Nonlinear Hadron Couplings from Divergence Conditions. II. ϱ Mesons, Pions, and Nucleons

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The interaction of the ρ field with pions and nucleons is investigated by extending our earlier treatment of the pion-nucleon system and imposing the requirement that the source function in the ρ-field equation have a vanishing divergence. This leads to a nonlinear equation for the ρ field without the use of either a broken symmetry or a correspondence between the neutral ρ field and the photon field. It is possible to develop our formalism with as well as without the axial-vector a field. However, the usual condition of partial conservation of axial-vector current (PCAC) is fulfilled only if the ρ field is accompanied by the a field, while a modified PCAC condition is found to hold in the absence of the a field.

I. INTRODUCTION

IN an earlier paper[1] it was shown that the nonlinear chiral-dynamical models for pion-nucleon couplings can be obtained simply by using an appropriate divergence condition for the pion-field equation. We shall now show that the interaction of the ρ field with other fields can also be treated by imposing a simple divergence condition which leads to a nonlinear equation for the ρ field. We would like to emphasize that *we shall not use either the broken chiral symmetry*[2,3] *or the broken Yang-Mills symmetry*[4] *or a correspondence between the neutral ρ field and the photon field*,[5] but shall derive our results solely by the use of the divergence condition described below.

It is well known that the free-field equation for ρ mesons is

$$\partial_\mu(\partial_\mu \varrho_\nu - \partial_\nu \varrho_\mu) - m_\rho^2 \varrho_\nu = 0. \quad (1.1)$$

We shall postulate that in the presence of interaction the ρ-field equation takes the form

$$\partial_\mu(\partial_\mu \varrho_\nu - \partial_\nu \varrho_\mu) - m_\rho^2 \varrho_\nu = -\mathbf{J}_\nu, \quad (1.2)$$

where the source function $\mathbf{J}_\nu$ satisfies the divergence condition

$$\partial_\nu \mathbf{J}_\nu = 0. \quad (1.3)$$

Note that the field equation (1.2) can be decomposed with the help of (1.3) into

$$(\Box^2 - m_\rho^2)\varrho_\nu = -\mathbf{J}_\nu, \quad (1.4)$$

$$\partial_\nu \varrho_\nu = 0, \quad (1.5)$$

so that (1.5) can be regarded as an alternative form of the divergence condition.

In Sec. II we shall treat the ρ-N system of fields by using the divergence condition (1.3), while Secs. III and IV will deal with the ρ-π-N system. As we point out in Sec. V, the axial-vector a field is not necessary in our formalism, but this field too can be treated by means of a divergence condition. The formalism of the PCAC condition with as well as without the a field will also be discussed.

At present the experimental situation regarding the a mesons is not clear.[6] If it is found that a mesons do not exist, it will create a serious difficulty in the chiral-dynamical approach,[7] while it will not give rise to any difficulty in our approach. On the other hand, if it is established that a mesons do exist, both approaches could be used, but our approach will have the advantage of combining the features of chiral dynamics and the PCAC condition by employing only the divergence conditions.

We shall follow the same notation as in Ref. 1 except that the pion coupling constants g and f will be denoted by g_π and f_π to distinguish them from the ρ coupling constant g_ρ.

II. LAGRANGIAN FORMALISM FOR NONLINEAR ϱ FIELD

We shall consider the Lagrangian formalism for the interaction of ρ mesons first with nucleons and then with an arbitrary field. The present approach is similar to that of Lee and Zumino[5] except that we shall derive the self-interaction terms for the ρ field without postulating any correspondence between the neutral ρ field and the electromagnetic field.

The linear Lagrangian density for a system of ρ mesons and nucleons with the simplest ρ-N coupling is given by

$$L_{\text{linear}} = -\tfrac{1}{4}(\partial_\mu \varrho_\nu - \partial_\nu \varrho_\mu)^2 - \tfrac{1}{2}m_\rho^2 \varrho_\mu^2 - \bar{N}(\gamma\cdot\partial + M)N + \tfrac{1}{2}g_\rho \bar{N} i\gamma_\mu \boldsymbol{\tau}\cdot\varrho_\mu N, \quad (2.1)$$

and it can be easily ascertained that the source function in the resulting ρ-field equation does not satisfy (1.3). Let us, therefore, try to obtain the desired form of the Lagrangian density by introducing nonlinearity in the

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[5] T. D. Lee and B. Zumino, Phys. Rev. **163**, 1667 (1967).
[6] Particle Data Group, Rev. Mod. Phys. **42**, 87 (1970).
[7] See, especially, Sec. VI of Gasiorowicz and Geffen, Ref. 3.

ρ field through the replacement of (2.1) by

$$L=-\tfrac{1}{4}(\partial_\mu \boldsymbol{\varrho}_\nu-\partial_\nu \boldsymbol{\varrho}_\mu)^2-\tfrac{1}{2}m_\rho^2\boldsymbol{\varrho}_\mu^2+g_1(\boldsymbol{\varrho}_\mu\times\boldsymbol{\varrho}_\nu)\cdot(\partial_\mu\boldsymbol{\varrho}_\nu-\partial_\nu\boldsymbol{\varrho}_\mu)+g_2(\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\varrho}_\mu)(\boldsymbol{\varrho}_\nu\cdot\boldsymbol{\varrho}_\nu)+g_3(\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\varrho}_\nu)(\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\varrho}_\nu)-\bar{N}(\gamma\cdot\partial+M)N+\tfrac{1}{2}g_\rho\bar{N}i\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\varrho}_\mu N, \quad (2.2)$$

which includes the simplest isospin-invariant self-interaction terms that are trilinear and quadrilinear in the ρ field.

