we first normalize the experimental differential cross section to the pole equation modified by appropriate form factors instead of to the pole equation alone; (b) in the analysis of the data for reaction (1a) we allow for nonvanishing contributions of the experimental differential cross section (e.g., " $t\sigma$ ") at $t=0$ in our fitting procedure. The results of using several t-dependent extrapolation functions to fit the data of reaction (1a) are somewhat ambiguous in that generally good fits are obtained to the "to" points, but the extrapolated $\pi^+\pi^-$ elastic scattering cross sections differ somewhat at the ρ -mass peak. Clearly an order-of-magnitude increase in the number of available $\pi^- p \rightarrow \pi^- \pi^+ n$ events is necessary in order to accurately determine the a and c parameters in the $a+bt+ct^2$ fits to "to" points which are evidently required for a more precise extrapolation.

We find, additionally, that the extrapolated $\pi^+\pi^$ elastic scattering cross sections obtained using each extrapolation function are similar for each c.m. energy region. The c.m.-energy-averaged results of reaction (1b) and the $a+bt$ fit results of reaction (1a) are consistent with each other, thus serving to verify the factorization hypothesis implicit in Eqs. (4) as well as the utility of the extrapolation procedure. Our crosssection results are also consistent with those values obtained from the plane-wave expansion for σ [Eq. (8)], which utilizes published values for the s-wave phase shifts.

ACKNOWLEDGMENT

We thank Professor Peter E. Schlein for helpful suggestions.

PHYSICAL REVIEW D

VOLUME 3, NUMBER 9

1 MAY 1971

Evidence for the Internal Structure of Hadrons Obtained from Multipion Production*

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The striking asymmetry observed in the longitudinal momentum distribution of produced pions essentially disappears when production is viewed in a coordinate system where the incident proton has a momentum equal to $\frac{3}{2}$ that of the incident pion. Interpretation of this result in the framework of a quark model appears to suggest some simple production rules as well as several new experiments.

N this paper we report on multipion production data I from 25-GeV/c $\pi^{-}p$ collisions in the 80-in. BNL hydrogen bubble chamber. Various general features of these data together with experimental details have been discussed in several sources previously.^{1,2}

The particular feature which we would like to call attention to here can best be seen by first studying the distribution of the longitudinal momentum p_L of pions in the $\pi^{-}p$ center-of-mass system. In Fig. 1 we have plotted the momentum component along the beam direction for each negative track from a large sample of events. Events containing strange particles with detectable decay modes have been eliminated, so that the negative charge ensures a rather pure sample of pions. There have been no kinematic constraints applied to any of the data plotted in this paper.

ration of Wis-
consin, 1970 (unpublished).
² J. W. Waters, W. D. Walker, A. R. Erwin, and J. W. Elbert,
Nucl. Phys. **B17**, 445 (1970); J. W. Waters, University of
Wisconsin Ph.D. thesis (unpublished).

Two obvious features of the plot are the elastic peak at¹3.5. GeV/ c and the gross asymmetry of the distribution about $p_L=0$. Because it is well studied, the elastic peak has been subtracted out, leaving the dashed histogram in the figure. The asymmetry persists in the

FIG. 1. Center-of-mass longitudinal momentum (c.m.s. p_L) distribution for all negative pions coming out of a $\pi^- p$ collision.
Subtracting out the elastic events results in the dashed histogram.

Example 1989
 EXECUTE: Work supported in part by the U.S. Atomic Energy Commission under Contract Nos. AT (11-1)-881 and COO-881-289.
 I, W. Elbert, A. R. Erwin, S. Mikamo, D. Reeder, Y. Y. Chen, W. D. Walker, and A Proceedings of the International Conference on Expectations for Particle Reactions at the New Accelerators, University of Wis-

remaining inelastic events. As is generally done, we conclude from this that many events have a "leading" π^- which remembers in some way the momentum vector of the incident π^- .

