where

$$\bar{f}_\pi^{\,2}=g_\rho^{\,2}/2m_\rho^{\,2}, \tag{6.3}$$

which is known as the KSRF relation.[11]

In the treatment of the interaction of the ρ field with other fields, we have introduced the simplest couplings consistent with the divergence condition for the ρ field. This divergence condition, of course, remains inviolate if for any isofield ψ we introduce additional coupling terms that are functions of $\boldsymbol{\varrho}_{\mu\nu}$, ψ, and $D_\mu\psi$, and therefore it is possible to introduce for the ρ-π-N system additional coupling terms such as

$$f_\rho\boldsymbol{\varrho}_{\mu\nu}\cdot(\bar{N}\sigma_{\mu\nu}\boldsymbol{\tau}N) \quad\text{or}\quad f_\rho'\boldsymbol{\varrho}_{\mu\nu}\cdot(D_\mu\boldsymbol{\pi}\times D_\nu\boldsymbol{\pi}). \tag{6.4}$$

However, in view of the experience in quantum electrodynamics, it seems that these couplings are not likely to be fundamental, although the higher-order effects will undoubtedly generate effective couplings of the above form.

When the ρ field is not accompanied by the a field, we have shown that the usual PCAC condition cannot be fulfilled, and it must be replaced by the covariant PCAC condition (3.12) or (3.13). Since the existence of the a mesons has not been clearly established by experiments, it would be desirable to explore the consequences of the covariant PCAC condition.

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Complex Regge Poles in the Amati-Bertocchi-Fubini-Stanghellini-Tonin Multiperipheral Model*

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The multiperipheral model of Amati, Bertocchi, Fubini, Stanghellini, and Tonin is used to study complex Regge poles. We use the trace approximation to solve the multiperipheral integral equation. The equation determining complex Regge poles for the forward-scattering case is derived. We explicitly solve for the locations of complex Regge poles and discuss their dependence on the pion mass.

I. INTRODUCTION

MULTIPERIPHERAL models[1,2] are very useful for describing general features of high-energy collisions. It is well known that they predict Regge asymptotic behavior for elastic amplitudes and total cross sections, a constant elasticity, a ln$s$ behavior for the multiplicity, and a small average transverse momentum for secondary particles produced in high-energy collisions.

Another group of useful models for studying Regge behavior are potential models. It is known that, in potential models, complex conjugate pairs of Regge poles may occur at energies below the physical threshold.[3] It is natural to ask whether this phenomenon is also present in a relativistic model like the multipheripheral model.

Recently, using a simplified multiperipheral model, Chew and Snider[4] illustrated the possibility of complex conjugate pairs of Regge poles.

This paper studies the problem of complex Regge poles in a more realistic multiperipheral model. The model we use is the Amati-Bertocchi-Fubini-Stanghellini-Tonin (ABFST) model.[1] We do not attempt to solve the ABFST integral equation exactly. Rather we use an approximation method. The method we adopt is the trace approximation, which has been employed by Chew, Rogers, and Snider[5] (CRS) in the case of the leading real Regge poles. With this approximation, the eigenvalue equation can be written down quite easily. The dependence of the locations of complex Regge poles on various physical quantities becomes transparent. In forward scattering, we explicitly solve for the locations of these complex Regge poles. The results agree with those of Misheloff,[6] who solved the ABFST integral equation exactly by a numerical method.

The plan of the paper is as follows. In Sec. II, we develop the trace approximation for the forward-

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scattering ABFST integral equation. The equation determining the locations of Regge poles is written down. In Sec. III, under a simplified assumption, the locations of the complex conjugate pairs of Regge poles are found. In Sec. IV we comment on the internal consistency of the trace approximation for the forward-scattering integral equation. Appendix A evaluates a mathematical expression [Eq. (2.12)] used in Sec. II. The problem of the locations of complex Regge poles in the limit of very small m^2/m_ρ^2 is considered in Appendix B.

II. TRACE APPROXIMATION FOR FORWARD-SCATTERING ABFST INTEGRAL EQUATION

The ABFST multiperipheral model was the first multiperipheral model. It is based on the pion dominance of the scattering amplitude.

