Current Algebra, Vector Dominance, and the Decay $K_L{}^0 \to \pi^+\pi^-\pi^0\gamma$

Gerald W. Intemann
Department of Physics, State University of New York, Binghamton, New York 13901
(Received 24 July 1970)

We have calculated the decay rate and photon energy spectrum for the decay mode $K_L{}^0 \to \pi^+\pi^-\pi^0\gamma$. Neglecting small CP-violating effects, we have related the $E1$ amplitude for $K_L{}^0 \to \pi^+\pi^-\pi^0\gamma$ to the amplitude for $K_L{}^0 \to \pi^0\gamma\gamma$ using current algebra and the hypothesis of a partially conserved axial-vector current. We have found for the branching ratio $\Gamma(K_L{}^0 \to \pi^+\pi^-\pi^0\gamma)/\Gamma(K_L{}^0 \to \pi^0\gamma\gamma) = 0.16\%$. Using the present experimental upper limit on $K_L{}^0 \to \pi^0\gamma\gamma$ we estimate $\Gamma(K_L{}^0 \to \pi^+\pi^-\pi^0\gamma)/\Gamma(K_L{}^0 \to \text{all}) < 3.7\times10^{-7}$. We have also determined the same branching ratio based on the ρ-dominance model, and we find good agreement with the corresponding result of current algebra.

I. INTRODUCTION

Over the last few years, the techniques of current albegra have been successful in describing various meson decays.[1] In particular, the radiative decays $\eta \to \pi^+\pi^-\gamma$, $\eta \to \pi^+\pi^-\pi^0\gamma$, and $K_L{}^0 \to \pi^+\pi^-\gamma$ have been studied,[2] and the current-algebra predictions are in good agreement with experiment.[3]

During the same time, there has also been a number of studies of these radiative decays based on vector-meson dominance.[4] In all cases there has been fairly good agreement with the results of current algebra. This rather remarkable agreement has led to further investigations[5] into the possibility of a general equivalence between current algebra and vector-meson dominance.

In this paper we shall study the radiative nonleptonic decay

$$K_L{}^0 \to \pi^+\pi^-\pi^0\gamma\,. \tag{1}$$

This decay along with the other radiative decays of $K_L{}^0$ are of interest since they involve electromagnetic interactions which have been suggested as a possible source of CP violation.[6] In looking for CP violation in K^0 decays other than $K^0 \to 2\pi$ or $K^0 \to 3\pi$, it is well known[7] that an observation of an interference effect between K_L and K_S decays in the partial decay rate would be direct evidence of CP violation. Thus, a study of $K_{L,S} \to \pi^+\pi^-\pi^0\gamma$ might, in principle, lead to new evidence for CP violation, especially if its origin is electromagnetic.

We shall examine decay (1) from both the approaches of current algebra and vector-meson dominance. We shall primarily be interested in studying the direct, CP-conserving $E1$ transition amplitude with the objective of estimating the size of this more interesting contribution to (1) as compared to the inner bremsstrahlung part. Also, by comparing our results from current algebra with those from vector-meson dominance, we will be in a position to further test the equivalence of these two approaches.

In Sec. II, using current algebra, the hypothesis of a partially conserved axial-vector current (PCAC) and Weinberg's T-product decomposition method,[8] we shall relate the parity-conserving (PC) amplitude for the decay (1) to the amplitude for the decay $K_L{}^0 \to \pi^0\gamma\gamma$. Using the experimental data on this latter decay, we will be able to obtain an upper limit for the $E1$ transition rate of (1). We shall also derive a single soft-pion theorem relating decay (1) to the parity-violating part of the decay $K_L{}^0 \to \pi^+\pi^-\gamma$. In Sec. III we shall present the vector-dominance calculation for the $E1$ transition of (1). In Sec. IV we discuss our results.

II. CURRENT-ALGEBRA CALCULATION OF $E1$ TRANSITION

Before presenting the details of the current-algebra calculation, let us discuss some of the kinematics of the decay (1) and the possible types of transitions. Throughout this paper we shall only consider the CP-invariant mode of $K_L{}^0 \to \pi^+\pi^-\pi^0\gamma$ (i.e., neglect the small effects due to CP violation).

Since the maximum photon momentum is low ($k_{\max} = 77$ MeV), we need to consider only the lowest multipoles. Let us classify the $\pi^+\pi^-\pi^0$ state by (L,l) where L is the angular momentum of relative motion

[1] For a comprehensive review see, e.g., S. L. Adler and R. Dashen, *Current Algebra and Applications to Particle Physics* (Benjamin, New York, 1968).

[2] $\eta \to \pi^+\pi^-\gamma$: J. Pasupathy and R. E. Marshak, Phys. Rev. Letters **17**, 888 (1966); M. Ademello and R. Gatto, Nuovo Cimento **44A**, 282 (1966). These authors predict the branching ratio $\Gamma(\eta \to \pi^+\pi^-\gamma)/\Gamma(\eta \to \gamma\gamma) = 0.19$. $\eta \to \pi^+\pi^-\pi^0\gamma$: For the $E1$ transition there have been two calculations, A. Q. Sarker, Phys. Rev. Letters **19**, 1261 (1967), and G. W. Intemann and I. R. Lapidus, Phys. Rev. **165**, 1650 (1968). These calculations yield the branching ratio $\Gamma(\eta \to \pi^+\pi^-\pi^0\gamma)/\Gamma(\eta \to \pi^0\gamma\gamma) \simeq 0.28\%$. For the $M1$ transition, A. Chatterjee [*ibid.* **174**, 1832 (1968)] predicts the same branching ratio to be 0.2%. $K_L{}^0 \to \pi^+\pi^-\gamma$: C. S. Lai and B. L. Young, Nuovo Cimento **52A**, 83 (1967), predict $\Gamma(K_L{}^0 \to \pi^+\pi^-\gamma)/\Gamma(K_L{}^0 \to \gamma\gamma) = 0.14$.

