Regge Cuts and Sum Rules for Meson-Nucleon Charge-Exchange Scattering*

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Pseudoscalar-meson–nucleon charge-exchange scattering is investigated in the Regge pole-cut model. Using a complex-conjugate-poles approximation, several sum rules for experimentally observable quantities are derived and compared with experimental results. It is emphasized that the measurement of the polarization in the K-nucleon charge-exchange scattering can decide whether the cuts are important in charge-exchange reactions, and which cut model provides the better description of these processes.

I. INTRODUCTION

DURING the last few years it has become evident that the Regge cuts in the complex-angular-momentum plane cannot be neglected. Models involving Regge cuts have been extensively discussed by several authors.[1]

Generally it is assumed that the leading cut corresponds to the simultaneous exchange of the Pomeranchuk pole and the leading Regge pole.[2] If the Pomeranchuk trajectory α_P has the value close to 1 at $t=0$, then the branch point in the J plane resulting from the exchange of the Regge trajectory α and the Pomeranchuk trajectory occurs at or near $J=\alpha(0)$ at $t=0$. If we take the cut along the negative real axis to the left from the branch point, then each Regge trajectory meets the corresponding Regge-Pomeranchuk cut at or near $t=0$.

It has been shown[3] that in a certain class of models, because of the collision of the Regge trajectory with the corresponding Regge-Pomeranchuk cut, either a new pole can come through the cut from the unphysical sheet of the angular-momentum plane, meet the original Regge pole and then form a pair of complex-conjugate poles on the physical sheet, or the Regge pole can disappear through the cut, meet the pole on the unphysical sheet and the pair of complex-conjugate poles is formed on the unphysical sheet of the J plane. Thus, for negative t, the leading J-plane singularities in the first case are the cut and the pair of complex-conjugate poles, while in the second case we are left with the cut only. In the following we will work inside this class of models and we will assume that we have a pair of complex-conjugate poles on either sheet of the J plane.

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[1] R. C. Arnold, Phys. Rev. **140**, B1022 (1956); S. Frautschi and B. Margolis, Nuovo Cimento **56A**, 1155 (1968); F. S. Henyey, G. L. Kane, J. Pumplin, and M. Ross, Phys. Rev. Letters **21**, 946 (1968); A. Krzywicki and J. Tran Thanh Van, Nuovo Cimento Letters **2**, 249 (1969); R. Carlitz and M. Kisslinger, Phys. Rev. Letters **24**, 186 (1970).

[2] If the Pomeranchuk trajectory passes through $J=1$ at $t=0$, then the leading cut can result from the combination of the leading Regge pole with an arbitrary number of Pomeranchuk poles.

[3] P. Kaus and F. Zachariasen, Phys. Rev. D **1**, 2962 (1970).

II. COMPLEX-CONJUGATE-POLES APPROXIMATION

The continued partial-wave amplitude containing a cut and a pair of complex-conjugate poles can be conveniently written in the form

$$T(J,t)=f(J,t)/(J-\alpha_+)(J-\alpha_-)\,, \qquad (1)$$

where $\alpha_\pm(t)=\alpha_R(t)\pm i\alpha_I(t)$ is a pair of complex-conjugate trajectories and $f(J,t)$ is a function which has a cut.

The scattering amplitude $T(s,t)$ can be written using the inverse Mellin transform as

$$T(s,t)=\frac{f(\alpha_+,t)}{\alpha_+-\alpha_-}s^{\alpha+}-\frac{f(\alpha_-,t)}{\alpha_+-\alpha_-}s^{\alpha-} -\frac{1}{\pi}\int_{-\infty}^{\alpha_c}\frac{\operatorname{Im}f(J,t)s^J}{(J-\alpha_R)^2+\alpha_I{}^2}dJ\,, \qquad (2)$$

where $\alpha_c(t)$ is the position of the branch point and $2i\operatorname{Im}f(J,t)$ is the discontinuity across the cut. If a pair of complex-conjugate poles is on the unphysical sheet, then $f(\alpha_\pm,t)=0$.