According to (2.2), the nucleon-field equations are

$$\gamma_\mu\partial_\mu N+MN=\tfrac{1}{2}g_\rho i\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\varrho}_\mu N,\qquad \partial_\mu\bar{N}\gamma_\mu-M\bar{N}=-\tfrac{1}{2}g_\rho\bar{N}i\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\varrho}_\mu, \quad (2.3)$$

while the ρ-field equation can be put in the form

$$\partial_\mu(\partial_\mu\boldsymbol{\varrho}_\nu-\partial_\nu\boldsymbol{\varrho}_\mu)-m_\rho^2\boldsymbol{\varrho}_\nu=-\mathbf{J}_\nu, \quad (2.4)$$

with

$$\mathbf{J}_\nu=-2g_1\boldsymbol{\varrho}_\mu\times(\partial_\mu\boldsymbol{\varrho}_\nu-\partial_\nu\boldsymbol{\varrho}_\mu)-2g_1\partial_\mu(\boldsymbol{\varrho}_\mu\times\boldsymbol{\varrho}_\nu)+4g_2(\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\varrho}_\mu)\boldsymbol{\varrho}_\nu+4g_3(\boldsymbol{\varrho}_\nu\cdot\boldsymbol{\varrho}_\mu)\boldsymbol{\varrho}_\mu+\tfrac{1}{2}g_\rho\bar{N}i\gamma_\nu\boldsymbol{\tau}N. \quad (2.5)$$

It follows from (2.5) that

$$\partial_\nu\mathbf{J}_\nu=-2g_1\boldsymbol{\varrho}_\mu\times\partial_\nu(\partial_\mu\boldsymbol{\varrho}_\nu-\partial_\nu\boldsymbol{\varrho}_\mu)+4g_2\partial_\nu[(\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\varrho}_\mu)\boldsymbol{\varrho}_\nu]+4g_3\partial_\nu[(\boldsymbol{\varrho}_\nu\cdot\boldsymbol{\varrho}_\mu)\boldsymbol{\varrho}_\mu]+\tfrac{1}{2}g_\rho\partial_\nu(\bar{N}i\gamma_\nu\boldsymbol{\tau}N)$$

or, with the help of (2.3) and (2.4),

$$\partial_\nu\mathbf{J}_\nu=-2g_1\boldsymbol{\varrho}_\mu\times\mathbf{J}_\mu+4g_2\partial_\nu[(\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\varrho}_\mu)\boldsymbol{\varrho}_\nu]+4g_3\partial_\nu[(\boldsymbol{\varrho}_\nu\cdot\boldsymbol{\varrho}_\mu)\boldsymbol{\varrho}_\mu]+\tfrac{1}{2}g_\rho^2\bar{N}i\gamma_\nu\boldsymbol{\tau}\times\boldsymbol{\varrho}_\nu N. \quad (2.6)$$

Moreover, since (2.5) also yields

$$\boldsymbol{\varrho}_\nu\times\mathbf{J}_\nu=-2g_1\partial_\nu[(\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\varrho}_\mu)\boldsymbol{\varrho}_\nu-(\boldsymbol{\varrho}_\nu\cdot\boldsymbol{\varrho}_\mu)\boldsymbol{\varrho}_\mu]-\tfrac{1}{2}g_\rho\bar{N}i\gamma_\nu\boldsymbol{\tau}\times\boldsymbol{\varrho}_\nu N,$$

(2.6) can be simplified as

$$\partial_\nu\mathbf{J}_\nu=4(g_1^2+g_2)\partial_\nu[(\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\varrho}_\mu)\boldsymbol{\varrho}_\nu]-4(g_1^2-g_3)\partial_\nu[(\boldsymbol{\varrho}_\nu\cdot\boldsymbol{\varrho}_\mu)\boldsymbol{\varrho}_\mu]+g_\rho(g_1+\tfrac{1}{2}g_\rho)\bar{N}i\gamma_\nu\boldsymbol{\tau}\times\boldsymbol{\varrho}_\nu N. \quad (2.7)$$

Evidently, the divergence condition (1.3) can be fulfilled provided that the coefficient of each of the three terms on the right-hand side of (2.7) vanishes, which gives

$$g_1=-\tfrac{1}{2}g_\rho,\quad g_2=-\tfrac{1}{4}g_\rho^2,\quad g_3=\tfrac{1}{4}g_\rho^2. \quad (2.8)$$

The Lagrangian density, obtained by substituting (2.8) into (2.2), can be put in the compact form

$$L=-\tfrac{1}{4}\boldsymbol{\varrho}_{\mu\nu}^2-\tfrac{1}{2}m_\rho^2\boldsymbol{\varrho}_\mu^2-\bar{N}(\gamma_\mu D_\mu+M)N, \quad (2.9)$$

where

$$\boldsymbol{\varrho}_{\mu\nu}=\partial_\mu\boldsymbol{\varrho}_\nu-\partial_\nu\boldsymbol{\varrho}_\mu+g_\rho\boldsymbol{\varrho}_\mu\times\boldsymbol{\varrho}_\nu \quad (2.10)$$

and

$$D_\mu=\partial_\mu-\tfrac{1}{2}ig_\rho\boldsymbol{\tau}\cdot\boldsymbol{\varrho}_\mu. \quad (2.11)$$