[3] From the compiled data on η decay experiments, $\Gamma(\eta \to \pi^+\pi^-\gamma)/\Gamma(\eta \to \gamma\gamma) = 0.14 \pm 0.02$ and $\Gamma(\eta \to \pi^+\pi^-\pi^0\gamma)/\Gamma(\eta \to \pi^0\gamma\gamma) < 0.6\%$; for $K_L{}^0$ decay, experiments have placed the upper limit $\Gamma(K_L{}^0 \to \pi^+\pi^-\gamma)/\Gamma(K_L{}^0 \to \gamma\gamma) < 0.77$. See A. Barbaro-Galtieri *et al.*, Rev. Mod. Phys. **42**, 87 (1970).

[4] L. Brown and P. Singer, Phys. Rev. Letters **8**, 460 (1962); S. Oneda, Y. S. Kim, and D. Korff, Phys. Rev. **136**, B1064 (1964); P. Singer, *ibid.* **154**, 1592 (1967); A. Chatterjee, *ibid.* **174**, 1832 (1968).

[5] K. R. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); J. J. Sakurai, Phys. Rev. **156**, 1508 (1967).

[6] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. **139**, B1650 (1965); S. Barshay, Phys. Letters **17**, 78 (1965); F. Salzman and F. Salzman, *ibid.* **15**, 91 (1965).

[7] L. M. Sehgal and L. Wolfenstein, Phys. Rev. **162**, 1362 (1967).

[8] S. Weinberg, Phys. Rev. Letters **17**, 336 (1966).

between the $\pi^+\pi^-$ pions and l is the angular momentum of the π^0 relative to the $\pi^+\pi^-$ system. For the (1,0) or (0,1) configuration one has either an electric dipole ($E1$) or magnetic dipole ($M1$) transition.

For the $E1$ transition, if CP invariance holds, the three-pion state has $J^P=1^+$, $I=0$ and is an eigenstate of the CP operator with $CP=-1$. If there exists an axial-vector meson with $I=0$ and $CP=-1$, it could mediate the decay. In view of the lack of experimental evidence for such a meson, the $E1$ transition might more likely be described by ρ dominance with the $\pi^+\pi^-$ state in a relative p wave.

In the case of $M1$ transition, the three-pion state has $J^P=1^-$, $I=1$, and $CP=-1$. The absence of a vector meson with such quantum numbers which could mediate this transition indicates that the $M1$ transition predominantely enters into the CP-violating amplitude in which the 3π state has $J^P=1^-$, $I=0$, and $CP=+1$. Since we are neglecting the small effects of CP violation, we shall in turn neglect the $M1$ transition as compared to $E1$.

We now begin our calculation by considering the two-point function

$$M_{\mu\nu}=\int dxdy\; e^{i(q_1\cdot x+q_2\cdot y)}\langle\pi^0(q_3),\gamma(k)|\times T(A_\mu^-(x)A_\nu^+(y)H_w(0))|K_L^0(p)\rangle\,, \quad (2)$$

where $A_\mu^\pm$ are the charged, $\Delta S=0$, axial-vector currents and H_w represents the weak nonleptonic Hamiltonian.

If one uses the standard current commutation relations[9]

$$[A_0^+(x),A_\nu^-(y)]\delta(x_0-y_0)=2V_\nu^3(y)\delta^4(x-y)\,, \quad (3)$$

$$[A_0^+(x),\partial_\mu A_\mu^-(y)]\delta(x_0-y_0)=2i\sigma(y)\delta^4(x-y)\,, \quad (4)$$

and the PCAC relation[10]

$$\partial_\mu A_\mu^\pm(x)=F_\pi\mu^2\phi_{\pi^\mp}(x)\,, \quad (5)$$

where F_π is the pion decay constant and μ represents the $\pi^\pm$ mass, one obtains after two partial integrations and a decomposition of the T product[8]

