effects the transformation from the H.T.P. *cum* H.R.P. to this I.T.P. *cum* I.R.P. will of course be the solution of (4.12) and (4.14) with $J^{(3)\mu\nu}=J^{\mu\nu}+x^\nu P^{(0)\mu}-x^\mu P^{(0)\nu}$.

Finally, it is hardly necessary to point out that the choice of rotation pictures is not limited to ones in which operators behave like scalars. One could, in principle, employ rotation pictures which impose any nominated tensorial or spinorial character on operators. The concept we have been discussing would thus appear to have some utility.

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Scale-Symmetry Breaking and Scaling Properties of Currents

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If δ [the term that breaks scale symmetry and not $SU(3)\otimes SU(3)$] is a c number, (or vanishes) all components of the $SU(3)\otimes SU(3)$ currents have the same dimension $l_0=l_k=3$. In other words, $l_0\neq l_k$ implies that a q number δ is present. Presence of a q number δ thus follows if the Schwinger term in the current commutators is a (finite) c number. We obtain the commutators of $J_0{}^a$ with the scale-symmetry-breaking operators δ, u, and $\theta_\mu{}^\mu$ in terms of $J_k{}^a$ and $\partial^\mu J_\mu{}^a$. As one of their consequences, continuity of the limit $\delta\to 0$ implies that $l_k=3$.

The problem of whether, in addition to the scale- and $SU(3)\otimes SU(3)$-symmetry-breaking term u, a q number δ breaking scale symmetry [and not $SU(3)\otimes SU(3)$] is present in the Hamiltonian density $\theta_{00}(x)$ of strong interactions, is of particular interest.[1] We connect in this paper properties and existence of δ with the scale-transformation property (a major problem itself) of the $J_k{}^a$ [the space components of the ($a=1$–16) $SU(3)\otimes SU(3)$ currents $J_\mu{}^a$]. As one of our main results, a q number δ must be present if the $J_k{}^a$ have no dimension or a dimension $l_k\neq 3$. In other words, if $\delta=0$ or a c number then $l_0=l_k=3$. Since it is known[2] that $l_k=3$ is incompatible with a (finite) c-number Schwinger term (ST), our result implies presence of a q number δ in this case. Therefore, evidence that the ST is a (finite) c number indicates that a q number δ is present.[3]

We will adopt in this paper the scheme of broken internal and scale symmetry as developed in Ref. 1. As has become customary, we denote by $\theta_{\mu\nu}$ the "new and improved" energy-momentum tensor of Ref. 4. Assuming that scale invariance implies conformal invariance, we may write the dilatation current D_μ as[4]

$$D_\mu(x)=x^\nu\theta_{\mu\nu}(x). \tag{1}$$

The dilatation charge Q_D is then given by

$$Q_D(x_0)=\int d^3x\, D_0(x). \tag{2}$$

In this paper we are concerned with the algebraic properties of the currents J_μ, D_μ, and their divergences. Thus, the particular mechanism of scale-symmetry breaking is of no importance to us. We note that the dilatation charge $Q_D(x_0)$ is time independent if and only if the divergence of the dilatation current

$$\partial^\mu D_\mu(x)=\theta_\mu{}^\mu(x) \tag{3}$$

vanishes. For basic canonical quantities $X(x)$, the definition in Eq. (1) implies that formally

$$[Q_D(x_0),X(x)]=-i(l_x+x^\mu\partial_\mu)X(x), \tag{4}$$

with $l_x=1$ ($\frac{3}{2}$) for bosons (fermions). Following Refs. 1, 5, and 6, we assume that Eq. (4) holds for some operators of physical interest such as, for example, the currents. (The precise assumptions will be given later.) However, we do not require that l_x is fixed by canonical arguments since interactions might change the dimension of a field without necessarily destroying the form of the commutator (4).