In this paper we restrict ourselves to the forward-scattering ABFST model. The model can be formulated in the form of an integral equation. We follow here closely the notation of CRS.[5] Our starting point is the diagonalized integral equation

$$F_I^\lambda(\tau,\tau') = F_{I,1}^\lambda(\tau,\tau') + \int_{-\infty}^0 d\tau'' F_I^\lambda(\tau,\tau'') K_I^\lambda(\tau'',\tau')\,, \quad (2.1)$$

where the partial wave F_I^λ is defined by

$$F_I^\lambda(\tau,\tau') \equiv \int_{4m^2}^\infty ds \frac{e^{-(\lambda+1)\eta(s,\tau,\tau')}}{\lambda+1} F_I^+(s,\tau,\tau')\,, \quad (2.2)$$

with

$$\cosh\eta(s,\tau,\tau') \equiv (s-\tau-\tau')/2(\tau\tau')^{1/2} \quad (2.3)$$

and

$$K_I^\lambda(\tau'',\tau') = \int_{4m^2}^\infty ds \frac{e^{-(\lambda+1)\eta(s,\tau,''\tau')}}{\lambda+1} \frac{C_I(s)}{(\tau''-m^2)^2}\,, \quad (2.4)$$

where m is the pion mass and $C_I(s)$ is proportional to the π-π elastic cross section.

Of course, $F_I^+(s,\tau,\tau')$ can also be expressed in terms of $F_I^\lambda(\tau,\tau')$. It is given by

$$2(\tau\tau')^{1/2} F_I^+(s,\tau,\tau') = \frac{1}{2\pi i}\int d\lambda(\lambda+1) \frac{e^{(\lambda+1)\eta(s,\tau,\tau')}}{\sinh\eta(s,\tau,\tau')} F_I^\lambda(\tau,\tau')\,, \quad (2.5)$$

where the integration is over a contour from $-i\infty$ to $i\infty$, passing to the right of all λ singularities of $F_I^\lambda(\tau,\tau')$ in the complex λ plane.

Although the ABSFT integral equation can be solved numerically by using computers, we show that using the trace approximation transforms the eigenvalue problem into solving a simple transcendental equation. This approximation also provides a more tractable mathematical expression, giving more insight into the physical aspects of the problem.

The trace approximation for the ABFST integral equation has been used by CRS. They found that it is a good approximation for the leading Regge pole. In this section we shall use the same trace approximation to solve the ABFST integral equation for the complex Regge poles.

The approximation amounts to replacing the Fredholm determinant by the trace of the kernel K_I^λ, i.e.,

$$D_I(\lambda) \approx 1 - \mathrm{Tr} K_I^\lambda = 1 - \int_{-\infty}^0 d\tau \frac{1}{(\tau-m^2)^2} \times \int_{4m^2}^\infty ds \frac{e^{-(\lambda+1)\eta(s,\tau)}}{\lambda+1} C_I(s)\,, \quad (2.6)$$

with

$$\cosh\eta(s,\tau) = 1 - s/2\tau\,. \quad (2.7)$$

The Regge poles are located at the zeros of the Fredholm determinant, which in the trace approximation becomes

$$1 - \mathrm{Tr} K_I^\lambda = 0\,. \quad (2.8)$$

It is more convenient to use η instead of τ as integration variable. Equation (2.6) can be expressed as

$$D_I(\lambda) = 1 - \frac{2}{\lambda+1}\int_{4m^2}^\infty ds \frac{C_I(s)}{s} \times \int_{\eta=0}^\infty d(\cosh\eta) \frac{e^{-(\lambda+1)\eta}}{[1+(2m^2/s)(\cosh\eta-1)]^2}\,. \quad (2.9)$$

Notice that the integral in Eq. (2.9) has meaning only for $\mathrm{Re}\lambda > -2$.

The explicit evaluation of the integral

$$\int_{\eta=0}^\infty d(\cosh\eta) \frac{e^{-(\lambda+1)\eta}}{[1+(2m^2/s)(\cosh\eta-1)]^2} = \left(\frac{s}{2m^2}\right)^2 \int_{\eta=0}^\infty d(\cosh\eta) \frac{e^{-(\lambda+1)\eta}}{[\cosh\eta + (s/2m^2) - 1]^2}$$

is given in Appendix A. It is sufficient to confine ourselves to the case of large s/m^2. In this case (for $\mathrm{Re}\lambda > -2$)

$$\int_{\eta=0}^\infty d(\cosh\eta) \frac{e^{-(\lambda+1)\eta}}{[1+(2m^2/s)(\cosh\eta-1)]^2} = \frac{1}{\lambda(\lambda+2)} - \frac{\pi(\lambda+1)}{2\sin\pi\lambda}\left(\frac{m^2}{s}\right)^\lambda + \frac{12}{\lambda(\lambda+2)(\lambda+3)(1-\lambda)}\frac{m^2}{s} + (\text{higher-order terms in } m^2/s)\,. \quad (2.10)$$