If the function $f(J,t)$ which has a cut in J is a logarithmic function, the integral in (2) can be evaluated analytically. If $f(J,t)$ is not a logarithmic function, we can approximate[4] the integral in (2) at high but not asymptotic energies by a similar integral in which $\operatorname{Im}f(J,t)$ is replaced by $\operatorname{Im}f(\alpha_R,t)$.

In any case, after integration we get

$$T(s,t)=\left[\frac{f(\alpha_+,t)}{\alpha_+-\alpha_-}-\frac{\operatorname{Im}f(\alpha_R,t)\operatorname{Ei}[(\alpha_c-\alpha_+)\ln s]}{2\pi i\alpha_I}\right]s^{\alpha+} +\left[\frac{f(\alpha_-,t)}{\alpha_--\alpha_+}+\frac{\operatorname{Im}f(\alpha_R,t)\operatorname{Ei}[(\alpha_c-\alpha_-)\ln s]}{2\pi i\alpha_I}\right]s^{\alpha-}. \qquad (3)$$

Assuming that the function $f(J,t)$ is real analytic, we can finally write

$$T(s,t)=C(t,s)s^{\alpha+(t)}+C^*(t,s)s^{\alpha-(t)}\,, \qquad (4)$$

[4] J. S. Ball, G. Marchesini, and F. Zachariasen, Phys. Letters **31B**, 583 (1970).

where

$$C(t,s)=\frac{i}{2\pi\alpha_I}\{\operatorname{Im}f(\alpha_R,t)\ \operatorname{Ei}[(\alpha_c-\alpha_+)\ln s]-\pi f(\alpha_+,t)\}.$$

If one includes the signature, then the final expression for the scattering amplitude in the discussed model can be written in the form[4]

$$T(s,t)=C(t,s)(1\pm e^{-i\pi\alpha_+})s^{\alpha_+(t)}+C^*(s,t)(1\pm e^{-i\pi\alpha_-})s^{\alpha_-(t)}. \quad (5)$$

We should remind the reader that the above amplitude includes the cut contribution as well as the contribution from a pair of complex-conjugate poles. Expression (5) is an approximation which is, however, expected to hold at presently accessible high energies.[4]

It is important to realize that because of the cut contribution, the new residue function $C(t,s)$ is also a function of s and is complex.

III. MESON-NUCLEON CHARGE-EXCHANGE SCATTERING

In the following we will use the amplitude of the form (5) to investigate pseudoscalar-meson–nucleon charge-exchange scattering. We will derive several relations (sum rules) among the observable quantities and we will compare these relations with experimental results as well as with similar relations which were derived before without the consideration of the Regge cuts.

We will consider the following charge-exchange reactions:

$$\pi^-p\to\pi^0n\,, \quad (6a)$$

$$\pi^-p\to\eta n\,, \quad (6b)$$

$$K^-p\to\bar{K}^0n\,, \quad (6c)$$

$$K^+n\to K^0p\,, \quad (6d)$$

and we will make the following assumptions.

(i) The signatured amplitude for a given process, containing the contribution from an appropriate Regge pole and Regge-Pomeranchuk cut, has a form given by (5).

(ii) We assume the $SU(3)$ symmetry for the Regge-pole couplings. Because the Pomeranchuk pole is an $SU(3)$ singlet, the Regge-Pomeranchuk cut contributions and therefore the whole amplitudes will be related by the same symmetry.

(iii) We assume the strong form of exchange degeneracy for the ρ and A_2 Regge trajectories. Consequently also the ρ-Pomeranchuk and A_2-Pomeranchuk branch-point trajectories will be exchange degenerate. The ρ-A_2 exchange degeneracy has to be considered as a convenient assumption which enables us to reduce the number of unknown parameters rather than as the consequence of some experimental evidence. The reason for this is that, as will be shown later, one of the best experimental tests of exchange degeneracy (equality of $K^-p\to\bar{K}^0n$ and $K^+n\to K^0p$ differential cross sections) does not apply to the case of complex-conjugate poles.