The above result suggests an obvious generalization for the treatment of the interaction of the ρ field with an arbitrary isofield represented by a one-column matrix ψ. Consider an infinitesimal isospin transformation

$$\delta\psi=ic_iT_i\psi, \quad (2.12)$$

where c_i are infinitesimal real constants and T_i are Hermitian matrices representing the isospin generators acting on ψ. It is well known, for instance, that

$$T_i=\tfrac{1}{2}\tau_i \quad (2.13)$$

for an isospin-$\frac{1}{2}$ field, while the matrix elements of T_i are

$$T_{i,jk}=-i\epsilon_{ijk} \quad (2.14)$$

for an isospin-1 field. We take the Lagrangian density for the ρ field interacting with the isofield ψ as[8]

$$L=L_1+L_2(\psi,D_\mu\psi), \quad (2.15)$$

where

$$L_1=-\tfrac{1}{4}\boldsymbol{\varrho}_{\mu\nu}^2-\tfrac{1}{2}m_\rho^2\boldsymbol{\varrho}_\mu^2, \quad (2.16)$$

and L_2 is obtained from the Lagrangian density of the free ψ field by replacing ∂_μ by the covariant derivative D_μ, defined as

$$D_\mu=\partial_\mu-ig_\rho\mathbf{T}\cdot\boldsymbol{\varrho}_\mu. \quad (2.17)$$

The invariance of (2.15) under the isospin transformations

$$\delta\rho_{\nu,j}=c_i\epsilon_{ijk}\rho_{\nu,k},\quad \delta\psi=ic_iT_i\psi, \quad (2.18)$$

gives the conserved isospin current

$$S_{\mu,i}=-g_\rho\left[\frac{\partial L}{\partial(\partial_\mu\rho_{\nu,j})}\epsilon_{ijk}\rho_{\nu,k}+i\frac{\partial L}{\partial(\partial_\mu\psi)}T_i\psi\right]$$

or

$$\mathbf{S}_\mu=-g_\rho\left[\boldsymbol{\varrho}_\nu\times\boldsymbol{\varrho}_{\mu\nu}+i\frac{\partial L_2}{\partial(D_\mu\psi)}\mathbf{T}\psi\right], \quad (2.19)$$

while the ρ-field equation resulting from (2.15) can be expressed in terms of the above isospin current as

$$\partial_\mu(\partial_\mu\boldsymbol{\varrho}_\nu-\partial_\nu\boldsymbol{\varrho}_\mu)-m_\rho^2\boldsymbol{\varrho}_\nu=-[\mathbf{S}_\nu+g_\rho\partial_\mu(\boldsymbol{\varrho}_\mu\times\boldsymbol{\varrho}_\nu)]. \quad (2.20)$$

The source function

$$\mathbf{J}_\nu=\mathbf{S}_\nu+g_\rho\partial_\mu(\boldsymbol{\varrho}_\mu\times\boldsymbol{\varrho}_\nu) \quad (2.21)$$

in (2.20) satisfies the divergence condition

$$\partial_\nu\mathbf{J}_\nu=\partial_\nu\mathbf{S}_\nu=0. \quad (2.22)$$

The ρ-field equation given by (2.20) and (2.19) can also be put in the alternative form

$$D_\mu\boldsymbol{\varrho}_{\mu\nu}-m_\rho^2\boldsymbol{\varrho}_\nu=-\boldsymbol{\mathfrak{J}}_\nu, \quad (2.23)$$

where $\boldsymbol{\mathfrak{J}}_\nu$, the ψ-dependent part of $\mathbf{J}_\nu$, is given by

$$\boldsymbol{\mathfrak{J}}_\nu=-ig_\rho\left[\frac{\partial L_2}{\partial(D_\mu\psi)}\right]\mathbf{T}\psi \quad (2.24)$$

and satisfies the covariant divergence condition

$$D_\nu\boldsymbol{\mathfrak{J}}_\nu=0. \quad (2.25)$$

III. ρ-π-N SYSTEM

It will be convenient for the treatment of the ρ-π-N system to establish an important property of the

[8] We treat ψ as a real field; the extension of the formalism to a complex field is self-evident.

covariant derivative of a product of isofields. Let ψ be an isoquantity that is a product of n isofields $\psi^{(1)},\psi^{(2)},\ldots,\psi^{(n)}$. Under an arbitrary infinitesimal isospin transformation,

$$\psi=\psi^{(1)}\psi^{(2)}\cdots\psi^{(n)} \tag{3.1}$$

yields

$$\begin{aligned}ic_iT_i\psi=(ic_iT_i^{(1)}\psi^{(1)})\psi^{(2)}\cdots\psi^{(n)}\\+\psi^{(1)}(ic_iT_i^{(2)}\psi^{(2)})\cdots\psi^{(n)}+\cdots\\+\psi^{(1)}\psi^{(2)}\cdots(ic_iT_i^{(n)}\psi^{(n)}),\end{aligned} \tag{3.2}$$

where the matrices $T_i^{(r)}$ represent the isospin generators acting on $\psi^{(r)}$. By dropping the arbitrary parameters c_i in (3.2) and multiplying by $\rho_{\mu,i}$, we obtain

$$\begin{aligned}i\mathbf{T}\cdot\boldsymbol{\rho}_\mu\psi=(i\mathbf{T}^{(1)}\cdot\boldsymbol{\rho}_\mu\psi^{(1)})\psi^{(2)}\cdots\psi^{(n)}\\+\psi^{(1)}(i\mathbf{T}^{(2)}\cdot\boldsymbol{\rho}_\mu\psi^{(2)})\cdots\psi^{(n)}+\cdots\\+\psi^{(1)}\psi^{(2)}\cdots(i\mathbf{T}^{(n)}\cdot\boldsymbol{\rho}_\mu\psi^{(n)}),\end{aligned}$$

