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Regge Trajectories in a Multichannel Quark Model*

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Meson trajectories are considered in a nonrelativistic quark model. The model, in a specific example involving the ρ trajectory, incorporates a $\pi\pi$ channel and two quark-antiquark $q\bar{q}$ (or nucleon-antinucleon) channels in spin triplet states. A harmonic-oscillator potential provides the attractive $q\bar{q}$ potential, and a Yukawa potential is used in the $\pi\pi$ channel. A short-range potential of exponential type couples the $\pi\pi$ and $q\bar{q}$ channels. The model gives infinitely rising trajectories with resonances of small finite width capable of decaying into pions, but not into quarks. The ρ trajectory deviates considerably from a straight line in the negative-energy region. The model is capable of imposing exchange degeneracy in a natural way.

I. INTRODUCTION

THE quark model has enjoyed considerable success in describing the meson spectrum.[1] In order to accommodate the observed regularly spaced resonances (i.e., linear Regge trajectories in the region of positive total energy ε_T) it is often assumed that the $q\bar{q}$ potential can be approximated by a harmonic force.[2–4] With this force law, the trajectory in the region of negative ε_T (or its relativistic analog s) is also linear.

In the real world, the $q\bar{q}$ states are coupled to observable decay channels (e.g., $\pi\pi$) which give finite widths to resonances. In this article we study the effects of such channels on the originally linear trajectories.[5]

To be specific, we consider the ρ-meson trajectory to be coupled to two spin triplet $q\bar{q}$ states of $L=J\pm1$, where $L(J)$ is the orbital (total) angular momentum. Nonrelativistic dynamics is used.[6] The decay channel is a $\pi\pi$ system interacting through a Yukawa potential and coupled to the $q\bar{q}$ channels through a short-range exponential potential. If the channels were decoupled, then a Yukawa trajectory asymptotic to $l=-1$ ($\varepsilon_T\to-\infty$) is the leading trajectory for large negative ε_T. When the coupling is turned on, the leading trajectory is still dominated by the $q\bar{q}$ (oscillator) states in the region $\varepsilon_T\gg0$, but changes character and becomes asymptotic to $l=-1$ in the region $\varepsilon_T<0$. We expect this type of "flattening" of the linear trajectories whenever potentials of a Yukawa or exponential type appear in channels *coupled to* the $q\bar{q}$ system.

This flattening is even more pronounced for the lower-lying trajectories. The first daughter trajectory (which is two units of angular momentum J below the leading trajectory in the decoupled case) is lifted by this mechanism to within one-half unit of J of the leading trajectory. We speculate that it may be natural to identify it with the ρ' trajectory used in Regge phenomonology.

In Sec. II we describe the model and discuss the quantum numbers. In Sec. III we give numerical results and a discussion. Appendix A describes the angular momentum decomposition of our equations, and Appendix B gives the numerical details.

II. DESCRIPTION OF MODEL

The $\pi\pi$ channel in a state with isospin 1 and odd angular momentum, satisfying Bose statistics, of course has the same quantum numbers as the ρ trajectory. The $q\bar{q}$ state has G parity $G=(-1)^{L+S+I}$, charge conjugation $C=(-1)^{L+S}$, and parity[7] $P=(-1)^{L+1}$. Thus the ρ trajectory is coupled to the spin triplet, $S=1$, $q\bar{q}$ states with even L and $J=L\pm1$. Now the ρ meson is expected to belong to $J=L+1$ state. However, in the present model, both the state $J=L+1$ and $J=L-1$ of the $q\bar{q}$ state are coupled to the $\pi\pi$ channel. Thus transitions from $J=L+1$ to $J=L-1$ state (via the $\pi\pi$ channel) are possible and we have a

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[1] For an extensive review, see J. J. J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969).

[2] R. H. Dalitz, in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics* (University of California Press, Berkeley, Calif., 1967), pp. 215–234 (reprinted in Ref. 1).