We next introduce our assumptions. We first assume that the charge densities $J_0{}^a$ have a fixed dimension,[1,5,6]

* Supported in part by the U. S. Atomic Energy commission.
[1] M. Gell-Mann, Caltech report (unpublished).
[2] H. Genz and J. Katz, Nuovo Cimento Letters **4**, 1103 (1970).
[3] See, e.g., R. Jackiw, R. van Royen, and J. West, Phys. Rev. D **2**, 2473 (1970), for a discussion of this evidence and its implications for Schwinger terms.
[4] C. Callan, S. Coleman, and R. Jackiw, Ann. Phys. (N. Y.) **59**, 42 (1970).
[5] G. Mack, Nucl. Phys. **B5**, 499 (1968); H. Kastrup, *ibid.* **B15**, 179 (1970); K. Wilson, Phys. Rev. **179**, 1499 (1969); D. Gross and J. Wess, Phys. Rev. D **2**, 753 (1970).
[6] M. dal Cin and H. Kastrup, Nucl. Phys. **B15**, 189 (1970).

i.e.,

$$\text{(i)}\quad [Q_D(0),J_0{}^a(x)]=-i(3+x^l\partial_l)J_0{}^a(x)\,. \tag{5}$$

In the above, we have inferred from charge algebra that the dimension of $J_0{}^a$ is 3. Namely, we assume that

$$\text{(ii)}\quad [Q^a(x_0),J_\mu{}^b(x)]=if^{abc}J_\mu{}^a(x) \tag{6}$$

for the 16 $SU(3)\otimes SU(3)$ currents $J_\mu{}^a$, $a=1$–16. (We use the notation $J_\mu{}^a=V_\mu{}^a$ for $a=1$–8 and $J_\mu{}^a=A_\mu{}^a$ for $a=9$–16.) It then follows by commuting Eq. (6) for $\mu=0$ with Q_D and using the Jacobi identity that the dimension of Q is 0 (and thus J_0 has dimension 3). Incidentally, it should also be noted at this point that it follows from Eq. (6) that if any one of the 16 currents $J_\mu{}^a$ has dimension $l_\mu{}^a$ then all the $J_\mu{}^a$ have this dimension, i.e., $l_\mu{}^a=l_\mu$.

Evidently, Eq. (1) simplifies for $x_0=0$. We therefore take all time components $x_0=\cdots=y_0=0$ throughout the remainder of the paper. Using[7] (compare also Ref. 8)

$$[Q_D(0),M_{0k}]=-i\int d^3x\, x_k\theta_\mu{}^\mu(x)\,, \tag{7}$$

it follows immediately that the operators S defined by

$$S_k{}^a=i\int d^3y\, y_k[J_0{}^a(0),\theta_\mu{}^\mu(y)] \tag{8}$$

have the property[2,7,9]

$$iS_k{}^a=[Q_D(0),J_k{}^a(0)]+3iJ_k{}^a(0)\,. \tag{9}$$

[Since $S_k{}^a=0$ if and only if $J_k{}^a$ has the unique dimension 3, it follows that Eqs. (8) and (9) provide an obvious criterion[2,7,9] for the dimension of $J_k{}^a$.] In order to obtain (8), one commutes (7) with $J_0{}^a$, uses the Jacobi identity, Eq. (5), and the covariance of $J_k{}^a$. Incidentally,[2,7] if $J_k{}^a$ has dimension l_k, one derives from Eq. (7) that

$$\int d^3y\, y_k[J_l{}^a(0),\theta_\mu{}^\mu(y)]=ig_{kl}(3-l_k)J_0{}^a(0)\,, \tag{10}$$

another criterion for l_k (since this ST vanishes if and only if $l_k=3$). We use Eq. (8) later on and determine in Eqs. (57)–(59) and (62) the equal-time commutator (ETC) of $J_0{}^a$ with u, δ, and $\theta_\mu{}^\mu$.[10]

We next assume that the divergences of the $SU(3)\otimes SU(3)$ currents may be written as ($\sum'$ denotes a finite sum)

$$\partial^\mu J_\mu{}^a(x)=\sum{}'\,\Phi^{a,n}(x)\,, \tag{11}$$

with

$$\text{(iii)}\quad [Q_D(0),\Phi^{a,n}(x)]=-i(l_{\Phi^{a,n}}+x^l\partial_l)\Phi^{a,n}(x)\,, \tag{12}$$

