that may be expected in a more realistic case, let us still use $b \simeq -1.32$ but take $W=1.7$. Then this becomes

$$-\gamma \sin\theta\,(1.18+0.54\gamma_5)\,.$$

The above treatment is of course crude; form factors have been neglected and (48) was assumed to be the only derivative-type interaction term. Nevertheless, the moral is clear; a reasonably large (30%) correction to the axial-vector part of the matrix element may be expected.

Still to be discussed are the nonderivative meson-baryon couplings; we postpone this to the following paper. Here we just remark that it is possible for all the different octet baryon masses to arise by the spontaneous breakdown mechanism from a chiral-invariant meson-baryon interaction, but that this requirement by itself is too weak to give us any additional information.

APPENDIX

Here we consider the $U(3)\times U(3)$ case. V_0 is no longer allowed to depend on I_4, so we must set $V_4=0$. Thus [when V_{SB} is given by (36)] (41) becomes the diagonal matrix

$$\begin{bmatrix} 2g_0/\alpha & 0 & 0 \\ 0 & 2g_0/\alpha & 0 \\ 0 & 0 & 2(g_0+g)/\alpha \end{bmatrix}. \qquad \text{(A1)}$$

The diagonal entries in (A1) are the π, η, and η' masses. Evidently the π and one of the isoscalars are degenerate in clear contradiction to nature. The same unfortunate conclusion holds if V_{SB} is modified to include any combination of the following terms:

$$(M_3{}^{\bar c}M_{\bar c}{}^3+M_{\bar 3}{}^cM_c{}^{\bar 3}),\quad (M_c{}^{\bar c}+M_{\bar c}{}^c)I_1,$$
$$(M_3{}^{\bar 3}+M_{\bar 3}{}^3)I_1,\quad (M_c{}^{\bar a}M_{\bar a}{}^bM_b{}^{\bar c}+M_{\bar c}{}^aM_a{}^{\bar b}M_{\bar b}{}^c)\,, \qquad \text{(A2)}$$
$$(M_3{}^{\bar a}M_{\bar a}{}^bM_b{}^{\bar 3}+M_{\bar 3}{}^aM_a{}^{\bar b}M_{\bar b}{}^3)\,.$$

It appears more promising to try for V_{SB} a form that looks something like I_4. Define the "dual" objects

$$T_a{}^{\bar b}=\epsilon_{amn}\epsilon^{\bar b\bar f\bar g}M_{\bar f}{}^mM_{\bar g}{}^n\,,$$
$$T_{\bar b}{}^a=(T_a{}^{\bar b})^\dagger\,, \qquad \text{(A3)}$$

and try

$$V_{\mathrm{SB}}=-\tfrac{1}{2}g_0'(T_c{}^{\bar c}+T_{\bar c}{}^c)-\tfrac{1}{2}g'(T_3{}^{\bar 3}+T_{\bar 3}{}^3)\,. \qquad \text{(A4)}$$

Using (A4), the squared pion and kaon masses come out to be

$$\pi^2=2g_0'W\,,$$
$$K^2=[2/(1+W)](2g_0'+g')\,, \qquad \text{(A5)}$$

while the squared masses of π, η, and η' are the roots of the secular equation of

$$2\begin{bmatrix} g_0'(1+W)+g' & g_0'+g' & g_0' \\ g_0'+g' & g_0'(1+W)+g' & g_0' \\ g_0' & g_0' & 2g_0'/W \end{bmatrix}. \qquad \text{(A6)}$$

The system (A5) and (A5) is more restrictive than the corresponding set (37), (38), and (41); it does not yield as convincing a solution. Specifically, the analog of (45) is here

$$W^4+2W^2\frac{\pi^2}{(\eta\eta')^2}(\pi^2-\eta^2-\eta'^2)+4\left(\frac{\pi^2}{\eta\eta'}\right)^2=0\,. \qquad \text{(A7)}$$

This gives $W=\pm0.35$ or ±0.21, none of which is consistent with the *usual* theory of weak interactions.

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Electromagnetic Perturbation of the Pseudoscalar-Mass Spectrum*

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Using the formalism developed in the preceding paper, we investigate electromagnetic perturbations in a rather general chiral $SU(3)\times SU(3)$ model of mesons. The meson and octet baryon mass shifts can be successfully correlated, and it is found that the electromagnetic breaking term in the Lagrangian may be of the same order of magnitude as the chiral-symmetry-breaking term. We also discuss the speculation that all strong symmetry breaking may be of electromagnetic and weak origin.