Since the dominating contribution of $C_I(s)$ comes from the ρ meson, we can make the following estimate of m^2/s:

$$m^2/s \approx m^2/m_\rho^2 \approx 0.03 .$$

We shall consequently neglect the terms inside the bracket on the right-hand side of Eq. (2.10) and keep only the first three terms.

The equation determining the locations of Regge poles is then

$$1=\frac{2}{1+\lambda}\int\frac{ds}{s}C_I(s)\left[\frac{1}{\lambda(\lambda+2)}-\frac{\pi(\lambda+1)}{2\sin\pi\lambda}\left(\frac{m^2}{s}\right)^\lambda+\frac{12}{\lambda(\lambda+2)(\lambda+3)(1-\lambda)}\frac{m^2}{s}\right]. \quad (2.11)$$

This is our essential result of the trace approximation for the forward-scattering ABFST integral equation.

We would like to make some pertinent remarks in passing.

(1) If we neglect m^2 in $\mathrm{Tr}K_I{}^\lambda$, we get

$$D_I(\lambda)=1-\frac{2}{\lambda(\lambda+1)(\lambda+2)}\int\frac{ds}{s}C_I(s).$$

This is the expression used by CRS for the leading Regge pole.

(2) It is quite obvious that in general we cannot just keep only the first term, especially when we are interested in the nonleading Regge poles.

(3) For determining the locations of complex Regge poles, the first two terms in Eq. (2.11) will be sufficient. However, if we are also interested in the region of $\lambda \approx 1$, we should use all three terms.

(4) From Eq. (2.10), we see clearly that the expression

$$\int_{\eta=0}^{\infty} d(\cosh\eta)\frac{e^{-(\lambda+1)\eta}}{[1+(2m^2/s)(\cosh\eta-1)]^2} \quad (2.12)$$

is not an analytic function of m^2 around $m^2=0$. The presence of the term

$$-\frac{\pi(\lambda+1)}{2\sin\pi\lambda}\left(\frac{m^2}{s}\right)^\lambda$$

is crucial for the problem of complex Regge poles.

III. SIMPLIFICATION AND NUMERICAL SOLUTIONS

In the previous section we developed the trace approximation for the ABFST multiperipheral model. We derived, in particular, Eq. (2.11), which can be used to determine the locations of Regge poles.

The function $C_I(s)$ is related to the π-π elastic cross section, consisting of the contributions from various resonant states. Since the most important resonant state is the ρ meson, it is reasonable to make a further simplification. We replace $C_I(s)$ by a δ function at $s=m_\rho^2$. With this simplification, Eq. (2.11) becomes

$$1=\frac{2}{1+\lambda}R_I\left[\frac{1}{\lambda(\lambda+2)}-\frac{\pi(\lambda+1)}{2\sin\pi\lambda}\left(\frac{m^2}{m_\rho^2}\right)^\lambda+\frac{12}{\lambda(\lambda+2)(\lambda+3)(1-\lambda)}\frac{m^2}{m_\rho^2}\right],$$

where

$$R_I\equiv\int\frac{ds}{s}C_I(s). \quad (3.1)$$

This can also be written as

$$\frac{1}{2R_I}=\left[\frac{1}{\lambda(\lambda+1)(\lambda+2)}-\frac{\pi}{2\sin\pi\lambda}\left(\frac{m^2}{m_\rho^2}\right)^\lambda+\frac{12}{\lambda(\lambda+1)(\lambda+2)(\lambda+3)(1-\lambda)}\frac{m^2}{m_\rho^2}\right]. \quad (3.2)$$

Before solving λ for given R_I, we discuss qualitatively the dependence of λ of the ratio m^2/m_ρ^2. It is clear that when λ is close to 1, the first term in Eq. (3.2) dominates over the sum of the second and third terms. In this region the dependence of λ on m^2/m_ρ^2 is weak. This is the reason that if the leading pole is close to 1, we may use

$$1=2R_I/\lambda(\lambda+1)(\lambda+2) ,$$

as in CRS. However, in the region where $\mathrm{Re}\lambda$ is close to zero or even less than zero, the first term is no longer the dominating term. The second term becomes important. Especially for $\mathrm{Re}\lambda<0$, the solution of Eq. (3.2) depends strongly on the ratio m^2/m_ρ^2 (we shall see later that the complex Regge poles exist in the region $\mathrm{Re}\lambda<0$). Therefore, their precise locations depend on the mass ratio m^2/m_ρ^2.

