PHYSICAL REVIEW D VOLUME 3, NUMBER 9 1 MAY 1971

Gauge Fields, Theory of Currents, and Vector Mesons*†

I. KIMEL‡
The Enrico Fermi Institute and the Department of Physics, The University of Chicago, Chicago, Illinois 60637
(Received 11 July 1970; revised manuscript received February 1971)

Various forms of the gauge-field model with broken symmetry are studied. It is shown that only symmetry breakings which originate in the kinetic terms lead to models consistent with the algebra of currents, and this requires, in particular, the ω-φ mixing to be of current type. Breaking in the mass terms lead to modifications of the algebra, the consequences of which are analyzed. A nonet symmetry model yielding inverse-mass-squared relations is presented, and Sugawara's theory of currents, in relation to the vector mesons, is also studied. The models treated in the paper are compared with experiment.

I. INTRODUCTION

IN the algebra of currents the underlying group structure is used otherwise than in the eightfold way.[1] According to the eightfold way, the multiplets of particles correspond to low-dimensional representations of the $SU(3)$ group and assumptions about the symmetry breaking directly led to relations among particles. In the current-algebra formalism[2] the physical states are connected to the group in a less direct way. The set of electromagnetic and weak currents is enlarged to complete a representation of the group, while the spin-0 and -1 particles enter the picture through the divergence-field identities or the current-field identities. In the first, divergences of axial-vector currents are substituted by pseudoscalar meson fields (PCAC),[3] and in the second, the divergenceless parts of the currents are assumed to be proportional to interpolating fields of spin-1 mesons (vector-meson dominance).[4–6]

Accepting vector-meson dominance as a working hypothesis, one can then try to translate the symmetry characteristics of the hadronic currents into properties of the vector mesons. As it turns out, the algebra of currents is not enough for that purpose, and one needs extra information which could (one might hope) be found in the broader frameworks of models which have been proposed for the currents.

The first model for the current algebra was, of course, the quark model[2] which, unfortunately, does not seem to be in complete harmony with the idea of vector-meson dominance. Schwinger terms, for instance, come out to be divergent in the quark model, whereas vector-meson dominance relates them to finite ratios of masses and coupling constants of physical particles.

In the present work we do not discuss further the quark model and restrict ourselves to two other models which have been proposed for the currents: the gauge-field algebra[7] and Sugawara's theory of currents.[8]

In the gauge-field algebra the currents are treated from the very beginning as canonical vector-meson fields. The algebra obeyed by the currents is, in fact, deduced from the canonical commutation relations for these fields. It is then not surprising that current-field identities (or vector-meson dominance) come out as a very natural feature of the model.

A completely different approach motivated Sugawara's theory of currents. Here the aim was to combine current algebra with dynamical information in the simplest possible way compatible with self-consistency. Specifically, an energy-momentum tensor is constructed solely from bilinear products of currents. But, as it was later shown,[9] despite the differences Sugawara's theory can be related through a singular limiting process to gauge-field models. One can then study the consequences of this limit on the current–vector-meson relationships.[10,11]

In the type of models we are considering, symmetry breakings will have their effects not only on the vector mesons but also on the algebra for the currents. As we will show, some types of symmetry breaking and mixings, which would otherwise be permissible, have to be excluded if one wants to maintain Gell-Mann's current algebra. In particular, symmetry breakings introduced through mass terms in a Lagrangian are ruled out.

Once the $SU(3)$ symmetry is broken, the isosinglet current of the octet in general undergoes mixing with a $SU(3)$ singlet corresponding to the baryon-number

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* Work supported in part by the U. S. Atomic Energy Commission.

† Submitted to the Department of Physics, The University of Chicago, in partial fulfillment of the requirements for the Ph.D. degree.

‡ Present address: Randal Laboratory of Physics, University of Michigan, Ann Arbor, Mich. 48104.

[1] M. Gell-Mann, Caltech Report No. CTSL-20, 1961 (unpublished); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961). Both papers are reprinted in M. Gell-Mann and Y. Ne'eman, *The Eightfold Way* (Benjamin, New York, 1964).

[2] Current algebra was first formulated in M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[3] For the different ways in which PCAC can be formulated, see Y. Nambu, Phys. Rev. Letters **4**, 380 (1960); M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960); J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, *ibid.* **17**, 757 (1960).

[4] J. J. Sakurai, Ann. Phys. (N.Y.) **11**, 1 (1960).

[5] M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).

[6] N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. **157**, 1376 (1967).

[7] T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).

[8] H. Sugawara, Phys. Rev. **170**, 1659 (1968). See also C. M. Sommerfield, *ibid.* **176**, 2019 (1968).

[9] K. Bardakci, Y. Frishman, and M. B. Halpern, Phys. Rev. **170**, 1353 (1968).

[10] H. Sugawara, Phys. Rev. Letters **21**, 772 (1968).

[11] I. Kimel, Phys. Rev. **181**, 2152 (1969).

current. As a result, the physical ω and φ particles are neither pure octet nor singlet but a mixture, which accounts for the failure of a naive application of Gell-Mann–Okubo mass formula for the vector mesons. The oldest and simplest way of treating the mixing is based on a nondiagonal mass term in the Lagrangian.[12] Along this line we consider two models, the mass-mixing as described by Kroll, Lee, and Zumino[6] and another scheme which leads to a quasi-orthogonal mixing of the vector mesons relative to the currents.

The mixings described in the last paragraph are shown to be in conflict with the algebra of currents.[13,14] The only type allowed is the current mixing which originates from nondiagonal kinetic terms in the Lagrangian. It is interesting that one of the few things one can say with certainty about Sugawara's theory is that precisely this mixing is excluded.

The paper is divided as follows: In Sec. II the gauge-field framework is presented and general expressions for current commutators and spectral function sum rules are given. The different types of mixing are treated in Sec. III and in the next section these are combined with octet breaking to produce the simplest possibilities of gauge-field models with broken symmetry. Sugawara's theory of currents is studied in Sec. V in relation to vector mesons. Consistency between symmetry breaking in gauge-field models and current algebra is treated in Sec. VI and in Sec. VII we explore the possibility of having a higher nonet symmetry for the vector mesons. Section VIII contains final remarks as well as comparison with experiments.

II. GAUGE-FIELD MODEL

The gauge-field model is based on a Yang-Mills Lagrangian. The motivation for the original work by Yang and Mills[15] was to generalize to more complicated groups the well-known procedure of introducing the electromagnetic field in order to preserve local gauge invariance in the Lagrangian. The prescription is to start from a Lagrangian[16] $\mathcal{L}_M(\psi,\partial_\mu\psi)$ for the fields ψ which transform according to representations of a compact semisimple Lie group with generators Q_α, and make everywhere in $\mathcal{L}_M$ the replacement

$$\partial_\mu\psi \to D_\mu\psi \equiv \partial_\mu\psi + ig_0 v^\alpha{}_\mu[Q_\alpha,\psi], \tag{2.1}$$

where the vectors $v^\alpha{}_\mu$ are usually called Yang-Mills or gauge fields. Then, add a Lagrangian for the v's of the form

$$\mathcal{L}_v = -\tfrac{1}{4}V^{\alpha\mu\nu}V^\alpha{}_{\mu\nu}, \tag{2.2}$$

with

$$V^\alpha{}_{\mu\nu} = \partial_\mu v^\alpha{}_\nu - \partial_\nu v^\alpha{}_\mu - ig_0 v^\beta{}_\mu[Q_\beta, v^\alpha{}_\nu]. \tag{2.3}$$

The combination (2.1) is gauge invariant if $v^\alpha{}_\mu$ transforms as

$$v^\alpha{}_\mu \to v^\alpha{}_\mu + \Lambda_\beta[Q_\beta, v^\alpha{}_\mu] - g_0^{-1}\partial_\mu\Lambda_\alpha, \tag{2.4}$$

where the Λ's are the gauge parameters and

$$[Q_\beta, v^\alpha{}_\mu] = if_{\beta\alpha\gamma}v^\gamma{}_\mu, \tag{2.5}$$

with the structure constants $f_{\alpha\beta\gamma}$ of the Lie group defined by

$$[Q_\alpha, Q_\beta] = if_{\alpha\beta\gamma}Q_\gamma. \tag{2.6}$$

It is easy to see that if $\mathcal{L}_M(\psi,\partial_\mu\psi)$ is invariant under global gauge transformation (Λ_α constant), then $\mathcal{L}_M(\psi,D_\mu\psi)+\mathcal{L}_v$ will be invariant under local gauge transformations where the Λ's are coordinate dependent.