I. INTRODUCTION

IN the preceding paper[1] (hereafter designated I) we dealt with symmetry breaking in a very general chiral $SU(3)\times SU(3)$ model of spin-0 mesons. A mass formula was derived which was true when the Lagrangian contained any chiral-invariant nonderivative part and some additional specific symmetry-breaking part. It was found that the resulting mass spectrum seemed in agreement with nature. For example, there was a striking tendency for the mass of the ninth pseudoscalar meson to come out right when a certain parameter W

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[1] J. Schechter and Y. Ueda, preceding paper, Phys. Rev. D 3, 168 (1971).

was about what was expected from weak-interaction theory.

In this paper we investigate electromagnetic effects on the mass spectrum. The mass formula of I is still applicable since we did not assume isotopic spin invariance in its derivation. All the notation of this paper is the same as in I.

The usual approach[2] to electromagnetic mass shifts evaluates them in terms of a "self-energy" part and also a "tadpole" part, which is dominant for all except the pion mass shift. Our procedure is similar to this, but differs in that we do not just add the "tadpole" contribution on an *ad hoc* basis. Our tadpole comes out naturally from the chiral Lagrangian used. Furthermore, the mass formula automatically takes into consideration the so-called "feedback" effect[3] of the tadpoles on the mass shifts. In this sense the present calculation is a "nonperturbative" one.[4,5]

Our main result is a relation between the K-meson mass shift and the strength of the part of the symmetry-breaking term due to electromagnetism [Eq. (21)]. It turns out that, in this model, the applied electromagnetic perturbation must be relatively large to produce the observed K-meson mass shift. In fact, the order of magnitude of the electromagnetic symmetry-breaking term is the same as that of the chiral-symmetry-breaking term, reinforcing the speculation that chiral-symmetry breaking is due to electromagnetism.

We have, in addition, used the value of the "tadpole" [Eq. (22)] deduced from the meson system together with a chiral-invariant meson-baryon Lagrangian to calculate the baryon mass shifts. A consistent picture is found when the parameter W is around the value we expected. The discussion of the meson-baryon interaction is given in the Appendix.

Finally, in Sec. IV we investigate the very speculative possibility that *all* symmetry breaking in strong-interaction physics is due to electromagnetic and weak perturbations.

II. MASS SPECTRUM WITH ELECTROMAGNETIC PERTURBATION

The Lagrangian of our model, in the notation of I, is

$$\mathcal{L}=-\tfrac{1}{2}\,\mathrm{Tr}(\partial_\mu M\partial_\mu M^\dagger)-V_0-V_{\mathrm{SB}}\,, \qquad (1)$$

where V_0 is the most general nonderivative chiral $SU(3)\times SU(3)$ invariant and V_{SB} is a symmetry-breaking term which also includes electromagnetic effects. The form of V_{SB} in I was

$$V_{\mathrm{SB}}=-g_0(M_1^{\bar{1}}+M_{\bar{1}}^{1}+M_2^{\bar{2}}+M_{\bar{2}}^{2})-(g+g_0)(M_3^{\bar{3}}+M_{\bar{3}}^{3})\,.$$

[2] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).
[3] R. Dashen, Phys. Rev. **183**, 1245 (1969).
[4] N. Cabibbo, Instituto di Fisica "G. Marconi," Universita di Roma, Nota Interna No. 141, 1967 (unpublished).
[5] H. Pagels (unpublished).

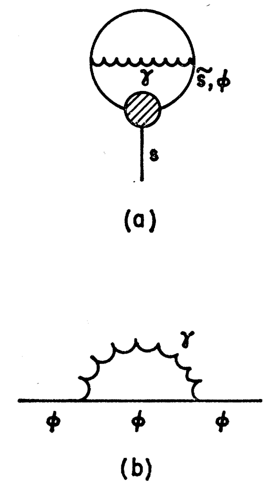

FIG. 1. Diagrams for electromagnetic perturbations.

To get an idea of the kind of electromagnetic perturbations to be added to this, we will take the usual approach of adopting Feynman diagrams as a guide. The kind of diagrams we have in mind are shown in Fig. 1. Evidently the actual computation of diagrams of these generic types is highly model dependent, so we shall do no more than regard them as general clues to the structure which can be expected. With this in mind, we adopt as the symmetry-breaking terms with electromagnetism the following:

$$V_{\mathrm{SB}}=-A_1(M_1^{\bar{1}}+M_{\bar{1}}^{1})-A_2(M_2^{\bar{2}}+M_{\bar{2}}^{2}) -A_3(M_3^{\bar{3}}+M_{\bar{3}}^{3})+d_\pi\phi_1^2\phi_2^1+d_K\phi_1^3\phi_3^1+\cdots\,, \qquad (2)$$

where A_1, A_2, A_3, d_π, and d_K are some real constants. The electromagnetic "tadpole" diagrams of Fig. 1(a) are expected to make A_1 different from A_2 (they are of course equal in the isotopic-spin-invariant limit), and may also give some equal part to all of A_1, A_2, and A_3. At the present time it is not necessary to specify the explicit choice of A_1, A_2, and A_3 or its origin. The "self-energy"-type diagram of Fig. 1(b) contributes to d_π and d_K. We are assuming that only charged particles get this kind of contribution.