$$\begin{aligned}-q_{2\nu}q_{1\mu}N_{\mu\nu}={}&F_\pi^2(\mu^2-q_1^2)(\mu^2-q_2^2)\int dxdy\; e^{i(q_1\cdot x+q_2\cdot y)}\langle\pi^0,\gamma|T(\phi_{\pi^+}(x)\phi_{\pi^-}(y)H_w(0))|K_L^0\rangle\\
&-i(q_2-q_1)_\mu\int dx\; e^{i(q_1+q_2)\cdot x}\langle\pi^0,\gamma|T(V_\mu^3(x)H_w(0))|K_L^0\rangle\\
&+\tfrac{1}{2}\int dxdy\; e^{i(q_1\cdot x+q_2\cdot y)}\langle\pi^0,\gamma|[A_0^-(x),[A_0^+(y),H_w(0)]]|K_L^0\rangle\delta(x_0)\delta(y_0)\\
&+\tfrac{1}{2}\int dxdy\; e^{i(q_1\cdot x+q_2\cdot y)}\langle\pi^0,\gamma|[A_0^+(y),[A_0^-(x),H_w(0)]]|K_L^0\rangle\delta(x_0)\delta(y_0)\\
&+F_\pi(\mu^2-q_1^2)\int dxdy\; e^{i(q_1\cdot x+q_2\cdot y)}\langle\pi^0,\gamma|T(\phi_{\pi^+}(x)[A_0^+(y),H_w(0)])|K_L^0\rangle\delta(y_0)\\
&+F_\pi(\mu^2-q_2^2)\int dxdy\; e^{i(q_1\cdot x+q_2\cdot y)}\langle\pi^0,\gamma|T(\phi_{\pi^-}(y)[A_0^-(x),H_w(0)])|K_L^0\rangle\delta(x_0)+(\sigma\text{ terms})\,,\end{aligned} \quad (6)$$

where $N_{\mu\nu}$ represents $M_{\mu\nu}$ after all the pion pole terms have been removed.

The weak Hamiltonian density $H_w(x)$ will be taken to be of the current-current form[11]

$$H_w(x)=gd_{6ij}J_\mu^i(x)J_\mu^j(x)=H_w^{\rm PC}+H_w^{\rm PV}\,, \quad (7)$$

where $J_\mu^i(x)=V_\mu^i(x)+A_\mu^i(x)$ is a sum of the vector and axial-vector weak hadron currents, the superscript i is an SU_3 index and $H_w^{\rm PC}$ and $H_w^{\rm PV}$ are the parity-conserving (PC) and parity-violating (PV) parts of H_w, respectively. If we assume CP invariance, then only $H_w^{\rm PC}$ contributes to the lowest multipole transition of (1).

With the weak Hamiltonian in the form of Eq. (7), assuming CP invariance, one can show that the terms containing the commutators $[A_0^\pm(x),H_w(0)]$ as well as the σ terms in Eq. (6) violate CP invariance and thus can be dropped. Furthermore, in the case of soft-photon emission we can neglect[12] the terms of the second order

[9] M. Gell-Mann, Phys. Rev. **125**, 1064 (1962). $\sigma(x)$ is a scalar density which in the σ model represents the σ-meson field. See M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960).

[10] In the presence of electromagnetism one should actually use, to first order in e, the modified form of PCAC, $(\partial_\mu\mp ie\mathcal{A}_\mu)A_\mu^\pm=F_\pi\mu^2\phi_{\pi^\mp}$, where $\mathcal{A}_\mu$ represents the electromagnetic field. However, since we are contracting both the π^+ and π^-, the contributions from the electromagnetic corrections to PCAC cancel.

[11] M. Gell-Mann, Phys. Rev. Letters **12**, 155 (1964); Y. Hara and Y. Nambu, *ibid.* **16**, 875 (1966).

[12] There is some question as to the validity of neglecting the nonpole term $q_{2\nu}q_{1\mu}N_{\mu\nu}$ since the $E1$ amplitude for $K_L^0\to\pi^+\pi^-\pi^0\gamma$ is itself quadratic in pion momenta. To be rigorous, it is necessary to make a model-dependent study of the nonpole terms in order to ascertain their degree of importance. See H. R. Rubinstein and S. Veneziano, Phys. Rev. Letters **18**, 411 (1967). However, in most previous calculations it appears that such nonpole terms are indeed negligible, and we will make this same assumption here.

TABLE I. Comparison of current-algebra predictions for $R(E1)$ and $R'(E1)$ based on various estimates for $\Gamma(K_L^0\to\pi^0\gamma\gamma)$.

$\Gamma(K_L^0\to\pi^0\gamma\gamma)/\Gamma(K_L^0\to\text{all})$	$R(E1)$	$R'(E1)$
2.2×10^{-7}–4.6×10^{-4} (current algebra)[a]	3.5×10^{-10}–7.3×10^{-7}	2.8×10^{-9}–5.7×10^{-6}
2.5×10^{-5} (η-pole model)[b]	$4.0\ \times10^{-8}$	$3.1\ \times10^{-7}$
6.11×10^{-9} (vector dominance)[c]	9.71×10^{-12}	7.66×10^{-11}
$<2.3\times10^{-4}$ (experiment)[d]	$<3.7\ \times10^{-7}$	$<2.9\times10^{-6}$

[a] A. Della Selva, A. De Rujula, and M. Mateev, Phys. Letters **24B**, 468 (1967).
[b] S. Oneda, Phys. Rev. **158**, 1541 (1967); G. Faldt, B. Petersson, and H. Pilkuhn, Nucl. Phys. **B3**, 234 (1967).
[c] G. Oppo and S. Oneda, Phys. Rev. **160**, 1397 (1967).
[d] See Ref. 13.