[3] S. Mandelstam [LRL Report No. UCRL-17250, 1966 (unpublished)] has argued that it is the inelastic multiparticle states which are responsible for rising trajectories. With this interpretation, the pathological harmonic-oscillator potential is an approximate way of summarizing the effects of such states. A more traditional interpretation of this model is that trajectories are dominated by a small number of channels (usually one). The linear rise of trajectories is due to pathological potentials (such as the present model and Ref. 4) or energy-dependent potentials whose strength increases with energy [see L. A. P. Balázs, Phys. Rev. **137**, B1510 (1965); **139**, B1646 (1965); U. Trivedi, *ibid.* **188**, 2241 (1969)]. Contrary to Trivedi's model in which multichannel effects are only a small perturbation, mixing effects are quite important in our model at low energies.

[4] See, e.g., G. Zweig, in *Meson Spectroscopy*, edited by C. Baltay and A. H. Rosenfeld (Benjamin, New York, 1968), p. 485.

[5] The dynamical effect of the $\pi\pi$ channel on the dominant $q\bar{q}$ structure of the ρ has been discussed by E. Squires in *Particle Interactions at High Energy* edited by T. W. Preist and L. L. J. Vick (Plenum, New York, 1967). The author uses relativistic N/D dynamics neglecting the effects of spin. The model does not give linearly rising trajectories at high energies.

[6] Various relativistic versions of this model are possible. One extension would be to use the Bethe-Salpeter equation to provide the underlying $q\bar{q}$ dynamics. See, e.g., the one-channel calculation of C. H. Llewellyn Smith, Ann. Phys. (N. Y.) **53**, 521 (1969). Another approach would be to interpret our equations as $O(4)$ oscillators. See L. Susskind, Phys. Rev. Letters **23**, 545 (1969). Both of these models (as well as the Schrödinger model presented here) suffer from the possible existence of ghosts in the region $E_T<0$ since unitarity is not imposed in the crossed channel.

[7] A. Ahmadzadeh and R. J. Jacob, Phys. Rev. **176**, 1719 (1968).

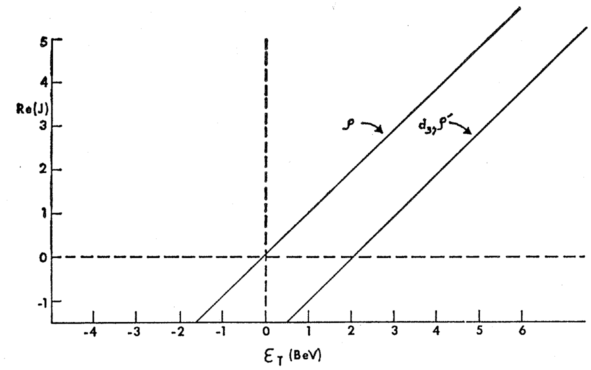

FIG. 1. Regge trajectories for the uncoupled system. The leading trajectory is nondegenerate; all lower trajectories are twofold degenerate. These trajectories belong to the quark-antiquark system. The Yukawa trajectory corresponding to the $\pi\pi$ channel has been omitted. As discussed in the text, one might interpret $2m_2\epsilon_t$ as total energy squared.

coupled three-channel problem. Starting from the two coupled equations ($q\bar{q}$ and $\pi\pi$) in the coordinate space, we make a separation of angular momentum and obtain the desired equations. The derivation is given in Appendix A. The resulting equations are

$$\left(\frac{1}{m_1}\frac{d^2}{dr^2}-\frac{J(J+1)}{m_1r^2}-V_\pi+E_1\right)\phi_\pi + V_c\left[-\left(\frac{J}{2J+1}\right)^{1/2}\phi_q^- + \left(\frac{J+1}{2J+1}\right)^{1/2}\phi_q^+\right]=0, \quad (2.1a)$$

