In the following we are interested in Eq. (3.2) only for Re$\lambda<0$. Let us define

$$\beta\equiv\ln(m_\rho^2/m^2)\,. \qquad (\text{B1})$$

We are interested in the large-β behavior of Eq. (3.2). It is convenient to introduce $\xi=\lambda\beta$. Equation (3.2) becomes

$$\frac{1}{2R_I}=\frac{1}{2(\xi/\beta)(1+\xi/\beta)(1+\xi/2\beta)}-\frac{\pi}{2\sin(\pi\xi/\beta)}e^{-\xi}+\frac{12e^{-\beta}}{(\xi/\beta)(1+\xi/\beta)(2+\xi/\beta)(3+\xi/\beta)(1-\xi/\beta)}$$

or

$$\frac{1}{R_I}\left(\frac{\xi}{\beta}\right)=\frac{1}{(1+\xi/\beta)(1+\xi/2\beta)}-\frac{\pi\xi/\beta}{\sin(\pi\xi/\beta)}e^{-\xi}+\frac{24e^{-\beta}}{(1+\xi/\beta)(2+\xi/\beta)(3+\xi/\beta)(1-\xi/\beta)}. \qquad (\text{B2})$$

We expand (B2) in a power series of ξ/β around the point zero and keep only the leading terms in $1/\beta$ (because eventually we will let β approach infinity). We obtain

$$\frac{1-e^{-\xi}}{\xi}=\frac{1}{\beta}\left(\frac{1}{R_I}+\frac{3}{2}\right). \qquad (\text{B3})$$

The solutions of Eq. (B3) can be found easily in the limit of large β. They are

$$\text{Re}\lambda=-\frac{1}{2}(2n\pi)^2\frac{1}{\beta^3}\left(\frac{1}{R_I}+\frac{3}{2}\right)^2+\cdots\,, \qquad (\text{B4})$$

$$\text{Im}\lambda=\pm(2n\pi)(1/\beta)+\cdots\,,$$

where $\cdots$ denotes higher-order terms in $1/\beta$ and n is any positive integer. It is clear that for large $\beta\equiv\ln(m_\rho^2/m^2)$, Re$\lambda$ can be larger than $-\frac{1}{2}$.

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Unit-Spin Propagation Functions and Form Factors*

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Source theory is used to derive a representation for the propagation function of a unit-spin meson. The asymptotic behavior for large momenta resembles that observed in electromagnetic form factors.

HADRONIC electromagnetic interactions are usually pictured as proceeding through the intermediary of the known 1^- mesons. In this model, electromagnetic form factors are linearly related to the meson propagation functions. Electron-positron colliding-beam experiments have confirmed the implied resonant behavior. But form-factor measurements for large momentum transfer seem to be at variance with the idea. Instead of the simple asymptotic dependence on momentum transfer, $1/p^2$, experiment indicates a more rapid decrease, $(1/p^2)^d$, $d\gtrsim2$. It is therefore particularly significant that the modified propagation functions derived from source theory[1] do show an asymptotic decrease that is at least as rapid as $(1/p^2)^2$, and can approach $(1/p^2)^3$.

To sketch how this comes about, consider ρ^0 for definiteness. The associated vector and tensor fields are designated as ρ_μ and $\rho_{\mu\nu}$, while the vectorial source is J_μ. The initial description of this particle, appropriate to such time intervals that its instability is not in evidence, is given by the vacuum amplitude expression

$$\langle 0_+|0_-\rangle^J=\exp[iW(J)]\,, \qquad (1)$$

where, stated in momentum space for convenience,

$$W(J)=\int\frac{(dp)}{(2\pi)^4}[\rho^\mu(-p)J_\mu(p)-\tfrac{1}{4}\rho^{\mu\nu}(-p)\rho_{\mu\nu}(p)-\tfrac{1}{2}m^2\rho^\mu(-p)\rho_\mu(p)]\,, \qquad (2)$$

$$\rho_{\mu\nu}(p)=ip_\mu\rho_\nu(p)-ip_\nu\rho_\mu(p)\,.$$

The action principle supplies the field equation

$$ip_\nu\rho^{\mu\nu}(p)+m^2\rho^\mu(p)=J^\mu(p)\,, \qquad (3)$$

which can be analyzed into longitudinal and transverse components relative to the momentum vector, as indicated symbolically by

$$\rho=\rho_T+\rho_L=\left(1-\frac{pp}{p^2}\right)\rho+\frac{pp}{p^2}\rho\,. \qquad (4)$$

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[1] Various aspects of source theory are described by J. Schwanger in (a) *Particles and Sources* (Gordon and Breach, New York, 1969), and (b) *Particles, Sources, and Fields* (Addison-Wesley, Reading, Mass., 1970).