which, together with the relation

$$\begin{aligned}\partial_\mu\psi=(\partial_\mu\psi^{(1)})\psi^{(2)}\cdots\psi^{(n)}+\psi^{(1)}(\partial_\mu\psi^{(2)})\cdots\psi^{(n)}+\cdots\\+\psi^{(1)}\psi^{(2)}\cdots(\partial_\mu\psi^{(n)}),\end{aligned}$$

gives

$$\begin{aligned}D_\mu\psi=(D_\mu\psi^{(1)})\psi^{(2)}\cdots\psi^{(n)}+\psi^{(1)}(D_\mu\psi^{(2)})\cdots\psi^{(n)}+\cdots\\+\psi^{(1)}\psi^{(2)}\cdots(D_\mu\psi^{(n)}),\end{aligned} \tag{3.3}$$

where

$$D_\mu\psi^{(n)}=(\partial_\mu-ig_\rho\mathbf{T}^{(n)}\cdot\boldsymbol{\rho}_\mu)\psi^{(n)}. \tag{3.4}$$

We observe in particular that the covariant derivative (3.4) gives, in view of (2.13) and (2.14),

$$\begin{aligned}D_\mu N&=\partial_\mu N-\tfrac{1}{2}ig_\rho\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu N,\\D_\mu\bar{N}&=\partial_\mu\bar{N}+\tfrac{1}{2}ig_\rho\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu,\\D_\mu\boldsymbol{\pi}&=\partial_\mu\boldsymbol{\pi}+g_\rho\boldsymbol{\rho}_\mu\times\boldsymbol{\pi},\end{aligned} \tag{3.5}$$

while $D_\mu=\partial_\mu$ for an isoscalar field. Similar relations apply to the covariant derivatives of isospinor, isovector, and isoscalar quantities consisting of products of field operators.

By following the procedure of Sec. II, the Lagrangian density for the ρ-π-N system can be expressed as

$$L=L_1+L_2, \tag{3.6}$$

where L_1 is given by (2.16) and L_2 is obtained from the Lagrangian density of the π-N system by replacing ∂_μ by D_μ. Thus,

$$L_2=L_{0\pi}'+L_{0N}'+L_{\pi\pi}'+L_{\pi N}', \tag{3.7}$$

with

$$\begin{aligned}L_{0\pi}'&=-\tfrac{1}{2}(D_\mu\boldsymbol{\pi}\cdot D_\mu\boldsymbol{\pi}+m_\pi^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}),\\L_{0N}'&=-\bar{N}(\gamma\cdot D+M)N,\\L_{\pi\pi}'&=a(\boldsymbol{\pi}^2)+b(\boldsymbol{\pi}^2)D_\mu\boldsymbol{\pi}\cdot D_\mu\boldsymbol{\pi}+c(\boldsymbol{\pi}^2)(\boldsymbol{\pi}\cdot D_\mu\boldsymbol{\pi})^2,\\L_{\pi N}'&=\alpha_1(\boldsymbol{\pi}^2)\bar{N}i\gamma_\mu\gamma_5\boldsymbol{\tau}\cdot D_\mu\boldsymbol{\pi}N\\&\quad+\alpha_2(\boldsymbol{\pi}^2)\bar{N}i\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\pi}\times D_\mu\boldsymbol{\pi}N\\&\quad+\alpha_3(\boldsymbol{\pi}^2)(\boldsymbol{\pi}\cdot D_\mu\boldsymbol{\pi})\bar{N}i\gamma_\mu\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}N,\end{aligned} \tag{3.8}$$

according to the general derivative π-N coupling of our earlier paper,[1] where the functions $a(\boldsymbol{\pi}^2)$, $b(\boldsymbol{\pi}^2)$, $c(\boldsymbol{\pi}^2)$, $\alpha_1(\boldsymbol{\pi}^2)$, $\alpha_2(\boldsymbol{\pi}^2)$, and $\alpha_3(\boldsymbol{\pi}^2)$ have been given explicitly for various special models. In the case of the nonderivative π-N coupling, L_2 takes a similar but simpler form.

The above Lagrangian density yields the same field equations for pions and nucleons as in the absence of the ρ field except that now D_μ appears in place of ∂_μ. Indeed, in view of the property (3.3) of the covariant derivative D_μ, the entire treatment of the π-N system in Ref. 1 is applicable here on replacing ∂_μ by D_μ everywhere.[9] It is especially important to recall that in the absence of the ρ field the pion-field equation is expressible in the form[1]

$$(\Box^2-m_\pi^2)\boldsymbol{\pi}=\partial_\mu\mathbf{J}_{\mu5}, \tag{3.9}$$

where the source function is a complete divergence. The corresponding pion-field equation in the presence of the ρ field is given by the modification of (3.9) as

$$(D_\mu^2-m_\pi^2)\boldsymbol{\pi}=D_\mu\mathbf{J}_{\mu5}' \tag{3.10}$$

or

$$\begin{aligned}[(\partial_\mu+g_\rho\boldsymbol{\rho}_\mu\times)(\partial_\mu+g_\rho\boldsymbol{\rho}_\mu\times)-m_\pi^2]\boldsymbol{\pi}\\=(\partial_\mu+g_\rho\boldsymbol{\rho}_\mu\times)\mathbf{J}_{\mu5}',\end{aligned} \tag{3.11}$$

