take the discontinuity of (4.12) with respect to $\alpha_M$ and then let $\bar{s}_i=s_i^*$. One could quote the explicit expression for $F$ in (4.12) which is in the form of a multiple integral, but this is not particularly illuminating. Furthermore, in order to get a convergent integral one must again subtract a real counterterm, as we did in Sec. III. There is nothing conceptually new, however, because of the fact that we detect several final particles instead of a single one.

Another possible outcome of the study of multiparticle distributions is the derivation of more consistency (bootstrap) conditions of the type discussed recently by one of us.[15] Such nonlinear conditions for the dual coupling constant are presently under investigation.

## V. CONCLUSIONS

The main point of this paper was to show the usefulness of considering inclusive-type experiments in the dual-model framework. Especially in view of the apparent success of dual phenomenology for exclusive experiments,[10] we have stressed the plausibility and the advantages of a dual phenomenology for inclusive reactions. Indeed, the theoretical predictions appear less model dependent in the latter than in the former case.

Following the guidelines of the work of Ref. 9, we have shown how the theoretical predictions can be reduced to rather straightforward computations of discontinuities of dual trees and loops. Furthermore, the multiparticle phase-space complications existing in other models are greatly simplified here by an appropriate use of duality.

We have thus examined total cross sections (mainly for completeness), single-particle distributions, and multiparticle distributions. However, we have limited ourselves to the calculations of the simplest contributions to the cross sections, those reducible to discontinuities of tree graphs. We find quite easily that scaling in the sense of Feynman[3] and of Benecke, Chou, Yang, and Yen[4] is obeyed by these contributions and we are able to predict the limiting distribution.

As a consequence of having neglected loop diagrams, we are quite strongly limited for two main reasons: First, we cannot take into account diffraction (Pomeranchukon exchange); second, we have to limit ourselves to the distribution of the fastest particles moving parallel to one of the incident particles. The positive results we obtain for the trees should, we believe, encourage further theoretical work on the dual factorized loops, and some workable technique should be found in order to compute their discontinuities in the relevant kinematic regions. In view of the recent progress in the theory of many loops,[13] this does not seem an impossible thing to ask for the near future. If such techniques were available, one could obtain the full single- (and multi-) particle distribution and investigate questions like pionization and average multiplicity.

Moreover, there is hope[22] that the multiloop calculation will yield a theory of diffraction in the duality framework. If this would turn out to be true, then also the first limitation of our present claculations could be overcome. If, however, it would be impossible (or simply too complicated) to obtain the Pomeranchukon in the dual framework, one will still have, as in the $B_5$ phenomenology,[10] two options open. One is to restrict to quantities which have no Pomeranchukon contribution. As an example, it appears that, if one considers $\sigma(\pi^- + p \rightarrow \pi^-(\text{fast}) + \text{anything}) - \sigma(\pi^- + n \rightarrow \pi^-(\text{fast}) + \text{anything})$, all the graphs of Fig. 3, except the ones we explicitly calculated, are unimportant. Therefore our result should apply directly to such single-particle distribution.

The second possibility would be to use a phenomenological expression for Pomeranchukon exchange as done in the $B_5$ phenomenological fits.[23]

Finally, one is faced with the problem of trajectories with $\alpha_0 > 0$. Again from the point of view of phenomenology one can try to use *ad hoc* modifications which hopefully should not alter too much the general features of dual amplitudes.[24]

Our results have also relevance to more theoretical questions, like that of implementing unitarity in the dual-model framework. It has already been noted[15] how one can get nonlinear bootstrap-type conditions on the dual coupling constant. Furthermore, the connection between dual models and other currently studied models such as the multiperipheral model,[25] the limiting fragmentation scheme,[4] and the statistical-thermodynamical model[19] seems now a bit easier to study.

We should remark, however, that, as usual, the most relevant comparison will occur through confrontation of the theory with the experimental high-energy data, which are expected to increase considerably within the next few years. Much more theoretical work is certainly needed before we shall be able to produce a useful and reliable scheme for phenomenology. Nevertheless, it does not seem impossible to us that such a scheme will indeed be available within the next couple of years.

## ACKNOWLEDGMENTS

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[22] See, e.g., D. J. Gross, A. Neveu, J. Scherk, and J. H. Schwazr, Phys. Rev. D **2**, 697 (1970); Phys. Letters **31B**, 592 (1970).

[23] S. Pokorski and H. Satz, Nucl. Phys. **B19**, 113 (1970).

[24] We are also neglecting problems connected with the presence of ghost states for $\alpha(0) \neq 1$. We are convinced, however, that the general conclusions we obtain should not be affected by this conceptual difficulty (see also Ref. 9).

[25] See, e.g., M. L. Goldberger, in *Proceedings of the Coral Gables Conference on Fundamental Interactions at High Energy, Coral Gables,* edited by T. Gudehus, G. Kaiser, and A. Perlmutter (Gordon and Breach, New York, 1969), Vol. I, p. 142.