To get the pseudoscalar meson mass spectrum for the V_{SB} given in (2) we may use the general mass formula (24) of I. Then the squared masses of the π^+, K^+, and K^0 are

$$\pi_+^2=2\left(\frac{A_1+A_2}{\alpha_1+\alpha_2}\right)+d_\pi\,,$$

$$K_+^2=2\left(\frac{A_1+A_3}{\alpha_1+\alpha_3}\right)+d_K\,, \qquad (3)$$

$$K_0^2=2\left(\frac{A_2+A_3}{\alpha_2+\alpha_3}\right),$$

where $\pi_{\pm,0}$ denotes the mass of $\pi^{\pm,0}$, and similarly for

$K_{\pm,0}$. The matrix with (ab) element $\langle\partial^2 V/\partial\phi_a{}^a\partial\phi_b{}^b\rangle_0$ is

$$\begin{bmatrix} \dfrac{2A_1}{\alpha_1}-12V_4\dfrac{\alpha_2\alpha_3}{\alpha_1} & -12V_4\alpha_3 & -12V_4\alpha_2 \\ -12V_4\alpha_3 & \dfrac{2A_2}{\alpha_2}-12V_4\dfrac{\alpha_1\alpha_3}{\alpha_2} & -12V_4\alpha_1 \\ -12V_4\alpha_2 & -12V_4\alpha_1 & \dfrac{2A_3}{\alpha_3}-12V_4\dfrac{\alpha_1\alpha_2}{\alpha_3} \end{bmatrix}. \quad (4)$$

The roots of the secular equation of (4) are the squared masses of π^0, η, and η'. Rather than solving the resulting cubic equation, we write down the following three sum rules:

$$\tfrac{1}{8}(\pi_0\eta\eta')^2=\frac{A_1A_2A_3}{\alpha_1\alpha_2\alpha_3}-6V_4\left(\frac{A_2A_3}{\alpha_1}+\frac{A_1A_3}{\alpha_2}+\frac{A_1A_2}{\alpha_3}\right),$$

$$\tfrac{1}{2}(\pi_0{}^2+\eta^2+\eta'^2)=\frac{A_1}{\alpha_1}+\frac{A_2}{\alpha_2}+\frac{A_3}{\alpha_3}-6V_4\alpha_1\alpha_2\alpha_3\left(\frac{1}{\alpha_1{}^2}+\frac{1}{\alpha_2{}^2}+\frac{1}{\alpha_3{}^2}\right), \quad (5)$$

$$\tfrac{1}{4}(\pi_0{}^2\eta^2+\pi_0{}^2\eta'^2+\eta^2\eta'^2)=\frac{A_1A_2}{\alpha_1\alpha_2}+\frac{A_1A_3}{\alpha_1\alpha_3}+\frac{A_2A_3}{\alpha_2\alpha_3}-6V_4\left[A_1\left(\frac{\alpha_2}{\alpha_3}+\frac{\alpha_3}{\alpha_2}\right)+A_2\left(\frac{\alpha_1}{\alpha_3}+\frac{\alpha_3}{\alpha_1}\right)+A_3\left(\frac{\alpha_1}{\alpha_2}+\frac{\alpha_2}{\alpha_1}\right)\right].$$

The quantity V_4 in (5) is unknown, so we eliminate it and get the following two equations:

$$\left[\tfrac{1}{8}(\pi_0\eta\eta')^2-\frac{A_1A_2A_3}{\alpha_1\alpha_2\alpha_3}\right]\left(\frac{1}{\alpha_1{}^2}+\frac{1}{\alpha_2{}^2}+\frac{1}{\alpha_3{}^2}\right)-\frac{1}{\alpha_1\alpha_2\alpha_3}\left(\frac{A_2A_3}{\alpha_1}+\frac{A_1A_3}{\alpha_2}+\frac{A_1A_2}{\alpha_3}\right)\times\left[\tfrac{1}{2}(\pi_0{}^2+\eta^2+\eta'^2)-\left(\frac{A_1}{\alpha_1}+\frac{A_2}{\alpha_2}+\frac{A_3}{\alpha_3}\right)\right]\equiv f_1=0, \quad (6)$$