i.e., $\partial^\mu J_\mu{}^a$ splits into a sum of scalars with the dimensions $l_{\Phi^{a,n}}$. Equations (11) and (12) are much weaker than the assumption that $\partial^\mu J_\mu{}^a$ itself has a fixed dimension, as is sometimes made. For a large class of $SU(3)\otimes SU(3)$-breaking mechanisms [which include the $(\bar{3},3)\oplus(3,\bar{3})$ model of Ref. 11 as well as the (8,8) model of Refs. 12 and 13], the latter assumption implies, as we shall soon see, that the $SU(3)\otimes SU(3)$ symmetry-breaking part u has a unique dimension. Since the question of whether or not u has a unique dimension is in fact a major problem itself, we prefer to use Eqs. (11) and (12) as well as the $SU(3)\otimes SU(3)$ symmetry-breaking assumption given in Eq. (25). Finally the assumptions on scale and $SU(3)\otimes SU(3)$ symmetry breaking fulfilled in Ref. 1 are introduced. According to these, θ_{00} may be split as

$$\text{(iv)}\quad \theta_{00}(x)=\hat{\theta}_{00}(x)+w(x)\,, \tag{13}$$

where

$$\int d^3y[Q_D(0),\hat{\theta}_{00}(y)]=-i\int d^3y\,\hat{\theta}_{00}(y) \tag{14}$$

and (we allow for different w_n with the same dimension)

$$w(x)=\sum{}'_n\, w_n(x)\,, \tag{15}$$

with

$$\int d^3y[Q_D(0),w_n(y)]=-i(l_{w_n}-3)\int d^3y\, w_n(y)\,. \tag{16}$$

Using next[14] (compare also Ref. 8)

$$[Q_D,H]=i\int d^3x\,\theta_\mu{}^\mu(x)-iH\,, \tag{17}$$

it follows[1] from this that

$$\theta_\mu{}^\mu(x)=\sum{}'(4-l_{w_n})w_n(x) \tag{18}$$

if $\sum'_n (4-l_{w_n})w_n(x)$ is assumed to be a scalar. To derive (18) it suffices to introduce the decomposition (13) into (17) and to use (14). Then the **x**-integrated equation (18) follows. Since $\theta_\mu{}^\mu$ is a scalar and since $\sum'_n (4-l_{w_n})w_n(x)$ is also a scalar by assumption, Eq. (18) then follows from a theorem stated in Ref. 1. Occasionally we will also use the expression

$$\theta_\mu{}^\mu(x)=4\theta_{00}(x)-i[Q_D(0),\theta_{00}(x)]+x^l\partial_l\theta_{00}(x) \tag{19}$$

[7] J. Katz, Nuovo Cimento (to be published).

[8] S. Coleman and R. Jackiw, MIT report (unpublished).

[9] After this work was completed we received a paper [M. Bég, J. Bernstein, D. Gross, R. Jackiw, and A. Sirlin, Phys. Rev. Letters **25**, 1231 (1970)] in which the criterion (9) for $l_0=l_k$ (Refs. 2 and 7) is also obtained. The result [Eq. (49)] that (19) and $j=0$ imply $\tau=0$ was also derived by Bég *et. al.* (see also Ref. 10).

[10] Our result in (46) includes determination of the term multiplying $\delta(\mathbf{x}-\mathbf{y})$ (i.e., of $[Q^a,\theta_\mu{}^\mu]$). For $l_k=3$ (as assumed in Ref. 6) our results in (44) and (47) agree with Ref. 6.

[11] M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

[12] K. Barnes and C. Isham, Nucl. Phys. **B17**, 267 (1970).

[13] H. Genz and J. Katz, Nucl. Phys. **B21**, 333 (1970).

[14] H. Genz, Phys. Letters **31B**, 146 (1970).

instead of (18). Evidently, (19) holds under the stronger assumptions of the local Eqs. (14) and (16), i.e.,

$$[Q_D(0),\hat{\theta}_{00}(x)]=-i(4+x^l\partial_l)\hat{\theta}_{00}(x) \tag{20}$$

and

$$[Q_D(0),w_n(x)]=-i(l_{w_n}+x^l\partial_l)w_n(x). \tag{21}$$

[The main conclusion on δ is independent of (20), (21), and (19).]

Equation (19) follows immediately from Eq. (18) upon making use of Eqs. (20) and (21). Alternatively, it may also be obtained by making use of the definition of Q_D and the equal-time commutators $i[\theta_{0k}(y),\theta_{00}(x)]$ provided that the contribution of possible noncanonical terms to the ETC $i[Q_D,\theta_{00}(x)]$ vanish (even if they are present in $i[\theta_{0k}(y),\theta_{00}(x)]$).