in the pion momenta. This approximation amounts to neglecting the quadrupole transitions. Thus, we find in the limit $q_1^2=q_2^2=0$

$$\begin{aligned}F_\pi^2\mu^4\int dxdy\, e^{i(q_1\cdot x+q_2\cdot y)}\langle\pi^0(q_3),\gamma(k)|&\\ \times T(\phi_{\pi^+}(x)\phi_{\pi^-}(y)H_w^{\mathrm{PC}}(0))|K_L^0(p)\rangle&\\ =i(q_2-q_1)_\mu\int dx\, e^{i(q_1+q_2)\cdot x}\langle\pi^0(q_3),\gamma(k)|&\\ \times T(V_\mu^3(x)H_w^{\mathrm{PC}}(0))|K_L^0(p)\rangle.&\qquad(8)\end{aligned}$$

The left-hand side of Eq. (8) is essentially the amplitude for $K_L^0\to\pi^+\pi^-\pi^0\gamma$ in the limit $q_1^2=q_2^2=0$. The right-hand side can be related to the amplitude for $K_L^0\to\pi^0\gamma\gamma$. Equation (8) may be written as

$$\begin{aligned}(2\pi)^{15/2}(32\omega_1\omega_2\omega_3\omega_K)^{1/2}&\\ \times\langle\pi^+(q_1)\pi^-(q_2)\pi^0(q_3)\gamma(k)|H_w^{\mathrm{PC}}|K_L^0(p)\rangle&\\ =(i/F_\pi^2)(2\pi)^6(16k'k\omega_3\omega_K)^{1/2}&\\ \times(q_2-q_1)_\mu M_\mu^3(k',k,q_3)\,,&\qquad(9)\end{aligned}$$

where

$$\begin{aligned}(2\pi)^{3/2}(2k')^{1/2}M_\mu^3=i\int dx\, e^{ik'\cdot x}\langle\pi^0(q_3)\gamma(k)|&\\ \times T(V_\mu^3(x)H_w^{\mathrm{PC}}(0))|K_L^0(p)\rangle,&\qquad(10)\end{aligned}$$

and $k'\equiv q_1+q_2$.

On grounds of general covariance we may write for the $E1$ amplitude of $K_L^0\to\pi^+\pi^-\pi^0\gamma$

$$\begin{aligned}(2\pi)^{15/2}(32\omega_1\omega_2\omega_3k\omega_K)^{1/2}\langle\pi^+\pi^-\pi^0\gamma|H_w^{\mathrm{PC}}|K_L^0\rangle&\\ =(eF/M_K^3)[k\cdot(q_1-q_2)\epsilon\cdot(q_1+q_2)&\\ -k\cdot(q_1+q_2)\epsilon\cdot(q_1-q_2)+k\cdot(q_3-q_1)\epsilon\cdot(q_3+q_1)&\\ -k\cdot(q_3+q_1)\epsilon\cdot(q_3-q_1)+k\cdot(q_2-q_3)\epsilon\cdot(q_2+q_2)&\\ -k\cdot(q_2+q_3)\epsilon\cdot(q_2-q_3)]\,,&\qquad(11)\end{aligned}$$

where F represents a form factor.

In a similar manner we have for the $K_L^0\to\pi^0\gamma\gamma$ amplitude

$$\begin{aligned}(2\pi)^6(16k'k\omega_3\omega_K)^{1/2}\langle\pi^0\gamma\gamma|H_w|K_L^0(p)\rangle&\\ =(e^2M/M_K\sqrt{2})[(k\cdot k')(\epsilon\cdot\epsilon')-(k'\cdot\epsilon)(k\cdot\epsilon')]\,,&\qquad(12)\end{aligned}$$

with M representing a form factor. Combining Eqs. (9)–(12) gives

$$F=eM_KM/F_\pi^2\,.\qquad(13)$$

This form-factor relation permits us to compare the decay rates of $K_L^0\to\pi^+\pi^-\pi^0\gamma$ and $K_L^0\to\pi^0\gamma\gamma$.

Calculating the decay rates, using Eq. (13), one finds for the $E1$ transition of $K_L^0\to\pi^+\pi^-\pi^0\gamma$

$$\Gamma_E(K_L^0\to\pi^+\pi^-\pi^0\gamma)=\frac{\alpha^2|M|^2}{2F_\pi^4(2\pi)^3M_K^4}\int_0^{k_{\max}}\frac{k^3dk}{(M_K-2k)}[2I(k)+I_0(k)]\,,\qquad(14)$$

where

$$\begin{aligned}I(k)=\int_\mu^{\omega_{\max}}qX(k,\omega)\{&-4(Q^2-\mu^2-\mu_0^2)(\omega E_K-2\omega^2-\tfrac{2}{3}q^2)-\tfrac{4}{3}(\mu^2+\mu_0^2)(4\omega^2-\mu^2)\\ &+[(E_K-\omega)^2+\tfrac{1}{3}q^2](Q^2+\mu^2-\mu_0^2)[1-\tfrac{1}{3}X^2(k,\omega)]\}d\omega\,,\qquad(15)\end{aligned}$$

$$\begin{aligned}I_0(k)=\int_{\mu_0}^{\omega_{0\,\max}}q_0X_0(k,\omega_0)\{&-4(Q_0^2-2\mu^2)(\omega_0E_K-2\omega_0^2-\tfrac{2}{3}q_0^2)-(8/3)\mu^2(4\omega_0^2-\mu_0^2)\\ &+[(E_K-\omega_0)^2+\tfrac{1}{3}q_0^2](Q_0^2+2\mu_0^2-2\mu^2)[1-\tfrac{1}{3}X_0^2(k,\omega_0)]\}d\omega_0\,,\qquad(16)\end{aligned}$$