In spite of its elegance, no use was made of the Yang-Mills theory in particle physics, mainly because in order to preserve the gauge invariance, the fields $v^\alpha{}_\mu$ have to be massless. Leaving aside this technical problem for a later solution, Sakurai,[4] inspired by the Yang-Mills work, proposed a theory of strong interactions in which the conserved hadronic currents are universally coupled to a set of vector mesons. Afterwards, as a solution to the mass problem, Schwinger[17] advanced the idea that the unrenormalized gauge fields are originally massless but acquire a finite mass due to the strong interactions and showed explicitly how this can happen in a field theory in two dimensions.

Interest in a massive Yang-Mills theory was recently revived by the work of Lee, Weinberg, and Zumino.[7] Their original gauge-field model is based on a symmetric Lagrangian. We will take symmetry-breaking effects into account from the beginning, but being mainly interested in the vector currents (and mesons), for the sake of simplicity we will not explicitly write the axial vectors in our Lagrangian. Let this Lagrangian, for an $SU(3)$ octet plus singlet of vectors, be[18]

$$\mathcal{L} = -\tfrac{1}{4}\mathbf{V}^T{}_{\mu\nu}K\mathbf{V}^{\mu\nu} - \tfrac{1}{2}\mathbf{v}^T{}_\mu M_0{}^2\mathbf{v}^\mu, \tag{2.7}$$

where we have employed a compact notation with vectors and matrices in a nine-dimensional space such that

$$\mathbf{v}_\mu = \begin{bmatrix} v^1{}_\mu \\ \vdots \\ v^8{}_\mu \\ v^0{}_\mu \end{bmatrix}, \tag{2.8}$$

and where the components of $\mathbf{V}_{\mu\nu}$ are

$$V^\alpha{}_{\mu\nu} = \partial_\mu v^\alpha{}_\nu - \partial_\nu v^\alpha{}_\mu - \tfrac{1}{2}g_0 f_{\alpha\beta\gamma}[v^\beta{}_\mu, v^\gamma{}_\nu]_+. \tag{2.9}$$

From Lagrangian (2.7) one can derive equations of motion like, for instance,

$$M_0{}^2\mathbf{v}_0 = \partial^k\boldsymbol{\pi}_k + g_0\mathbf{F}, \tag{2.10}$$

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[12] J. J. Sakurai, Phys. Rev. **132**, 434 (1963); S. L. Glashow, Phys. Rev. Letters **11**, 48 (1963).

[13] I. Kimel, Phys. Rev. Letters **21**, 177 (1968).

[14] K. Kang, Phys. Rev. **177**, 2439 (1969).

[15] C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954).

[16] The Greek letters α, β, γ, δ, ϵ, and λ are used as unitary symmetry indices and μ, ν, τ, and σ as Lorentz indices, while the Latin letters i, j, and k label space components only. We always sum over repeated indices except when the contrary is stated.

[17] J. Schwinger, in *Theoretical Physics*, edited by A. Salam (IAEA, Vienna, 1963).

[18] We denote transposition either by a superscript T or a tilde.

where $\boldsymbol{\pi}_k$, the canonical momentum corresponding to $\mathbf{v}_k$, is given by

$$\boldsymbol{\pi}_k = K\mathbf{V}_{k0}\,, \tag{2.11}$$

while the components of $\mathbf{F}$ are

$$F^\alpha = f_{\alpha\beta\gamma}\pi^\beta{}_k v^\alpha{}_k\,. \tag{2.12}$$

The hadronic currents are assumed to be linear combinations of these fields according to

$$\mathbf{j}_\mu = R\mathbf{v}_\mu\,, \tag{2.13}$$

where the matrix R is constant but not necessarily diagonal. So, for the charge densities we have

$$\mathbf{j}_0 = RM_0{}^{-2}\partial^k\boldsymbol{\pi}_k + g_0 RM_0{}^{-2}\mathbf{F}\,, \tag{2.14}$$

from which we can immediately calculate the equal-time commutators

$$[j^\alpha{}_0(x), j^\beta{}_0(y)] = ig_0(RM_0{}^{-2})_{\alpha\gamma}(RM_0{}^{-2})_{\beta\lambda} \times f_{\gamma\lambda\delta}(M_0{}^2R^{-1})_{\delta\epsilon}\, j^\epsilon{}_0(x)\delta^3(x-y) \tag{2.15}$$

and

$$[j^\alpha{}_0(x), j^\beta{}_i(y)] = ig_0(RM_0{}^{-2})_{\alpha\gamma}R_{\beta\lambda}f_{\gamma\lambda\delta}R^{-1}{}_{\delta\epsilon}\, j^\epsilon{}_i(x)\delta^3(x-y) + i(RM_0{}^{-2}\tilde{R})_{\alpha\beta}\partial_i\delta^3(x-y)\,. \tag{2.16}$$

We can now introduce fields Φ_μ corresponding to the vector-meson particles in terms of the v's and a matrix S,[19]

$$\mathbf{v}_\mu = S\boldsymbol{\Phi}_\mu\,. \tag{2.17}$$

The currents and the vector mesons are then related by

$$\mathbf{j}_\mu = RS\boldsymbol{\Phi}_\mu \tag{2.18}$$

in such a way that the two-point functions for the Φ's are diagonal.

From Eq. (2.11) we get the relation

$$\partial_k\mathbf{j}_0 - \partial_0\mathbf{j}_k = RK\boldsymbol{\pi}_k + \mathbf{F}'\,, \tag{2.19}$$

where $\mathbf{F}'$ is bilinear in the currents and will not contribute to the two-point functions.[20]

By commuting Eq. (2.19) with $\mathbf{j}_i(y)$ at equal times and taking the vacuum expectation value, we get

$$\langle[\partial_0 j^\alpha{}_k(x), j^\beta{}_i(y)]\rangle = -i(RK^{-1}\tilde{R})_{\alpha\beta}\delta_{ki}\delta^3(x-y) + i(RM_0{}^{-2}\tilde{R})_{\alpha\beta}\partial_k\partial_i\delta^3(x-y)\,. \tag{2.20}$$

This commutator can also be written in a Lehmann-Källén representation in terms of spin-1 ($\rho^{(1)}$) and spin-0 ($\rho^{(0)}$) spectral functions as

$$-i\int d\mu^2\left\{\rho_{\alpha\beta}{}^{(1)}(\mu^2)\left(\delta_{ik} - \frac{\partial_i\partial_k}{\mu^2}\right) - \rho_{\alpha\beta}{}^{(0)}\partial_i\partial_k\right\}\delta^3(x-y)\,. \tag{2.21}$$

For the vector currents the spin-0 spectral functions $\rho_{\alpha\beta}{}^{(0)}$ are zero for $\alpha, \beta = 1, 2, 3, 8$, and 0. For $\alpha, \beta = 4$, 5, 6, and 7 they are of second order in $SU(3)$ breaking and can be neglected. Thus, by equating terms with and without derivatives of the δ function in Eqs. (2.20) and (2.21), we get the two sum rules[21]

$$\int d\mu^2\mu^{-2}\rho_{\alpha\beta}{}^{(1)} = (RM_0{}^{-2}\tilde{R})_{\alpha\beta} \tag{2.22}$$

and

$$\int d\mu^2\rho_{\alpha\beta}{}^{(1)} = (RK^{-1}\tilde{R})_{\alpha\beta}\,. \tag{2.23}$$

From Eqs. (2.18) and (2.20) we get for the Φ's

$$\langle[\partial_0\Phi^\alpha{}_k(x), \Phi^\beta{}_i(y)]\rangle_{x_0=y_0} = -i(S^{-1}K^{-1}\tilde{S}^{-1})_{\alpha\beta}\delta^3(x-y) + i(S^{-1}M_0{}^{-2}\tilde{S}^{-1})_{\alpha\beta}\partial_k\partial_i\delta^3(x-y)\,. \tag{2.24}$$

A proper normalization of the vector-meson fields requires

$$\tilde{S}KS = I \quad (I \text{ the identity matrix}) \tag{2.25}$$

and

$$\tilde{S}M_0{}^2S = M^2\,, \tag{2.26}$$

where M is a diagonal matrix whose elements are the vector-meson masses.

In closing this section we point out that the sum rules (2.22) and (2.23) when applied to the components of the electromagnetic current give useful relations among the widths of the neutral vector mesons decaying into lepton pairs. From (2.22) one gets

$$\tfrac{1}{3}m_\rho\Gamma(\rho\to l^+l^-) = \frac{(RM_0{}^{-2}R)_{33}}{(RM_0{}^{-2}R)_{88}} \times[m_\omega\Gamma(\omega\to l^+l^-) + m_\varphi\Gamma(\varphi\to l^+l^-)] \tag{2.27}$$

and from (2.22)

$$\tfrac{1}{3}m_\rho{}^3\Gamma(\rho\to l^+l^-) = \frac{(RK^{-1}R)_{33}}{(RK^{-1}R)_{88}} \times[m_\omega{}^3\Gamma(\omega\to l^+l^-) + m_\varphi{}^3\Gamma(\varphi\to l^+l^-)]\,. \tag{2.28}$$

III. TYPES OF MIXING

The present section essentially follows the analysis of Kroll, Lee, and Zumino[6] except that besides mass and current mixing we consider a third possibility which we call orthogonal mixing. Since we do not take into account here weak or electromagnetic interactions, the only mixing to be considered is between the hypercharge

[19] We use whenever possible the notation of Ref. 6. The matrices S, $M_0{}^2$, K, T, g_D, g_0, and M^2 are essentially the same as there but, in general, extended to the nine-dimensional space.

