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Measurement of $T=2$ Elastic $\pi\pi$ Cross Sections*

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Experimental differential cross sections from data on the reaction $\pi^-p \to (\pi^-\pi^-)\Delta^{++}$ at beam momenta of 2.7, 3.0, 3.2, 3.9, and 4.2 GeV/c have been extrapolated to the one-pion-exchange pole to obtain the $\pi^-\pi^-$ elastic scattering cross section. An attempt is made to correct for background due to kinematic overlap with the competing one-pion-exchange process $\pi^-p \to \rho^0(\pi^-p)$. Analyses done independently on the data in two beam-momentum groupings at ~3.0 and ~4.0 GeV/c give consistent results of a roughly constant 7–11-mb cross section for $\pi^-\pi^- \to \pi^-\pi^-$ over the dipion mass range 440–750 MeV. Our results are compared with available results from other analyses and with several theoretical predictions for the $T=2$ s-wave phase shift δ_0^2.

IN this paper we present a determination of the $\pi^-\pi^-$ elastic scattering cross section by means of a modified Chew-Low[1] extrapolation to the pion-exchange pole in the reaction

$$\pi^-p \to (\pi^-\pi^-)\Delta^{++}(1238) \tag{1}$$

for $\pi^-\pi^-$ mass below 0.75 GeV.

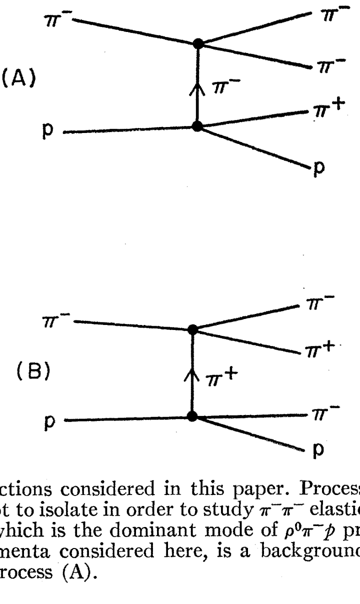

FIG. 1. Reactions considered in this paper. Process (A) is the one we attempt to isolate in order to study $\pi^-\pi^-$ elastic scattering. Process (B), which is the dominant mode of $\rho^0\pi^-p$ production at the beam momenta considered here, is a background source for the study of process (A).

It is thereby assumed that the process depicted in Fig. 1(A) [referred to hereafter as process (A)] plays a significant role in the over-all reaction. Since finite statistics, background, and other exchanges (e.g., A_2) complicate the analysis of the data for reaction (1), an appropriate extrapolation procedure must be carried out in order to isolate and determine the magnitude of the $\pi^-\pi^-$ elastic scattering cross section. The extrapolation procedure which we use has been shown[2] to successfully yield π^+p elastic scattering cross sections in the region of the $\Delta^{++}(1238)$ resonance from peripheral data on the reaction $pp \to (p\pi^+)n$ at 6.6-GeV/c incident laboratory beam momentum. The method first involves normalizing the experimental differential cross section ($d\sigma/dt$) at fixed $\pi^-\pi^-$ effective mass to a function which behaves similarly with t, but which reduces to the required value at the pion-exchange pole. The normalized data are then fitted with a low-order polynominal in t. The pole value of the polynomial with best-fit parameters is the $\pi^-\pi^-$ elastic scattering cross section. The procedure does not require extrapolation of quantities with poles. In addition, background and other exchanges are allowed for in the extrapolations. These components are assumed to vanish at the pion-exchange pole.

The data for reaction (1) have been obtained from a total of 21 941 events of the type

$$\pi^-p \to \pi^-\pi^-\pi^+p \tag{2}$$

having beam momenta from 2.7 to 4.2 GeV/c. The data were separated into two sets: 10733 events at 2.7, 3.0,

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[1] G. F. Chew and F. E. Low. Phys. Rev. 113, 1640 (1959).
[2] Z. Ming Ma *et al.*, Phys. Rev. Letters 23, 342 (1969).