We would like to distinguish as much as possible between the "leading" pions and the "produced" pions. As a first approximation we make the assumption that π^- mesons with negative p_L are all representative of produced pions in the backward hemisphere. We cannot safely use positive tracks for the backward hemisphere, since we might thus include a large sample of protons as a result of well-known identification problems at high energy.

However, we know experimentally that protons are unlikely to appear in the forward hemisphere for lowmultiplicity events, since for low multiplicity they can usually be identified. Although they are not as frequently identified for high-multiplicity events, we will nevertheless assume they are at least heavily outnumbered by π^{+} 's in the forward hemisphere. Thus our second approximation will be that produced pions are well represented by positive tracks for positive values of p_L .

Figure 2 shows the distribution of p_L for produced pions that results from our two assumptions. Data in both forward and backward hemispheres can be fit well to simple exponentials. There is no indication of a dip at $p_L = 0$, as would be suggested by a simple fireball model, and both hemispheres extrapolate to the same value at $p_L=0$. A few typical error bars are shown on the data points.

The most striking feature of Fig. 2 is the asymmetry about $p_L = 0$ for the produced pions. Since each component p_L also has a companion transverse component p_T , it is not obvious a priori how the p_L distribution will be affected if one does a Lorentz transformation to another system. We have nevertheless tried to reduce the asymmetry by transforming along the beam direction to another system. The parameter used to characterize a particular system is p_p/p_π , the ratio of the incident proton momentum to the incident pion

FIG. 2. Center-of-mass longitudinal momentum distribution for all produced pions. The solid lines are exponential fits to the data.

FIG. 3. χ^2 probability that forward and backward produced pions come from the same distribution in $/p_L$. The probability is plotted as a function of the ratio of incident proton momentum to pion momentum in the system for which it is calculated. The error bar indicates typical fluctuations in calculated probability as data are transformed to successive systems. We attach no particular significance to the dip in the probability.

momentum as seen in that system. This ratio is 1.0 for the center-of-mass system.

As a figure of merit for each system, we calculate a X^2 probability $p(\chi^2)$ that the data in both the backward and forward hemispheres are samples from the same distribution in $|p_L|$. The χ^2 function was calculated for $0 \leq |p_L| \leq 2.0$ GeV/c using bins 0.1 GeV/c wide by

$$
\chi^2 = \sum_{i=1}^{20} \frac{(\sigma_i^+ - \sigma_i^-)^2}{(\delta_i^+)^2 + (\delta_i^-)^2}.
$$
 (1)

The quantities σ_i^+ and σ_i^- are the cross sections for production of π^+ and π^- in the *i*th interval of $|p_L|$, respectively, and δ_i^+ and δ_i^- are the calculated statistical errors of σ_i^+ and σ_i^- , respectively.

Figure 3 shows that there is a system having $p_p/p_\pi \approx \frac{3}{2}$ in which the produced pions of the forward and the backward hemisphere both have a good probability of coming from the same distribution in $|p_L|$. The dis-

FIG. 4. Longitudinal momentum distribution of produced pions in the quark-quark center-of-mass system.

FIG. 5. Plots of the average transverse momentum \bar{p}_T for produced pions as a function of the longitudinal momentum p_L in three different reference frames. $P(\chi^2)$ is the χ^2 probability that the distribution is symmetric about $p_L=0$.

tribution, in fact, is still exponential as can be seen in the semilogarithmic plot of Fig. 4. We have designate the system having $p_p/p_{\pi} = \frac{3}{2}$ as the Q system and labeled the abscissa p_z to distinguish it from the p_L usually used for longitudinal momentum in the center-of-mass system.