[20] The contribution of $\mathbf{F}'$ to Eq. (2.20), for instance, will be proportional to the VEV of two currents at the same space-time point. The usual assumption made in relation to gauge-field models, that these VEV's vanish, is also adopted in the present work. For a discussion of this point see Ref. 7.

[21] Spectral function sum rules for the hadronic currents were first obtained by S. Weinberg, Phys. Rev. Letters **18**, 507 (1967). We refer to Eqs. (2.22) and (2.23) as Weinberg's first and second sum rules whether the right-hand sides are $SU(3)$ symmetric or not.

and baryon-number currents. K and $M_0{}^2$ will then be diagonal except for 2×2 blocks.

The current-field identity relation (2.18) is usually written in a different way. The matrix product RS has, of course, dimensions of mass squared and it is convenient to factor out the mass matrix M^2,

$$RS=g^{-1}M^2=g_D{}^{-1}TM^2\,, \qquad (3.1)$$

where g_D is diagonal and T is of the form[22]

$$T=\begin{bmatrix}1 & & & \\ & \ddots & & \\ & & \cos\theta_Y & -\sin\theta_Y \\ & & \sin\theta_B & \cos\theta_B\end{bmatrix}. \qquad (3.2)$$

To treat the mixing[23] we have to distinguish between mixing due to $M_0{}^2$ and mixing originating in K.[24] In the first case K is diagonal and Eq. (2.25) is solved with

$$\tilde{S}^{-1}=K^{1/2}\mathcal{O}\,, \qquad (3.3)$$

where $\mathcal{O}$ is an orthogonal matrix.

With regard to the current–vector-meson relations, the mixing coming from $M_0{}^2$ can be further subdivided, according to the form of R, into mass mixing and orthogonal mixing.

a. Mass mixing. R is taken to be

$$R=g_0{}^{-1}M_0{}^2\,, \qquad (3.4)$$

with g_0 proportional to the unit matrix.

From Eqs. (2.26), (3.1), (3.3), and (3.4), we get

$$T=g_Dg_0{}^{-1}K^{1/2}\mathcal{O}\,. \qquad (3.5)$$

With $(g_Dg_0{}^{-1}K^{1/2})$ being diagonal, Eq. (3.5) implies

$$K^{1/2}=g_0g_D{}^{-1}\,,\quad T=\mathcal{O}\,. \qquad (3.6)$$

That is, T is an orthogonal matrix with $\theta_Y=\theta_B=\theta$. In this case the vector-meson mass relations can be summarized in the matrix equation

$$TM^2T^{-1}=K^{-1/2}M_0{}^2K^{-1/2}\,. \qquad (3.7)$$

In the case we have just described, the v fields are mixed relative to the j's. That means that $v^8{}_\mu$ contains both $j^8{}_\mu$ and $j^0{}_\mu$ and the same goes for $v_\mu{}^0$. We might consider the alternative of having the following situation.

b. Orthogonal mixing. The mixing is still due to nondiagonal elements in $M_0{}^2$ but now R is diagonal and in the 8-0 sector K is proportional to the unit matrix, $K=\kappa^2I$. Then S is quasi-orthogonal,

$$S=\kappa^{-1}\mathcal{O}=\kappa^{-1}\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}, \qquad (3.8)$$

and from Eqs. (3.1), (3.2), and (3.8) we get

$$(m_\varphi{}^2/m_\omega{}^2)\tan\theta_B=(m_\omega{}^2/m_\varphi{}^2)\tan\theta_Y=\tan\theta\,, \qquad (3.9)$$

which can also be seen from the fact that with R and K diagonal, the 8-0 element of the second sum rule (2.23) is zero.[25] One also obtains

$$R=\kappa g_D{}^{-1}\begin{pmatrix}m_\varphi{}^2\cos\theta_Y/\cos\theta & 0\\ 0 & m_\omega{}^2\cos\theta_B/\cos\theta\end{pmatrix}. \qquad (3.10)$$

That is, with the notation of the Appendix, we have the following relations between the hypercharge and baryon-number currents and the ω, φ vector mesons:

$$j_\mu{}^Y=\frac{m_\varphi{}^2\cos\theta_Y}{f_Y\cos\theta}(\cos\theta\,\varphi_\mu-\sin\theta\,\omega_\mu)\equiv\frac{m_\varphi{}^2\cos\theta_Y}{f_Y\cos\theta}\omega^8{}_\mu \qquad (3.11)$$

and

$$j_\mu{}^B=\frac{m_\omega{}^2\cos\theta_B}{f_B\cos\theta}(\cos\theta\,\omega_\mu+\sin\theta\,\varphi_\mu)\equiv\frac{m_\omega{}^2\cos\theta_B}{f_B\cos\theta}\omega^0{}_\mu\,. \qquad (3.12)$$

This is as far as one can go in the way of connecting the ω and φ vector mesons to the currents (actually to $\omega_\mu{}^8$ and $\omega_\mu{}^0$) by an orthogonal transformation which is the reason for the name orthogonal mixing.

c. Current mixing. Here both $M_0{}^2$ and R, which can be related as in Eq. (3.4), are diagonal. Combining Eqs. (2.26) and (3.1), we see that both sides of

$$g_DRM^{-2}Rg_D=TM^2\tilde{T} \qquad (3.13)$$

have to be diagonal, which leads to

$$(m_\varphi/m_\omega)\tan\theta_B=(m_\omega/m_\varphi)\tan\theta_Y=\tan\theta\,. \qquad (3.14)$$

Alternatively, we see from Eq. (2.26) that we can define an orthogonal matrix

$$\mathcal{O}\equiv M_0SM^{-1}=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix} \qquad (3.15)$$

with which we have

$$g_DRM_0{}^{-1}=TM\mathcal{O}^{-1}\,. \qquad (3.16)$$

The vanishing of the off-diagonal elements of the right-hand side of Eq. (3.16) again gives Eq. (3.14), which justifies the identification of the angles in Eqs. (3.14) and (3.15).

[22] Our notation is further clarified in the Appendix.

[23] From here to the end of Sec. III we work in the 8-0 subspace and, consequently, all matrices are 2×2.

[24] Symmetry breaking and mixing in the kinetic matrix were first used by S. Coleman and H. J. Schnitzer, Phys. Rev. **134**, B863 (1964).

[25] In connection with an $SU(3)$-symmetric Weinberg second sum rule, the angle relation (39) was also considered by M. Gourdin, in lectures delivered at the International Conference on Quark Models, Wayne State University, Detroit, 1969 (unpublished). This angle relation, however, only depends on the vanishing of the 8-0 element of the sum rule and neither of the two simple octet breakings we consider in Sec. IV leads to Gourdin's model.

For current mixing, the mass relations are contained in

$$\mathcal{O}M^{-2}\mathcal{O}^{-1}=M_0^{-1}KM_0^{-1}, \tag{3.17}$$

which is easily obtained from Eqs. (2.25) and (3.15).