where $\mathbf{J}_{\mu5}'$ is obtained from $\mathbf{J}_{\mu5}$ by the replacement of ∂_μ by D_μ. Thus, the pion-field equation maintains an elegant form in the presence of the ρ field, but the source function is no longer expressible as a complete divergence. The conventional PCAC condition also breaks down in the presence of the ρ field because (3.10) gives the modified PCAC condition

$$D_\mu\mathbf{J}_{\mu5}''=-m^2\boldsymbol{\pi} \tag{3.12}$$

or

$$\partial_\mu\mathbf{J}_{\mu5}''=-m^2\boldsymbol{\pi}-g_\rho\boldsymbol{\rho}_\mu\times\mathbf{J}_{\mu5}'', \tag{3.13}$$

where

$$\mathbf{J}_{\mu5}''=\mathbf{J}_{\mu5}'-D_\mu\boldsymbol{\pi} \tag{3.14}$$

can be regarded with appropriate normalization as the PCAC current.

The ρ-field equation too can be obtained from (3.7) and (3.8) by using the relations (2.23) and (2.24), which give

$$D_\mu\boldsymbol{\rho}_{\mu\nu}-m_\rho^2\boldsymbol{\rho}_\nu=-\boldsymbol{\mathfrak{J}}_\nu, \tag{3.15}$$

with

$$\begin{aligned}\boldsymbol{\mathfrak{J}}_\nu=(\tfrac{1}{2}+\alpha_2\boldsymbol{\pi}^2)g_\rho\bar{N}i\gamma_\nu\boldsymbol{\tau}N-g_\rho\alpha_1\bar{N}i\gamma_\nu\gamma_5\boldsymbol{\tau}\times\boldsymbol{\pi}N\\-g_\rho\alpha_2\bar{N}i\gamma_\nu(\boldsymbol{\tau}\cdot\boldsymbol{\pi})\boldsymbol{\pi}N-g_\rho(1-2b)\boldsymbol{\pi}\times D_\nu\boldsymbol{\pi}.\end{aligned} \tag{3.16}$$

Substitution of the values of the functions α_1, α_2, and b,

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[9] There are no complications resulting from the fact that D_μ and D_ν do not commute since in the treatment under consideration the second covariant derivatives occur only in the form $D_\mu D_\mu$.

given in Ref. 1, shows that

$$\mathfrak{J}_\nu = \tfrac{1}{2}g_\rho\{\cos(2f_\pi\sqrt{\boldsymbol{\pi}^2})\bar{N}i\gamma_\nu\boldsymbol{\tau}N - (g_\pi/f_\pi\sqrt{\boldsymbol{\pi}^2})\sin(2f_\pi\sqrt{\boldsymbol{\pi}^2})\bar{N}i\gamma_\nu\gamma_5\boldsymbol{\tau}\times\boldsymbol{\pi}N + (1/\boldsymbol{\pi}^2)[1-\cos(2f_\pi\sqrt{\boldsymbol{\pi}^2})]\bar{N}i\gamma_\nu(\boldsymbol{\tau}\cdot\boldsymbol{\pi})\boldsymbol{\pi}N - (1/2f_\pi^2\boldsymbol{\pi}^2)\sin^2(2f_\pi\sqrt{\boldsymbol{\pi}^2})\boldsymbol{\pi}\times D_\nu\boldsymbol{\pi}\} \quad (3.17)$$

for Model A',

$$\mathfrak{J}_\nu = \tfrac{1}{2}g_\rho\{(1-4f_\pi^2\boldsymbol{\pi}^2)^{1/2}\bar{N}i\gamma_\nu\boldsymbol{\tau}N - 2g_\pi\bar{N}i\gamma_\nu\gamma_5\boldsymbol{\tau}\times\boldsymbol{\pi}N + (1/\boldsymbol{\pi}^2)[1-(1-4f_\pi^2\boldsymbol{\pi}^2)^{1/2}]\bar{N}i\gamma_\nu(\boldsymbol{\tau}\cdot\boldsymbol{\pi})\boldsymbol{\pi}N - 2\boldsymbol{\pi}\times D_\nu\boldsymbol{\pi}\} \quad (3.18)$$

for Model B', and

$$\mathfrak{J}_\nu = \tfrac{1}{2}g_\rho\{[2(1+f_\pi^2\boldsymbol{\pi}^2)^{-1}-1]\bar{N}i\gamma_\nu\boldsymbol{\tau}N - 2g_\pi(1+f_\pi^2\boldsymbol{\pi}^2)^{-1}\bar{N}i\gamma_\nu\gamma_5\boldsymbol{\tau}\times\boldsymbol{\pi}N + 2f_\pi^2(1+f_\pi^2\boldsymbol{\pi}^2)^{-1}\bar{N}i\gamma_\nu(\boldsymbol{\tau}\cdot\boldsymbol{\pi})\boldsymbol{\pi}N - 2(1+f_\pi^2\boldsymbol{\pi}^2)^{-2}\boldsymbol{\pi}\times D_\nu\boldsymbol{\pi}\} \quad (3.19)$$

for Model C'. The terms involving the nucleon field in (3.17)–(3.19) should be replaced by $\frac{1}{2}g_\rho\bar{N}i\gamma_\nu\boldsymbol{\tau}N$ for the simpler nonderivative π-N-coupling models A, B, and C.