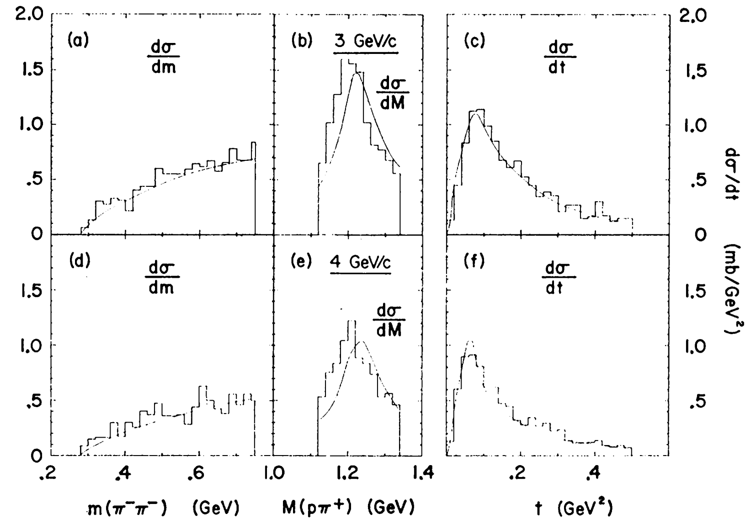

FIG. 2. Experimental differential cross sections: (a) $d\sigma/dm$, where m is $\pi^-\pi^-$ mass; (b) $d\sigma/dM$, where M is π^+p mass; (c) $d\sigma/dt$, where t is the momentum transfer to the π^+p system for the data at 3 GeV/c. (d)–(f) are the same for the data at 4 GeV/c. The smooth curves drawn are the predictions of the strict Dürr-Pilkuhn OPE model described in the text.

and 3.2 GeV/c (Refs. 3, 4, and 5, respectively; referred to hereafter as the 3-GeV/c data) and 11 168 events at 3.9 and 4.2 GeV/c (Refs. 6 and 5, respectively; referred to hereafter as the 4-GeV/c data). Similar investigations were independently performed on each set.

The peripheral sample of data corresponding to reaction (1) is obtained by applying the simultaneous cuts:

$$\begin{aligned} m&<0.75\ \text{GeV},\\ 1.12<M&<1.34\ \text{GeV},\\ t&<0.5\ \text{GeV}^2, \end{aligned} \tag{3}$$

where t is the square of the momentum transfer from the target proton to the outgoing π^+p system (we take t to be positive in the physical region), and m (M) denotes the $\pi^-\pi^-$ (π^+p) invariant mass. We restrict the discussion henceforth to those events which satisfy the restrictions of Eqs. (3): 1334 at 3 GeV/c and 978 at 4 GeV/c. The differential distributions $d\sigma/dm$, $d\sigma/dM$, and $d\sigma/dt$ are presented separately for the 3- and 4-GeV/c data in Figs. 2(a)–2(f). The smooth curves drawn through the data in Fig. 2 are the results of a one-pion-exchange-model calculation described below which includes the effect of background due to the process shown in Fig. 1(B) [referred to hereafter as process (B)].

The experimental $d\sigma/dt$ for reaction (1) have been extrapolated to the one-pion-exchange (OPE) pole for four different $\pi^-\pi^-$ mass regions at each beam momentum following the procedure of Ma *et al.*[2] This procedure differs from the traditional Chew-Low procedure in that $(d\sigma/dt)_{\text{expt}}$ is normalized to the pole equation[7] modified by a form factor $F(m,M,t)$:

$$\frac{d^3\sigma}{dt\,dm\,dM}=\frac{1}{4\pi^3 m_p{}^2 P_L{}^2}\frac{1}{(t+\mu^2)^2}\frac{m^2 q(m)\sigma(m)}{(\hbar c)^2}\times M^2 Q(M)\sigma(M)\,F(m,M,t)\,, \tag{4}$$

instead of to the pole equation alone. In Eq. (4), μ (m_p) is the pion (proton) rest mass, P_L is the laboratory beam momentum, and q (Q) are the magnitudes of the momenta in the $\pi^-\pi^-$ (π^+p) rest frames. The σ functions are the on-mass-shell vertex elastic scattering cross sections. All quantities have the units GeV or mb, except F which is dimensionless.

The form factor F can be any smooth function which reduces to unity at the pion-exchange pole. We use the phenomenological Dürr-Pilhuhn (DP)[8] or Benecke-Dürr[9] factors which have been shown[10–12] to summarize approximately the experimental Chew-Low distributions for strong-interaction reactions of the classes $Xp\to X\pi^+n$ and $Xp\to X\pi^-\Delta^{++}$ over a large range of beam momenta. The usefulness of this procedure lies in the fact that the complexity of the t dependence of the function to be extrapolated is minimized, thereby decreasing the order of the polynomial necessary to fit the experimental points.