IV. SIMPLE GAUGE-FIELD MODELS AND VECTOR MESONS

As we have seen, symmetry breaking for the set of nine vector mesons (and currents) involves two processes, the breaking of the octet symmetry and singlet octet mixing. The simplest possibilities for octet breaking are breaking in the kinetic matrix[24] with $K_{\alpha\beta}\propto\delta_{\alpha\beta}+Dd_{8\alpha\beta}$ and breaking in the mass matrix with $(M_0^2)_{\alpha\beta}\propto\delta_{\alpha\beta}+D'd_{8\alpha\beta}$. The combination of these breakings with the three types of mixing described in Sec. III yield six models, the first three of which are "pure" in the sense that breaking and mixing originate in only one of the matrices K or M_0^2. The other three are hybrid combinations with one of the matrices responsible for the octet breaking and the other for the mixing. These combinations are:

(1) *Octet breaking in K plus current mixing*. The mass matrix is

$$(M_0^2)_{\alpha\beta}=\delta_{\alpha\beta}m^2+\delta_{\alpha0}\delta_{\beta0}(n^2-m^2), \tag{4.1}$$

while for the kinetic matrix we have

$$K=\begin{bmatrix}(1+Dd_{811}) & & & \\ & \ddots & & \\ & & (1+Dd_{888}) & \epsilon\kappa \\ & & \epsilon\kappa & \kappa^2\end{bmatrix}. \tag{4.2}$$

This results in the following sum rules [from Eqs. (2.22) and (2.23)]:

$$\int d\mu^2\mu^{-2}\rho_{\alpha\beta}{}^{(1)}=g_0^{-2}[\delta_{\alpha\beta}m^2+\delta_{\alpha0}\delta_{\beta0}(n^2-m^2)] \tag{4.3}$$

and

$$\int d\mu^2\rho_{\alpha\beta}{}^{(1)}=\frac{m^2[\delta_{\alpha\beta}(1+\delta_{\alpha0}Dd_{888})-\epsilon(\delta_{\alpha8}\delta_{\beta0}+\delta_{\alpha0}\delta_{\beta8})]}{g_0^2[1+\delta_{\alpha0}(\kappa-1)][1+\delta_{\beta0}(\kappa-1)][1+Dd_{8\alpha\alpha}-\delta_{\alpha0}/\sqrt{3}-\epsilon^2(\delta_{\alpha8}+\delta_{\alpha0})]}\quad\text{(no sum over }\alpha\text{ or }\beta). \tag{4.4}$$

A model based essentially on these sum rules was first proposed by Oakes and Sakurai (OS).[26] From the first sum rule (4.3), or more directly from Eq. (2.27), one can derive the following width relation:

$$\Sigma=\tfrac{1}{3}\Gamma(\rho\to e^+e^-)-[(m_\omega/m_\rho)\Gamma(\omega\to e^+e^-)+(m_\varphi/m_\rho)\Gamma(\varphi\to e^+e^-)]=0. \tag{4.5}$$

With the angle θ defined in Eq. (3.14), the masses can be related as

$$\tfrac{1}{3}(4m_{K^*}{}^{-2}-m_\rho{}^{-2})=m_\varphi{}^{-2}\cos^2\theta+m_\omega{}^{-2}\sin^2\theta \tag{4.6}$$

and the coupling-constant ratios can be obtained from

$$m_\rho{}^2f_\rho{}^{-2}=m_{K^*}{}^2f_{K^*}{}^{-2}=(9/4)f_Y{}^{-2}/(4m_{K^*}{}^{-2}-m_\rho{}^{-2}), \tag{4.7}$$

while f_B depends on the as yet free parameter κ.

(2) *Octet breaking in M_0^2 and mass mixing*. This is the second type of mass mixing of Kroll, Lee, and Zumino except that they did not get the relations among coupling constants or widths. Here the kinetic matrix is simply

$$K_{\alpha\beta}=\delta_{\alpha\beta}+\delta_{\alpha0}\delta_{\beta0}(\kappa^2-1), \tag{4.8}$$

while the mass matrix is of the form

$$M_0^2=\begin{bmatrix}m^2(1+D'd_{811}) & & & \\ & \ddots & & \\ & & m^2(1+D'd_{888}) & m_{80}{}^2 \\ & & m_{80}{}^2 & n^2\end{bmatrix}. \tag{4.9}$$

From Eq. (3.6) we see that the coupling constants are simply related as

$$f_\rho=f_{K^*}=(2/\sqrt{3})f_Y=(\sqrt{\tfrac{2}{3}})\kappa f_B, \tag{4.10}$$

while for the masses we get

$$\tfrac{1}{3}(4m_{K^*}{}^2-m_\rho{}^2)=m_\varphi{}^2\cos^2\theta+m_\omega{}^2\sin\theta, \tag{4.11}$$

where $\theta=\theta_Y=\theta_B$.

The radiative width relation we obtain here is the one first proposed by Sugawara,

$$\Sigma=\tfrac{1}{3}\Gamma(\rho\to e^+e^-)-\frac{m_\rho{}^2}{\tfrac{1}{3}(4m_{K^*}{}^2-m_\rho{}^2)}\times\left[\frac{m_\omega}{m_\rho}\Gamma(\omega\to e^+e^-)+\frac{m_\varphi}{m_\rho}\Gamma(\varphi\to e^+e^-)\right]=0. \tag{4.12}$$

(3) *Octet breaking in M_0^2 plus orthogonal mixing*. In this case we have a mass matrix like Eq. (4.9) while K has to be proportional to the unit matrix, $K=\kappa^2I$, and R is as in Eq. (3.10). We get the same mass relation (4.11) as in model (2) but with the angle θ defined in (3.9).

The coupling constants are related by

$$f_\rho{}^{-2}=f_{K^*}{}^{-2}=\tfrac{3}{4}f_Y{}^{-2}\frac{m_\varphi{}^4}{[\tfrac{1}{3}(4m_{K^*}{}^2-m_\rho{}^2)]^2}\frac{\cos^2\theta_Y}{\cos^2\theta}, \tag{4.13}$$

[26] R. J. Oakes and J. J. Sakurai, Phys. Rev. Letters 19, 1266 (1967).

and from Eq. (2.28) we get

$$\Sigma=\tfrac{1}{3}\Gamma(\rho\to e^+e^-)-\left[\frac{m_\rho^2}{\frac{1}{3}(4m_{K^*}^2-m_\rho^2)}\right]^2 \times\left[\frac{m_\omega^3}{m_\rho^3}\Gamma(\omega\to e^+e^-)+\frac{m_\varphi^3}{m_\rho^3}\Gamma(\varphi\to e^+e^-)\right]=0. \quad (4.14)$$

(4) *Octet breaking in K and mass mixing*. This corresponds to the first case of mass mixing treated by Kroll, Lee, and Zumino. The kinetic and mass matrices are here

$$K_{\alpha\beta}=\delta_{\alpha\beta}+Dd_{8\alpha\beta}+\delta_{\alpha0}\delta_{\beta0}(\kappa^2-1) \quad (4.15)$$

and

$$M_0^2=\begin{bmatrix} m^2 & & & \\ & \ddots & & \\ & & m^2 & m_{80}^2 \\ & & m_{80}^2 & n^2 \end{bmatrix}. \quad (4.16)$$

The mass formula is now

$$[\tfrac{1}{3}(4m_{K^*}^{-2}-m_\rho^{-2})]^{-1}=m_\varphi^2\cos^2\theta+m_\omega^2\sin^2\theta, \quad (4.17)$$

where $\theta=\theta_Y=\theta_B$, and the coupling constants can be obtained from Eq. (4.7). In this model the right-hand side of the first sum rule (2.22) is the same for $\alpha=\beta=8$ as for $\alpha=\beta=3$ and so relation (4.5) is also valid here.

(5) *Octet breaking in M_0^2 and current mixing*. The elements of the mass matrix are

$$(M_0^2)_{\alpha\beta}=m^2(\delta_{\alpha\beta}+Dd_{8\alpha\beta})+\delta_{\alpha0}\delta_{\beta0}(n^2-m^2) \quad (4.18)$$

while the kinetic matrix contains the mixing parameter ϵ,

$$K=\begin{bmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \epsilon\kappa \\ & & \epsilon\kappa & \kappa^2 \end{bmatrix}. \quad (4.19)$$

We get here

$$[\tfrac{1}{3}(4m_{K^*}^2-m_\rho^2)]^{-1}=m_\varphi^{-2}\cos^2\theta+m_\omega^{-2}\sin^2\theta \quad (4.20)$$

with θ defined in Eq. (3.14), while the coupling constants satisfy Eq. (4.10). We also have in this model Sugawara's width relation (4.12).

(6) *Octet breaking in K plus orthogonal mixing*. The kinetic matrix is in this model,

$$K_{\alpha\beta}=\delta_{\alpha\beta}+Dd_{8\alpha\beta}+\delta_{\alpha0}\delta_{\beta0}Dd_{888}, \quad (4.21)$$

while the mass matrix is given by Eq. (4.16) and R is as in Eq. (3.10). From Eq. (2.28) we now immediately get

$$\Sigma=\tfrac{1}{3}\Gamma(\rho\to e^+e^-)-\tfrac{1}{3}\left(4\frac{m_\rho^2}{m_{K^*}^2}-1\right) \times\left[\frac{m_\varphi^2}{m_\rho^3}\Gamma(\varphi\to e^+e^-)+\frac{m_\omega^3}{m_\rho^3}\Gamma(\omega\to e^+e^-)\right]=0. \quad (4.22)$$

The masses are related as in Eq. (4.17) but with the angle θ defined in (3.9). The coupling-constant ratios can be obtained from

$$f_\rho^{-2}m_\rho^2=f_{K^*}^{-2}m_{K^*}^2=f_Y^{-2}m_\varphi^4(m_{K^*}^{-2}-\tfrac{1}{4}m_\rho^{-2}). \quad (4.23)$$

Besides the above simple models we would like to comment on a different proposal which appeared in the literature.