These components are

$$m^2\rho_L=J_L\,,\quad \rho_L=(1/m^2)J_L\,,$$
$$(p^2+m^2)\rho_T=J_T\,,\quad \rho_T=\Delta_+J_T\,,\tag{5}$$

where the zero-momentum value of the transverse propagation function,

$$\Delta_+(0)=1/m^2\,,\tag{6}$$

prevents a spurious singularity at $p^2=0$ from appearing in (4).

More generally, two causually arranged, extended ρ-particle sources (those for which $-p^2\neq m^2$, preventing real particle emission) can exchange multiparticle excitations of variable mass M with total quantum numbers equivalent to those of the ρ. The basic physical picture of ρ dynamics is introduced by asserting that the primitive interaction at work couples the vector ρ field to a conserved current. The equivalent gauge-invariance property implies that the effective coupling explicitly involves the *tensor* ρ field, as indicated in the following contribution to the vacuum amplitude:

$$\int dM^2a(M^2)d\omega_p\tfrac{1}{2}\rho_1{}^{\mu\nu}(-p)\rho_{2\mu\nu}(p)\,.\tag{7}$$

Here

$$d\omega_p=\frac{(d\mathbf{p})}{(2\pi)^3}\frac{1}{2p^0},\quad p^0=(\mathbf{p}^2+M^2)^{1/2}\tag{8}$$

is the invariant momentum space measure on the surface $-p^2=M^2$, the two fields are associated with the respective emission (J_2) and absorption (J_1) sources, and all detailed dynamics is contained in the real non-negative weight function $a(M^2)$. [These general characteristics of $a(M^2)$ become perspicuous in the structure of the modified propagation function.] The space-time extrapolation procedure of source theory is introduced by recognizing that

$$\int \tfrac{1}{2}\rho_1{}^{\mu\nu}(-p)id\omega_p\rho_{2\mu\nu}(p)=\int(dx)(dx')\tfrac{1}{2}\rho_1{}^{\mu\nu}(x)\times\left[i\int d\omega_pe^{ip(x-x')}\right]\rho_{2\mu\nu}(x')\tag{9}$$

contains the appropriate causal form of the propagation function $\Delta_+(x-x',M^2)$:

$$\Delta_+(x-x',M^2)=i\int d\omega_pe^{ip(x-x')},\quad x^0>x^{0\prime}.\tag{10}$$

It is then replaced by the general structure

$$\int\frac{(dp)}{(2\pi)^4}\frac{e^{ip(x-x')}}{p^2+M^2-i\epsilon}+(\text{contact term}),\tag{11}$$

or, in effect,

$$\int id\omega_p\rightarrow\int\frac{(dp)}{(2\pi)^4}\left(\frac{1}{p^2+M^2-i\epsilon}+\text{c.t.}\right).\tag{12}$$

Contact terms (c.t.) are multiples of $\delta(x-x')$, or a finite number of its derivatives, which are left undetermined by the nonoverlapping causal arrangement. They are to be fixed by consulting physical *normalization* requirements. The momentum-space version is a polynomial in momentum. Deferring for a moment the question of its determination, we observe that the vacuum amplitude (7) is implied by the additional action term

$$\int\frac{(dp)}{(2\pi)^4}dM^2a(M^2)(-\tfrac{1}{4})\rho^{\mu\nu}(-p)\times\left(\frac{1}{p^2+M^2-i\epsilon}+\text{c.t.}\right)\rho_{\mu\nu}(p)\,,\tag{13}$$

which is to be added to the initial action (2).