IV. TRANSFORMATION OF π-N COUPLING IN PRESENCE OF ρ FIELD

It is known[10] that the nonderivative π-N coupling in the nonlinear models can be transformed into the derivative π-N coupling. Thus, the Lagrangian density for the π-N system, given by

$$L_{\pi+N} = -\tfrac{1}{2}(\partial_\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}+m_\pi^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}) - \bar{N}(\gamma\cdot\partial+M)N - 2f_\pi M\bar{N}i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}N + O(f_\pi^2), \quad (4.1)$$

can also be expressed as

$$L_{\pi+N}' = -\tfrac{1}{2}(\partial_\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}+m_\pi^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}) - \bar{N}(\gamma\cdot\partial+M)N + f_\pi\bar{N}i\gamma_\mu\gamma_5\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi}N + O(f_\pi^2). \quad (4.2)$$

When the ρ-field interaction is introduced into the π-N system through the replacement of ∂_μ by D_μ, then (4.1) yields the coupling terms

$$L_{\rm int} = -g_\rho\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi} + \tfrac{1}{2}g_\rho\bar{N}i\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\varrho}_\mu N - 2f_\pi M\bar{N}i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}N + O(f_\pi^2,g_\rho^2), \quad (4.3)$$

while (4.2) yields

$$L_{\rm int}' = -g_\rho\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi} + \tfrac{1}{2}g_\rho\bar{N}i\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\varrho}_\mu N + f_\pi\bar{N}i\gamma_\mu\gamma_5\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi}N + f_\pi g_\rho\bar{N}i\gamma_\mu\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\varrho}_\mu\times\boldsymbol{\pi}N + O(f_\pi^2,g_\pi^2). \quad (4.4)$$

Although (4.3) and (4.4) are expected to be equivalent on general theoretical grounds, such equivalence is somewhat surprising in view of the fact that the ρ-field interaction specifically depends on the appearance of the derivatives in the Lagrangian density. It would, therefore, be reassuring to verify by an appropriate application that (4.3) and (4.4) lead to the same result. For this purpose, we shall consider the scattering process $\pi+N\to\rho+N$ described by the diagrams in Fig. 1, where p and p' denote the propagation four-vectors of the initial and final nucleons, and k and k' denote those of the π and ρ mesons, respectively. It should be observed that the coupling terms (4.3) and (4.4) do not lead to identical diagrams, because Fig. 1(d) can arise only from (4.4).

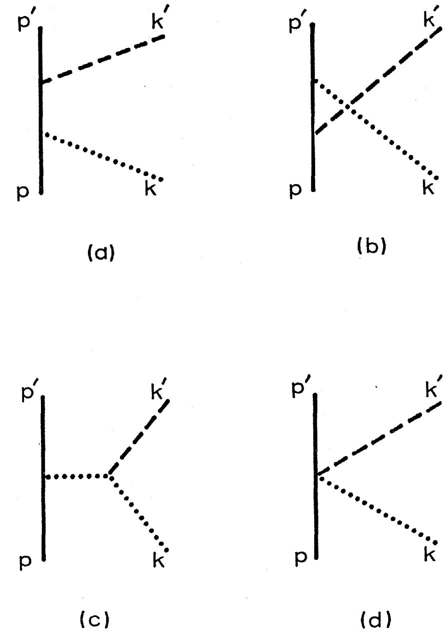

FIG. 1. Second-order diagrams for scattering process $\pi+N\to\rho+N$.

According to (4.3), the scattering operator for Figs. 1(a)–1(c) is

$$S_2 = -i(2\pi)^4\delta(p+k-p'-k')f_\pi g_\rho\bar{N}^-(\mathbf{p}') \times[A+B+C]N^+(\mathbf{p})\rho_{\mu,i}^-(\mathbf{k}')\pi_j^+(\mathbf{k}), \quad (4.5)$$

where, after simplification with the help of the relations

$$\bar{N}^-(\mathbf{p}')(i\gamma\cdot p'+M)=0, \quad (i\gamma\cdot p+M)N^+(\mathbf{p})=0, \quad k_\mu'\rho_{\mu,i}^-(\mathbf{k}')=0, \quad (4.6)$$

we find

$$A = \tau_i\tau_j M\gamma_5(2ip_\mu'-\sigma_{\mu\nu}k_\nu')/(k^2+2k\cdot p), \quad B = \tau_j\tau_i M\gamma_5(2ip_\mu-\sigma_{\mu\nu}k_\nu')/(k^2-2k\cdot p'), \quad C = 4\epsilon_{ijk}\tau_k M\gamma_5(p_\mu-p_\mu')/[(p'-p)^2+m_\pi^2]. \quad (4.7)$$

Similarly according to (4.4), the scattering operator for Figs. 1(a)–1(d) is found to be

$$S_2' = -i(2\pi)^4\delta(p+k-p'-k')f_\pi g_\rho\bar{N}^-(\mathbf{p}') \times[A'+B'+C'+D']N^+(\mathbf{p})\rho_{\mu,i}^-(\mathbf{k}')\pi_j^+(\mathbf{k}), \quad (4.8)$$

[10] See S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177, 2239 (1969), and earlier papers quoted there.

where

$$A'=\tau_i\tau_j[-\tfrac{1}{2}\gamma_5\gamma_\mu+M\gamma_5(2ip_\mu'-\sigma_{\mu\nu}k_\nu')/(k^2+2k\cdot p)],$$
$$B'=\tau_j\tau_i[\tfrac{1}{2}\gamma_5\gamma_\mu+M\gamma_5(2ip_\mu-\sigma_{\mu\nu}k_\nu')/(k^2-2k\cdot p')],\qquad(4.9)$$
$$C'=4\epsilon_{ijk}\tau_kM\gamma_5(p_\mu-p_\mu')/[(p'-p)^2+m_\pi^2],$$
$$D'=i\epsilon_{ijk}\tau_k\gamma_5\gamma_\mu.$$

It follows from the above results that

$$S_2'=S_2,\qquad(4.10)$$

which establishes the equivalence of the couplings (4.3) and (4.4) for the process under consideration.