(7) *Das, Mathur, and Okubo* (DMO) *model*. These authors have proposed a model[27] based on a symmetric Weinberg first sum rule and a second sum rule with octet breaking

$$\int d\mu^2\rho_{\alpha\beta}^{(1)}=\frac{m^4}{g_0^2}(\delta_{\alpha\beta}+Dd_{8\alpha\beta}) \quad (\alpha,\beta=1,\ldots,8). \quad (4.24)$$

In the framework of gauge-field models, as pointed out by Kang,[14] this situation arises when the inverse of the kinetic matrix for the octet is proportional to the right-hand side of Eq. (4.24). For the complete set of nine currents the inverse of the kinetic matrix is

$$K^{-1}=\begin{bmatrix} 1+Dd_{811} & & & \\ & \ddots & & \\ & & 1+Dd_{888} & \epsilon \\ & & \epsilon & \kappa^2 \end{bmatrix}. \quad (4.25)$$

In this model the widths are related as in Eq. (4.5) and the masses as in Eq. (4.11) but with the angle θ satisfying the current-mixing relation (3.14). The coupling-constant ratios can be obtained from

$$f_\rho^{-2}m_\rho^2=f_{K^*}^{-2}m_{K^*}^2 =\tfrac{3}{4}f_Y^{-2}\frac{m_\varphi^2m_\omega^2}{m_\varphi^2+m_\omega^2-\frac{1}{3}(4m_{K^*}^2-m_\rho^2)}. \quad (4.26)$$

V. VECTOR MESONS IN SUGAWARA THEORY OF CURRENTS

In Sec. IV we studied models which can be obtained by breaking the symmetry in simple ways in the gauge-field theory. A seemingly different type of theory for the currents proposed by Sugawara[8] was shortly afterwards shown to be obtainable as a singular limit of a gauge-field model.[9] It is our purpose in this section to see what implications Sugawara's theory of currents would have on the vector mesons.

Sugawara based his theory on current commutation relations as given by gauge-field models and on an energy-momentum tensor constructed solely in terms of currents (with no derivatives allowed).

For exact $SU(3)$ symmetry, Schwinger's condition

$$[\Theta_{00}(x),\theta_{00}(y)]_{x_0=y_0} =-i\{\Theta_{0i}(x)+\Theta_{0i}(y)\}\partial_i\delta^3(x-y) \quad (5.1)$$

restricts the energy-momentum tensor to be of the

[27] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters 19, 470 (1967).

form[28]

$$\Theta_{\mu\nu}=(1/2C)\{([j^\alpha{}_\mu,j^\alpha{}_\nu]_+-g_{\mu\nu}j^\alpha{}_\tau j^{\alpha\tau})-(j\to j_5)\}. \quad (5.2)$$

There is, besides, a consistency condition: In order for $\int\Theta_{0\mu}(x)d^3x$ to be a good Poincaré generator, the following (Sugawara) equations of motion have to hold:

$$\partial_\mu j^\alpha{}_\nu-\partial_\nu j^\alpha{}_\mu=(1/2C)f_{\alpha\beta\gamma}\{[j^\beta{}_\mu,j^\gamma{}_\nu]_++(j\to j_5)\} \quad (5.3)$$

and

$$\partial_\mu j^\alpha{}_{5\nu}-\partial_\nu j^\alpha{}_{5\mu} =(1/2C)f_{\alpha\beta\gamma}\{[j^\beta{}_{5\mu},j^\gamma{}_\nu]_++(j\rightleftarrows j_5)\}, \quad (5.4)$$

where C is the coefficient of the Schwinger terms in the current commutators.

As Bardakci, Frishman, and Halpern[9] have shown, the model we just sketched can be obtained by taking the limit

$$m\to 0,\quad g_0\to 0,\quad m^2/g_0{}^2\to C \quad (5.5)$$

in a symmetric massive Yang-Mills model. To see how that comes about and in order to allow for symmetry breaking, we consider the general relation between the tensors $V^\alpha{}_{\mu\nu}$ [in Eq. (2.9)] and the currents

$$R_{\alpha\beta}V^\beta{}_{\mu\nu}=\partial_\mu j^\alpha{}_\nu-\partial_\nu j^\alpha{}_\mu-\tfrac{1}{2}g_0R_{\alpha\beta}f_{\beta\gamma\lambda}R_{\gamma\gamma'}{}^{-1}R_{\lambda\lambda'}{}^{-1} \times\{[j^{\gamma'}{}_{\mu'}j^{\lambda'}{}_\nu]_++(j\rightleftarrows j_5)\}. \quad (5.6)$$

$V_{\mu\nu}$ is contained quadratically in the energy-momentum tensor and thus cannot be divergent in the limit (5.5). On the other hand R is proportional to m^2/g_0 and vanishes in that limit. That gives zero for the left-hand side of Eq. (5.6) which then becomes Sugawara's equation of motion (the more careful analysis of Ref. 9 confirms this result)

$$\partial_\mu j^\alpha{}_\nu-\partial_\nu j^\alpha{}_\mu=\tfrac{1}{2}g_0R_{\alpha\alpha'}f_{\alpha'\gamma\lambda}R^{-1}{}_{\gamma\gamma'}R^{-1}{}_{\lambda\lambda'} \times\{[j_\mu{}^{\gamma'},j_\nu{}^{\lambda'}]_++(j\to j_5)\}. \quad (5.7)$$

To obtain the current commutators in the present model, one has to substitute, in Eqs. (2.15) and (2.16), $g_0m^{-2}R=m^{-2}M_0{}^2$ by its (finite) value in the limit (5.5). By the same procedure, we get from Eq. (2.22) the sum rule

$$\int d\mu^2\mu^{-2}\rho_{\alpha\beta}{}^{(1)}=\lim[g_0{}^{-2}(M_0{}^2)_{\alpha\beta}], \quad (5.8)$$

the right-hand side of which is, of course, also the coefficient of the Schwinger terms.

Sugawara[10] has proposed a model with $M_0{}^2$ as given in Eq. (4.9), where m_{80} and n go to zero as m in the limit (5.5), and also assumed a second sum rule of the form

$$\int d\mu^2\rho_{\alpha\beta}=\text{const}\times\lim[g_0{}^{-4}(M_0{}^4)_{\alpha\beta}]. \quad (5.9)$$

With regard to the vector mesons, such a proposal leads to the same results as model (2) in Sec. IV.

[28] The j_5's designate the axial-vector currents.

But looking at Eq. (2.23) we see that in the limit (5.5), instead of a sum rule like (5.9), we have a trivial one (the right-hand side is zero). So, for Sugawara's model to have any content, terms which were assumed to vanish in deriving Eq. (2.23) ought to be allowed to contribute here. If we put $\mu=0$, $\nu=i$ in Eq. (5.7), commute it with $j^\beta{}_k$, and take the vacuum expectation value (VEV), the result is

$$\begin{aligned}\langle[\partial_0 j^\alpha{}_i(x)-\partial_i j^\alpha{}_0(x),\, j^\beta{}_k(y)]\rangle &=-i\delta_{ik}\int d\mu^2\rho_{\alpha\beta}\delta^3(x-y)\\ &=ig_0{}^2R_{\alpha\alpha'}f_{\alpha'\gamma\lambda}R^{-1}{}_{\lambda\lambda'}(M_0{}^{-2})_{\gamma\gamma'}R_{\beta\beta'}f_{\gamma'\beta'\epsilon}R^{-1}{}_{\epsilon\epsilon'}\\ &\quad\times\{\langle j^{\epsilon'}{}_k(x)j^{\lambda'}{}_i(x)\rangle+\langle j^{\epsilon'}{}_{5k}(x)j^{\lambda'}{}_{5i}(x)\rangle\}\delta^3(x-y)\\ &=-ig_0{}^2(M_0{}^2)_{\alpha\alpha'}(M_0{}^{-2})_{\lambda\lambda'}(M_0{}^{-2})_{\gamma\gamma'}(M_0{}^2)_{\beta\beta'}(M_0{}^{-2})_{\epsilon\epsilon'}\\ &\quad\times f_{\alpha'\gamma\lambda}f_{\beta'\gamma'\epsilon}\{\langle j^{\epsilon'}{}_k(x)j^{\lambda'}{}_i(x)\rangle\\ &\qquad+\langle j^{\epsilon'}{}_{5k}(x)j^{\lambda'}{}_{5i}(x)\rangle\}\delta^3(x-y). \quad (5.10)\end{aligned}$$

So, actually, owing to the ambiguous character of the VEV of two currents at the same point, the symmetry properties of Eq. (5.10) are at best an extra assumption one has to introduce into the model. If (5.10) is assumed to be like Eq. (5.9) then, as we already said, one gets relations (4.10)–(4.12). One could instead take a symmetric second Weinberg sum rule and reproduce the results from the orthogonal mixing-model (3) of Sec. IV.