The scattering amplitudes for the reactions under consideration can then be written at high energies in the form

$$T_\pm(\pi^-p\to\pi^0n)=\sqrt{2}[C_\pm(s,t)(1-e^{-i\pi\alpha_+})s^{\alpha_+}+C_\pm^*(s,t)(1-e^{-i\pi\alpha_-})s^{\alpha_-}], \quad (7a)$$

$$T_\pm(\pi^-p\to\eta n)=-(\sqrt{\tfrac{2}{3}})[C_\pm(s,t)(1+e^{-i\pi\alpha_+})s^{\alpha_+}+C_\pm^*(s,t)(1+e^{-i\pi\alpha_-})s^{\alpha_-}], \quad (7b)$$

$$T_\pm(K^-p\to\bar{K}^0n)=-2[C_\pm(s,t)e^{-i\pi\alpha_+}s^{\alpha_+}+C_\pm^*(s,t)e^{-i\pi\alpha_-}s^{\alpha_-}], \quad (7c)$$

$$T_\pm(K^+n\to K^0p)=-2[C_\pm(s,t)s^{\alpha_+}+C_\pm^*(s,t)s^{\alpha_-}], \quad (7d)$$

where $T_\pm$ are the helicity-nonflip and helicity-flip amplitudes and $C_\pm(s,t)$ are unknown residue functions.

Eliminating unknown residue and trajectory functions, one gets the following relations between differential cross sections and polarizations:

$$\frac{d\sigma}{dt}(\pi^-p\to\pi^0n)+3\frac{d\sigma}{dt}(\pi^-p\to\eta n)=\frac{d\sigma}{dt}(K^-p\to\bar{K}^0n)+\frac{d\sigma}{dt}(K^+n\to K^0p)\,, \quad (8)$$

$$P(K^+n\to K^0p)=0\,, \quad (9)$$

$$P(K^-p\to\bar{K}^0n)\frac{d\sigma}{dt}(K^-p\to\bar{K}^0n)=P(\pi^-p\to\pi^0n)\frac{d\sigma}{dt}(\pi^-p\to\pi^0n)+3P(\pi^-p\to\eta n)\frac{d\sigma}{dt}(\pi^-p\to\eta n)\,. \quad (10)$$

IV. DISCUSSION

Relation (8) is the well-known sum rule derived several times[5] before using only the Regge poles. It should be no surprise that this sum rule has been rederived in the model taking into account the cut effect. For its derivation it is sufficient to consider the signs of the contributions from the vector and tensor exchanges to individual amplitudes and $SU(3)$ symmetry. As far as these two features remain valid,[6] the sum rule (8) will not be affected by the presence of the cuts. Rela-

[5] A. Ahmadzadeh and C. N. Chan, Phys. Letters **22**, 692 (1966); V. Barger and D. Cline, Phys. Rev. **156**, 1522 (1967).

[6] The necessary assumptions for the derivation of the sum rule (8) would not be valid if the Regge-Regge cuts were important.

tion (8) has been compared with experimental data and a good agreement has been found.[5]

The polarization sum rule (10) differs sharply from the Regge-pole predictions.[7] It is known that the model using only Regge poles predicts zero polarizations in $\pi^- p \to \pi^0 n$ and $\pi^- p \to \eta n$ because only the ρ trajectory can be exchanged in the first reaction and only A_2 in the second one. Furthermore, because of the strong ρ-A_2 exchange degeneracy also, the polarizations in $K^- p \to \bar{K}^0 n$ and $K^+ n \to K^0 p$ are predicted to be zero.