and

$$\begin{aligned}&E_K=(M_K^2-2M_Kk)^{1/2}, &&X(k,\omega)=\left(\frac{Q^2-2\mu^2-2\mu_0^2}{Q^2}\right)^{1/2},\\ &Q^2=E_K(E_K-2\omega)+\mu^2, &&X_0(k,\omega)=\left(\frac{Q_0^2-4\mu^2}{Q_0^2}\right)^{1/2},\\ &Q_0^2=E_K(E_K-2\omega_0)+\mu_0^2, &&\omega_{\max}=\frac{E_K^2+\mu^2-(\mu+\mu_0)^2}{2E_K},\\ &\omega_{0\,\max}=\frac{E_K^2+\mu_0^2-4\mu^2}{2E_K}, &&k_{\max}=\frac{M_K^2-(2\mu+\mu_0)^2}{2M_K},\end{aligned}\qquad(17)$$

and μ, μ_0 are the masses of $\pi^\pm$, π^0, respectively.

Similarly, for $K_L{}^0 \to \pi^0\gamma\gamma$,

$$\Gamma(K_L{}^0 \to \pi^0\gamma\gamma) = \frac{\alpha^2|M|^2\mu_0{}^4}{96\pi M_K{}^3}\left\{3\lambda^2\left(1+\frac{1}{\lambda^2}\right)\right.$$
$$\times\left[\left(\frac{\lambda^4-1}{4\lambda^2}\right)-\ln\lambda\right]+24\left(\frac{\lambda^4-1}{16\lambda}\right)\left(\frac{\lambda^2-1}{2\lambda}\right)^2$$
$$\left.+3\left(\frac{\lambda^4-1}{4\lambda}\right)-3\ln\lambda-\left(1+\frac{1}{\lambda^2}\right)\frac{(\lambda^2-1)^3}{\lambda^2}\right\}, \quad (18)$$

where $\lambda \equiv M_K/\mu_0$.

Equations (14) and (18) yield

$$\frac{\Gamma_E(K_L{}^0 \to \pi^+\pi^-\pi^0\gamma)}{\Gamma(K_L{}^0 \to \pi^0\gamma\gamma)} = 1.59\times10^{-3}. \quad (19)$$

It is not possible at the present time to directly compare this current-algebra prediction with experiment since there are no published experimental data on $K_L{}^0 \to \pi^+\pi^-\pi^0\gamma$. However, recently Banner *et al.*[13] have reported an upper limit on $K_L{}^0 \to \pi^0\gamma\gamma$, namely, $\Gamma(K_L{}^0 \to \pi^0\gamma\gamma)/\Gamma(K_L{}^0 \to \text{all}) < 2.3\times10^{-4}$ with 90% confidence. Combining this datum with Eq. (19), we then find for the $E1$ transition the branching ratio

$$R(E1) = \frac{\Gamma_E(K_L{}^0 \to \pi^+\pi^-\pi^0\gamma)}{\Gamma(K_L{}^0 \to \text{all})} < 3.7\times10^{-7}. \quad (20)$$

There have also been a number of theoretical estimates for $K_L{}^0 \to \pi^0\gamma\gamma$. In Table I we compare the predictions for $R(E1)$ based on these theoretical estimates. Equation (14) also permits us to determine the photon energy spectrum for the $E1$ transition of $K_L{}^0 \to \pi^+\pi^-\pi^0\gamma$. This spectrum is shown in Fig. 1.

From our results we see that current algebra predicts a very small rate for the direct $E1$ transition of $K_L{}^0 \to \pi^+\pi^-\pi^0\gamma$ *if* the decay $K_L{}^0 \to \pi^0\gamma\gamma$ is as rare as it is generally believed. Another way, perhaps, in which to see this suppression of the $E1$ transition is by considering a single soft-pion limit of its amplitude.

If one considers the quantity

$$M_\mu = \int dx\, e^{iq_3\cdot x}\langle\pi^+\pi^-\gamma|T(A_\mu{}^3(x)H_w{}^{\text{PC}}(0))|K_L{}^0\rangle, \quad (21)$$

integrates by parts, using PCAC and the equal-time commutation relation

$$[A_0{}^3(x), H_w{}^{\text{PC}}(0)]\delta(x_0) = \tfrac{1}{2}H_w{}^{\text{PV}}(0)\delta^4(x), \quad (22)$$

one finds in the soft-pion limit $q_3 \to 0$

$$(F_\pi/\sqrt{2})\langle\pi^+(q_1)\pi^-(q_2)\pi^0(q_3)\gamma(k)|H_w{}^{\text{PC}}|K_L{}^0(p)\rangle$$
$$\times(2\pi)^{3/2}(2\omega_3)^{1/2}$$
$$= -\tfrac{1}{2}\langle\pi^+(q_1)\pi^-(q_2)\gamma(k)|H_w{}^{\text{PV}}|K_L{}^0(p)\rangle. \quad (23)$$

[13] M. Banner *et al.*, Phys. Rev. **188**, 2033 (1969).