The only restriction seems to be that it is not possible to have current mixing in Sugawara's theory. If $(M_0{}^2)_{80}=0$ as is required in order to have the 8-0 element of Eq. (5.8) vanishing (necessary condition for current mixing), then regardless of whatever else one assumes of Eq. (5.10), its 8-0 element is also zero, and one does not have any mixing at all. With regard to vector mesons, then, the only sure statement one can make about Sugawara's theory is that it does not allow for current mixing.

VI. CONSISTENCY BETWEEN SYMMETRY BREAKING AND CURRENT ALGEBRA

Of the different types of mixing and symmetry breaking in a Lagrangian model, the ones coming from the mass matrix seem to be the simplest. They were, in fact, the only types used until Coleman and Schnitzer[24] called attention to another kind originating in the kinetic matrix.

When it became necessary to break the symmetry in the Weinberg sum rules, again the mass type of breaking was the first to be invoked by Okubo *et al.*[27] Besides simplicity, a physical argument can be given for that choice. As Okubo pointed out,[29] since the mass terms in

[29] S. Okubo, in *Proceedings of the International Theoretical Physics Conference on Particles and Fields, Rochester, N. Y., 1967*, edited by C. R. Hagen *et al.* (Interscience, New York, 1968).

a Yang-Mills Lagrangian violate gauge invariance it seems natural to blame them for the symmetry breaking also.

But, as we show in this section, in order to be consistent with Gell-Mann's algebra of currents, a gauge-field model has to have a mass matrix which does not contain symmetry breaking or mixing. Of all the models studied in Sec. IV, only the first (Oakes-Sakurai) and and the seventh (Das-Mathur-Okubo) satisfy this requirement.

To see how symmetry breaking and current algebra are related, we first restrict ourselves to the currents belonging to the octet only (i.e., ignore the mixing with the singlet) and start by checking the better-defined time-time commutators. From Eq. (2.15) we see that for these commutators to have the usual form R has to be

$$R=g_0^{-1}M_0^2. \tag{6.1}$$

With this substitution the time-space commutators, Eq. (2.15), become

$$[j^\alpha{}_0(x),j^\beta{}_i(y)]_{x_0=y_0}=i(M_0^2)_{\beta\gamma}(M_0^{-2})_{\lambda\epsilon}\times f_{\alpha\gamma\lambda}j^\epsilon{}_i(x)\delta^3(x-y)+ig_0^{-2}(M_0^2)_{\alpha\beta}\partial_i\delta^3(x-y). \tag{6.2}$$

In order to recover Gell-Mann's type of commutator from Eq. (6.2), we see that M_0^2 has to be proportional to the unit matrix, $M_0^2=m^2I$.[13]

As we already pointed out, what would seem to be the simplest assumption, i.e., symmetry breaking in the mass terms, has to be ruled out unless one is willing to modify the algebra for the currents. On the other hand, the algebra remains unchanged when the symmetry breaking is introduced in the kinetic matrix K, for which the simplest possibility is, of course,

$$K_{\alpha\beta}=\delta_{\alpha\beta}+Dd_{8\alpha\beta}.$$

We proceed now to show that current algebra and 8-0 mixing in the mass matrix cannot coexist in a gauge-field model. In order to do that, we consider a Lagrangian with a mass matrix which in the 8-0 sector is of the form

$$M_0^2=\begin{pmatrix} m_8^2 & m_{80}^2 \\ m_{80}^2 & n^2 \end{pmatrix}. \tag{6.3}$$

To have the correct time-time commutators, R still has to be given by Eq. (6.1), which means that orthogonal mixing has to be ruled out. To stress that point, we remark that for orthogonal mixing, $R_{\alpha\beta}=R_\alpha\delta_{\alpha\beta}$ substituted into Eq. (2.15) leads, for instance, to

$$[Q^0,Q^6]\sim m_{80}^2Q^7, \tag{6.4}$$

which clearly contradicts the original assumption of Q^0 being an $SU(3)$ singlet (unless $m_{80}^2=0$).

Mass mixing also leads to bad results. From Eq. (2.16) with R as given in Eq. (3.4), we obtain

$$[Q^\alpha,j^0{}_i(x)]=i\sum_\beta(M_0^2)_{80}(M_0^{-2})_{\beta\beta}f_{\alpha 8\beta}j^\beta{}_i(x) \tag{6.5}$$

and

$$[Q^6,j^7{}_i(x)]=i(M_0^2)_{77}\{\tfrac{1}{2}\sqrt{3}[(M_0^{-2})_{88}j^8{}_i(x)+(M_0^{-2})_{80}j^0{}_i(x)]-\tfrac{1}{2}(M_0^{-2})_{33}j^3{}_i(x)\}, \tag{6.6}$$

which are inconsistent with $j^0{}_i$ being a singlet when $(M_0^2)_{80}=m_{80}^2\neq 0$.

The mixing problem can also be analyzed starting from the sum rules. Weinberg's first sum rule can be extended to the nine vector currents in the form

$$\int d\mu^2\mu^{-2}\rho_{\alpha\beta}{}^{(1)}=s_1\delta_{\alpha\beta}+s_1'\delta_{\alpha 0}\delta_{\beta 0}+s_1''(\delta_{\alpha 0}\delta_{\beta 8}+\delta_{\alpha 8}\delta_{\beta 0}), \tag{6.7}$$

where s_1'' is proportional to the VEV of the crossed 8-0 Schwinger term. It is easy to show that the following three assumptions lead to unphysical results: (i) $s_1''\neq 0$ in Eq. (6.7), (ii) $j^0{}_\mu$ transforms as an authentic singlet, and (iii) all Schwinger terms are C numbers (as is usually assumed in gauge-field models). Under the conditions stated above, the right-hand side of the following equal-time Jacobi identity[30] vanishes:

$$[j^0{}_0(x),[Q^6,j^7{}_i(y)]]=[Q^6,[j^0{}_0(x),j^7{}_i(y)]]+[j^7{}_i(y),[Q^6,j^0{}_0(x)]]. \tag{6.8}$$

So, from the left-hand side we get

$$\tfrac{1}{2}\sqrt{3}i[j^0{}_0(x),j^8{}_i(y)]-\tfrac{1}{2}i[j^0{}_0(x),j^3{}_i(y)]=0, \tag{6.9}$$

according to which if $s_1''\neq 0$, there has to be also a nonvanishing 0-3 Schwinger term whose effect would be to add to Eq. (6.7) still another term of the form $s_1^{03}(\delta_{\alpha 0}\delta_{\beta 3}+\delta_{\alpha 3}\delta_{\beta 0})$ giving rise to medium-strong ω-ρ and φ-ρ mixing. To avoid this, both terms in Eq. (6.9) (and s_1'') have to be zero and as Oakes and Sakurai[26] first pointed out, $s_1''=0$ necessarily implies ω-φ current mixing. Saturation of $\rho_{80}{}^{(1)}$ by ω and φ gives, with the notation of the Appendix,

$$\frac{3}{2\sqrt{2}f_Yf_B}(m_\varphi^2\cos\theta_Y\sin\theta_B-m_\omega^2\sin\theta_Y\cos\theta_B)=s_1''=0, \tag{6.10}$$

from which the current mixing relation (3.14) follows immediately.

As we can see from Eqs. (6.5) and (6.6), for mass mixing condition (ii) listed above is not satisfied. While $j^0{}_0$ transforms as a unitary singlet, the space components $j^0{}_i$ do not. In this way $s_1^{03}\neq 0$ can be avoided in a mass mixing model. That this possibility be taken seriously has been advocated in a recent paper by Cremmer.[31] It is not clear whether this proposal, as well

[30] It has been found that Jacobi identities for equal-time commutators of current densities might not hold in some cases. See F. Bucella *et al.*, Phys. Rev. **149**, 1268 (1966); K. Johnson and F. E. Low, Progr. Theoret. Phys. (Kyoto) Suppl. **37-38**, 74 (1966). On the other hand, it is perfectly correct to use a Jacobi identity like Eq. (6.8) involving a charge, a charge density, and the space component of a current.

[31] E. Cremmer, Nucl. Phys. **B15**, 131 (1970).

as Sugawara's[10] modification of the commutation relations [which arise, for instance, from a model like (2) in Sec. IV]

$$[Q^\alpha, j^\beta{}_\mu(x)] = if_{\alpha\beta\gamma}\left[\delta_{\mu\nu}+\left(1-\frac{1+D'd_{8\beta\beta}}{1+D'd_{8\gamma\gamma}}\right)\delta_{\mu i}\delta_{\nu i}\right]j^\gamma{}_\nu(x)\,, \quad (6.11)$$

will lead to noncovariance of measurable quantities. We think that these points warrant further investigation. What we can say now is that, as was first shown in Ref. 13 and in more detail in this section, symmetry breaking in the mass terms of a gauge-field Lagrangian leads to modification of the algebra for the currents. Conversely, if one wants to preserve the algebra as postulated by Gell-Mann, the symmetry breaking has to be introduced in the kinetic terms, which means that of all the models of Sec. IV one has to choose either (1) or (7).