The function which we extrapolate to the pole for

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[3] P. R. Klein *et al.*, Phys. Rev. **150**, 1123 (1966).

[4] A. W. Key *et al.*, Phys. Rev. **166**, 1430 (1968).

[5] S. U. Chung, LRL Report No. UCRL-16881-Rev., 1967 (unpublished); see also S. U. Chung, O. I. Dahl, J. Kirz, and D. H. Miller, Phys. Rev. **165**, 1491 (1968).

[6] K. Abe *et al.*, Phys. Rev. Letters **22**, 251 (1969).

[7] See, e.g., E. Ferrari and F. Selleri, Nuovo Cimento Suppl. **24**, 453 (1962).

[8] H. P. Dürr and H. Pilkuhn, Nuovo Cimento **40**, 899 (1965).

[9] J. Benecke and H. P. Dürr, Nuovo Cimento **56**, 269 (1968). At the low values of t used in this analysis, these form factors are numerically almost identical to the Dürr-Pilkuhn factors.

[10] G. Wolf, Phys. Rev. Letters **19**, 925 (1967).

[11] P. Schlein, in *Meson Spectroscopy*, edited by C. Baltay and A. H. Rosenfeld (Benjamin, New York, 1968), p. 161; and in *Proceedings of the Conference on ππ and Kπ Interactions* (Argonne National Laboratory, Argonne, Ill., 1969), pp. 1 and 446.

[12] G. Wolf, Phys. Rev. **182**, 1538 (1969).

TABLE I. Results of fits of the experimental solid "σ" points shown in Fig. 3 to the assumed forms "σ" $= a+bt$.

Data set	Fit quantities	$\pi^-\pi^-$ mass range (GeV) 0.28–0.44	0.44–0.55	0.55–0.65	0.65–0.75
3 GeV/*c*	Con. lev. (%)	84	45	93	78
	a (mb)	3.4±2.0	6.6 ±1.8	9.7±1.9	11.9±2.1
	b (mb/GeV^2)	231.1±33.2	128.2 ±19.7	63.1±13.7	29.9±12.1
	σ_{extrap} (mb)	−1.1±2.5	4.1 ±2.2	8.4±2.1	11.3±2.3
	δ_0^2 (deg)	0[a]	−7.7 ±2.0	−14.3±1.8	−20.2±2.1
4 GeV/*c*	Con. lev. (%)	17	71	85	29
	a (mb)	4.6±2.6	6.5 ±2.0	10.7±1.8	7.0±1.8
	b (mb/GeV^2)	279.4±45.1	169.5 ±24.0	55.2±14.4	58.9±14.1
	σ_{extrap} (mb)	−0.9±3.4	3.2 ±2.3	9.6±2.0	5.9±2.1
	δ_0^2 (deg)	0[a]	−6.7 ±2.4	−15.3±1.7	−14.5±2.6
Average value of the two sets	Con. lev. (%)	50	58	89	53
	a (mb)	3.8±1.6	6.55±1.3	10.2±1.3	9.1±1.4
	b (mb/GeV^2)	248.5±26.8	145.0 ±15.2	59.5±9.9	42.3±9.2
	σ_{extrap} (mb)	−1.0±2.0	3.7 ±1.6	9.0±1.4	8.4±1.5
	δ_0^2 (deg)	0[a]	−7.3 ±1.5	−14.8±1.2	−18.0±1.6

[a] Since $\sigma_{\text{extrap}} \propto \sin^2\delta_0^2$ and σ_{extrap} was less than zero, the phase shift has been set to zero.

each specified Δm interval is

$$``\sigma" = \frac{(d\sigma/dt)_{\text{expt}}}{(d\sigma/dt)_{\text{DP-OPE}}}, \tag{5}$$

where $(d\sigma/dt)_{\text{DP-OPE}}$ is (4) after integration over m and M. The on-shell $\pi^-\pi^-$ cross section σ is set equal to 1 mb in calculating the denominator, so that at the pole $\sigma^{\pi^-\pi^-}{}_{\text{on-shell}} =$ "σ". A polynomial in t is then fitted to the experimental "σ" points. If $(d\sigma/dt)_{\text{DP-OPE}}$ had precisely the same t dependence as $(d\sigma/dt)_{\text{expt}}$, then "$\sigma$" would be independent of t. Thus the presence of linear or higher terms in the polynomial fit allows for departures of $(d\sigma/dt)_{\text{DP-OPE}}$ from $(d\sigma/dt)_{\text{expt}}$. The DP factors used in calculating $(d\sigma/dt)_{\text{DP-OPE}}$ have the form[10,11]