For any given R_I, the determination of the solutions of Eq. (3.2) is easy. The solutions are tabulated in Table I.

These results are in good agreement with those of Misheloff,[6] who numerically solved the ABFST integral

TABLE I. Positions of leading pole and complex Regge poles.

Leading real Regge pole	First pair of complex Regge poles	Second pair of complex Regge poles	$1/2R_I$
0.1	$-1.03\pm1.40i$	$-1.90\pm2.38i$	1.32
0.2	$-0.97\pm1.47i$	$-1.83\pm2.45i$	0.835
0.3	$-0.93\pm1.52i$	$-1.78\pm2.50i$	0.609
0.4	$-0.91\pm1.56i$	$-1.74\pm2.54i$	0.467
0.5	$-0.89\pm1.59i$	$-1.71\pm2.58i$	0.370
0.6	$-0.87\pm1.63i$	$-1.68+2.61i$	0.298
0.7	$-0.85\pm1.65i$	$-1.65\pm2.65i$	0.244
0.8	$-0.84\pm1.69i$	$-1.62\pm2.70i$	0.203
0.9	$-0.83\pm1.70i$	$-1.60\pm2.72i$	0.171
1.0	$-0.82\pm1.72i$	$-1.59\pm2.74i$	0.154

equation. We also find that Reλ remains less than $-\frac{1}{2}$ even when R_I approaches infinity. This last statement depends on the value of $m^2/m_\rho{}^2$, as explained below.

In Appendix B we study Eq. (3.2) in the limit that $m^2/m_\rho{}^2$ approaches zero. In this limiting case we can solve analytically for the locations of complex Regge poles. There we shall see that for very small values of $m^2/m_\rho{}^2$, complex Regge poles can exist in the region $\mathrm{Re}\lambda > -\frac{1}{2}$.

We recall that although in potential theory one can prove that the complex Regge poles can exist only for $\mathrm{Re}\lambda < -\frac{1}{2}$,[7] no such proof exists based on the general principles of S-matrix theory.

IV. DISCUSSION

We would like to make the following remarks on the approximation method used in this paper. From the practical point of view, the work of CRS has established the usefulness of the trace approximation for the leading Regge pole. The agreement of our results with those of Misheloff extends the usefulness of the approximation to the problem of the complex Regge poles.

In what follows, we try to answer from another point of view the question: Is the trace approximation a good approximation for the locations of the complex Regge poles?

It is well known that the trace approximation can be used in solving the Fredholm integral equation in two limiting cases. In the first case, the kernel is very weak (weak-coupling limit); in the second case, the kernel of the integral equation is approximately factorizable. It is the latter limit that we adopt in this paper. We now present some qualitative arguments in support of the trace approximation by showing that our kernel is approximately factorizable.

First we show that the average momentum transfer square is of the order of $-m^2$, i.e., $\langle\tau\rangle = O(-m^2)$. After we succeed in establishing this fact, we then show the approximate factorizability of the kernel.

The first argument comes from the self-consistency of our model. In the trace approximation the following integral appears:

$$I = \int_{\eta=0}^{\infty} d(\cosh\eta) \frac{e^{-(\lambda+1)\eta}}{[1+(2m^2/s)(\cosh\eta-1)]^2}.$$

In the region $\mathrm{Re}\lambda<0$, if $m^2=0$, I approaches infinity. It is precisely the factor

$$\frac{1}{[1+(2m^2/s)(\cosh\eta-1)]^2}$$

which prevents the integral from divergence. It is easy to show that the most important contribution to I comes from the region $(2m^2/s)(\cosh\eta-1) = O(1)$, i.e., $\cosh\eta-1 = O(s/2m^2)$. We have by definition $\cosh\eta-1 = -s/2\tau$. Therefore, $\langle\tau\rangle = O(-m^2)$.