Since transverse momentum is also required for the complete description of a particle and is even expected to be independent of p_L in simple statistical models, it probably is worth noting that there is some indication of a Q system in the behavior of p_T . The effect is not as pronounced as the one exhibited by p_L alone, but it is nevertheless significant. Figure 5 shows plots of the average transverse momentum \bar{p}_T as a function of p_L . Such plots are generally shown for the center-of-mass system. Statistics on the average value of p_T are invari-

FIG. 6. Typical diagrams to show how the quark picture might be used to explain leading particles. (a) A leading π^- and neutron. be used to explain leading particles. (a) A leading π and neutron.
(b) A leading π and proton. n^+, n^- , and n^0 are, respectively, the
number of produced positive, negative, and neutral pions.

ably good near $p_L=0$, and one usually sees a dip near that point. We note in Fig. 5 that the dip becomes symmetric about $p_L=0$ for produced pions when we transform to a system with $p_p/p_\pi \approx \frac{3}{2}$. Thus both p_T and p_L have some symmetry in the Q system.

The Q system is, of course, the system suggested by the triplet³ quark model. In that system one speculates that all five incident quarks have the same average value of $|p_L|$. Thus the Q system is the quark-quar center-of-mass system for any quark collisions that take place. The relevance of similar systems has been pointed place. The relevance of similar systems has been pointed out previously by other authors^{4,5} in slightly different contexts.

To extract more information from the data it will be useful to consider diagrams of a type suggested by Satz.⁴ Two typical diagrams are illustrated in Fig. 6. We will interpret these to depict multipion production as originating from the collision of two quarks, one from the pion and one from the proton, in an impulse approximation. The other three quarks continue on as spectators. The colliding quarks are "picked up" again after the collision by the spectators to form the leading particles. Such a picture conveniently explains the necessity for distinguishing a leading pion from a produced one.

The asymmetry caused by the leading π^- is shown again in the histogram of forward hemisphere π^{-2} s plotted in Fig. 7. Since the data are shown in the ^Q system, we may easily subtract out the produced pion distribution in p_z as represented by the dashed exponential. This leaves the distribution of longitudinal

FIG. 7. Histogram of the forward-hemisphere longitudinal momentum distribution in inelastic events for all negative pions in the quark-quark center-of-mass system. The dashed curve is the fitted distribution for produced pions.

' M. Gell-Mann, Phys. Letters 8, 214 (1964); G. Zweig, CERN Report Nos. Th 401 and 412, 1964 (unpublished). 4H. Satz, Phys. Rev. Letters 19, 1453 (1967); Phys. Letters

2SB, 220 (1967). '

¹⁶ P. B. James and H. D. D. Watson, Phys. Rev. Letters 18, 179 (1967); J. Benecke, T. T. Chou, C. N. Yang, and E. Yen, Phys. Rev. 188, 2159 (1969).

momenta for leading pions as seen in the Q system and plotted in Fig. 8.

The first conclusion to be drawn from Fig. 8 is that the inelastic cross section is almost independent of the longitudinal momentum transfer to the leading pion in the Q system. A similar conclusion has been noted by others^{6,7} for the leading proton in the p - p center-of-mas system. For p - p collisions the center-of-mass system is identical with the Q system.

The second feature to note is that the elastic scattering (dashed histogram) is a highly favored pickup process. This is to be expected when the momentum of the picked-up quark overlaps the Fermi momentum distribution of the parent hadron. With a knowledge of quark scattering amplitudes and Fermi momentum distributions for the proton and pion quarks, one might even be able to calculate $d\sigma/dt$ for elastic scattering in a somewhat more expanded model.

In contrast to the elastic events, the inelastic events seem to obey a less stringent set of rules for pickup. The reasoning is as follows: The average p_z value of the leading negative pion for the inelastic events plotted in Fig. 8 is $1.30 \text{ GeV}/c$. (Note that our operation for defining the lending pion does not allow much probability for a negative p_z .) The average momentum per quark of the initial state in the Q system for our beam momentum is $1.38 \text{ GeV}/c$. Thus, the leading pion appears to retain, on the average, only the momentum of its spectator quark. In other words, the quark which is picked up by the spectator quark to form the lending

FtG. 8. Longitudinal momentum distribution for the leading pion in the quark-quark center-of-mass system. The dashed histogram shows the location and relative size of the elastic peak.