$$\left[\tfrac{1}{4}(\pi_0{}^2\eta^2+\pi_0{}^2\eta'^2+\eta^2\eta'^2)-\left(\frac{A_1A_2}{\alpha_1\alpha_2}+\frac{A_1A_3}{\alpha_1\alpha_3}+\frac{A_2A_3}{\alpha_2\alpha_3}\right)\right]\times\left(\frac{1}{\alpha_1{}^2}+\frac{1}{\alpha_2{}^2}+\frac{1}{\alpha_3{}^2}\right)-\frac{1}{\alpha_1\alpha_2\alpha_3}\left[\tfrac{1}{2}(\pi_0{}^2+\eta^2+\eta'^2)-\left(\frac{A_1}{\alpha_1}+\frac{A_2}{\alpha_2}+\frac{A_3}{\alpha_3}\right)\right]\left[A_1\left(\frac{\alpha_2}{\alpha_3}+\frac{\alpha_3}{\alpha_2}\right)+A_2\left(\frac{\alpha_1}{\alpha_3}+\frac{\alpha_3}{\alpha_1}\right)+A_3\left(\frac{\alpha_1}{\alpha_2}+\frac{\alpha_2}{\alpha_1}\right)\right]\equiv f_2=0. \quad (7)$$

For convenience we have defined the left-hand side of (6) to be f_1, and the left-hand side of (7) to be f_2.

It is also helpful to note that by inverting (3), the A_i can be expressed as linear functions of the α_i:

$$\begin{bmatrix}A_1\\A_2\\A_3\end{bmatrix}=\tfrac{1}{4}\begin{bmatrix}\tilde{\pi}_+{}^2+\tilde{K}_+{}^2 & \tilde{\pi}_+{}^2-K_0{}^2 & \tilde{K}_+{}^2-K_0{}^2\\ \tilde{\pi}_+{}^2-\tilde{K}_+{}^2 & \tilde{\pi}_+{}^2+K_0{}^2 & K_0{}^2-\tilde{K}_+{}^2\\ \tilde{K}_+{}^2-\tilde{\pi}_+{}^2 & K_0{}^2-\tilde{\pi}_+{}^2 & \tilde{K}_+{}^2+K_0{}^2\end{bmatrix}\begin{bmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{bmatrix}, \quad (8)$$

where $\tilde{\pi}_+{}^2=\pi_+{}^2-d_\pi$ and $\tilde{K}_+{}^2=K_+{}^2-d_K$.

Now Eqs. (6)–(8) constitute an algebraically complicated set of five equations for the six masses and other parameters of our theory. We could attempt to solve these by computer, but shall instead try to make some physical sense out of them by using a perturbation approach. Our basic assumption in this procedure is that the charged particles (π^+ and K^+) change their masses from their neutral values (π_0 and K_0), and the parameters α_i change to $\alpha_i+\delta\alpha_i$ as a result of the electromagnetic perturbation which distinguishes the 1 direction in unitary space (nonsinglet part). Formally this means that corresponding to a physically reasonable δA_i, the α_i's change by $\delta\alpha_i$, the π^+ particle changes its squared mass from $\pi_0{}^2$ to $\pi_+{}^2-d_\pi$, and the K^+ particle changes its squared mass from $K_0{}^2$ to $K_+{}^2-d_K$. Note that the δA_i are responsible for only a portion of the mass shift; the rest is made up by the constant terms d_π and d_K. Thus the interesting parameters are

$$\delta\tilde{\pi}^2\equiv\pi_+{}^2-\pi_0{}^2-d_\pi, \quad \delta\tilde{K}^2\equiv K_+{}^2-K_0{}^2-d_K. \quad (9)$$

It is also convenient to introduce the dimensionless parameters ϵ_i,

$$\delta\alpha_i=\epsilon_i\alpha_i \quad (i=1,2,3;\text{ no sum}). \quad (10)$$

From (8) we have

$$\begin{aligned}4\delta A_1&=2\alpha\delta\tilde{\pi}^2+(\alpha+\alpha_3)\delta\tilde{K}^2+(\pi_0{}^2+K_0{}^2)\alpha\epsilon_1+(\pi_0{}^2-K_0{}^2)\alpha\epsilon_2,\\ 4\delta A_2&=2\alpha\delta\tilde{\pi}^2-(\alpha+\alpha_3)\delta\tilde{K}^2+(\pi_0{}^2-K_0{}^2)\alpha\epsilon_1+(\pi_0{}^2+K_0{}^2)\alpha\epsilon_2, \quad (11)\\ 4\delta A_3&=-2\alpha\delta\tilde{\pi}^2+(\alpha+\alpha_3)\delta\tilde{K}^2+(K_0{}^2-\pi_0{}^2)(\alpha\epsilon_1+\alpha\epsilon_2)+2K_0{}^2\alpha_3\epsilon_3.\end{aligned}$$

Our main problem is the handling of (6) and (7). Let us formally rewrite them as follows, displaying their dependence on the charged masses and the α_i's:

$$f_1(\pi_+{}^2,K_+{}^2;\alpha_1,\alpha_2,\alpha_3)=0, \quad (6')$$

$$f_2(\pi_+{}^2,K_+{}^2;\alpha_1,\alpha_2,\alpha_3)=0. \quad (7')$$

[Originally (6) and (7) depended on the A_i's and the α_i's, but the A_i's depend on the masses as well as the α_i's by (8).] Before the perturbation, δA_i, we have

$$[f_{1,2}]_0\equiv f_{1,2}(\pi_0{}^2,K_0{}^2;\alpha,\alpha,\alpha_3)=0. \quad (12)$$