$$\left(\frac{1}{m_2}\frac{d^2}{dr^2}-\frac{(J+1)(J+2)}{m_2r^2}-V_q+E_2\right)\phi_q^+ + V_c\left(\frac{J+1}{2J+1}\right)^{1/2}\phi_\pi=0, \quad (2.1b)$$

$$\left(\frac{1}{m_2}\frac{d^2}{dr^2}-\frac{(J-1)J}{m_2r^2}-V_q+E_2\right)\phi_q^- - V_c\left(\frac{J}{2J+1}\right)^{1/2}\phi_\pi=0, \quad (2.1c)$$

which define the analytic continuation from odd J values. In the above equation, m_1 (m_2) is the pion (quark) mass, V_π and V_q are the potentials in the $\pi\pi$ and $q\bar{q}$ channels, respectively, and finally V_c is the potential coupling the $\pi\pi$ and $q\bar{q}$ channels. In our present model, the potentials are given by

$$V_\pi=-g_1e^{-ar}/r, \quad (2.2a)$$

$$V_q=g_2r^2, \quad (2.2b)$$

$$V_c=g_3e^{-\beta r^4}. \quad (2.2c)$$

The exact form of V_c is not crucial, so long as it is of short range. With this choice of V_c Eqs. (2.1) decouple as $r\to\infty$. The numerical values of the parameters in Eqs. (2.2) are given by

$$g_1=12.86,\quad a=1,\quad \beta=0.01,$$
$$g_2=0.25,\quad g_3=3.0,\quad m_1=0.14\text{ GeV}, \quad (2.3)$$
$$m_2=1.0\text{ GeV}.$$

The ratio g_2/m_2 is fixed by the trajectory slope, while g_3 is determined by the ρ width. The parameter β was chosen somewhat arbitrarily to give the coupling potential a range of approximately 0.5 F. In principle it could be fixed by the widths of higher resonances on the trajectory. The direct $\pi\pi$ potential range a approximates that of ρ exchange. These parameter values are meant only to be representative and are used to make the model definite. In addition, the total energy ε_T is related to E_1 and E_2 by

$$\varepsilon_T=E_1+2m_1=E_2+2m_2+U, \quad (2.4)$$

with $U=2.31$, where we have used an extra constant potential in the $q\bar{q}$ channels, adjustable to make the ρ trajectory pass through $J=1$ at $\varepsilon_T=m_\rho$. Note that for the uncoupled harmonic oscillator the trajectory is given by

$$\alpha_0=(m_2/2g_2)^{1/2}E_2-\tfrac{3}{2}-2k \quad (k=0,1,\ldots). \quad (2.5)$$

Therefore, $(m_2/2g_2)^{1/2}$ approximately determines the slope of the trajectory at high energies (see Fig. 1).

Let us now make a few remarks about $SU(3)$ symmetry and exchange degeneracy. The model in its present form is not $SU(3)$ symmetric because the $K\bar{K}$ channel has not been included. Ignoring the pseudoscalar octet mass splittings, we can replace ϕ_π in Eqs. (2.1) by the wave function for the isotriplet member of the antisymmetric octet 8_a obtained from the direct product of the pseudoscalar octet with itself. If we allow for mass splittings within the pseudoscalar octet then we must include, in addition to the $\pi\pi$ channel, the $K\bar{K}$ channel. Similarly, for the even-J analytic continuation we would have instead of Eqs. (2.1) a new set of equations in which ϕ_π would be replaced by the wave function for the isotriplet member of the symmetric octet 8_s of $P_8\times P_8$. This even-J analytic continuation would thus give the corresponding model

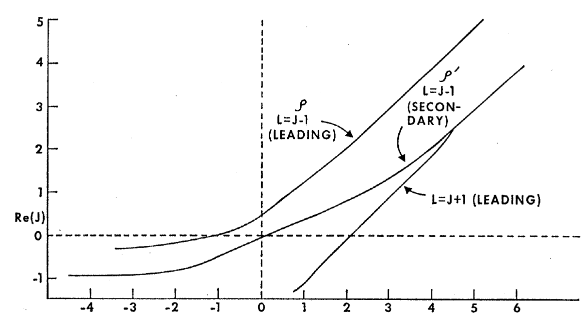

FIG. 2. Regge trajectories for the coupled system: Re(J) vs ε_T.