V. AXIAL-VECTOR MESONS

In chiral dynamics an axial-vector a field is introduced along with the vector ρ field and, further, these fields are believed to satisfy the divergence conditions

$$\partial_\mu \boldsymbol{\rho}_\mu=0,\quad \partial_\mu \mathbf{a}_\mu=\text{const}\,\boldsymbol{\pi}\qquad(5.1)$$

in the presence of the pion field. It is possible to include the a field in our formalism by regarding the divergence conditions (5.1) as basic postulates but without employing the broken chiral symmetry. However, while the chiral dynamics demands the existence of the a field, our formalism can accommodate this field but does not necessarily require its inclusion. We shall confine ourselves to a brief consideration of the π-ρ-a system.

By introducing the simplest coupling terms with undetermined coefficients for the interaction of the a field with the π-ρ system and imposing the requirements (5.1), it can be shown that the Lagrangian density for the π-ρ-a system can be expressed as

$$L=-\tfrac{1}{4}(\boldsymbol{\rho}_{\mu\nu}'^2+\mathbf{a}_{\mu\nu}'^2)-\tfrac{1}{2}m_\rho^2(\boldsymbol{\rho}_\mu^2+\mathbf{a}_\mu^2)+\bar{L},\qquad(5.2)$$

where

$$\boldsymbol{\rho}_{\mu\nu}'=\partial_\mu\boldsymbol{\rho}_\nu-\partial_\nu\boldsymbol{\rho}_\mu+g_\rho\boldsymbol{\rho}_\mu\times\boldsymbol{\rho}_\nu+g_\rho\mathbf{a}_\mu\times\mathbf{a}_\nu,$$
$$\mathbf{a}_{\mu\nu}'=\partial_\mu\mathbf{a}_\nu-\partial_\nu\mathbf{a}_\mu+g_\rho\boldsymbol{\rho}_\mu\times\mathbf{a}_\nu+g_\rho\mathbf{a}_\mu\times\boldsymbol{\rho}_\nu,\qquad(5.3)$$

while $\bar{L}$, which depends on the pion models described in Ref. 1, is given by

$$\bar{L}=-\frac{1}{2}\left[\frac{\sin(2f_\pi\sqrt{\boldsymbol{\pi}^2})}{2f_\pi\sqrt{\boldsymbol{\pi}^2}}(\partial_\mu\boldsymbol{\pi}+g_\rho\boldsymbol{\rho}_\mu\times\boldsymbol{\pi})+\frac{g_\rho}{2f_\pi}\cos(2f_\pi\sqrt{\boldsymbol{\pi}^2})\mathbf{a}_\mu\right]^2$$
$$-\frac{1}{2}\left[\left(\frac{1}{\sin(2f_\pi\sqrt{\boldsymbol{\pi}^2})}-\frac{\cos(2f_\pi\sqrt{\boldsymbol{\pi}^2})}{2f_\pi\sqrt{\boldsymbol{\pi}^2}}\right)\frac{(\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi})}{\sqrt{\boldsymbol{\pi}^2}}+g_\rho\frac{\sin(2f_\pi\sqrt{\boldsymbol{\pi}^2})}{2f_\pi\sqrt{\boldsymbol{\pi}^2}}(\boldsymbol{\pi}\cdot\mathbf{a}_\mu)\right]^2$$
$$+\frac{1}{2}\left[\left(\cot(2f_\pi\sqrt{\boldsymbol{\pi}^2})-\frac{1}{2f_\pi\sqrt{\boldsymbol{\pi}^2}}\right)\frac{(\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi})}{\sqrt{\boldsymbol{\pi}^2}}\right]^2-\tfrac{1}{2}m_\pi^2\boldsymbol{\pi}^2\qquad(5.4)$$

for Model A,

$$\bar{L}=-\tfrac{1}{2}[\partial_\mu\boldsymbol{\pi}+g_\rho\boldsymbol{\rho}_\mu\times\boldsymbol{\pi}+(g_\rho/2f_\pi)(1-4f_\pi^2\boldsymbol{\pi}^2)^{1/2}\mathbf{a}_\mu]^2$$
$$-\tfrac{1}{2}[2f_\pi(1-4f_\pi^2\boldsymbol{\pi}^2)^{-1/2}(\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi})+g_\rho(\boldsymbol{\pi}\cdot\mathbf{a}_\mu)]^2$$
$$-(m_\pi^2/4f_\pi^2)[1-(1-4f_\pi^2\boldsymbol{\pi}^2)^{1/2}]\qquad(5.5)$$

for Model B, and

$$\bar{L}=-\tfrac{1}{2}[(1+f_\pi^2\boldsymbol{\pi}^2)^{-1}(\partial_\mu\boldsymbol{\pi}+g_\rho\boldsymbol{\rho}_\mu\times\boldsymbol{\pi})$$
$$+(g_\rho/2f_\pi)(1-f_\pi^2\boldsymbol{\pi}^2)(1+f_\pi^2\boldsymbol{\pi}^2)^{-1}\mathbf{a}_\mu]^2$$
$$-\tfrac{1}{2}[f_\pi(1+f_\pi^2\boldsymbol{\pi}^2)^{-1}(\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi})+g_\rho(1+f_\pi^2\boldsymbol{\pi}^2)^{-1}(\boldsymbol{\pi}\cdot\mathbf{a}_\mu)]^2$$
$$+\tfrac{1}{2}[f_\pi(1+f_\pi^2\boldsymbol{\pi}^2)^{-1}(\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi})]^2$$
$$-(m_\pi^2/2f_\pi^2)\ln(1+f_\pi^2\boldsymbol{\pi}^2)\qquad(5.6)$$

for Model C.