This leads to the equation

$$W^2(8p+K_0{}^2-2sK_0{}^2+K_0{}^4)+W(K_0{}^2-1)(1-2s+2K_0{}^2)+(4p-s+2+K_0{}^4-2K_0{}^2)=0, \quad (13)$$

where we have adopted units so that $\pi_0=1$ and have introduced the abbreviations

$$p=\tfrac{1}{8}(\pi_0\eta\eta')^2\,,\qquad s=\tfrac{1}{2}(\pi_0^2+\eta^2+\eta'^2)\,. \tag{14}$$

As in I,

$$W=\alpha_3/\alpha\,.$$

Equation (13) is of course the same as (45) of I. After the perturbation, δA_i, we have from (6′)

$$\delta f_1=\left(\frac{\partial f_1}{\partial\pi_+^2}\right)_0\delta\tilde{\pi}^2+\left(\frac{\partial f_1}{\partial K_+^2}\right)_0\delta\tilde{K}^2+\sum_{i=1}^{3}\left(\frac{\partial f_1}{\partial\alpha_i}\right)_0\delta\alpha_i=0\,, \tag{15}$$

and a similar equation for f_2. Evaluating (15) is a straightforward but lengthy task and gives

$$\begin{aligned}(\epsilon_1+\epsilon_2)[&W(K_0^2-1)(2K_0^2-2s+1)+2(K_0^2-1)^2\\&+8p+2-2s]+2\epsilon_3[W^2(K_0^4+K_0^2-2K_0^2s+8p)\\&-(K_0^2-1)^2-1+s-4p]+\delta\tilde{\pi}^2 2[W^2(K_0^2-8p)\\&+W(2s-K_0^2-2)+4-2K_0^2-s-4p]+\delta\tilde{K}^2(W+1)\\&\times[W(2K_0^2+1-2s)+2(K_0^2-1)]=0\,.\end{aligned} \tag{16}$$

The equation representing $\delta f_2=0$ is the same as the above except that the coefficient of $\delta\tilde{\pi}^2$ is

$$2[W^2(K_0^2-2s+2)+W(2s-K_0^2-2)+5-2s-2K_0^2]\,.$$

Taking the last two equations together and using (13) gives

$$\delta\tilde{\pi}^2=0\,, \tag{17}$$

$$(\epsilon_1+\epsilon_2-2\epsilon_3)(x+Wy)=-\delta\tilde{K}^2(1+W)\frac{[yW+(K_0^2-1)^2]}{(K_0^2-1)}\,, \tag{18}$$

where

$$x=1+4p-s+(K_0^2-1)^2\,,\qquad y=(K_0^2-1)[\tfrac{1}{2}+K_0^2-s]\,.$$

Equation (17) means that, in this model, all of the pion electromagnetic mass shift comes from the self-energy-type diagram of Fig. 1(b). This is in accord with expectations (since the pion mass splitting is a $\Delta I=2$ object and our δA_i contains $\Delta I=0$ and 1 terms only), and with the usual calculations.[2]

Equation (18) contains the main content of this calculation. It is clear that for a given $\delta\tilde{K}^2$ there are many choices of ϵ_1, ϵ_2, and ϵ_3 that will satisfy it. In order to choose among these, we may impose the requirement that the resulting set of δA_i [see Eq. (11)] have a "physically reasonable" form. The δA_i that we shall choose is the most standard one, corresponding to the T_1^1 component of an octet:

$$\begin{bmatrix}\delta A_1\\\delta A_2\\\delta A_3\end{bmatrix}=g_e\begin{bmatrix}2\\-1\\-1\end{bmatrix}\,. \tag{19}$$

This is to be added to the symmetry-breaking form given in I:

$$\begin{bmatrix}A_1\\A_2\\A_3\end{bmatrix}_0=g_0\begin{bmatrix}1\\1\\1\end{bmatrix}+g\begin{bmatrix}0\\0\\1\end{bmatrix}\,. \tag{20}$$

[The quantities $A_1=(A_1)_0+\delta A_1$, etc., are the ones that appear in Eq. (2).] From (18), (11), and (9) we are able to predict[6] g_e/g_0 in terms of $\delta\tilde{K}^2$ and W:

$$\frac{g_e}{g_0}=-\frac{\delta\tilde{K}^2(1+W)}{1+K_0^2(1+W)}\times\left\{1+\frac{K_0^2W[yW+(K_0^2-1)^2]}{(K_0^2-1)(x+yW)}\right\}\,. \tag{21}$$

[We have used $\pi_0^2=2g_0/\alpha$ to get (21).]