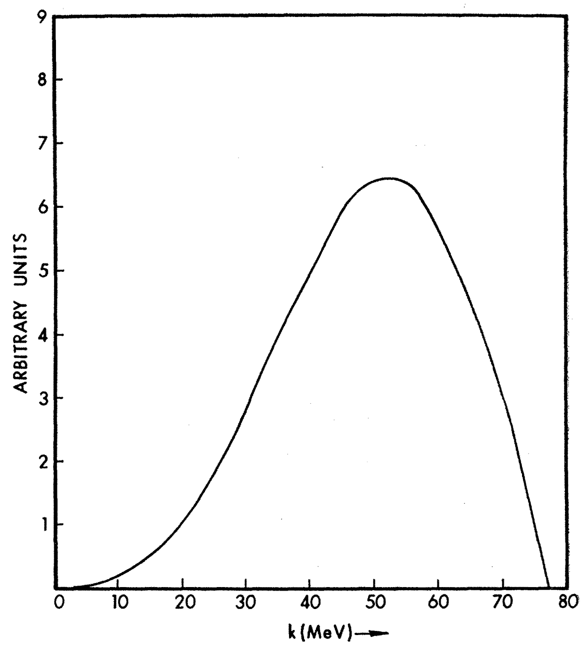

FIG. 1. Plot of photon energy spectrum for the $E1$ transition of $K_L{}^0 \to \pi^+\pi^-\pi^0\gamma$ in the $K_L{}^0$ rest frame.

Thus, we have related the parity-conserving $E1$ transition of $K_L{}^0 \to \pi^+\pi^-\pi^0\gamma$ to the parity-violating amplitude of $K_L{}^0 \to \pi^+\pi^-\gamma$. This PV amplitude contributes to $E1$ CP-violating and $E2$ CP-conserving radiation and is expected to be small compared to the PC amplitude of $K_L{}^0 \to \pi^+\pi^-\gamma$ which contributes to $M1$ CP-conserving radiation.

The PV amplitude for $K_L{}^0 \to \pi^+\pi^-\gamma$ can be written as

$$(2\pi)^6(16\omega_1\omega_2k\omega_K)^{1/2}\langle\pi^+\pi^-\gamma|H_w{}^{\text{PV}}|K_L{}^0\rangle$$
$$= (ef/M_K{}^3)[(p\cdot k)(\epsilon\cdot q_1)-(q_1\cdot k)(\epsilon\cdot p)]. \quad (24)$$

Thus, in the soft-pion limit we have, after combining Eqs. (11), (23) and (24), the form-factor relation

$$F = f/2\sqrt{2}F_\pi. \quad (25)$$

The form factor f has been estimated[14] to be $f \approx 6.35\times10^{-8}$. We thus find from Eq. (25)

$$M_KF \approx 8.3\times10^{-8}. \quad (26)$$

This estimate for F yields for the $E1$ transition rate $\Gamma_E(K_L{}^0 \to \pi^+\pi^-\pi^0\gamma) \approx 8.4\times10^{-2}\ \text{sec}^{-1}$, which results in the branching ratio

$$R(E1) \approx 4.5\times10^{-9}, \quad (27)$$

which is consistent with our previous current-algebra result combined with the experimental upper limit on $K_L{}^0 \to \pi^0\gamma\gamma$. Combining Eqs. (19) and (27), one is led to the prediction

$$\frac{\Gamma(K_L{}^0 \to \pi^0\gamma\gamma)}{\Gamma(K_L{}^0 \to \text{all})} \approx 2.8\times10^{-6}. \quad (28)$$

[14] D. P. Majumdar and J. Smith, Phys. Rev. **187**, 2039 (1969).

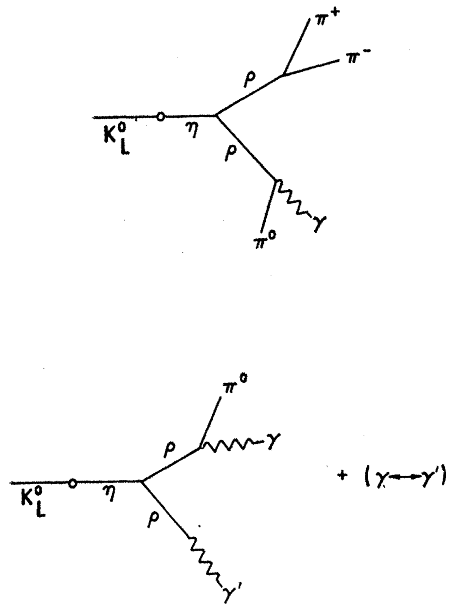

FIG. 2. Feynman diagrams for $K_L^0 \to \pi^+\pi^-\pi^0\gamma$ and $K_L^0 \to \pi^0\gamma\gamma$ in the ρ-dominance model.

In accepting this result it should be remarked that Eqs. (19) and (27) were obtained by means of different soft-pion limits. Thus, the validity of combining these results rests primarily on the smoothness properties of the various form factors involved.

III. VECTOR-DOMINANCE CALCULATION OF $E1$ TRANSITION

In this section we will determine the magnitude of the $E1$ transition contribution to $K_L^0 \to \pi^+\pi^-\pi^0\gamma$ by means of vector-meson dominance. Neglecting small CP-violating effects, if the photon in the decay is emitted through an $E1$ transition, the 3π state has $I=0$ and $J^P=1^+$. Thus the π^0 will be in an s state relative to the $\pi^+\pi^-$ and the decay is described by the ρ-dominance model. The principal Feynman diagram is shown in Fig. 2. We have only included the η-pole contribution, neglecting the effects of a π^0 or X^0 intermediate state. Since the η mass is so close in value to the K_L^0 mass, the η-pole contribution should dominate in this decay. The effects of η-X^0 mixing should only affect our results by no more than 10% for a mixing angle of $\pm 10°$.