We have explicitly given the results for three different models because only one of these models seems to have been explored previously for the π-ρ-a system.[3] Each model involves a mixing of the a and π fields. Therefore, in order to obtain the coupling terms for physical fields, it is necessary to carry out the decomposition of the a field followed by the renormalization of π, f_π, and m_π in the usual manner.

VI. CONCLUDING REMARKS

We conclude with some comments on the fundamental couplings of the π, N, ρ, and a fields.

We have observed that the nonlinear pion-nucleon couplings in general involve two couplings constant g_π and f_π. The divergence condition for the pion field is unable to yield the relationship between g_π and f_π, but the following two cases seem worthy of notice because of their simplicity:

(a) The case $g_\pi=f_\pi$ has the advantage that the derivative pion-nucleon coupling can be transformed entirely into the nonderivative form. In this case the small difference in the phenomenological values of g_π and f_π for the low-energy π-N scattering can be attributed to renormalization effects.

(b) The case $f_\pi=0$ has the advantage that all models for pion couplings become linear and identical with each other, because $L_{\pi N}$ and $L_{\pi\pi}$ reduce to

$$L_{\pi N}=g_\pi\bar{N}i\gamma_\mu\gamma_5\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi}N,\quad L_{\pi\pi}=0.\qquad(6.1)$$

In this case the low-energy π-N scattering can be accounted for reasonably well by taking into consideration not only the fundamental π-N coupling (6.1) but also the effective ρ-exchange coupling, given in the static limit by

$$L_{\text{eff}}=-\bar{f}_\pi^2\bar{N}i\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}N,\qquad(6.2)$$

where

$$\bar{f}_\pi^2 = g_\rho^2/2m_\rho^2, \tag{6.3}$$

which is known as the KSRF relation.[11]

In the treatment of the interaction of the ρ field with other fields, we have introduced the simplest couplings consistent with the divergence condition for the ρ field. This divergence condition, of course, remains inviolate if for any isofield ψ we introduce additional coupling terms that are functions of $\boldsymbol{\varrho}_{\mu\nu}$, ψ, and $D_\mu\psi$, and therefore it is possible to introduce for the ρ-π-N system additional coupling terms such as

$$f_\rho \boldsymbol{\varrho}_{\mu\nu}\cdot(\bar{N}\sigma_{\mu\nu}\boldsymbol{\tau}N) \quad \text{or} \quad f_\rho' \boldsymbol{\varrho}_{\mu\nu}\cdot(D_\mu\boldsymbol{\pi}\times D_\nu\boldsymbol{\pi}). \tag{6.4}$$

However, in view of the experience in quantum electrodynamics, it seems that these couplings are not likely to be fundamental, although the higher-order effects will undoubtedly generate effective couplings of the above form.

When the ρ field is not accompanied by the a field, we have shown that the usual PCAC condition cannot be fulfilled, and it must be replaced by the covariant PCAC condition (3.12) or (3.13). Since the existence of the a mesons has not been clearly established by experiments, it would be desirable to explore the consequences of the covariant PCAC condition.

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Complex Regge Poles in the Amati-Bertocchi-Fubini-Stanghellini-Tonin Multiperipheral Model*

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The multiperipheral model of Amati, Bertocchi, Fubini, Stanghellini, and Tonin is used to study complex Regge poles. We use the trace approximation to solve the multiperipheral integral equation. The equation determining complex Regge poles for the forward-scattering case is derived. We explicitly solve for the locations of complex Regge poles and discuss their dependence on the pion mass.

I. INTRODUCTION

Multiperipheral models[1,2] are very useful for describing general features of high-energy collisions. It is well known that they predict Regge asymptotic behavior for elastic amplitudes and total cross sections, a constant elasticity, a ln$s$ behavior for the multiplicity, and a small average transverse momentum for secondary particles produced in high-energy collisions.

Another group of useful models for studying Regge behavior are potential models. It is known that, in potential models, complex conjugate pairs of Regge poles may occur at energies below the physical threshold.[3] It is natural to ask whether this phenomenon is also present in a relativistic model like the multiperipheral model.

Recently, using a simplified multiperipheral model, Chew and Snider[4] illustrated the possibility of complex conjugate pairs of Regge poles.

This paper studies the problem of complex Regge poles in a more realistic multiperipheral model. The model we use is the Amati-Bertocchi-Fubini-Stanghellini-Tonin (ABFST) model.[1] We do not attempt to solve the ABFST integral equation exactly. Rather we use an approximation method. The method we adopt is the trace approximation, which has been employed by Chew, Rogers, and Snider[5] (CRS) in the case of the leading real Regge poles. With this approximation, the eigenvalue equation can be written down quite easily. The dependence of the locations of complex Regge poles on various physical quantities becomes transparent. In forward scattering, we explicitly solve for the locations of these complex Regge poles. The results agree with those of Misheloff,[6] who solved the ABFST integral equation exactly by a numerical method.

The plan of the paper is as follows. In Sec. II, we develop the trace approximation for the forward-

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