VII. BROKEN NONET SYMMETRY

Okubo has shown[32] how the vector mesons satisfy a (broken) nonet symmetry when $SU(3)$ medium-strong effects are contained in the mass terms of the Lagrangian. Since we concluded in Sec. VI that on theoretical grounds models with breaking in the kinetic terms are to be preferred, we show here that model (1) in Sec. IV also comes close to obeying a nonet symmetry.

The closest one can get to a nonet symmetry is with $n=m$ in Eq. (4.1) and the kinetic matrix (4.2) reducing to the form

$$K_{\alpha\beta}=\delta_{\alpha\beta}+Dd_{8\alpha\beta} \quad (\alpha,\beta=1,\ldots,8,0)\,, \quad (7.1)$$

where $d_{0\alpha\beta}=(\sqrt{\tfrac{2}{3}})\delta_{\alpha\beta}$. In this situation symmetry breaking and mixing are simply related by $U(3)$ Clebsch-Gordan coefficients. From such a higher symmetry one can derive the following relations [besides Eqs. (4.5) and (4.6)]:

$$\tan\theta=1/\sqrt{2}\Rightarrow\theta=35.26°\,, \quad (7.2)$$

$$m_\omega=m_\rho\,, \quad (7.3)$$

and

$$m_\varphi{}^{-2}+m_\omega{}^{-2}=2m_{K^*}{}^{-2}\,. \quad (7.4)$$

The first two are the same as in Okubo's nonet model[32] while the last equality, independent of the mixing angle, is experimentally satisfied even better than the corresponding (Okubo) expression for the direct masses squared. In BeV^{-2}, the left-hand side of Eq. (7.4) is 2.59 while the right-hand side is 2.51.

Since Eqs. (7.2)–(7.4) are not far from reality, we only have to add to $K_{\alpha\beta}$ in Eq. (7.1) small amounts of further symmetry breaking and it is convenient to do it in the following way:

$$K_{\alpha\beta}=\delta_{\alpha\beta}+D\{d_{8\alpha\beta}+\tfrac{1}{2}\sqrt{3}\eta[\delta_{\alpha 0}\delta_{\beta 0} -(\sqrt{\tfrac{1}{2}})(1+\Omega)(\delta_{\alpha 8}\delta_{\beta 0}+\delta_{\alpha 0}\delta_{\beta 8})]\}\,, \quad (7.5)$$

[32] S. Okubo, Phys. Letters 5, 165 (1963).

where η and Ω are simply related to the violations of Eqs. (7.2) and (7.3) according to

$$\eta=\frac{m_\varphi{}^{-2}+m_\omega{}^{-2}-2m_{K^*}{}^{-2}}{m_\rho{}^{-2}-m_{K^*}{}^{-2}}\approx 0.16 \quad (7.6)$$

and

$$\Omega=\frac{3}{2}\,\frac{m_\rho{}^{-2}-m_\omega{}^{-2}}{m_\varphi{}^{-2}+m_\omega{}^{-2}-2m_{K^*}{}^{-2}}\approx 1.12\,. \quad (7.7)$$

The angle θ and these symmetry-breaking parameters are connected by

$$\tan 2\theta=2\sqrt{2}[1-\tfrac{3}{4}(3+\Omega)\eta]\,. \quad (7.8)$$

Our purpose in this section was to point out that the approximate validity of Okubo's nonet scheme cannot be taken as an indication that direct-mass-squared relations are to be used for vector mesons. With the same ease and with as good results one can formulate a nonet-symmetry model leading to inverse-mass-squared formulas.

VIII. COMPARISON WITH EXPERIMENT AND FINAL COMMENTS

With regard to the vector mesons, the consequences of the models we have studied are reflected in the values of coupling constants and mixing angles. Experiments dealing with $e^+e^-\rightarrow$ hadrons provide one of the best ways of measuring these quantities and so in this section we compare the predictions of the different models with the results from the colliding-beam experiments performed by the Orsay Storage Ring Group (OSRG).[33–35]

From the radiative widths of φ and ω the angle θ_Y can be obtained but it is much harder to get an experimental value for θ_B. OSRG lists θ_B derived from $\Gamma(\varphi\rightarrow K_SK_L)$ but, as pointed out by Gourdin,[36] this width actually gives a measure of θ instead of θ_B. The width for $\varphi\rightarrow K\bar{K}$ is given by

$$\Gamma(\varphi\rightarrow K\bar{K})=\frac{1}{12}\frac{f_Y{}^2}{4\pi}\frac{\cos^2\theta_B}{\cos^2(\theta_Y-\theta_B)}m_\varphi\left(1-\frac{4m_K{}^2}{m_\varphi{}^2}\right)^{3/2} =\frac{1}{12}\frac{f_Y{}^2}{4\pi}\frac{m_\varphi(1-4m_K{}^2/m_\varphi{}^2)^{3/2}}{\cos^2\theta_Y(1+\tan\theta_Y\tan\theta_B)^2}\,. \quad (8.1)$$

In the three types of mixing we have considered (mass, orthogonal, and current) there is a sort of universal

[33] J. E. Augustin *et al.*, Phys. Letters 28B, 503 (1969).

[34] J. E. Augustin *et al.*, Phys. Letters 28B, 508 (1969); 28B, 513 (1969); and 28B, 517 (1969); J. E. Augustin *et al.*, Nuovo Cimento Letters 2, 214 (1969); J. Haissinski, in Proceedings of the Conference on $\pi\pi$ and $K\pi$ Interactions, Argonne, 1969 (unpublished).

[35] J. Perez-y-Jorba, in *Proceedings of the Fourth International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969*, edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970).

[36] See Gourdin's lectures cited in Ref. 25.

TABLE I. Comparison of theoretical models and experiment.

	θ_Y (deg)	θ (deg)	θ_B (deg)	f_ρ^{-2}	g_ω^{-2}	g_φ^{-2}	f_Y^{-2}	Σ (keV)
ORSAY expt.	41.6±3.0	29.8±4.4	21.2±1.0	9	1.20±0.24	1.56±0.15	11.1±1.2	
(1) (OS)	34.8	28.1	22.4	9	0.65	1.34	7.92	−0.59±0.52
(2)	39.7	39.7	39.7	9	1.23	1.77	12.0	0.37±0.40
(3)	53.4	39.7	26.1	9	2.33	1.31	14.58	0.26±0.39
(4)	34.8	34.8	34.8	9	0.65	1.34	7.92	−0.59±0.52
(5)	39.8	32.6	26.2	9	1.23	1.77	12.0	0.37±0.40
(6)	49.6	34.8	22.4	9	1.35	0.98	9.28	−0.62±0.48
(7) (DMO)	47.2	39.7	32.5	9	1.17	1.01	8.75	−0.59±0.52

relation among the angles [see Eqs. (3.9) and (3.15)]

$$\tan\theta_Y \tan\theta_B = \tan^2\theta \tag{8.2}$$

and so, for all types of mixing,

$$\Gamma(\varphi \to K\bar{K}) = \frac{1}{12}\frac{f_Y^2}{4\pi} m_\varphi \left(1-\frac{4m_K^2}{m_\varphi^2}\right)^{3/2} \frac{\cos^4\theta}{\cos^2\theta_Y}$$
$$= \frac{\cos^4\theta\, m_\varphi(1-4m_K^2/m_\varphi^2)^{3/2}}{4\pi\times 48 g_\varphi^{-2}}, \tag{8.3}$$

where we have employed the usual definitions

$$g_\omega^{-2} = \tfrac{1}{4} f_Y^{-2}\sin^2\theta_Y, \quad g_\varphi^{-2} = \tfrac{1}{4} f_Y^{-2}\cos^2\theta_Y. \tag{8.4}$$

Thus from a knowledge of $\Gamma(\varphi \to K\bar{K})$ and g_φ^{-2} one can obtain θ. But there is another way of getting θ which is independent of g_φ^{-2} and that is through the cross section for $e^+e^- \to K\bar{K}$ at the φ mass,

$$\sigma_{s=m_\varphi^2}(e^+e^- \to K\bar{K}) = \frac{12\pi}{m_\varphi^2}\frac{\Gamma(\varphi \to e^+e^-)\Gamma(\varphi \to K\bar{K})}{\Gamma_\varphi^2}, \tag{8.5}$$

where one can substitute for radiative width the first of

$$\begin{aligned}\Gamma(\varphi \to e^+e^-) &= \tfrac{4}{3}\pi\alpha^2 g_\varphi^{-2} m_\varphi,\\ \Gamma(\omega \to e^+e^-) &= \tfrac{4}{3}\pi\alpha^2 g_\omega^{-2} m_\omega,\end{aligned} \tag{8.6}$$

and get

$$\sigma_{s=m_\varphi^2}(e^+e^- \to K\bar{K}) = \frac{\pi}{12}\frac{\alpha^2}{\Gamma_\varphi^2}\cos^4\theta\left(1-\frac{4m_K^2}{m_\varphi^2}\right)^{3/2}. \tag{8.7}$$

We have calculated θ from Eq. (8.3) with $\Gamma(\varphi \to K^+K^-)$ and g_φ^{-2} of Ref. 39 and also from Eq. (8.7). Both ways gave the same result which we list as the experimental θ in Table I. Let us emphasize again that from the OSRG experiments θ_B cannot be obtained independently of θ_Y and θ. Thus, we calculated θ_B from Eq. (8.2) in terms of the experimental θ and θ_Y and obtained the value listed in the table which agrees with the one in Ref. 33 but not with the newest value quoted in Ref. 35.