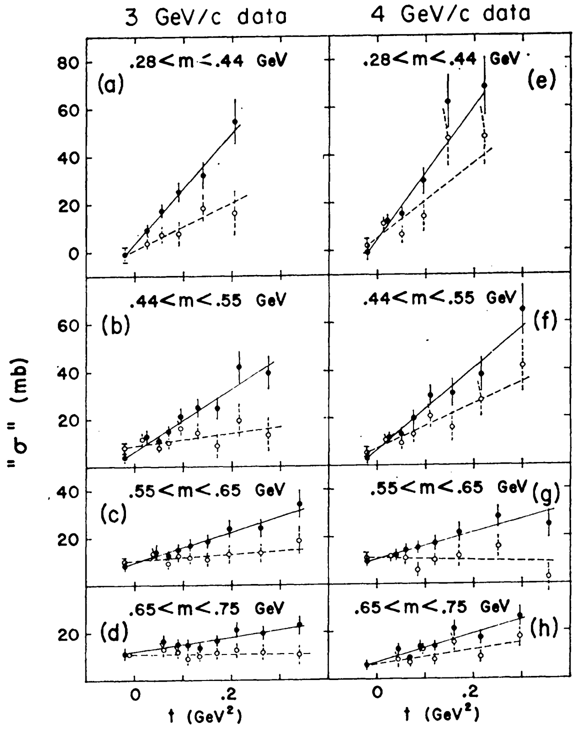

FIG. 3. Experimental "σ" values plotted as a function of t. The solid data points are calculated using Eq. (5) whereas the open-circle data points are corrected for background with the use of Eq. (8) as described in the text. The solid and dashed lines represent the best fits of the linear expression "σ" $= a+bt$ to the data. The extrapolated cross sections at the left side of each plot are the values of this best-fit function at $t = -\mu^2$.

$$F(m,M,t) = \left(\frac{2.3-\mu^2}{2.3+t}\right)^2 \left(\frac{Q_t}{Q}\right)^2 \left(\frac{1+16Q^2}{1+16Q_t^2}\right) \times \left[\frac{(M+m_p)^2+t}{(M+m_p)^2-\mu^2}\right], \tag{6}$$

where Q_t is the momentum of the incoming target proton in the Δ^{++} rest frame. No correction is made to the $\pi^-\pi^-$ vertex factor. This last assumption (valid according to DP for s-wave vertices) is in disagreement with expected off-shell effects near threshold.[13] By allowing "σ" to depend on t in the extrapolation fits, any objections to the use of this form factor near threshold are satisfied. See further comments on this point below.

The experimental "σ" values given by Eq. (5) are shown as the solid dots in Figs. 3(a)–3(d) and Figs. 3(e)–3(h) for four different dipion mass ranges in the 3- and 4-GeV/c data, respectively. The linear extrapolation function "σ" $= a+bt$, shown as the solid lines in Fig. 3, has been fitted to the data in each case. The resulting confidence levels, and the best-fit values for a and b as well as the extrapolated on-shell cross sections ($\sigma^{\pi^-\pi^-}{}_{\text{on-shell}}$) are presented in Table I. All of the fits are seen to yield acceptable confidence levels. Also

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[13] C. Lovelace, in *Proceedings of the Conference on ππ and Kπ Interactions* (Argonne National Laboratory, Argonne, Ill., 1969), p. 562; see also the Comments on p. 18.

TABLE II. Results of fits of the experimental open-circle "σ" points shown in Fig. 3 to the assumed form "σ" $=a+bt$.

