perturbation theory can be removed by a counterterm originating from the invariant functional measure. The counterterm vanishes in the coordinate system uniquely defined by the condition $g=1$. The question of renormalizability is not affected by this result. The arguments against renormalizability[5] remain untouched.

PHYSICAL REVIEW D VOLUME 3, NUMBER 8 15 APRIL 1971

Closed-Loop Calculations Using a Chiral-Invariant Lagrangian: An Addendum

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The significance of the condition on the metric $\det g_{ij}=1$, found in an earlier paper to lead to the elimination of the most divergent parts of the amplitudes, is explained on the basis of a paper by Salam and Strathdee.

IN a recent paper[1] we reported the result of application of naive Feynman rules derived from the chiral-invariant Lagrangian density for zero-mass pions,

$$L=\tfrac{1}{2}\partial_\mu\phi_i g_{ij}(\phi)\partial_\mu\phi_j\,, \tag{1}$$

to closed-loop contributions to invariant amplitudes. We remarked on the presence of contributions which appeared to violate the equivalence theorem, in that they depended explicitly on the choice of pion "gauge," or Weinberg's function $f(\phi^2)$. The contributions are the most divergent parts of the amplitudes. They also fail to vanish in the soft-pion limit, in violation of Adler's condition. It was shown by explicit calculation that for the choice of gauge[2]

$$f(\phi^2)=f_\pi\left[1-\frac{2}{5}\left(\frac{\phi^2}{f_\pi^2}\right)-\frac{9}{175}\left(\frac{\phi^2}{f_\pi^2}\right)^2 -\frac{184}{15\,750}\left(\frac{\phi^2}{f_\pi^2}\right)^3+\cdots\right], \tag{2}$$

these contributions vanished, and the Adler conditions were satisfied.

In a note added in proof we remarked that the same choice of gauge leads to the condition

$$g\equiv\det g_{ij}=1 \tag{3}$$

on the metric, although we could offer no interpretation of this condition.

We have since seen a paper by Salam and Strathdee[3] in which may be found the missing explanation. Suppose that for a general interaction one starts with a generating functional given in a canonical formulation by

$$Z(I,J)=\int(d\phi)(d\pi)\times\exp\left\{i\int dx[\pi\dot\phi-H(\phi,\nabla\phi,\pi)+I\phi+J\pi]\right\}, \tag{4}$$

where I and J are external source functions, and ϕ and π are a set of fields and canonically conjugate momenta. Then, in passing to the manifestly covariant Lagrangian formulation by setting $J=0$ and performing the functional integration over π, there results

$$Z(I)=\int(d\phi)M(\phi)\exp\left\{i\int dx[L(\phi,\partial\phi)+I\phi]\right\}, \tag{5}$$

in which occurs a factor $M(\phi)$ which is not usually included. This factor is given explicitly by

$$M(\phi)=\int(du)\exp\left\{-i\int dx[H(\phi,\nabla\phi,\pi_0+u)-H(\phi,\nabla\phi,\pi_0)]-u\frac{\partial H(\phi,\nabla\phi,\pi_0)}{\partial\pi_0}\right\}, \tag{6}$$

and in fact is unity for theories in which the interaction has no more than one field derivative.

As shown by Salam and Strathdee, in the case of the chiral-invariant theory with Lagrangian density as in (1), the functional integration in (6) is Gaussian and leads to

$$M(\phi)=\exp\left[\tfrac{1}{2}\delta^{(4)}(0)\int dx\ln g(\phi)\right]. \tag{7}$$

In fact, it plays the role of a measure on the space of the fields, so that $(d\phi)M(\phi)$ is a chiral invariant.

Now it is clear that the generating functional becomes

$$Z(I)=\int(d\phi)\exp\left\{i\int dx[\bar L(\phi,\partial\phi)+I\phi]\right\}, \tag{8}$$

[1] J. M. Charap, Phys. Rev. D 2, 1554 (1970).
[2] An algebraic error in Ref. 1 has been corrected to give the new coefficient of the last term. We are grateful to J. Honerkamp and K. Meetz for pointing out our error.
[3] A. Salam and J. Strathdee, Phys. Rev. D 2, 2869 (1970).

where we have introduced

$$\bar{L}=L-\tfrac{1}{2}i\delta^{(4)}(0)\ln g(\phi). \quad (9)$$

The usual derivation of Feynman rules from the generating functional integral would treat $\bar{L}$ rather than L as the Lagrangian density, and so would include additional highly singular contact interactions contained in the term proportional to $\ln g(\phi)$. These terms, although Lorentz invariant, are not chiral invariant but they are needed to make the theory chiral invariant. The situation is similar to that which was described by Matthews[4] for gradient couplings, where the presence of non-Lorentz-invariant contact interactions is necessary to maintain Lorentz covariance of the theory.