for the A_2 trajectory. Again, with the octet mass splittings we would have to include the $K\bar{K}$ and $\pi\eta$ channels instead of 8_s. Note that this procedure of including 8_a and 8_s automatically gives exchange degenerate ρ and A_2 trajectories. One could also obtain the K^* and $K^{*\prime}$ trajectories by a rotation in the $SU(3)$ space.

III. NUMERICAL RESULTS AND DISCUSSION

The results of a numerical solution of Eqs. (2.1) are summarized in Figs. 2 and 3, where the three top-ranking trajectories are shown. The highest is identified as the ρ trajectory. The harmonic-oscillator strength g_2 and the constant potential U have been adjusted to give the ρ trajectory its approximate physical intercept and slope. The ρ-meson width may be fixed by adjusting the strength of the coupling potential. As is common in nonrelativistic quark models, one may heuristically replace $2m_2\varepsilon_T$ by the invariant energy squared for a relativistic interpretation.

For the ρ trajectory the wave function ϕ_q^- has no radial nodes. This trajectory (designated as a_3 in Ref. 6) asymptotically becomes linear and coincides with the corresponding decoupled ($V_c=0$) case given in Fig. 2. As the energy ε_T decreases, the second quark channel (ϕ_q^+) and the pion channel (ϕ_π) mix strongly and the trajectory becomes quite different from its uncoupled counterpart. Since the leading trajectory for negative ε_T is that of the Yukawa potential, this is not surprising. As ε_T becomes negative, the slope gets smaller and the trajectory becomes quite flat by $\varepsilon_T \simeq -3$.

The secondary trajectory corresponding to the "first excited" trajectory with $L=J-1$ is also dramatically different from the corresponding uncoupled ($V_c=0$) case. For the "first excited" $L=J-1$ trajectory the wave function ϕ_q^- has one radial node. We call this trajectory ρ'. As ε_T decreases below 5 GeV, the trajectory deviates quickly from the uncoupled case, differing by about two units of J by the time $\varepsilon_T=0$ is reached. The point $J=0$ on this trajectory will be discussed later. We identify this trajectory with the conspiring ρ' trajectory (called c_3' in Ref. 7). This trajectory also flattens rapidly as ε_T becomes negative. In the uncoupled problem, the leading trajectory of the $q\bar{q}$ channel with $L=J+1$ lies two units below the leading trajectory of the $L=J-1$ channel. In the notation of Ref. 7, the leading $L=J+1$ trajectory is called d_3. The real part of this trajectory is only slightly different from the uncoupled ($V_c=0$) case, although a large imaginary parts for the trajectory is generated in the region below $\mathrm{Re}J=0$.

The imaginary parts of these trajectories are shown in Fig. 3. Note that $\mathrm{Im}J$ for the ρ' trajectory is about ten times larger than that of the ρ trajectory. Thus the resonances on the ρ' trajectory would have very large total widths. Also for the d_3 trajectory $\mathrm{Im}J$ changes sign as the real part of the trajectory passes through zero.

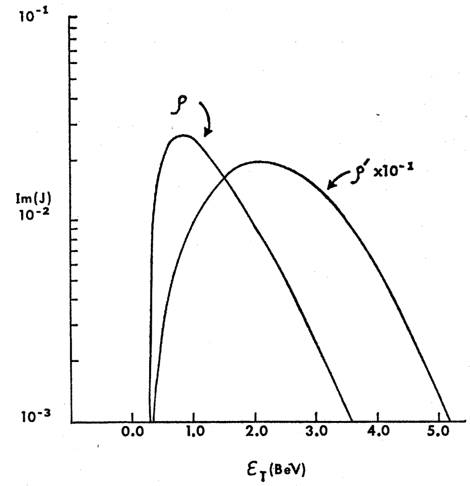

FIG. 3. Regge trajectories for the coupled system: Im(J) vs ε_T.