Fio. 9. Probability of backward-forward symmetry for produced pions of each charge multiplicity as a function of the system of observation. Error bars indicates typical fluctuations of the probability as the system is changed. The statistical significance of the 2-, 4-, and 12-prong samples is considerably less than the total sample used in Fig. 3.

pion carries, on the average, a net momentum of about 0.0 GeV/ c in the Q system.

Although we cannot generally observe the leading proton in this experiment, we can infer its average behavior from momentum conservation. Owing to the apparent symmetry of the produced particles in the Q system, their average momentum is zero. The over-all momentum vector in the Q system is 1.38 GeV/c in the direction of the incident proton. To balance momentum after the collision, the leading proton must have $1.38+1.30=2.68$ GeV/c on the average, approximately the momentum of two incoming quarks. This agrees with the assumption that the two spectator quarks in the nucleon neither gain nor lose momentum on the average by the addition of the pickup quark.

It seems that the leading nucleon does not quite have a uniform $d\sigma/dp_z$ in the interval $0 \ge p_z \ge -4.14 \text{ GeV}/c$, where $-4.14 \text{ GeV}/c$ is the kinematic limit. This is apparent because the estimated average of p_z for the nucleon is -2.68 GeV/c, which is significantly more negative than -2.07 GeV/c, the average which would result from a uniform distribution in the specified interval.

In the spirit of the model, one might explain the zero average momentum of the picked-up quark by assuming that quarks coming from inelastic quark-quark collisions are symmetric in the forward and backward direction. Then such an average momentum might come about at present beam energies if the pickup process is relatively insensitive to momentum, accepting negative and positive quark momenta with almost equal probability.

Another simple feature seems to emerge from a study of Figs. 7 and 8. If one assumes each event has only one leading pion, the total cross section for inelastic events with leading *negative* pions is estimated to be 8.13 ± 0.24 mb. This is $\frac{1}{2}$ the total inelastic cross section of 16.8 ± 0.3

⁶ E. W. Anderson, E. J. Blesser, G. B. Collins, T. Fujii, J. Menes, F. Turkot, R. A. Carrigan, R. M. Edelstein, N. C. Hien, T. J. McMahon, and I. Nadelhaft, Phys. Rev. Letters 19, 198 (1967).

⁷ M. A. Abolins, G. A. Smith, Z. Ming Ma, E. Gellert, and A. 8. Wicklund, Phys. Rev. Letters 25, ¹²⁶ (1970).

mb for events without strange particles. The implication would seem to be that $\frac{1}{2}$ of the inelastic events have leading *neutral* particles, which is consistent with the statistical assumpton that the leading particle occurs with equal probability in each of the possible charge states (negative or neutral) allowed to it in this model. Since the proton is difficult to identify for high multiplicity events in a bubble chamber, we are unable in this experiment to observe what fraction of the leading baryons is charged.

The preceding discussion is evidence in support of a quark model for the pion production processes. It tacitly assumes that the quark pickup process predominantly yields leading pions and nucleons rather than meson resonances and isobars. A simple extension of the model would be to assume that events of each charge multiplicity have a symmetrical p_z distribution for produced particles in the Q system. Figure 9 shows the result of an effort to find a symmetrical system for produced pions for each charge multiplicity. Larger samples of 6-, 8-, and 10-prong events were used for this search than the one shown in Fig. 3. As a result it is only the remaining multiplicities that begin to suffer from insufficient statistics when the data are divided according to multiplicity.

If one takes the \bar{x}^2 probability seriously, a systematic trend seems to be evident. Four-prong events are symmetrical in a system with $p_p/p_\pi \approx 2.0$. Higher multiplicities favor successively lower values of this ratio.

It is not difficult to see how this behavior might have been anticipated within the framework of the model. Obviously if one only considered elastic two-prong events, the ratio of p_p/p_π necessary to produce a symmetric π distribution in p_L would be much greater than $\frac{3}{2}$. One can imagine that for elastic events this comes about, as mentioned earlier, because of the severe restrictions on the overlap of quark momentum distributions, limited two-body phase space, and over-all conservation of energy and momentum. Elastic events, however, have not been included in our searches because we did not use events with forward π ^{-'}s.