Taking as the invariant couplings for the $\rho\pi\gamma$, $\eta\rho\rho$, $\rho\pi\pi$, and K_L^0-η vertices

$$(f_{\rho\pi\gamma}/\mu)\epsilon_{\mu\beta\gamma\alpha}q_{3\mu}k_\beta\epsilon_\gamma\rho_\alpha\,, \quad f_{\rho\pi\pi}\rho_\nu(q_1-q_2)_\nu\,, \quad (f_{\eta\rho\rho}/\mu)\epsilon_{\nu\sigma\kappa\delta}p_{1\nu}\rho_{1\sigma}p_{2\kappa}\rho_{2\delta}\,, \quad \sqrt{2}M_K^2 f_{K_L{}^0\eta}\,, \tag{29}$$

where p_1, p_2, ρ_1, ρ_2 represent ρ-meson momenta and polarizations, respectively, the matrix element for the $E1$ transition is given by

$$\mathfrak{M}_E(K_L^0 \to \pi^+\pi^-\pi^0\gamma) = \frac{\sqrt{2}f_{K_L{}^0\eta}f_{\rho\pi\pi}f_{\rho\pi\gamma}f_{\eta\rho\rho}M_K^2}{\mu^2(M_K^2-m_\eta^2)[(q_1+q_2)^2-m_\rho^2][(q_3+k)^2-m_\rho^2]}A_E\,, \tag{30}$$

where A_E is defined as

$$\begin{aligned} A_E = {} & q_3\cdot(q_1+q_2)[(\epsilon\cdot q_3)k\cdot(q_1-q_2)-(k\cdot q_3)\epsilon\cdot(q_1-q_2)] \\ & +q_3\cdot(q_3+k)[\epsilon\cdot(q_1-q_2)k\cdot(q_1+q_2) \\ & -k\cdot(q_1-q_2)\epsilon(q_1+q_2)]+q_3\cdot(q_1-q_2) \\ & \times[\epsilon\cdot(q_1+q_2)(k\cdot q_3)-k\cdot(q_1+q_2)(\epsilon\cdot q_3)]\,. \end{aligned} \tag{31}$$

The $E1$ transition rate is then given by

$$\Gamma_E(K_L^0 \to \pi^+\pi^-\pi^0\gamma) = \frac{1}{32(2\pi)^8 M_K}\int \frac{d^3q_1}{\omega_1}\frac{d^3q_2}{\omega_2}\frac{d^3q_3}{\omega_3}\frac{d^3k}{k} \times |\mathfrak{M}_E|^2\delta^4(p-q_1-q_2-q_3-k)\,. \tag{32}$$

Similarly, describing $K_L^0 \to \pi^0\gamma\gamma$ with the ρ-dominance model and evaluating its Feynman diagram in Fig. 2, leads to the decay rate

$$\Gamma(K_L^0 \to \pi^0\gamma\gamma) = \frac{1}{16(2\pi)^5 M_K}\int \frac{d^3k}{k}\frac{d^3k'}{k'}\frac{d^3q}{\omega} \times |\mathfrak{M}(K_L^0 \to \pi^0\gamma\gamma)|^2\delta^4(p-q-k-k')\,, \tag{33}$$

where

$$\mathfrak{M}(K_L^0 \to \pi^0\gamma\gamma) = \frac{\sqrt{2}f_{K_L{}^0\eta}f_{\rho\pi\gamma}f_{\eta\rho\rho}f_{\rho\gamma}M_K^2}{\mu^2 m_\rho^2(m_K^2-m_\eta^2)[(q+k)^2-m_\rho^2]}B+(k\leftrightarrow k') \tag{34}$$

and

$$\begin{aligned} B = {} & (q\cdot k')[(\epsilon\cdot q)(k\cdot\epsilon')-(k\cdot q)(\epsilon\cdot\epsilon')] \\ & +q\cdot(q+k')\{(\epsilon\cdot\epsilon')(k\cdot k')-(k\cdot\epsilon')(k'\cdot\epsilon)\} \\ & +(q\cdot\epsilon')\{(k'\cdot\epsilon)(k\cdot q)-(k\cdot k')(q\cdot\epsilon)\}\,. \end{aligned} \tag{35}$$

Combining Eqs. (32) and (33) and using $f_{\rho\gamma}=em_\rho^2/f_{\rho\pi\pi}$, $f_{\rho\pi\pi}^2/4\pi=2.5$, we find

$$\Gamma_E(K_L^0 \to \pi^+\pi^-\pi^0\gamma)/\Gamma(K_L^0 \to \pi^0\gamma\gamma) = 1.18\times10^{-3}\,, \tag{36}$$

in fairly good agreement with the current-algebra prediction, Eq. (19).