The last assumption concerns the splitting of w into an $SU(3)\otimes SU(3)$-breaking and an $SU(3)\otimes SU(3)$-invariant term, i.e.,

$$\text{(v)}\quad w(x)=\delta(x)+u(x), \tag{22}$$

where the scalars δ and u have the properties

$$[Q^a(0),\delta(x)]=0, \tag{23}$$

$$[iu(x),Q^a(0)]=\partial^\mu J_\mu{}^a(0), \tag{24}$$

and, for certain numbers c,

$$u(x)=\sum{}'_b c_b\partial^\mu J_\mu{}^b(x)+\sum{}'_{a,b} c_{ab}[Q^a(0),\partial^\mu J_\mu{}^b(x)] +\sum{}'_{a,b,c} c_{abc}[Q^a(0),[Q^b(0)\partial^\mu J_\mu{}^c(x)]]+\cdots. \tag{25}$$

This possibility of constructing u from the current divergences certainly holds in the $(\bar{3},3)\oplus(3,\bar{3})$ model of Ref. 11 and the (8,8) model of Refs. 12 and 13. However, it is much more general since, for example, it also holds for any irreducible representation of $SU(3)\otimes SU(3)$.

We shall then see from the above that if δ is a c number then $S_k{}^a=0$, i.e., the space components of the currents have dimension 3. We observe to this end that u splits into terms with fixed dimension α, i.e., $u(x)=\sum'_\alpha u_\alpha(x)$, where

$$[Q_D(0),u_\alpha(x)]=-i(\alpha+x^l\partial_l)u_\alpha(x). \tag{26}$$

This follows since [using (5), (11), and (12)] each one of the terms $c_b\partial^\mu J_\mu{}^b$, $c_{ab}[Q^a,\partial^\mu J_\mu{}^b],\ldots$ (and therefore their sum, i.e., u) evidently is a sum of terms with fixed dimensions. Adding in this expansion of u all terms with the same dimension α, the unique labeling by dimension [as assumed in (26)] is achieved. After splitting w into $\sum'_\beta w_\beta$ (with β the dimension of w_β) we may write

$$w(x)=\sum{}'_\lambda \delta_\lambda(x)+\sum{}'_\lambda u_\lambda(x) \tag{27}$$

and

$$\delta(x)=\sum{}'_\lambda \delta_\lambda(x). \tag{28}$$

In the above, λ is any one of the α or β and we have defined $\delta_\lambda(x)=w_\lambda-u_\lambda$ with $w_\lambda=0$ ($u_\lambda=0$) if no w_α (u_β) with dimension λ exists. For later purposes let us note at this point that all $q_\lambda{}^a$ defined by

$$q_\lambda{}^a=[Q^a,\delta_\lambda(x)] \tag{29}$$

have different dimensions for different λ (namely, λ) or vanish. Likewise, all Δ defined by

$$\Delta_{k,u_\lambda}{}^a=i\int d^3y\, y_k[J_0{}^a(0),u_\lambda(y)] \tag{30}$$

have either dimension $\lambda-1$ or vanish. [To check this, commute (29) and (30) with Q_D and use the Jacobi identity.] Since [from (23) and (28)] $\sum'_\lambda q_\lambda=0$, it then follows that

$$q_\lambda{}^a(x)=0, \tag{31}$$

since all q_λ have different dimensions. Likewise the condition

$$\int d^3y\, y_k[J_0{}^a(0),u(y)]=0 \tag{32}$$

is sufficient to ensure

$$\Delta_{k,u_\lambda}{}^a=0 \tag{33}$$

for all a, k, and λ.

From (27) it follows that (18) may be written as

$$\theta_\mu{}^\mu(x)=\sum{}'_\lambda (4-\lambda)\delta_\lambda(x)+\sum{}'_\lambda (4-\lambda)u_\lambda(x). \tag{34}$$

The reader should notice that (also for the q-number parts)

$$\sum{}'_\lambda (4-\lambda)\delta_\lambda(x)\neq 0 \quad\text{implies}\quad \delta\neq 0. \tag{35}$$