It is also important to record the formula for $\epsilon_1-\epsilon_2$ which is found from (11) and (19):

$$\epsilon_1-\epsilon_2=\frac{3}{K_0^2}\frac{g_e}{g_0}-\frac{(W+1)}{K_0^2}\delta\tilde{K}^2\,. \tag{22}$$

III. DISCUSSION OF RESULTS

The main result is Eq. (21). The quantity $\delta\tilde{K}^2$ represents the portion of the K-meson squared mass shift that does *not* arise from the "self-energy"-type diagram. This part is crucial physically, since the self-energy-type diagram by itself gives the wrong sign. To find the numerical value of $\delta\tilde{K}^2$ we must, according to (9), subtract out the self-energy contribution d_K. We may attempt to get an idea of d_K by comparing it to d_π. Since $\delta\tilde{\pi}^2=0$, the whole of the pion electromagnetic shift is due to d_π and we find

$$d_\pi=\pi_+^2-\pi_0^2\simeq 0.069\,,$$

in units of π_0^2. Possibly the most reasonable assumption (which follows from U-spin invariance) is

$$d_\pi=d_K\,. \tag{23}$$

[6] If a general δA_i, rather than (19), is used we would have instead of (21) the following:

$$2[(\delta A_1+\delta A_2)(K_0^2+WK_0^2-1)-2\delta A_3]=-\delta\tilde{K}^2\alpha(1+W)\left(1+\frac{K_0^2W}{(x+yW)(K_0^2-1)}[Wy+(K_0^2-1)^2]\right).$$

From this equation we may easily see that not all δA_i are consistent with our original assumptions. For example, if we choose the isospin-invariant perturbation $\delta A_1=\delta A_2=0$, $\delta A_3\neq 0$ we see that, in general, $\delta\tilde{K}^2\neq 0$, which is absurd. We interpret this to mean that our assumption, that the neutral particles do not have their masses changed as a result of an isospin-invariant perturbation, is not correct.

One special case is a pure isovector perturbation, $\delta A_1=-\delta A_2$, $\delta A_3=0$. Then the left-hand side above is zero, and we get either $\delta\tilde{K}^2=0$, or $W=1.58$ with $\delta\tilde{K}^2$ arbitrary. The analogous equation to (22) is

$$\epsilon_1-\epsilon_2=(2/\alpha K_0^2)(\delta A_1-\delta A_2)-(1/K_0^2)(1+W)\delta\tilde{K}^2,$$

which shows that, in this case, the quantity $\delta A_1-\delta A_2$ cannot be predicted uniquely in terms of $\delta\tilde{K}^2$.

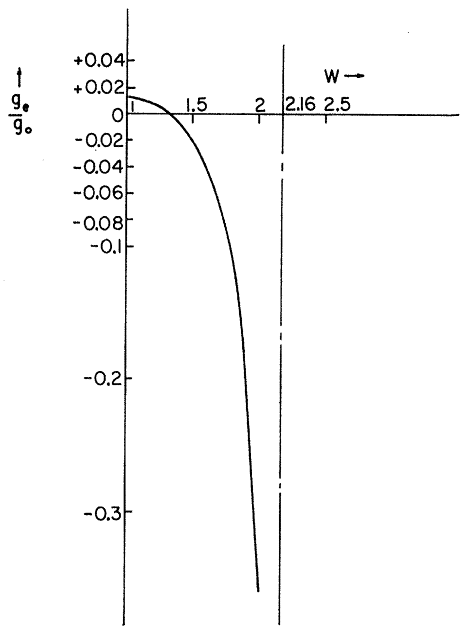

FIG. 2. g_e/g_0 as a function of W.

This leads to

$$\delta\tilde{K}^2=K_+{}^2-K_0{}^2-d_K=-0.29\,, \tag{24}$$

in units of $\pi_0{}^2$. Another possibility is to take the literal form of Fig. 1(b) very seriously and, assuming that π, K, and the photon are the only particles in nature, to use dimensional analysis to get

$$d_K=(K_0/\pi_0)^2 d_\pi\,. \tag{25}$$

This leads to $\delta\tilde{K}^2=-1.17$ in $\pi_0{}^2$ units.

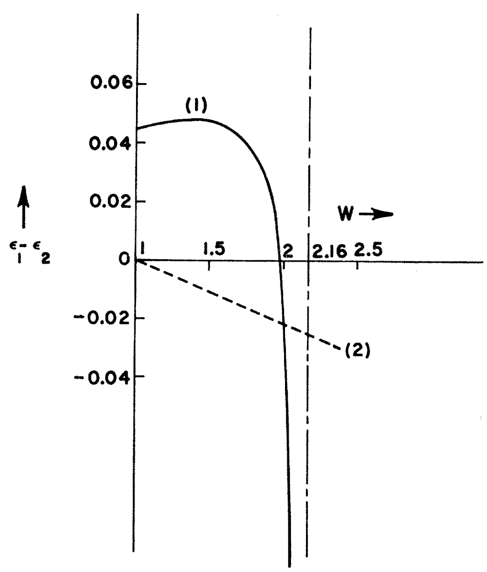

FIG. 3. $\epsilon_1-\epsilon_2$ as a function of W. Curve (1) is the prediction from the mesons; curve (2) is the prediction from the baryons.