As can be seen from Table I, we have adopted the convention of normalizing the coupling constants to $f_\rho^{-2}=9$.

Σ in the table refers to width relations like Eq. (4.5). In any given model there is a particular combination of the radiative widths of the neutral vector mesons which theoretically should vanish. The numbers in the Σ column measure the departure of the experimental from the theoretical (zero) value of these combinations.

It has been stated[36] that the radiative widths found by OSRG favor mass mixing models. Actually, from Table I we see that the current mixing model (5) fits the radiative widths very well [as well as the mass mixing model (2)]. The real test for the type of mixing is whether θ and θ_Y are equal or not and experimentally they seem to be different, favoring current mixing. For another comparison between these models and experiment, we can plug into Eq. (8.7) the theoretical θ and experimental[35] $\Gamma_\varphi = 4.24\pm0.28$ MeV, which give $\sigma(e^+e^- \to K^+K^-) = (2.28\pm0.30)\times10^{-30}$ cm^2 for model (5) and $(1.59\pm0.21)\times10^{-30}$ cm^2 for model (2), as compared with the experimental $(2.53\pm0.16)\times10^{-30}$ cm^2.[35] Had we used for Γ_φ the world average[37] 3.9 ± 0.4 MeV we would have obtained $(2.70\pm0.54)\times10^{-30}$ cm^2 for model (5) and $(1.88\pm0.38)\times10^{-30}$ cm^2 for model (2).

Model (5), which comes closest to the OSRG experimental results, is a current mixing model which agrees with our findings in Sec. IV that only the current mixing is consistent with Gell-Mann's algebra.

On the other hand, the octet breaking in model (5) originates in the mass matrix, with the consequence that some current commutators are of the form (6.11). These conclusions are, of course, valid in the framework of gauge-field models. Then, in a sense, experiments like the one by OSRG test the combination of gauge-field algebra and vector-meson–dominance hypothesis. It would be very interesting to see whether further (higher-precision) experiments still favor a model like (5) over the Oakes-Sakurai model (1) or Das-Mathur-Okubo model (7), which as we have shown in Sec. VI should be preferred on theoretical grounds. Note for instance that the largest discrepancy between the Oakes-Sakurai model (1) and the experimental results is in the radiative decay of ω, for which the world average is reported with an error as large as 27%.[37]

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ACKNOWLEDGMENTS

The author wishes to express his deep appreciation to Professor J. J. Sakurai for many helpful suggestions and continued guidance. Conversations with Professor Y. Nambu are also gratefully acknowledged.

APPENDIX: NOTATION FOR VECTOR-MESON DOMINANCE

Operator relations like $\mathbf{j}_\mu = RS\mathbf{\Phi}_\mu$ used in the text are rigorously valid in the sense

$$\langle 0|\, j^\alpha{}_\mu(x)\,|\,\mathrm{v.m.}\rangle = (RS)_{\alpha\beta}\langle 0|\,\Phi^\beta{}_\mu(x)\,|\,\mathrm{v.m.}\rangle\,,$$

where $|\mathrm{v.m.}\rangle$ is a vector-meson state. For the elements of $RS = g_D{}^{-1}TM^2$ [Eq. (3.1)], we employ the notation

$$\langle 0|\, j^3{}_\mu(0)\,|\,\rho^0\rangle = m_\rho{}^2 f_\rho{}^{-1}\epsilon_\mu\,,$$
$$\langle 0|\, j^6{}_\mu(0)\,|\,(\sqrt{\tfrac{1}{2}})(K^{*0}+\bar{K}^{*0})\rangle = m_{K^*}{}^2 f_{K^*}{}^{-1}\epsilon_\mu\,,$$
$$\langle 0|\, j^8{}_\mu(0)\,|\,\varphi\rangle = \tfrac{1}{2}\sqrt{3}\langle 0|\, j^Y{}_\mu(0)\,|\,\varphi\rangle = \tfrac{1}{2}\sqrt{3}m_\varphi{}^2 f_Y{}^{-1}\cos\theta_Y\ \epsilon_\mu\,,$$
$$\langle 0|\, j^8{}_\mu(0)\,|\,\omega\rangle = \tfrac{1}{2}\sqrt{3}\langle 0|\, j^Y{}_\mu(0)\,|\,\omega\rangle = -\tfrac{1}{2}\sqrt{3}m_\omega{}^2 f_Y{}^{-1}\sin\theta_Y\ \epsilon_\mu\,,$$
$$\langle 0|\, j^0{}_\mu(0)\,|\,\varphi\rangle = \sqrt{\tfrac{3}{2}}\langle 0|\, j^B{}_\mu(0)\,|\,\varphi\rangle = \sqrt{\tfrac{3}{2}}m_\varphi{}^2 f_B{}^{-1}\sin\theta_B\ \epsilon_\mu\,,$$
$$\langle 0|\, j^0{}_\mu(0)\,|\,\omega\rangle = (\sqrt{\tfrac{3}{2}})\langle 0|\, j^B{}_\mu(0)\,|\,\omega\rangle = (\sqrt{\tfrac{3}{2}})m_\omega{}^2 f_B{}^{-1}\cos\theta_B\ \epsilon_\mu\,,$$

where ϵ_μ is the polarization vector of the corresponding vector meson; $j^Y{}_\mu$ and $j^B{}_\mu$ are the hypercharge and baryon-number currents.

PHYSICAL REVIEW D VOLUME 3, NUMBER 9 1 MAY 1971

Consistency of Hard-Pion Theorems in K_{l3} Decays*

ROBERT OLSHANSKY AND KYUNGSIK KANG

Physics Department, Brown University, Providence, Rhode Island 02912

(Received 30 November 1970)

The K_{l3} scalar form factor is studied by the hard-pion method in the framework of a $(3,3^*)+(3^*,3)$-symmetry-breaking model using a modified pole-dominance approximation. A set of consistency relations is found which provides a test of the reliability of the quadratic-smoothness assumption as well as of the symmetry-breaking model. In particular, we find that the solution for the symmetry-breaking parameters which fit the data is inconsistent with the quadratic-smoothness assumption. In addition, the status of other theoretical models is briefly reviewed.

I. INTRODUCTION

THE semileptonic decays of the K meson have been the subject of much attention[1] both because of their accessibility to experiment and because they provide a simple process for testing the ideas of current algebra, pole dominance, and symmetry breaking. Owing to some experimental uncertainties concerning the determination of the parameter $\xi(0)$ as well as uncertainty about the existence of the κ meson,[2,3] an adequate theoretical understanding of these decays has not yet been achieved.

In a recent survey of the experimental situation, Gaillard and Chounet[1] have found that a world average on K^+ decays gives $\xi(0) = -0.85\pm 0.20$. Should this value for $\xi(0)$ survive additional measurements, a discrepancy will exist between it and the predictions of the conventional theoretical models.[4]

In Sec. II, the results of these models are briefly reviewed. It is argued that within a dispersion-theory approach the experimental result for $\xi(0)$ implies that the idea of κ dominance must be modified. The predictions of current-algebra calculations are dependent on the pole-dominance assumptions for current divergences as well as the type of $SU(3)\times SU(3)$-symmetry-breaking interactions that are assumed.[5] Again the most straightforward approach based on pole-dominance approximations gives results in contradiction with the experimental value of $\xi(0)$.

Accordingly in Sec. III, we carry out a study of the hard-pion method[6] which provides a means for including

* Supported in part by the U. S. Atomic Energy Commission (Report No. NYO-2262TA-233).

[1] See, e.g., M. K. Gaillard and L. M. Chounet, CERN Report No. 70-14 (unpublished); M. K. Gaillard and L. M. Chounet, Phys. Letters **32B**, 505 (1970).

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