The modified action implies a modified field equation:

$$\left[1+\int dM^2a(M^2)\left(\frac{1}{p^2+M^2-i\epsilon}+\text{c.t.}\right)\right]ip_\nu\rho^{\mu\nu}(p)+m^2\rho^\mu=J^\mu(p)\,.\tag{14}$$

Its decomposition into longitudinal and transverse components gives

$$\rho_L=(1/m^2)J_L\,,\quad \rho_T=\bar{\Delta}_+J_T\,,\tag{15}$$

where

$$\bar{\Delta}_+(p)=\left[p^2+m^2+p^2\int dM^2a(M^2)\times\left(\frac{1}{p^2+M^2-i\epsilon}+\text{c.t.}\right)\right]^{-1}\tag{16}$$

continues to have the necessary property,

$$\bar{\Delta}_+(0)=1/m^2\,.\tag{17}$$

The instability of the particle will find its expression in the imaginary term of $\bar{\Delta}_+{}^{-1}$, which is correctly associated with real processes ($-p^2=M^2$) if $a(M^2)$ is a real number (as are the numerical coefficients in the contact term). We write

$$-\text{Im}\bar{\Delta}_+(p)^{-1}|_{-p^2=M^2>0}=\pi M^2a(M^2)=m\Gamma(M^2)\,,\tag{18}$$

which also indicates that $a(M^2)$ is a positive quantity, for only then is

$$|\langle 0_+|0_-\rangle^J|^2=\exp\left[-\int\frac{(dp)}{(2\pi)^4}|\rho_T(p)|^2(-\text{Im})\bar{\Delta}_+(p)^{-1}\right]\leq 1\,.\tag{19}$$

The nominal width of the resonance is

$$\Gamma=\Gamma(m^2)=\pi m a(m^2)\,. \qquad (20)$$

The contact terms will now be picked to guarantee what we have just taken for granted. The mass of the unstable particle, initially measured as m for short time intervals, must retain that value as the description is extended to longer times [certainly to relative order Γ/m; the mass of an unstable particle is intrinsically ambiguous to the order $(\Gamma/m)^2$, as reflected in the difficulty of assigning the ρ mass with that degree of accuracy]. One must therefore choose the contact term to make the real part of the spectral integral in (16) vanish at $-p^2=m^2$, as expressed by the appearance of a p^2+m^2 factor. But just one such factor will not do, since it alters the normalization of the propagation function in the neighborhood $-p^2\sim m^2$, which has already been specified in describing the situation where instability plays an unimportant role. Accordingly, two factors are needed, as exhibited in

$$\frac{1}{p^2+M^2-i\epsilon}-\frac{1}{M^2-m^2}+\frac{p^2+m^2}{(M^2-m^2)^2}=\left(\frac{p^2+m^2}{M^2-m^2}\right)^2\frac{1}{p^2+M^2-i\epsilon}. \qquad (21)$$

The result is

$$\bar{\Delta}_+(p)=\left[p^2+m^2+(p^2+m^2)^2p^2\frac{d}{dm^2}P\times\int dM^2\frac{a(M^2)}{M^2-m^2}\frac{1}{p^2+M^2-i\epsilon}\right]^{-1}, \qquad (22)$$

where we have also introduced, in the principal-value sign, the prescription for relaxing the temporary exclusion of $M^2\sim m^2$. It is verified by confirming that the initial short-time description is indeed maintained.[2]

The structure of the modified propagation function has now been determined, without reference to the requirement of existence. That is assured if the weight function $a(M^2)$ increases less rapidly than $(M^2)^2$ as $M^2\to\infty$. If we make the stronger assumption that $a(M^2)$ increases less rapidly than M^2, we infer the following asymptotic behavior for large spacelike values of p:

$$\bar{\Delta}_+(p)\sim(1/p^2)^2. \qquad (23)$$

The more general situation indicated by

$$a(M^2)\sim(M^2)^{2-\alpha},\quad 0<\alpha<1 \qquad (24)$$

leads to

$$\bar{\Delta}_+(p)\sim(1/p^2)^{3-\alpha}. \qquad (25)$$

[2] Another example is discussed in Ref. 1(b), Sec. 3–16.