We can also give a second argument. In the integral equation, we have τ'' integration. For τ'' small, we have

$$\int d\tau'' \frac{(\tau''/s)^{\lambda+1}}{(\tau''^2-m^2)^2}.$$

In the region $\mathrm{Re}\lambda>0$, we may neglect m^2. However, in the region $\mathrm{Re}\lambda<0$, we cannot. In fact it can be shown that for $\mathrm{Re}\lambda<0$, $\langle\tau''\rangle = O(-m^2)$. We consider these two arguments to be sufficient to support $\langle\tau\rangle = O(-m^2)$ when $\mathrm{Re}\lambda<0$.

Now we turn our attention to the problem of the factorizability of the kernel. Our kernel is given by

$$K_I{}^\lambda(\tau'',\tau) = \frac{1}{(\tau''-m^2)^2} \int_{4m^2}^{\infty} ds \frac{e^{-(\lambda+1)\eta(s,\tau'',\tau')}}{\lambda+1} C_I(s),$$

with

$$\cosh\eta(s,\tau'',\tau') = \frac{s-\tau''-\tau'}{2(\tau''\tau')^{1/2}}.$$

It is quite obvious that the factorizability of $K_I{}^\lambda$ depends on that of $e^{-(\lambda+1)\eta(s,\tau'',\tau')}$. The expression $(e^{-\eta(s,\tau'',\tau')})^{\lambda+1}$ is factorizable in the limit of large s/τ'', s/τ'. As we have argued previously, for $\mathrm{Re}\lambda<0$, $\langle\tau''\rangle, \langle\tau'\rangle = O(-m^2)$. Since $s\approx m_\rho{}^2$, $\langle\tau''\rangle/s \approx \langle\tau'\rangle/s \approx m^2/m_\rho{}^2 = 0.03$. Thus, to a very good accuracy, the trace approximation can be used in solving the integral equation.

We want to emphasize again that the argument to justify the trace approximation is not the weakness of the coupling, but is the approximate factorizability of the kernel.

Before concluding this section, we would like to mention another application of the trace approximation. It can also be used to study the Regge poles for the problem of nonforward scattering. In particular, the behavior of the complex Regge poles as t approaches the threshold $4m^2$ can be studied. It can be shown that an infinite number of complex Regge poles accumulates at $\mathrm{Re}\lambda = -\frac{1}{2}$, in agreement with the result of Desai and Newton,[8] and Gribov and Pomeranchuk.[9] We will not discuss $\lambda(t)$ here after further. The nonforward scattering problem will be left for a future publication.

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APPENDIX A: EVALUATION OF EQ. (2.12)

Before evaluating the integral appearing in our trace approximation, we consider first the integral

$$I_1 \equiv \int_0^\infty \frac{e^{-(\lambda-1)\eta}}{e^\eta+b}d\eta = \int_0^1 \frac{x^{\lambda-1}}{1+bx}dx, \quad (A1)$$

where $x=e^{-\eta}$ has been used.

If $|b|<1$, we can expand $1/(1+bx)$ as a power series in x, and we end up with

$$I_1 = \sum_{\gamma=0}^{\infty} (-)^\gamma \frac{b^\gamma}{\lambda+\gamma}, \quad |b|<1. \quad (A2)$$

If $b>1$, we use Cauchy's theorem to evaluate I_1. The contour we choose is shown in Fig. 1. We obtain

$$\int_0^1 \frac{x^{\lambda-1}}{1+bx}dx[1-e^{2\pi i(\lambda-1)}] + \int_0^{2\pi} \frac{(e^{i\theta})^{\lambda-1}}{1+be^{i\theta}} ie^{i\theta}d\theta = 2\pi i \frac{1}{b}\left(\frac{1}{b}\right)^{\lambda-1} e^{i\pi(\lambda-1)}. \quad (A3)$$

Now expand

$$\frac{1}{1+be^{i\theta}} = \frac{1}{be^{i\theta}}\left(\frac{1}{1+b^{-1}e^{-i\theta}}\right)$$

in a power series of $b^{-1}e^{-i\theta}$. From Eq. (A3), we obtain

$$I_1 = \frac{1}{b}\sum_{r=0}^{\infty}(-)^r \frac{1}{b^r}\frac{1}{\lambda-1-r} + \left(\frac{1}{b}\right)^\lambda \frac{\pi}{\sin\pi\lambda}, \quad b>1. \quad (A4)$$