		$\pi^-\pi^-$ mass range (GeV)			
Data set	Fit quantities	0.28–0.44	0.44–0.55	0.55–0.65	0.65–0.75
3 GeV/c	Con. lev. (%)	80	40	72	91
	a (mb)	1.2 ± 2.4	8.9 ± 1.9	10.4 ± 1.9	10.9 ± 2.1
	b (mb/GeV^2)	97.5 ± 36.7	27.0 ± 20.5	13.5 ± 14.0	0.4 ± 12.3
	σ_{extrap} (mb)	-0.7 ± 3.0	8.4 ± 2.2	10.1 ± 2.1	10.9 ± 2.3
	δ_0^2 (deg)	0^{a}	-11.0 ± 1.5	-15.7 ± 1.7	-19.9 ± 2.2
4 GeV/c	Con. lev. (%)	2	65	29	18
	a (mb)	5.1 ± 2.7	7.2 ± 2.0	10.9 ± 1.9	6.1 ± 2.0
	b (mb/GeV^2)	155.3 ± 47.1	91.0 ± 24.6	-5.9 ± 15.0	29.3 ± 15.0
	σ_{extrap} (mb)	2.1 ± 3.5	5.4 ± 2.4	11.1 ± 2.1	5.5 ± 2.2
	δ_0^2 (deg)	-3.3 ± 2.8	-8.7 ± 1.9	-16.5 ± 1.6	-14.0 ± 2.9
Average value of the two sets	Con. lev. (%)	41	52.5	50.5	54.5
	a (mb)	2.9 ± 1.8	8.1 ± 1.4	10.7 ± 1.4	8.4 ± 1.4
	b (mb/Gev^2)	119.5 ± 29.0	53.2 ± 15.7	4.5 ± 10.2	12.1 ± 9.5
	σ_{extrap} (mb)	0.5 ± 2.3	7.0 ± 1.6	10.6 ± 1.5	8.1 ± 1.6
	δ_0^2 (deg)	-1.3 ± 2.3	-10.1 ± 1.2	-16.1 ± 1.2	-17.8 ± 1.8

[a] Since $\sigma_{\text{extrap}}\propto\sin^2\delta_0^2$ and $\sigma_{\text{extrap}}<0$, the phase shift has been set to zero.

given in Table I are the $T=2$ s-wave $\pi\pi$ phase shifts[14] (δ_0^2) obtained from

$$\sigma^{\pi^-\pi^-}{}_{\text{on-shell}}=8\pi\lambda^2\sin^2\delta_0^2\,, \tag{7}$$

in which it is assumed that only the s-wave contribution is significant for $m<0.75$ GeV. In Eq. (7) the quantity λ^2 is calculated at the central value of the $\pi^-\pi^-$ mass bin. Since the results at both beam momenta are similar, the average cross sections for each dipion mass range are also presented in Table I.

We turn now to the subject of background[15] from process (B) and its effects upon the pole extrapolation. For events satisfying the cuts (3), we display in Fig. 4 the differential distributions of $d\sigma/dm_{\pi^-\pi^+}$, $d\sigma/dM_{\pi^-p}$, and $d\sigma/dt_{\pi^-p}$ for each beam momentum. Two combinations are plotted for each event. The curves drawn through the data in Fig. 4 are similar to those presented in Fig. 2 and are discussed below. The strong peripheral $\rho^0(765)$ component which is observed in Figs. 4(a) and 4(d) constitutes evidence for the process (B) [see Fig. 1(B)]. With the use of the DP-OPE description of this process, which has been demonstrated[10–12] to summarize rather well the Chew-Low distributions of available processes of this type, we attempt to subtract this background contribution and redo the pole extrapolation analysis described above.

Assuming only π exchange and neglecting interference terms between the competing processes, the background contribution to a $d\sigma/dt$ point is given by $\Delta\sigma_B/\Delta t$, where $\Delta\sigma_B$ is the cross-section contribution from process (B) subject to the cuts on m, M, and t, specified in Eq. (3). The width Δt is the width of the t bin in question. The calculation of $\Delta\sigma_B$ is discussed in a brief appendix to this paper. The function which is extrapolated to the pole for each specfied Δm interval is, therefore,

$$\text{“}\sigma\text{”}=\frac{(d\sigma/dt)_{\text{expt}}-\Delta\sigma_B/\Delta t}{(d\sigma/dt)_{\text{DP-OPE}}}\,. \tag{8}$$

As before, $\sigma(m)$ is set equal to 1 mb in calculating the denominator, so that at the pole $\sigma^{\pi^-\pi^-}{}_{\text{on-shell}}=$ "σ".