The condition, as may be seen from (25),

$$\int d^3y\, y_k[J_0{}^a(0),\partial^\mu J_\mu{}^b(y)]=0, \tag{36}$$

implies (repeatedly using the Jacobi identity) Eq. (32) and therefore (33). Since (36) can be found in the literature[15,16] (for the reader's convenience we present the derivation of Ref. 16 below), it remains to be seen that (33) and $S_k{}^a\neq 0$ implies that δ is a q number. This follows since, upon substituting (34) into the definition of S in Eq. (8), the only surviving term is

$$S_k{}^a=i\int d^3y\, y_k[J_0{}^a(0),\sum{}'_\lambda (4-\lambda)\delta_\lambda(y)]. \tag{37}$$

Thus, if $J_k{}^a$ has no dimension or a dimension different from 3, $\sum'_\lambda (4-\lambda)\delta_\lambda(y)$ has a nonvanishing q-number part. From this it then follows that δ is a q number.

To derive (36) we note that covariance and locality imply

$$[i\theta_{00}(x),J_0{}^a(y)]=\partial^\mu J_\mu{}^a(x)\delta(\mathbf{x}-\mathbf{y})+J_k{}^a(x)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}) +\sum_{\rho=2}{}' j_{k_1\cdots k_\rho}{}^a(y)\frac{\partial}{\partial x_{k_1}}\cdots\frac{\partial}{\partial x_{k_\rho}}\delta(\mathbf{x}-\mathbf{y}). \tag{38}$$

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[15] D. Gross and R. Jackiw, Phys. Rev. **163**, 1688 (1967); R. Jackiw, *ibid.* **175**, 2058 (1968).

[16] H. Genz and J. Katz, Phys. Rev. D **2**, 2225 (1970).

In the above relation,

$$\textstyle\sum'_{\rho}\, \partial^{k_1}\cdots\partial^{k_\rho} j_{k_1\cdots k_\rho}{}^a(x)=0 \qquad (39)$$

is obtained from (13), (23), and (24) assuming

$$[Q^a(0),\hat{\theta}_{00}(x)]=0\,. \qquad (40)$$

Incidentally it should be noticed that $j=0$ has been obtained for canonical currents,[15–18] using the associative law[16,18] or Schwinger's action principle.[15,17] Commuting (38) with $J_0{}^b$ and using commutators of charges with charge densities, we get (compare Refs. 15 and 16)

$$\begin{aligned}&[J_0{}^a(y),\partial^\mu J_\mu{}^b(x)]-[Q^b(0),\partial^\mu J_\mu{}^a(x)]\delta(\mathbf{x}-\mathbf{y})\\&-if^{abc}\partial^\mu J_\mu{}^c(z)\delta(\mathbf{x}-\mathbf{y})=[Q^b(0),J_k{}^a(x)]\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y})\\&\qquad -if^{abc}J_k{}^c(x)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y})+\int Z(x,y,z)d^3z\,, \qquad (41)\end{aligned}$$

where Z denotes the contributions of the j's in (38) and

$$\int d^3x\, Z(x,y,z)=\int d^3x\, x_k Z(x,y,z)=0 \qquad (42)$$

from covariance. It should also be noted that in order to derive Eq. (41) one does not need to make any assumptions about the existence of the ETC $[J_0{}^a(x), J_k{}^b(y)]$. Thus the preceding derivation holds even when infinities arise in the ETC. This is the case, for example, in scale-invariant theories. (Recall that it may be shown that the vacuum expectation value of the Schwinger term must be infinite in scale-invariant theories.[2,7]) Multiplication of (41) by $(x-y)_m$ and integration over $\mathbf{x}$ then leads to

$$\begin{aligned}&[Q^b(0),J_m{}^a(0)]\\&\qquad =if^{bac}J_m{}^c(0)-\int d^3z\, z_m[J_0{}^a(0),\partial^\mu J_\mu{}^b(z)]\,, \qquad (43)\end{aligned}$$

from which Eq. (36) is obtained by use of charge-current commutators.

In what follows, commutators involving $J_0{}^a$ are calculated. In order to obtain reasonable simple expressions, we assume for the moment that $\partial^\mu J_\mu{}^a$ and $J_k{}^a$ have fixed dimensions $l_d{}^a$ and l_k, respectively. Then, commuting (17) with $J_0{}^a$ and calculating the left-hand side using the Jacobi identity, one obtains

$$\begin{aligned}&i\left[\int d^3x\,\theta_\mu{}^\mu(x),J_0{}^a(0)\right]\\&\qquad =(4-l_d{}^a)\partial^\mu J_\mu{}^a(0)+(l_k-3)\partial^k J_k{}^a(0)\,. \qquad (44)\end{aligned}$$

[17] J. Schwinger, Phys. Rev. **130**, 406 (1963); Nuovo Cimento **30**, 278 (1963).