Both of the above approaches lead to the same sign for $\delta\tilde{K}^2$. Fortunately the magnitude of $\delta\tilde{K}^2$ will not play a very important role in our discussion.

A plot of Eq. (21) is given in Fig. 2, where Eq. (24) has been used. The outstanding feature of this curve is that near $W=2$ the magnitude $|g_e/g_0|$ is about unity, and as W increases to 2.16, $g_e/g_0\to-\infty$. This situation has extremely interesting physical consequences which will be discussed later.

From I we have an idea of what W should be. However, it is interesting to try to deduce this value from electromagnetic considerations only. At a given value of W we may find from (22), with the aid of (21), the value of $\epsilon_1-\epsilon_2$. This is shown as curve (1) of Fig. 3. Another independent determination of $\epsilon_1-\epsilon_2$ may be made from the octet baryon electromagnetic mass splitting on the assumption that the meson-baryon interaction is chiral invariant. This is, of course, the usual argument that leads to the Goldberger-Treiman relation in chiral theories of this type. For electromagnetic splittings this procedure leads to an approach similar to the one of Coleman and Glashow.[2] In the Appendix, we find

$$\epsilon_1-\epsilon_2=(W-1)(\Sigma^+-\Sigma^-)/(\Xi^0-n)\,. \tag{26}$$

This equation, under the simplifying assumption that the shift $\Sigma^+-\Sigma^-$ is completely due to the tadpole-type mechanism, is plotted as curve (2) in Fig. 3. We see that the two curves intersect at $W\simeq2$, for which Fig. 2 shows that $g_e/g_0\simeq-0.35$. If we were to use (25) instead of (24), we would still find $W\simeq2$ but now $g_e/g_0\simeq-1.4$. Thus we see that the value of W is essentially independent of the magnitude of $\delta\tilde{K}^2$. Furthermore, the above determination[7] was found to be not very sensitive to the particular form of meson-baryon interaction, although we shall not give details.

In I it was noted that reasonably good values of η'^2 were obtained for the range of W, $1.5\to3$, and that $1.0\to2$ was what we might expect from the theory of weak interactions. It is interesting that the value of W obtained from comparison of baryon and meson mass shifts is roughly consistent with this. If additional particles (e.g., spin-1 mesons) are included in the theory the situation may change slightly, but we expect the over-all features to be the same.

Now let us discuss the fact that $|g_e/g_0|$ comes out to be of order of magnitude unity in this theory. This means that we are in a region where electromagnetic effects in symmetry breaking are comparable with chiral-symmetry breaking. Furthermore, since the mech-

[7] For the actual numerical evaluation of (21) and (22), we used the *physical* value of η'. From a mathematical viewpoint it is better to use the value of η' appropriate to each value of W as given in I. This has a relatively minor effect on our conclusions, since, in the range of interest, $\eta'(W)$ is always close to the physical value (see Fig. 1 of I). What happens is that the value of W at which g_e/g_0 diverges is increased somewhat, and the value of W that correlates meson and baryon electromagnetic shifts is about 2.4. The order of magnitude of $|g_e/g_0|$ remains around unity. Also, for small W, the quantity g_e/g_0 remains negative.

anism responsible for the electromagnetic perturbation —diagrams of the generic type of Fig. 1(a)—also make some contribution to g_0, it becomes plausible that they make *all* the contribution to g_0. In other words, all of the chiral-symmetry breaking of the strong Lagrangian may be due to electromagnetism. This point of view has been previously advocated by a number of authors.[4,5,8,9] Note that according to Eq. (37) of I the pion mass would be zero if g_0 were zero (before introducing electromagnetism according to this point of view). The evaluation of g_e/g_0 from some set of diagrams[5] like Fig. 1(a) may be interesting but is probably too model dependent to be reliable.

We should point out that the main new conclusion about electromagnetism from this model is that a mechanism exists whereby the g_e term can be very large. On the other hand, deciding whether or not the exact value of W is the right one obviously requires further work. We note that a consistent situation can also be achieved for a slightly smaller value of W, when we have the special case of footnote 6. However, the consequences of this case are not so exciting.