The experimental "σ" points, calculated using Eq. (8), are displayed in Fig. 3 as the open-circle points. The dashed lines and the open-circle extrapolated cross sections at $t=-\mu^2$ are the results of fits of "σ" $=a+bt$ to these points. The parameters and cross-section results are presented in Table II. The cross sections in the mass range 440–750 MeV are seen to have a more or less consistent value of 7–11 mb.[16]

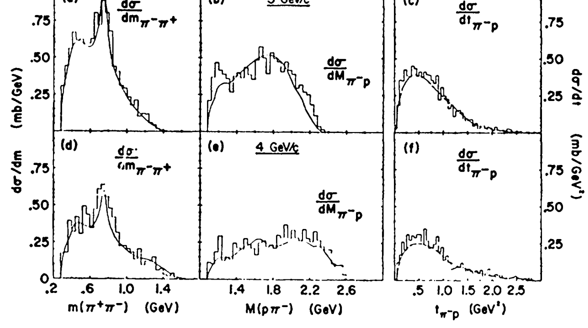

FIG. 4. Experimental differential cross sections for events satisfying the selection criteria defined in Eq. (3) for the data at 3 GeV/c: (a) $d\sigma/dm_{\pi^-\pi^+}$, (b) $d\sigma/dM_{\pi^-p}$, (c) $d\sigma/dt_{\pi^-p}$. (d)–(f) are the same for the 4-GeV/c data. Two combinations are plotted for each event. The smooth curves are the predictions of the strict Dürr-Pilkuhn OPE model described in the text.

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[14] For the sign of the phase shift see, e.g., J. P. Baton and G. Laurens, in *Proceedings of the Conference on $\pi\pi$ and $K\pi$ Interactions* (Argonne National Laboratory, Argonne, Ill., 1969), p. 131.

[15] P. G. Wohlmut *et al.*, Nucl. Phys. **B18**, 505 (1970); see also Ref. 6.

[16] We note that these numbers are in agreement with the preliminary value of ~10 mb quoted by E. Malamud and P. E. Schlein, in *Proceedings of the Conference on $\pi\pi$ and $K\pi$ Interactions* (Argonne National Laboratory, Argonne, Ill., 1969), p. 93. In the extraction of the phase shift δ_0^2 at the K mass from the cross sections, a factor of 2 was inadvertently left out in their article, causing our value of $|\delta_0^2|$ at the K mass to come out $(17\pm3)°$ instead of the value $(12\pm3)°$, which more properly goes with a $\pi^-\pi^-$ cross section of 10 mb.

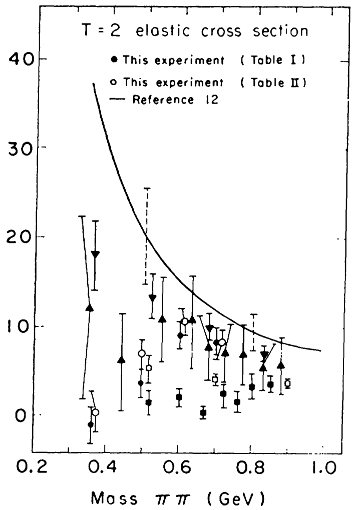

FIG. 5. Comparison between $\sigma^{\pi^-\pi^-}{}_{\text{on-shell}}$ determined in this analysis with (open-circle points) and without (solid dots) the background subtraction procedure described in the text, and other published values. The solid curve is from Ref. 12 and the symbols ■, □, ▼, and ▲ refer to data from Refs. 17–20, respectively. In those cases where δ_0^2 values only are published (Refs. 17 and 20), the corresponding cross sections are derived using Eq. (7).

The results in Table II indicate that large positive b coefficients are still required in the 4-GeV/c data for dipion mass $m<0.55$ GeV. If we assume that the background subtraction has been properly done and that interference effects are insignificant, the necessity for the nonzero b parameters in the fits to the background subtracted "σ" vs t points in this mass range indicate that DP-OPE is a poor approximation to the t distribution for $\pi\pi$ masses just above threshold, as suggested by Lovelace.[13]

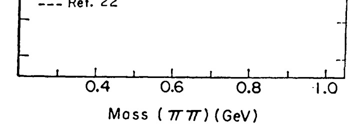

FIG. 6. Comparison between δ_0^2 determined in this analysis with (open-circle points) and without (solid dots) the background subtraction procedure described in the text, and other published values. The solid and dashed curves are theoretical predictions from Refs. 21 and 22, respectively. The symbols ■, □, ▼, and ▲ refer to data from Refs. 17–20, respectively. In the case where only $\sigma^{\pi^-\pi^-}{}_{\text{on-shell}}$ values are published (Ref. 19), the corresponding phase shifts are derived using Eq. (7).