[18] H. Genz and J. Katz, Nucl. Phys. **B13**, 401 (1969).

Since Eq. (9) under our present assumptions reads

$$S_m{}^a=(3-l_k)J_m{}^a(0)\,, \qquad (45)$$

we may combine (8), (44), and (45) in writing

$$\begin{aligned}&[i\theta_\mu{}^\mu(x),J_0{}^a(y)]\\&\quad =(4-l_d{}^a)\partial^\mu J_\mu{}^a(x)\delta(\mathbf{x}-\mathbf{y})-(l_k-3)J_k{}^a(x)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y})\\&\qquad +\sum_{\rho=2}{}' \tau_{k_1\cdots k_\rho}{}^a(y)\frac{\partial}{\partial x_{k_1}}\cdots\frac{\partial}{\partial x_{k_\rho}}\delta(\mathbf{x}-\mathbf{y})\,. \qquad (46)\end{aligned}$$

The reader should notice the strong similarity between Eqs. (38) and (46), which are—possibly except for the j and τ [see (49)]—identical if $l_d{}^a=3$ and $l_k=2$.

Because of (46)

$$[Q^a(0),\theta_\mu{}^\mu(x)]=(4-l_d{}^a)i\partial^\mu J_\mu{}^a(x) \qquad (47)$$

is equivalent to

$$\sum_{\rho=2}{}' \partial^{k_1}\cdots\partial^{k_\rho}\tau_{k_1\cdots k_\rho}{}^a(x)=0\,. \qquad (48)$$

Under the additional assumptions leading to (19), we easily see that (48) [and thus (47)] is implied by (39) [i.e., (40)]. Namely, commuting (38) for $y=0$ with Q_D we recover the form (46) and, as we already know, the terms multiplying $\delta(\mathbf{x})$ and $\partial_k\delta(\mathbf{x})$ [with $\delta(\mathbf{x})$ at present the three-dimensional δ function] must cancel upon comparison [use $x^n\partial_n\partial_k\delta(\mathbf{x})\propto\partial_k\delta(x)$]. The only ST of at least second order we get come from the j's themselves and from the j's in $[(4+x^n\partial_n)\theta_{00}(x),J_0{}^a(0)]$. These therefore yield the terms involving τ. We then have (comparing the coefficients of the derivatives of $\delta(\mathbf{x})$ and performing a translation)

$$\begin{aligned}[Q_D(0),j_{k_1\cdots k_\rho}{}^a(y)]=-i[&\tau_{k_1\cdots k_\rho}(y)\\&+(-4+\rho+y^n\partial_n)j_{k_1\cdots k_\rho}{}^a(y)]\,. \qquad (49)\end{aligned}$$

Thus $j=0$ implies $\tau=0$.[9] In order to get (48) from (39), we need only know that $\sum'_{\rho=2}\partial^{k_1}\cdots\partial^{k_\rho}y^n\partial_n j_{k_1\cdots k_\rho}(y)=0$, as is easily seen. Therefore Eq. (47) (which is often assumed in the literature) is derived here [using (40)] even if $l_k\neq 3$ and ST of higher order are present in (38) [and (46)].

Finally we calculate [independent of (20)] Δ defined by

$$\Delta_k{}^a=i\int d^3y\, y_k[J_0{}^a(0),w(y)] \qquad (50)$$

in terms of $J_k{}^a$. We need here (21) for all w_β except for w_4 or, equivalently, for all δ_λ except for δ_4 (if any)

$$i[Q_D(0),\delta_\lambda(x)]=(\lambda+x^n\partial_n)\delta_\lambda(x)\,. \qquad (51)$$