Other two-body processes such as production of ρN , ρN^* , and A_2N have been included in the symmetry searches. These two-body processes are most strongly represented in the inelastic 2-prong and 4-prong events. They are also events with no produced pions at the quark vertex. Their broader mass widths make somewhat more phase space available for energy and momentum conservation than for the elastic events, but one should still find that the presence of the leading resonances requires a large p_p/p_π to symmetrize the p_L distributions. Although our data may seem to suggest that leading resonances are not as favored by the pickup process as leading pions and nucleons, selection of 2- and 4-prong events tends to contaminate the sample of produced pions with decay products from the leading resonances that do exist.

As one considers higher charge multiplicities, two effects are likely to appear which can be expected to reduce the ratio of p_p/p_π towards 1.0. The most obvious is phase space. When the multiplicity approaches the kinematic limit sufficiently, the only system where p_L can be symmetric is the center-of-mass system which has $p_p/p_{\pi} = 1.0$. Invariant phase-space restrictions begin to become important when there are as few as 12 finalstate particles.⁸ Thus final states with 8 charged prongs might reflect these restrictions.

The other effect is multiple quark scattering in a single $\pi\phi$ collision. This possibility has been considered in the literature' to explain breaks in elastic scattering at large t and finite cross sections for the forbidden "exotic" exchanges. These studies would lead us to expect that when the cross section for a high multiplicity is less than a few percent of the most dominant multiplicity, we may find a significant contamination of events in which the exchanged quark has scattered more than once. If pions are produced in the first quark scatter, then the second scatter occurs at less than full energy, reducing the ratio p_p/p_π needed for the symmetry of these events.

If this simple quark model is in any way a valid description, the following picture should begin to emerge as the incident beam energy increases. A number of the higher multiplicities will begin to converge on a particular value of p_p/p_π , say $\frac{3}{2}$, which yields a symmetry for the produced pions. The value of p_p/p_π for which this occurs should be crucial in determining the exact quark composition of hadrons in nature.

The admissibility of $p_p/p_\pi \approx 2.0$ for the 4-prong events leaves open some possibility for a quartet quark
model.¹⁰ One such model constructs baryons containing model.¹⁰ One such model constructs baryons containin four quarks, e.g., three triplet quarks plus one single quark. Mesons can be constructed as usual from a triplet plus an antitriplet quark. In such a model the Q system would have $p_p/p_{\pi} = 2.0$.

An attractive feature of one of the quartet models is that it is the only one of the four parton models which can give the correct sign for the neutron-proton mass
difference.¹¹ Except for the fact that the quarks have difference.¹¹ Except for the fact that the quarks have

⁸ R. Honecker, B. Junkmann, R. Schulte, R. Steinberg, Tsanos, J. Klugow, S. Nowak, E. Ryseck, M. Walter, K. Böckmann, H. Drevermann, K. Sternberger, B. Wagini, W. Johnssen, H. Böttcher, V. T. Cocconi, J. D. Hansen, G. Kellner, D. R. O.
Morrison, K. Paler, A. Mihul, V. Moskale observed p_T for pions with phase-space predictions as a function of multiplicity.

⁹ E. Schrauner, L. Benofy, and D. W. Cho, Phys. Rev. 177, 2590 (1969); 181, 1930 (1969); N. W. Dean, Nucl. Phys. **B4**, 534 (1968); D. R. Harrington and A. Pagnamenta, Phys. Rev. Letters 18, 1147 (1967); D. Rarrington and

^{&#}x27;0 For a review of quark models, see T. D. Lee, Xuovo Cimento 35, 933 (1965).

ⁱ¹ Taizo Muto, Progr. Theoret. Phys. (Kyoto) 44, 1022 (1970).

integer charge, many other aspects of the familiar triplet model would appear to remain unchanged. It may, therefore, be important to understand the Q -system dependence on multiplicity as a means of selecting the correct parton model.