As a means of illustrating the degree of over-all fit quality of a strict DP-OPE model to the data (*which we stress is not assumed in the actual extrapolations, where we permit the linear coefficient b to be nonzero*), we show in Figs. 2 and 4 curves calculated assuming an incoherent sum of processes (A) and (B). The contribution for process (A) is assumed given by Eqs. (4) and (6) with $\sigma(m)=10$ mb for $m<0.75$ GeV. The contribution from process (B) is described in the Appendix. The integrated theoretical cross sections [for process (A)+process (B) subject to the same experimental cuts (3)] of 0.21 and 0.15 mb for the 3- and 4-GeV/c data are to be compared with the experimental values of 0.24 ± 0.01 mb and 0.17 ± 0.01 mb, respectively. The integrated theoretical cross sections from process (A) account for 55 and 59% of the total, respectively, at the two momenta. The curves describe the data rather well in Figs. 2 and 4, especially in the m and t distributions; however, the position of the $\Delta^{++}(1238)$ is shifted to lower mass in the data and the curve does not adequately reproduce the data in the $\Delta^0(1238)$ region of the π^-p mass spectrum of the 3-GeV/c data.

In conclusion we compare our results for $\sigma^{\pi^-\pi^-}{}_{\text{on-shell}}$ with previous determinations.[12,17–20] Figure 5 shows most available values. The dashed error bars in Fig. 5 represent the uncertainties in the smooth curve at the $\pi\pi$ mass value in question. In those cases in which only δ_0^2 was given (e.g., Baton *et al.*[17]), $\sigma^{\pi^-\pi^-}{}_{\text{on-shell}}$ was calculated using Eq. (7). Similarly we present in Fig. 6 the set of related δ_0^2 values as well as the prediction (solid curve) of Arnowitt[21] from current algebra and the prediction (dashed curve) of Wagner,[22] who utilized a unitarized Veneziano formula. While our results are in good agreement with theoretical expectations, the rather large divergence of available experimental results suggests that unknown systematic uncertainties exist in many determinations. A high-statistics electronics experiment on the more background-free $\pi^+p\rightarrow\pi^+\pi^+n$ reaction at higher beam momentum should permit a more reliable determination of δ_0^2 and of the as yet unknown contributions of d-wave and higher angular momentum states.

APPENDIX

The calculation of the curves shown in Figs. 2 and 4 was performed with a Monte Carlo program.[23] The integrations were taken over the full kinematic range

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[17] J. P. Baton, G. Laurens, and J. Reignier, Nucl. Phys. **B3**, 349 (1967).

[18] W. M. Katz, T. Ferbel, P. F. Slattery, and H. Yuta, in *Proceedings of the Conference on ππ and Kπ Interactions* (Argonne National Laboratory, Argonne, Ill., 1969), p. 300.

[19] G. V. Beketov *et al.*, Institute for Theoretical and Experimental Physics Report No. ITEP-767, 1970 (unpublished).

[20] W. D. Walker *et al.*, Phys. Rev. Letters **18**, 630 (1967).

[21] R. Arnowitt, in *Proceedings of the Conference on ππ and Kπ Interactions* (Argonne National Laboratory, Argonne, Ill., 1969), p. 619.

[22] F. Wagner, Nuovo Cimento **64A**, 189 (1969).

[23] J. H. Friedman, J. Computational Phys. (to be published).

of variables subject to the cuts of Eq. (3). In order to calculate the reflection of process (B) on the histograms relevant to process (A) and vice versa, it is necessary to include information about the angular distribution in each vertex c.m. system. In all cases this was approximated by the on-shell angular distribution.[24] The π-proton angular distributions were calculated from the CERN phase-shift analysis[25] and the $\pi\pi$ angular distributions reconstructed from the phase-shift analysis of Malamud and Schlein[26] (the results are insensitive to the choice of solution) for $m<1$ GeV and for $m>1$ GeV from Wolf.[12] The DP correction at the π^-p vertices are identical to those used by Colton *et al.*[27] in an analysis of $pp \rightarrow (p\pi^-)(p\pi^+)$, in which it was demonstrated that these corrections are unnecessary for $M \geq 1.6$ GeV.