If $\delta_4\neq 0$ we define a new $\hat{\theta}_{00}(w)$ by $\hat{\theta}_{00}+\delta_4(w-\delta_4)$ without changing our assumptions [e.g., (23) holds—use (31)—also for the new δ]. Upon calculating Δ for this splitting with $\delta_4=0$, no part with dimension 3 will be found. We write $\theta_\mu{}^\mu$ under our present assumptions as

$$\theta_\mu{}^\mu(x)=4\delta(x)-i[Q_D,\delta(x)]+x^n\partial_n\delta(x)+4u(x)-i[Q_D,u(x)]+x^n\partial_n u(x) \quad (52)$$

and get from (52) and (8)

$$iS_k{}^a=[Q_D(0),\Delta_k{}^a]+3i\Delta_k{}^a. \quad (53)$$

Therefore, using (9),

$$[Q_D(0),\,\Delta_k{}^a-J_k{}^a(0)]=-3i(\Delta_k{}^a-J_k{}^a(0)), \quad (54)$$

i.e., $\Delta_k{}^a-J_k{}^a(0)$ has either dimension 3 (notice that Δ and S are local operators) or vanishes. For our new splitting, Δ itself has no part with dimension 3. Defining $\Delta_{k,\lambda}{}^a=i\int d^3y\, y_k[J_0{}^a(0),\delta_\lambda(y)]$, we have, in the splitting of Δ [use (33)], $\sum'_\lambda \Delta_{k,\lambda}{}^a=\Delta_k{}^a$ at most as many terms with different dimensions (i.e., $\lambda-1$) as there are δ_λ. Thus [Eq. (54)] $\Delta_k{}^a$ precisely cancels all parts of $J_k{}^a$ not having dimension 3. In the particular simple case that the $J_k{}^a$ have a fixed dimension l_k, the result is

$$i\int d^3x\, x_k[\delta(x),J_0{}^a(0)]=0 \qquad \text{for } l_k=3 \quad (55)$$

$$=-J_k{}^a(0) \quad \text{for } l_k\neq 3. \quad (56)$$

Thus, from the splitting $\delta=\sum'_\lambda \delta_\lambda$, at most the term with $\lambda=l_k+1$ (there is none with $l_k=4$) contributes in the above equations. [In addition it should also be noted that Eq. (56) is independent of $\delta_4=0$.]

Assuming, for simplicity of writing, the absence of ST of at least second order in the ETC involving $J_0{}^a$, we collect here the ETC obtained in the second half of this paper [choosing $\delta_4=0$ and using Eq. (14)]

$$[iu(x),J_0{}^a(y)]=\partial^\mu J_\mu{}^a(x)\delta(\mathbf{x}-\mathbf{y}), \quad (57)$$

$$[i\delta(x),J_0{}^a(y)]=J_k{}^a(x)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}) \quad \text{for } l_k\neq 3 \quad (58)$$

$$=0 \qquad \text{for } l_k=3 \quad (59)$$

$$i[\hat{\theta}_{00}(x),J_0{}^a(y)]=0 \qquad \text{for } l_k\neq 3 \quad (60)$$

$$=J_k{}^a(x)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}) \quad \text{for } l_k=3, \quad (61)$$

and

$$[i\theta_\mu{}^\mu(x),J_0{}^a(y)]=(4-l_d{}^a)\partial^\mu J_\mu{}^a(x)\delta(\mathbf{x}-\mathbf{y})-(l_k-3)J_k{}^a(x)\frac{\partial}{\partial x_k}\delta(\mathbf{x}-\mathbf{y}). \quad (62)$$

The last relation should be compared with the commutator $[i\theta_{00}(x),J_0{}^a(y)]$ noted in Eq. (38). If scale invariance is broken as proposed in Ref. 1 [assuming Eqs. (51)], the origin of the two remaining terms in (38) (setting $j=0$ in this equation) can be read off from Eqs. (13), (22), and (57)–(61).

Note added in proof. Let us now summarize our main conclusions. We have shown that if δ is absent or is a c number it then follows that $l_k=3$. Conversely, if $l_k\neq 3$, a q number δ must be present. Moreover, we have derived $[Q^a(x_0),\theta_\mu{}^\mu(x)]=(4-l_d)i\partial^\mu J_\mu{}^a(x)$ (which is frequently assumed in the literature) even if $l_k\neq 3$ and higher order Schwinger terms are present in Eqs. (38) and (46). [If they are absent in (38) then they are also absent in (46).] The reader should also note our results in Eqs. (57)–(62). It follows from these equations that if $l_k=3$ then $\hat{\theta}_{00}$ must be present. Furthermore, if the limit $\delta\to 0$ is continuous then $l_k=3$.

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