We have checked that these extra terms do indeed cancel explicitly the most divergent parts of the amplitudes we had calculated previously. The equivalence theorem is also restored since the amplitudes after this cancellation do not depend on $f(\phi^2)$. And the Adler conditions are satisfied because, after this removal of their most divergent parts, the amplitudes vanish in the soft-pion limit.[5]

It is a pleasure to thank Professor Salam for drawing my attention to Ref. 3.

[4] P. T. Matthews, Phys. Rev. **76**, 684(L) (1949); **76**, 1419(L) (1949); **76**, 1657 (1949).

[5] *Note added in proof.* Similar results to those described in this addendum have been independently derived by other authors. At the time of writing I am aware of the following papers: J. Honerkamp and K. Meetz, preceding paper, Phys. Rev. D **3**, 1998 (1971); I. S. Gerstein, R. Jackiw, B. W. Lee, and S. Weinberg, Phys. Rev. D **3**, 2076 (1971); cf. also, D. G. Boulware, Ann. Phys. (N. Y.) **56**, 140 (1970); V. Ya. Fainberg and R. E. Kallosh, P. N. Lebedev, Phys. Inst. Report No. 170, Moscow, 1970 (unpublished).

PHYSICAL REVIEW D VOLUME 3, NUMBER 8 15 APRIL 1971

Rapidly Rotating Pulsars and Jacobi Ellipsoids*

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In connection with the evolution of a rotating Jacobi ellipsoid through the emission of gravitational radiation, we discuss the possibility that rapidly rotating pulsars can assume such triaxial, nonaxisymmetrical configurations.

THE evolution of a rotating Jacobi ellipsoid by gravitational radiation has recently been considered by Chandrasekhar[1,2] and the possible bearing of the results on gravitational collapse and pulsars has been discussed.[3] We have also looked into the problem[4] and besides obtaining similar conclusions with regard to time scales and possible sources of energy dissipation (i.e., just rotational kinetic energy and gravitational potential energy in the present context, though of course internal energy will also contribute for the more realistic case), we show it is highly likely that rapidly rotating pulsars can actually assume a triaxial Jacobi configuration.

Classically, it is known that break-up of an object by fission or equatorial mass loss could set in before the object can attain a large enough angular momentum for the triaxial configuration to be energetically favorable. While such factors as viscosity, differential rotation, etc., complicate the problem tremendously, it turns out that for the case of a uniformly rotating, inviscid gaseous mass with the pressure P and density ρ obeying a polytropic relation $P\propto\rho^n$ (the constant n is called the polytropic index), a simple answer is available.[5] There exists a critical index $n_c\sim0.8$ such that for objects with $n<n_c$, a triaxial configuration can be realized. With $n>n_c$, on the other hand, break-up would set in first.

Neutron stars probably cannot be described in terms of a complete polytrope (a single n for the whole object), but we should still be able to gain some insight by assigning an effective polytropic index. We have done this by taking the recently published equations of state describing matter at nuclear densities[6] and trying to fit a curve of the form $P=A\rho^B$ to the values of P and

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[1] S. Chandrasekhar, Phys. Rev. Letters **24**, 611 (1970).

[2] S. Chandrasekhar, Astrophys. J. **161**, 571 (1970).

[3] For studies on the relation between gravitational radiation and neutron stars, see, e.g., J. A. Wheeler, Ann. Rev. Astron. and Astrophys. **4**, 428 (1966); W. Y. Chau, Astrophys. J. **147**, 664 (1967); W. Y. Chau and R. N. Henriksen, *ibid.* **161**, L137 (1970).

[4] P. Srulovicz, Queen's University Research report, Astronomy Group, Physics Department, 1970 (unpublished); see also P. Srulovicz, M.Sc. thesis, Queen's University, 1970 (unpublished).

[5] R. A. James, Astrophys. J. **140**, 552 (1964).

[6] J. M. Cohen, W. D. Langer, L. C. Rosen, and A. G. W. Cameron, Astrophys. and Space Sci. **6**, 228 (1970); B. K. Harrison, K. S. Thorne, M. Wakano, and J. A. Wheeler, *Gravitational Theory and Gravitational Collapse* (Chicago U. P., Chicago, 1965); W. D. Langer and L. C. Rosen, Astrophys. and Space Sci. **6**, 217 (1970).