There is another series of trajectories which in the uncoupled limit are given by the Yukawa potential. We have not calculated these. With our choice of strength of the weak Yukawa potential (for the direct $\pi\pi$ interaction), the uncoupled Yukawa trajectory does not intrude into the physical region of $J>0$. Thus we conjecture that this would also be the case in the present model. It is perhaps worth mentioning that our results are most sensitive to the harmonic-oscillator and the coupling potentials. In fact we can make the Yukawa potential arbitrarily weak without any significant changes in the qualitative result.

Certain points in the J plane require special attention. At the point $J=0$ one of the quark channels becomes unphysical, corresponding to $L=-1$. In the usual parlance, $J=0$ is a *nonsense point* of any trajectory coupled to this channel. The factor $(J)^{1/2}$ in V_c decouples this unphysical channel from the remaining two. Thus at $J=0$ there are two types of solutions: (1) those for which ϕ_q^+ and ϕ_π vanish and the trajectory chooses nonsense, and (2) those for which ϕ_q^- vanishes and the trajectory chooses sense. An example of the second type of solution is the point ($\varepsilon_T \simeq -1, J=0$) on the ρ trajectory.

It is interesting to note that, although the channel $L=J-1$ dominates this trajectory at high energies, decrease of the energy leads to large admixtures of the other channels until at $J=0$ the $L=J-1$ channel is completely absent.

An example of the first type of solution is the point ($\varepsilon_T=0.28, J=0$) on the ρ' trajectory. The system of equations now reduce to a single equation. Note that this point indeed lies on an uncoupled trajectory (i.e., the *leading* trajectory of Fig. 2).

We would like to emphasize that one should not take the *details* of this model too seriously. It has, however, several interesting features.

(1) The ρ and ρ' trajectories have considerable mixtures of $L=J\pm1$ $q\bar{q}$ states in the region $\varepsilon_T<2$ even though the coupling (through the $\pi\pi$ channel) is quite weak.

(2) The ρ' trajectory which is often used in the phenomonology of meson-nucleon charge-exchange reactions[8] can naturally be identified with the present ρ'. Our present ρ' trajectory also passes through $\varepsilon_T=0$ approximately one-half unit of J below the ρ trajectory and has roughly the same slope. Were it not for the channel coupling, this trajectory would lie two units of J below the ρ trajectory. Furthermore, at $J=0$ the ρ' trajectory has a nonsense wrong-signature point (i.e., $L=-1$), while the ρ trajectory chooses sense. The analogous A_2' trajectory obtained from the even-J analytic continuation of a similar set of equations has a right-signature point at $J=0$. The model predicts no unwanted particle there because the trajectory chooses nonsense.

(3) The ρ and ρ' trajectories are "lifted" in the negative ε_T region. The ρ trajectory still passes through $J=0$ at approximately $\varepsilon_T=-1.0$. The trajectories flatten considerably to the left of this point, eventually behaving similarly to a fixed pole for $\varepsilon_T<-3$.

(4) The width of the ρ can be made arbitrarily narrow in this type of model by reducing V_c.

(5) With the interpretation $s=2m_2\varepsilon_T$, the mass of ρ' particle will turn out to be about 2.3 BeV. However, as we have mentioned, $\mathrm{Im}(J)$ for the ρ' meson is about ten times larger than that of the ρ meson (see Fig. 3). Therefore, the ρ' "particle" (with the present choice of parameters and coupling potential) would have a very large width and would thus not be observed as a resonance experimentally.

(6) In our model, the widths of the resonances decrease as we move up along the trajectory. In particular, the width of first ρ recurrence turns out to be considerably smaller than that of the ρ meson. The numerical values of these widths strongly depend on our choice of parameters and the form of coupling potential, however.