As a purely mathematical example, we take $(\alpha=\frac{1}{2})$

$$a(M^2)=\frac{1}{\pi}\lambda\left(\frac{M^2-m_0^2}{m^2}\right)^{3/2},\quad \lambda=\frac{\Gamma}{m}\left(1-\frac{m_0^2}{m^2}\right)^{-3/2},\quad M>m_0 \qquad (26)$$

which gives, for $-p^2>m_0^2$,

$$\bar{\Delta}_+(p)=\left[p^2+m^2-i\lambda(-p^2)\left(-\frac{p^2+m_0^2}{m^2}\right)^{3/2}\right]^{-1} \qquad (27)$$

and, for $p^2>-m_0^2$,

$$\bar{\Delta}_+(p)=\left[p^2+m^2+\lambda p^2\left(\frac{p^2+m_0^2}{m^2}\right)^{3/2}\right]^{-1}\sim\frac{1}{\lambda m^2}\left(\frac{m^2}{p^2}\right)^{5/2}. \qquad (28)$$

Attention should be directed to the decisive role played by the factor p^2. If it were replaced by the constant $-m^2$, which maintains the meaning of λ in (26), the spacelike behavior of $\bar{\Delta}_+(p)$ would be determined by the function

$$p^2+m^2-\lambda m^2\left(\frac{p^2+m_0^2}{m^2}\right)^{3/2}, \qquad (29)$$

which changes sign with increasing p^2. The resulting $p^2>0$ singularity of the propagation function is completely unacceptable.[3]

To these results we add some remarks.

(1) Since the electromagnetic field that probes a hadron proceeds through the intermediary of a 1^- meson, the observed electromagnetic form factors compound the meson propagation functions with the meson-hadron vertex form factors. If the large-momentum-transfer behavior of $\bar{\Delta}_+(p)$, as illustrated in (28), can reproduce the trend of the data, we must conclude that the vertex form factors have attained transitory constant limits in the high-energy region now under exploration.

(2) The steeper decrease of the 1^--meson propagation functions with increasing momenta removes the apparent divergence in calculations of electromagnetic mass splittings such as $\pi^\pm$-π^0, and thereby substantially reduces the motivation for recent attempts[4] to modify electrodynamics. Similar comments apply to 1^+ mesons in relation to the weak-interaction properties of hadrons, and, at another level of energy (and speculation), to the hypothetical W, X, or Z boson and leptonic couplings.[5]

[3] Of course, p^2 is not unique. The more general possibilities that are roughly suggested by the addition of a suitably bounded constant to $-p^2$ can have the same features, and doubtless are more realistic.

[4] T. D. Lee and G. C. Wick, Nucl. Phys. **B9**, 209 (1969).

[5] Compare M. Gell-Mann, M. L. Goldberger, N. M. Kroll, and F. R. Low, Phys. Rev. **179**, 1518 (1969). Other references can be found in S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 1285 (1970).

(3) The asymptotic behavior of $\bar{\Delta}_+(p)$ is at variance with the standard spectral form

$$\bar{\Delta}_+(p)=\int dM^2\frac{A(M^2)}{p^2+M^2-i\epsilon},\quad A(M^2)\geq 0 \qquad (30)$$

which forbids a decrease faster than $1/p^2$. Looking at the example of (27), we see that $\bar{\Delta}_+(p)$, considered as a function of the complex variable $-p^2$, possesses not only a branch line beginning at $m_0{}^2$ but also a pair of complex poles, leading to the representation

$$\bar{\Delta}_+(p)=\int dM^2\frac{A(M^2)}{p^2+M^2-i\epsilon} -\tfrac{1}{2}A\left(\frac{1}{p^2+M_1{}^2-iM_1\Gamma_1}+\frac{1}{p^2+M_1{}^2+iM_1\Gamma_1}\right). \qquad (31)$$