It is not difficult to suggest better experiments for investigating this model than the one presented here. An experiment using π^+p collisions could assume all negative particles are produced pions in both hemispheres.¹² A K^+p experiment would be still better, for spheres.¹² A $K^+\rho$ experiment would be still better, for one could identify uniquely the leading particle, assuming the K^+ could be identified. In addition, the neutral leading particles can be observed easily through K_1^0 decay modes.

Experiments designed to search for leading π^{0} 's, η^{0} 's, and ρ 's may provide information on selection rules for the quark pickup process. With sufficient statistics it should be possible to learn to what extent the production of strange particles in the final state might call for modifications of the model. Any experiments that permit identification of the leading particles can pose a

wide variety of questions which recognize the three distinct subsystems in the final state.

In seeking new insights into hadronic structure, one might consider repeating the present experiment for $p\bar{p}$ collisions. The p_z distribution of the leading particles may well be sensitive to any special conditions or structure that lead preferentially to annihilation rather than the usual final-state pickup. For example, one might find that events with small p_z and large longitudinal momentum transfer preferentially annihilate, giving very few events with leading particles near $p_z = 0$.

We realize that the specific analysis of our data in terms of a quark model is highly speculative. However, we believe the data do indicate that a very real difference between the pion and proton structure is easily seen in multiparticle production. The particular analysis used in this paper is a sensitive method for demonstrating this difference in a way which is completely independent of the quark mnemonic used to introduce it. Because two incident hadrons are involved, the interpretation may be more difficult, but the problem is free of the complications of the radiative corrections to be found in deep-inelastic electron scattering experiments.

We would like to acknowledge many useful discussions of this experiment with our colleagues, especially Dr. George Collins and Dr. Lubbo von Lindern.

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Study of the Two-Charged-Particle Final States of 3.9-GeV/ $c \pi^{\pm}p$ Interactions Including a Longitudinal-Momentum Analysis of the One-Pion-Production Channels

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> We have analyzed the two-prong final states in π^+p interactions at 3.9 GeV/c. Our result for elastic scattering is σ (elastic) = 6.50 \pm 0.1 mb (statistical error only). We find the elastic slope to be 6.61 \pm 0.14 (GeV/c)⁻². We find the elastic forward cross section to be 40.0 ± 1.4 mb/(GeV/c)². We have applied a longitudinalmomentum analysis to the one-pion-production channel. We find the cross section for the reaction $\pi^+ + \rho \rightarrow$ $\pi^++\pi^0+p$ to be 2.30 \pm 0.06 mb and that for $\pi^++p \rightarrow \pi^++\pi^++n$ to be 1.45 \pm 0.05 mb. For resonanceproduction cross sections in these channels we find $\Delta(1236)=0.60\pm0.07$ mb, $\rho(760)=0.86\pm0.06$ mb, and diffraction dissociation = 1.69 ± 0.11 mb. We find that we can satisfactorily fit all distributions in the onepion-production channel without assuming any phase-space production. In the missing-mass channel we observe dominant Δ^{++} (1236) production plus evidence for A_2^+ production.

I. PRELIMINARY DISCUSSION

'N this paper we discuss the analysis of the final states I that have two charged particles in $\pi^+ p$ interactions at 3.9 GeV/ c . In particular, we present our results for

* Work supported in part through funds provided by the U. S. Atomic Energy Commission under Contract No. AT(30-1)2098.

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the elastic-scattering process, the one-pion-production process and those interactions where more than one neutral particle is produced. We also describe a new technique used to analyze the one-pion-production channels.

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¹² In $\pi^+\rho$ interactions at 18.5 GeV/c the π^- distribution shows the same qualitative asymmetry we present in Fig. 2. See N. N.
Biswas, N. M. Cason, M. S. Farber, V. P. Kenney, J. S. Poirier,
J. T. Powers, O. R. Sander, and W. D. Shephard, in *Proceedings of* the Fifteenth International Conference on High-Energy Physics, Kiev, 1970 (Academy of Science, IUPAP, Moscow, 1970).