Combining Eqs. (A2) and (A4), one obtains

$$\int_0^\infty \frac{e^{-\lambda\eta}d\eta}{\cosh\eta+\cosh\alpha} = \frac{1}{\sinh\alpha}\left(\frac{1}{\lambda} - e^{-\lambda\alpha}\frac{\pi}{\sin\pi\lambda} + \sum_{r=1}^{\infty}(-e^{-\alpha})^r \frac{2\lambda}{\lambda^2-r^2}\right). \quad (A5)$$

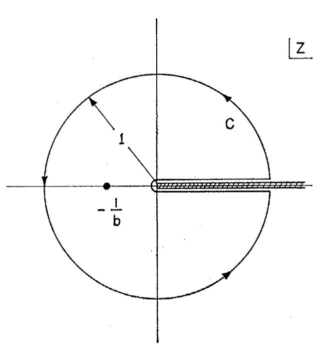

FIG. 1. Contour of integration in the complex z plane. The branch cut for $z^{\lambda-1}$ is along the positive real axis.

Therefore,

$$\int_{\eta=0}^\infty d(\cosh\eta)\frac{e^{-(\lambda+1)\eta}}{(\cosh\eta+\cosh\alpha)^2}$$
$$= \int_{\eta=0}^\infty e^{-(\lambda+1)\eta} d\left(\frac{-1}{\cosh\eta+\cosh\alpha}\right)$$
$$= \frac{-1}{\cosh\eta+\cosh\alpha}e^{-(\lambda+1)\eta}\Big|_0^\infty - \int_{\eta=0}^\infty \frac{-1}{\cosh\eta+\cosh\alpha}de^{-(\lambda+1)\eta}$$
$$= \frac{1}{1+\cosh\alpha} - \frac{\lambda+1}{\sinh\alpha}\left(\frac{1}{\lambda+1} + e^{-(\lambda+1)\alpha}\frac{\pi}{\sin\pi\lambda} + \sum_{r=1}^{\infty}(-e^{-\alpha})^r\frac{2(\lambda+1)}{(\lambda+1)^2-r^2}\right). \quad (A6)$$

Using Eq. (A6) and upon identifying $\cosh\alpha=-1+s/2m^2$, we immediately find the desired result, which in the limit of large $s/2m^2$ becomes

$$\int_{\eta=0}^\infty d(\cosh\eta)\frac{e^{-(\lambda+1)\eta}}{[1+(2m^2/s)(\cosh\eta-1)]^2}$$
$$= \frac{1}{\lambda(\lambda+2)} - \frac{\pi(\lambda+1)}{2\sin\pi\lambda}\left(\frac{m^2}{s}\right)^\lambda + \frac{12}{\lambda(\lambda+2)(\lambda+3)(1-\lambda)}\frac{m^2}{s}$$
$$+(\text{higher-order terms in } m^2/s), \quad \text{for } \operatorname{Re}\lambda>-2, \quad (A7)$$

which is the result we used in Eq. (2.10).

APPENDIX B: COMPLEX REGGE POLES IN LIMIT $m^2/m_\rho^2 \to 0$

In this appendix we consider the limit of Eq. (3.2) as $m^2/m_\rho^2 \to 0$. Equation (3.2) is

$$\frac{1}{2R_I} = \frac{1}{\lambda(\lambda+1)(\lambda+2)} - \frac{\pi}{2\sin\pi\lambda}\left(\frac{m^2}{m_\rho^2}\right)^\lambda + \frac{12}{\lambda(\lambda+1)(\lambda+2)(\lambda+3)(1-\lambda)}\frac{m^2}{m_\rho^2}. \quad (3.2)$$

It is easy to see that for $\operatorname{Re}\lambda>0$, Eq. (3.2) becomes

$$\frac{1}{2R_I} = \frac{1}{\lambda(\lambda+1)(\lambda+2)} \quad \text{as} \quad m^2/m_\rho^2 \to 0.$$