APPENDIX A: DERIVATION OF COUPLED EQUATIONS

In the following, α is a channel index defined to be 1 for the $\pi\pi$ and 2 for the qq systems. The state vector,

$$|\Psi\rangle=\begin{pmatrix}|\Psi_1\rangle\\|\Psi_2\rangle\end{pmatrix},$$

satisfies the Schrödinger equation

$$(\mathcal{H}_0+\mathcal{V}_c)|\Psi\rangle=E|\Psi\rangle,$$

where $\mathcal{H}_0$ and E are diagonal; $\mathcal{V}_c$ couples the channels.

[8] A. Ahmadzadeh and J. C. Jackson, Phys. Rev. **187**, 2078 (1969); A. Ahmadzadeh and W. B. Kaufmann, *ibid.* **188**, 2438 (1969).

The diagonal elements of the operator $\mathcal{H}_0-E$ are $(m_\alpha^{-1}\nabla^2+\mathcal{V}_\alpha)$, where m_α is the particle mass and $\mathcal{V}_\alpha$ is the potential energy for channels α. The $\pi\pi$ potential is of the Yukawa form and the $q\bar{q}$ potential is a harmonic oscillator. The state vectors are expanded in angular momentum eigenstates $|\alpha;JMj_aj_b\rangle$, where $j_a\equiv l(\pi\pi)=l$ and $j_b\equiv S(\pi\pi)=0$ if $\alpha=1$, and $j_a\equiv l(q\bar{q})=L$ and $j_b\equiv S(q\bar{q})=1$ if $\alpha=2$.

The interaction potential between channels is taken to be

$$H_{\mathrm{int}}=\mathbf{Y}_1\cdot\mathbf{S}f(r),$$

where $\mathbf{S}$ is the total spin operator, $\mathbf{Y}_1$ is the tensor operator constructed from the first-order spherical harmonics (i.e., the relative coordinate vector $\mathbf{r}$), and $f(r)$ is a scalar under rotations. The operator $\mathbf{Y}_1$ acts only on spatial coordinates and $\mathbf{S}$ acts only on the spin coordinates. This interaction is odd under the *spatial* parity operation since the $\pi\pi$ and $q\bar{q}$ systems have opposite intrinsic parity. Thus H_{int} has no diagonal matrix elements. The off-diagonal matrix elements are

$$\begin{aligned}V_L{}^J&=\langle 2;JML1|H_{\mathrm{int}}|1;JMl0\rangle\\&=(-)^{L-J}W(l0L1;J1)[3(L+1)]^{1/2}(L\|Y_1\|J)(0\|S\|1).\end{aligned}$$

In accord with the conservation of angular momentum, the Racah coefficient W is zero unless $l=J$ and $L=J\pm1$. This demonstrates that our system has three channels: two $q\bar{q}$ channels with $L=J\pm1$ and one $\pi\pi$ channel with $l=J$. Using

$$(J\|Y_1\|L)=-C(L1J;000)[3(2L+1)/4\pi(2J+1)]^{1/2}$$

and the definition

$$V_c(r)=(3/4\pi)(0\|S\|1)f(r),$$

we find

$$\begin{aligned}V_L{}^J(r)&=-V_c(r)\left(\frac{J}{2J+1}\right)^{1/2}&&\text{if } L=J-1,\\&=0&&\text{if } L=J,\\&=+V_c(r)\left(\frac{J+1}{2J+1}\right)^{1/2}&&\text{if } L=J+1.\end{aligned}$$