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[24] Colton *et al.* (Ref. 27) have shown in a series of papers on pp studies that this is a rather good approximation for peripherally produced π-proton systems. See also E. Colton and P. Schlein, in *Proceedings of the Conference on $\pi\pi$ and $K\pi$ Interactions* (Argonne National Laboratory, Argonne, Ill., 1969), p. 1.

[25] A. Donnachie, R. G. Kirsopp, and C. Lovelace, CERN Report No. CERN-TH 838, Addendum, 1967 (unpublished).

[26] See Ref. 16.

[27] E. Colton, P. E. Schlein, E. Gellert, and G. A. Smith, Phys. Rev. D **3**, 1063 (1971).

PHYSICAL REVIEW D VOLUME 3, NUMBER 9 1 MAY 1971

Pole Extrapolation of the Reactions $\pi^-p \rightarrow \pi^-\pi^+n$ and $\pi^+p \rightarrow \pi^+\pi^-\Delta^{++}(1238)$*

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Experimental differential cross sections from data on the reactions $\pi^-p \rightarrow \pi^-\pi^+n$ and $\pi^+p \rightarrow \pi^+\pi^-\Delta^{++}(1238)$ have been extrapolated to the one-pion-exchange pole to obtain the $\pi^+\pi^-$ elastic scattering cross section from threshold to 1.4 GeV. Consistent results are obtained in three c.m. energy ranges for both reactions. The data have been fitted to several t-dependent extrapolation functions, and the results of the fits are tabulated and plotted as a function of $\pi^+\pi^-$ effective mass. In particular, we find cross sections of approximately 25 and 125 mb, respectively, at the K and central $\rho^0(765)$ mass positions.

I. INTRODUCTION

THE proper extraction of $X\pi$ elastic scattering cross sections from data on the reactions $Xp \rightarrow X\pi^+n$ and $Xp \rightarrow X\pi^-\Delta^{++}(1238)$ [for $X=\pi$, K, p] has been the goal of many experiments since the work of Goebel[1] and Chew and Low.[1] Many analyses, which include the fits of experimental differential cross sections to various theoretical formulas as well as numerous extrapolation procedures, have either assumed or attempted to show that single-pion exchange is dominant in the above reactions in the intermediate energy region. More recently, however, Kane[2] has pointed out that other exchanges (e.g., ρ, A_2) are just as, if not more, important in the region of small momentum transfer. Thus it appears that if pion exchange is indeed present in a reaction, an appropriate extrapolation procedure must be carried out in order to isolate and determine its magnitude.

In this paper we present a determination of the $\pi^+\pi^-$ elastic scattering cross section by means of a modified Chew-Low[1] extrapolation to the pion-exchange pole in the reactions

$$\pi^-p \rightarrow (\pi^-\pi^+)n \tag{1a}$$

and

$$\pi^+p \rightarrow (\pi^+\pi^-)\Delta^{++}(1238). \tag{1b}$$

Pole-extrapolation analyses[3,4] of reaction (1a) which yield similar results have been performed previously. This analysis, however, reports the first high-statistics pole extrapolation of reaction (1b) using data at different beam energies. We show below that our results from reactions (1a) and (1b) are consistent with each other and with earlier determinations,[3,4] thus supporting the one-pion-exchange (OPE) assumption and the validity of our pole-extrapolation procedure.

We assume (for small momentum transfer) that the processes depicted in Figs. 1(a) and 1(b) play significant roles in reactions (1a) and 1(b), respectively. The extrapolation procedure which we use has been shown[5] to successfully yield π^+p elastic scattering cross sections in the region of the $\Delta^{++}(1238)$ resonance from peripheral data on the reaction $pp \rightarrow (p\pi^+)n$ at 6.6 GeV/c incident laboratory beam momentum. The method in-

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[1] C. Goebel, Phys. Rev. Letters **1**, 337 (1958); G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959).

[2] G. L. Kane, in *Experimental Meson Spectroscopy 1970*, edited by C. Baltay and A. H. Rosenfeld (Columbia U. P., New York, 1971), p. 1.

[3] S. Marateck *et al.*, Phys. Rev. Letters **21**, 1613 (1968).

[4] J. P. Baton *et al.*, Phys. Letters **33B**, 525 (1970).