On the other hand, in the discussed model only one of the K-nucleon charge-exchange reactions is predicted to have no polarization. The polarization in the second K-nucleon charge-exchange reaction is determined by the sum rule (10). Furthermore, by writing explicitly the expressions for individual polarizations,

$$P(K^- p \to \bar{K}^0 n)\frac{d\sigma}{dt}(K^- p \to \bar{K}^0 n) \sim 2|C_+ C_-| \sin(\varphi_+ - \varphi_-)[e^{2\pi\alpha_I} - e^{-2\pi\alpha_I}], \quad (11a)$$

$$P(\pi^- p \to \pi^0 n)\frac{d\sigma}{dt}(\pi^- p \to \pi^0 n) \sim |C_+ C_-| \sin(\varphi_+ - \varphi_-)[e^{2\pi\alpha_I} - e^{-2\pi\alpha_I} - 2(e^{\pi\alpha_I} - e^{-\pi\alpha_I}) \cos\pi\alpha_R], \quad (11b)$$

$$P(\pi^- p \to \eta n)\frac{d\sigma}{dt}(\pi^- p \to \eta n) \sim \tfrac{1}{3}|C_+ C_-| \sin(\varphi_+ - \varphi_-)[e^{2\pi\alpha_I} - e^{-2\pi\alpha_I} + 2(e^{\pi\alpha_I} - e^{-\pi\alpha_I}) \cos\pi\alpha_R], \quad (11c)$$

it can be seen that whenever the polarization in the KN charge-exchange scattering goes through zero, the polarizations in $\pi^- p \to \pi^0 n$ and $\pi^- p \to \eta n$ also have to go through zero. In relations (11) the notation $C_\pm(s,t) = |C_\pm(s,t)| e^{i\varphi_\pm}$ was used.

The predictions concerning the polarizations are the most significant results of the discussed Regge pole-cut model. They are different from the usual Regge-pole predictions (which are known to disagree with experiment unless hypothetical trajectories ρ' and A_2' are introduced) and they are also different from the prediction of the absorptive-cut model.[8] Unfortunately, only $\pi^- p \to \pi^0 n$ polarization data are available.[9] There are insufficient data[10] for $\pi^- p \to \eta n$ and no polarization data exist for the KN charge-exchange reactions. Therefore, we can only once more emphasize the importance of the corresponding experiments.

Finally, we would like to make a short comment on the relation which has been suggested as a test of ρ-A_2 exchange degeneracy.[5] The Regge-pole model with exchange-degenerate ρ-A_2 trajectories leads to the equality of the differential cross sections[5,11]

$$\frac{d\sigma}{dt}(K^- p \to \bar{K}^0 n) = \frac{d\sigma}{dt}(K^+ n \to K^0 p). \quad (12)$$

This relation, which seems to be satisfied within experimental error at 5.5 GeV/c[12] as well as at 12 GeV/c,[13] is supposed to provide the test of the ρ-A_2 exchange degeneracy. We would like to point out that unlike the sum rule (8), relation (11) is changed by the presence of the cuts. In our model the difference between the two differential cross sections is proportional to

$$\frac{d\sigma}{dt}(K^- p \to \bar{K}^0 n) - \frac{d\sigma}{dt}(K^+ n \to K^0 p) \sim (|C_+|^2 + |C_-|^2)(e^{2\pi\alpha_I} + e^{-2\pi\alpha_I} - 2) \sim 4\pi^2 \alpha_I^2 (|C_+|^2 + |C_-|^2).$$

We see that the difference is proportional to α_I^2 and therefore it can be smaller than the present experimental error [$\alpha_I(t)$ itself is assumed to be small]. However, the future possible deviation from equality (12) would not necessarily substantiate the case against the ρ-A_2 exchange degeneracy; it can indicate the presence of the Regge-Pomeranchuk cuts.

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[7] See, for example, V. Barger and D. Cline, *Phenomenological Theories of High Energy Scattering* (Benjamin, New York, 1969).

[8] T. Roth and G. H. Renninger, Phys. Rev. D **2**, 1293 (1970).

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[12] D. Cline, J. Matos, and D. D. Reeder, Phys. Rev. Letters **23**, 1318 (1969).

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