Simplifying the model by the restriction $m_0{}^2\ll m^2$, $\lambda\ll 1$, we have

$$A=\tfrac{4}{3},\quad \left(\frac{M_1}{m}\right)^2=\frac{1}{2\lambda^{2/3}},\quad \frac{\Gamma_1}{M_1}=\sqrt{3}, \qquad (32)$$

with

$$A(M^2)=\frac{1}{\pi}\frac{(\lambda/m^3)(M^2)^{5/2}}{(M^2-m^2)^2+(\lambda^2/m^6)(M^2)^5}, \qquad (33)$$

and one can verify the sum rules

$$\int dM^2A(M^2)=A,\quad \int dM^2M^2A(M^2)=M_1{}^2A, \qquad (34)$$

which ensure that $\bar{\Delta}_+(p)$ decreases faster than $(1/p^2)^2$. The unusual supplementary term in (31) seems not to violate any general principle of source theory (which does not have to contend with the restrictions imposed by an assumed underlying operator field structure, nor with preconceptions about analyticity). Since the additional term is real, it makes no contribution in the vacuum persistence probability and therefore does not describe particle processes. Correspondingly, the contribution of this term to the time development of the propagation function is rapidly extinguished and plays no role in any causal arrangement. But, naturally, the new direction thus opened by source theory demands further intensive scrutiny to test its complete physical acceptability.[6]

Notes added in proof

1. More reasonable than the possibility suggested in Ref. 3 is the following. The presence of nonconserved currents is signaled by the explicit appearance of the scalar field $\partial_\mu\rho^\mu$ rather than the vector field ρ^μ. Such terms do not affect the transverse propagation function and accordingly (22) is an entirely general form. The corresponding general form of the modified longitudinal propagation function is

$$\bar{\Delta}_L(p)=\left[m^2+(p^2+m^2)p^2P\times\int dM^2\frac{b(M^2)}{M^2-m^2}\frac{1}{p^2+M^2-i\epsilon}\right]^{-1}, \qquad (35)$$

where $b(M^2)$ is real and positive. For large p^2 this function decreases as least as rapidly as $(p^2)^{-1}$, but not as rapidly as $(p^2)^{-2}$.

2. The close relationship between strong interactions and the massive nature of the $1^\pm$ particles should be stressed. Thus, in the analogous photon (electron charge) discussion, which persists without change up to Eq. (16), with $m^2=0$, the contact term is only required to produce an additional p^2 factor in the spectral integral, which yields

$$\bar{D}_+(p)=\left[p^2-(p^2)^2\int dM^2\frac{a(M^2)}{M^2}\frac{1}{p^2+M^2-i\epsilon}\right]^{-1}. \qquad (36)$$

If spacelike singularities are to be avoided, this $a(M^2)$ must vanish as $M^2\to\infty$, and be sufficiently limited in magnitude that

$$\int dM^2\frac{a(M^2)}{M^2}<1. \qquad (37)$$

The comparison with the behavior illustrated in Eq. (26) indicates that, in contrast to massive particles, a massless particle cannot be associated with strong interactions.

3. In the analogous discussion of $0^\pm$ particles, applicability to strong interactions seems to demand both the massive nature of the particles and the dominance of (pseudo) vector couplings over (pseudo) scalar couplings. If the particles were massless, the form (36) would apply, with its implied exclusion of strong interactions. A massive particle, with couplings represented by its field rather than a derivative of the field, has a modified propagation function of a form that is derived from (22) by removing the factor $-p^2$. Again there are magnitude restrictions on the weight function $a(M^2)$ if spacelike singularities are to be avoided. But the strong interaction form (22) is appropriate (with the generalization of Ref. 3) if the dominant dynamics involves the gradient of the field. Here is a suggestion of the partial chiral symmetry that does seem to govern the dynamics of 0^- (0^+?) particles. It should be emphasized that all these considerations assume that only the contact terms required by the evident physical normalization requirements are to be used.

[6] It may be that the theory is correctly formulated only in Euclidean space [see the discussion of the Euclidean postulate in Ref. 1(b)], with the final results of calculation transformed back into Minkowski space for physical interpretation. (A Euclidean calculation is implicit in the statement concerning electromagnetic mass splittings, for example.) That could produce a limitation in the applicability of the conventional space-time description for very small time intervals, with possible repercussions in the structure of forward-scattering dispersion relations.