The coupled Schrödinger equations are

$$\left(\frac{1}{m_1}\frac{d^2}{dr^2}-\frac{J(J+1)}{m_1r^2}-V_\pi+E_1\right)\phi_\pi+V_c\left[-\left(\frac{J}{2J+1}\right)^{1/2}\phi_q{}^-+\left(\frac{J+1}{2J+1}\right)^{1/2}\phi_q{}^+\right]=0,$$

$$\left(\frac{1}{m_2}\frac{d^2}{dr^2}-\frac{(J+1)(J+2)}{m_2r^2}-V_q+E_2\right)\phi_q{}^++V_c\left(\frac{J+1}{2J+1}\right)^{1/2}\phi_\pi=0,$$

$$\left(\frac{1}{m_2}\frac{d^2}{dr^2}-\frac{(J-1)J}{m_2r^2}-V_q+E_2\right)\phi_q^- -V_c\left(\frac{J}{2J+1}\right)^{1/2}\phi_\pi=0,$$

where $r\phi_\pi$ is the $\pi\pi$ wave function and $r\phi_q^\pm$ are the $q\bar{q}$ wave functions with $L=J\pm1$. The total energy ε_T is related to the nonrelativistic energy E_α and m_α by

$$\varepsilon_T=E_1+2m_1=E_2+2m_2+U\,,$$

where U is a constant potential which is adjusted to make the ρ trajectory pass through m_ρ at $J=1$.

APPENDIX B: NUMERICAL METHODS

1. Methods of Solution

The following methods are commonly in use for the calculation of Regge trajectories.

i. Direct integration of differential equation.[9] Boundary conditions at the origin and infinity are connected by a numerical solution of the differential equation. The method is relatively slow, but accurate and straightforward. It is in principle applicable to the entire J plane. It has the advantage of providing accurate wave functions automatically. It allows the calculation of the complete scattering amplitude. Regge residues, widths of resonances, etc.

ii. Inversion of integral kernel.[10] The Green's function of the Lippmann-Schwinger equation is approximated by a sum of dyads, yielding an algebraic eigenvalue problem. Alternatively, one could approximate the integral by a sum over mesh points. This again leads to a matrix eigenvalue problem. This method has the difficulty of requiring large matrix inversions for multichannel problems. Special treatment is also required for negative Re(J), where the integrals must be analytically continued.

iii. Variational methods.[11] The Rayleigh-Ritz and Schwinger variational quotients may be converted into algebraic systems of equations if trial functions of the form $\sum_i e_i\phi_i$ are chosen (where ϕ_i is a function which satisfies the boundary conditions). The resulting matrix elements are then analytically continued into the complex J plane. The method is quite accurate and rapid, but is somewhat more difficult to set up than method i.

Because of its ease of formulation, and since we require only the first few trajectories, we have chosen method i.

2. Numerical Details

For a given value of ε_T we construct an algorithm to determine J, the corresponding point on the Regge trajectory.

Since the differential equations decouple at $r=\infty$ [because of the $\exp(-\beta r^4)$ form of V_c], it is relatively easy to calculate asymptotic forms for the wave functions for large r. The over-all normalization coefficient of one wave function may be fixed arbitrarily, but the remaining two normalization coefficients together with J are undetermined functions of ε_T. Initial values of these normalization coefficients and J are chosen, and the system of differential equations is integrated from the asymptotic region to a small value of r.

A power-series solution is next developed about $r=0$. The normalization coefficients for the power series are fixed by equating the series with the integrated solution. The differences between the three derivatives of the series and the integrated solution at the matching point defines three (complex) functions of J and the two normalization coefficients. Simultaneous zeros of these functions are then found through a Newton-type iteration scheme. We have found that this method converges rapidly, seldom requiring more than 12–15 integrations per point on the trajectory each time the energy is suitably incremented.

[9] A. Ahmadzadeh, P. Burke, and C. Tate, Phys. Rev. **131**, 1315 (1963); A. Ahmadzadeh, thesis, University of California, 1964 (unpublished).

[10] A. Ahmadzadeh and V. Chung, Phys. Rev. **161**, 1602 (1967).

[11] C. Schwartz, Phys. Rev. **141**, 1468 (1966); W. B. Kaufmann, *ibid.* **164**, 1991 (1967).