In the following we are interested in Eq. (3.2) only for $\mathrm{Re}\lambda<0$. Let us define

$$\beta\equiv\ln(m_\rho^2/m^2)\,. \tag{B1}$$

We are interested in the large-β behavior of Eq. (3.2). It is convenient to introduce $\xi=\lambda\beta$. Equation (3.2) becomes

$$\frac{1}{2R_I}=\frac{1}{2(\xi/\beta)(1+\xi/\beta)(1+\xi/2\beta)}-\frac{\pi}{2\sin(\pi\xi/\beta)}e^{-\xi}+\frac{12e^{-\beta}}{(\xi/\beta)(1+\xi/\beta)(2+\xi/\beta)(3+\xi/\beta)(1-\xi/\beta)}$$

or

$$\frac{1}{R_I}\left(\frac{\xi}{\beta}\right)=\frac{1}{(1+\xi/\beta)(1+\xi/2\beta)}-\frac{\pi\xi/\beta}{\sin(\pi\xi/\beta)}e^{-\xi}+\frac{24e^{-\beta}}{(1+\xi/\beta)(2+\xi/\beta)(3+\xi/\beta)(1-\xi/\beta)}. \tag{B2}$$

We expand (B2) in a power series of ξ/β around the point zero and keep only the leading terms in $1/\beta$ (because eventually we will let β approach infinity). We obtain

$$\frac{1-e^{-\xi}}{\xi}=\frac{1}{\beta}\left(\frac{1}{R_I}+\frac{3}{2}\right). \tag{B3}$$

The solutions of Eq. (B3) can be found easily in the limit of large β. They are

$$\mathrm{Re}\lambda=-\frac{1}{2}(2n\pi)^2\frac{1}{\beta^3}\left(\frac{1}{R_I}+\frac{3}{2}\right)^2+\cdots, \tag{B4}$$

$$\mathrm{Im}\lambda=\pm(2n\pi)(1/\beta)+\cdots,$$

where $\cdots$ denotes higher-order terms in $1/\beta$ and n is any positive integer. It is clear that for large $\beta\equiv\ln(m_\rho^2/m^2)$, $\mathrm{Re}\lambda$ can be larger than $-\frac{1}{2}$.

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Unit-Spin Propagation Functions and Form Factors*

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Source theory is used to derive a representation for the propagation function of a unit-spin meson. The asymptotic behavior for large momenta resembles that observed in electromagnetic form factors.

HADRONIC electromagnetic interactions are usually pictured as proceeding through the intermediary of the known 1^- mesons. In this model, electromagnetic form factors are linearly related to the meson propagation functions. Electron-positron colliding-beam experiments have confirmed the implied resonant behavior. But form-factor measurements for large momentum transfer seem to be at variance with the idea. Instead of the simple asymptotic dependence on momentum transfer, $1/p^2$, experiment indicates a more rapid decrease, $(1/p^2)^d$, $d\gtrsim 2$. It is therefore particularly significant that the modified propagation functions derived from source theory[1] do show an asymptotic decrease that is at least as rapid as $(1/p^2)^2$, and can approach $(1/p^2)^3$.

To sketch how this comes about, consider ρ^0 for definiteness. The associated vector and tensor fields are designated as ρ_μ and $\rho_{\mu\nu}$, while the vectorial source is J_μ. The initial description of this particle, appropriate to such time intervals that its instability is not in evidence, is given by the vacuum amplitude expression

$$\langle 0_+|0_-\rangle^J=\exp[iW(J)]\,, \tag{1}$$

where, stated in momentum space for convenience,

$$W(J)=\int\frac{(dp)}{(2\pi)^4}[\rho^\mu(-p)J_\mu(p)-\tfrac{1}{4}\rho^{\mu\nu}(-p)\rho_{\mu\nu}(p)-\tfrac{1}{2}m^2\rho^\mu(-p)\rho_\mu(p)], \tag{2}$$

$$\rho_{\mu\nu}(p)=ip_\mu\rho_\nu(p)-ip_\nu\rho_\mu(p)\,.$$

The action principle supplies the field equation

$$ip_\nu\rho^{\mu\nu}(p)+m^2\rho^\mu(p)=J^\mu(p)\,, \tag{3}$$

which can be analyzed into longitudinal and transverse components relative to the momentum vector, as indicated symbolically by

$$\rho=\rho_T+\rho_L=\left(1-\frac{pp}{p^2}\right)\rho+\frac{pp}{p^2}\rho. \tag{4}$$

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[1] Various aspects of source theory are described by J. Schwanger in (a) *Particles and Sources* (Gordon and Breach, New York, 1969), and (b) *Particles, Sources, and Fields* (Addison-Wesley, Reading, Mass., 1970).