IV. DISCUSSION AND CONCLUSIONS

We have seen that current algebra and PCAC yield specific predictions for the CP-conserving $E1$ transition of $K_L^0 \to \pi^+\pi^-\pi^0\gamma$. First, by applying a momentum expansion technique of Weinberg, we calculated the branching ratio $\Gamma_B(K_L{}^0 \to \pi^+\pi^-\pi^0\gamma)/\Gamma(K_L{}^0 \to \pi^0\gamma\gamma) = 0.16\%$. Using the present experimental upper limit on $K_L{}^0 \to \pi^0\gamma\gamma$ we then found $R(E1) < 3.7\times10^{-7}$. Then by considering a single soft-pion limit, we found, using an estimate for the parity-violating amplitude of $K_L{}^0 \to \pi^+\pi^-\gamma$, the value $R(E1) = 4.5\times10^{-9}$.

We have further calculated the $E1$ transition by means of a vector-meson-dominance model. We found that the description of the $E1$ transition by the ρ-dominance model gives results consistent with current algebra.

Finally, it remains for us to compare the size of the direct-emission contribution to the inner bremsstrahlung. Unlike the decay $K_L{}^0 \to \pi^+\pi^-\gamma$, CP invariance does not prohibit bremsstrahlung from taking place in $K_L{}^0 \to \pi^+\pi^-\pi^0\gamma$. One can in fact easily estimate its size compared to the CP-conserving nonradiative process to be

$$R'(B) \equiv \frac{\Gamma_B(K_L{}^0 \to \pi^+\pi^-\pi^0\gamma)}{\Gamma(K_L{}^0 \to \pi^+\pi^-\pi^0)} = 9.8\times10^{-4}, \quad k > 10 \text{ MeV}.$$

In Table I we have listed the estimates for $R'(E1)$. We find $R'(E1) \leq 2.9\times10^{-6}$. Based on these calculations, we conclude that the direct-emission contribution is smaller by at least two orders of magnitude than the inner-bremsstrahlung contribution. As a result, the possibilities of observing CP-violating effects in $K_L{}^0 \to \pi^+\pi^-\pi^0\gamma$ would appear quite remote in the foreseeable future.

On the other hand, our calculations have given further support to the belief in a general equivalence between current algebra and vector-meson dominance.

PHYSICAL REVIEW D VOLUME 3, NUMBER 1 1 JANUARY 1971

Backward πN Scattering: A Regge-Pole–Cut Model without Parity Doubling*

LOYAL DURAND III AND HUBERT M. LIPINSKI†
Department of Physics, University of Wisconsin, Madison, Wisconsin 53706
(Received 9 September 1970)

We present a very simple model in which the amplitude for backward πN scattering is described in terms of the exchange of the N_α and Δ_δ Regge trajectories and associated Regge cuts. The trajectory functions $\alpha(u)$ and the positions of the branch points $\alpha_c(u)$ are linear functions of u, but there are no odd-parity resonances associated with the resonances on the N_α and Δ_δ trajectories (no parity doubling). The model provides a good description of the data on backward $\pi^\pm p$ scattering, including the differential charge exchange cross section and the polarization observed in π^+p scattering.

IN a recent paper,[1] Carlitz and Kislinger showed that it is possible to construct simple Regge-type models for boson-fermion scattering amplitudes which do not contain parity doublets. In their model, the fermion trajectory functions are linear functions of $u = W^2$, but the positive-parity resonances at $\alpha^+(W) = \frac{1}{2}, \frac{3}{2}, \ldots$ do not have odd-parity partners as would seem to be required by the MacDowell symmetry relation $\alpha^-(W) = \alpha^+(-W)$. This model is potentially of great interest, since it provides a natural explanation of the absence of odd-parity states corresponding to the states on the nucleon and Δ trajectories, despite the near linearity of those trajectories. In the Carlitz-Kislinger model, the odd-parity poles required by the MacDowell symmetry are on the second sheet of the j plane. These poles can only be reached by passing through a fixed cut with a branch point at $j = \alpha(0)$, and consequently do not lead to physical resonances. Despite these desirable properties, the Carlitz-Kislinger model is not entirely satisfactory. First, the fact that the cut is a fixed cut rather than a moving (Regge) cut—while not precluded by any general argument—is not in accord with the usual ideas about the structure of scattering amplitudes in the j plane. Second, and more important, the discontinuity across the cut diverges at the branch point, in contradiction to a result derived by Bronzan and Jones[2] using the unitarity relations for the partial-wave amplitudes.

In the present paper, we discuss a simple model of the Carlitz-Kislinger type in which the foregoing difficulties are eliminated. In this model, the only singularities of the scattering amplitude in the j plane are moving poles and moving cuts, with pole and branch point trajectories which are linear functions of $u = W^2$,

$$\begin{aligned} \alpha_p{}^+(W) &= \alpha_0 + \alpha_p{}' W^2, \\ \alpha_c{}^+(W) &= \alpha_0 + \alpha_c{}' W^2, \quad \alpha_c{}' < \alpha_p{}'. \end{aligned} \tag{1}$$

There are no (physical) parity doublets in the model,

* Supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation, and in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-881, #C00-881-288.

† Present address: Department of Engineering–Economic Systems, Stanford University, Stanford, Calif. 94305.

[1] R. Carlitz and M. Kislinger, Phys. Rev. Letters 24, 186 (1970).

[2] J. B. Bronzan and C. E. Jones, Phys. Rev. 160, 1494 (1967).