We remark that in the presence of electromagnetic interactions it is possible to calculate the squared mass of the ϵ_+ (isovector) scalar meson from (25) of I. This could *not* be done in the isotopic-spin limit since (I22′) gives no nontrivial information when $\alpha_1=\alpha_2$. On the other hand, when electromagnetic breaking is included and $\alpha_1\neq\alpha_2$, we have [neglecting diagrams like Fig. 1(b)]

$$\epsilon_+{}^2=2\frac{A_1-A_2}{\alpha_1-\alpha_2}=\frac{(3g_e/g_0)}{\epsilon_1-\epsilon_2}\pi_0{}^2.$$

This formula is interesting in that it determines the ratio of the two "electromagnetic" quantities, g_e/g_0 and $\epsilon_1-\epsilon_2$, in terms of the "strong" masses. Since $\epsilon_+{}^2$ is positive, we see that g_e/g_0 and $\epsilon_1-\epsilon_2$ must have the same sign, in agreement with our previous results. Numerically this formula gives $\epsilon_+{}^2\approx 50$ (200) when $d_K=d_\pi$ $(d_K=(K_0{}^2/\pi_0{}^2)d_\pi)$, which is a reasonable order of magnitude. For the two squared κ masses, we have

$$\kappa_+{}^2=2\left(\frac{A_1-A_3}{\alpha_1-\alpha_3}\right),\quad \kappa_0{}^2=2\left(\frac{A_2-A_3}{\alpha_2-\alpha_3}\right),$$

which evidently satisfy the positivity conditions in our model.

IV. SPECULATION ON ORIGIN OF SYMMETRY BREAKING

Here we will give arguments to support the following postulate.

The strong-interaction Lagrangian is exactly chiral $SU(3)\times SU(3)$ invariant. What we mean is that all symmetry breaking—both chiral breaking *and* $SU(3)$ breaking—is of combined electromagnetic and weak origin. The inspiration for this assumption is evidently the results of Sec. III, where it was shown that we seem to be in a region of nature where relatively large electromagnetic "tadpoles" are needed to produce the relatively small electromagnetic mass shifts. In the usual picture, $SU(3)$ breaking is expected to come from some new "medium-strong" force. It is clearly more economical to use the already discovered weak interaction for this purpose.

Undoubtedly the reader will have some objections to this. Let us state the two most likely objections and attempt to answer them.

Objection A: The weak and electromagnetic interactions are of too small magnitude to account for strong effects.

Answer A: The diagrams we have in mind are those like Fig. 1(a) with both photons and intermediate bosons crossing the "tadpole." According to ordinary methods of calculation, these diagrams are divergent, so it is conceivable that they could be large if form factors or cutoffs are introduced to make them finite. As a further guide to our thinking, and as a help in making contact with other work, we point out that our tadpoles may be identified as being proportional to the "quark mass terms" investigated by Gatto, Sartori, and Tonin[9] and by Cabibbo and Maiani.[10] We shall make use of this identification in what follows.

Objection B: Granted that you can somehow evaluate divergent diagrams to give large finite results, why are not the electromagnetic mass shifts large, and why do not the weak decays proceed faster than they do?

Answer B: The first part of B was already answered; namely, in our model, we *need* large electromagnetic perturbations to give the relatively small electromagnetic mass shifts. To explain why the weak decays do not proceed much faster than they do, we may note that the type of tadpoles which contribute to weak decays are off-diagonal in the $SU(3)$ octet space, while the tadpoles which contribute to strong symmetry breaking are diagonal in the $SU(3)$ space. It has been shown by Bouchiat *et al.*,[11] using the Bjorken[12] technique, that matrix elements of the "most divergent" contribution of the off-diagonal terms can be expressed as matrix elements of a four-divergence, and are hence zero. This does not apply to matrix elements of the diagonal terms.

We now point out that our speculation is similar in spirit to the (also highly speculative) approach of Cabibbo and Maiani.[10] Adopting their evaluation of the most divergent weak and electromagnetic contributions

[8] Y. Nambu and G. Jona Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).

[9] R. Gatto, G. Sartori, and M. Tonin, Phys. Letters **28B**, 128 (1968).

[10] N. Cabibbo and L. Maiani, Phys. Letters **28B**, 131 (1968).

[11] C. Bouchiat, J. Iliopoulos, and J. Prentki, CERN Report No. TH908, 1968 (unpublished).

[12] J. D. Bjorken, Phys. Rev. **188**, 1467 (1966).

would give us

$$\begin{aligned} A_1 &= tA_1 + Z_1, \\ A_2 &= tA_2 \cos^2\theta + Z_2, \\ A_3 &= tA_3 \sin^2\theta + Z_2, \end{aligned} \tag{27}$$

where θ is the Cabibbo angle, t is a cutoff-dependent quantity involving the weak coupling constant, and Z_1, Z_2 represent electromagnetic effects which distinguish the 1 direction in $SU(3)$ space. A_1, A_2, and A_3 are known from this paper and from I. It should be stressed that in the previous work we made no commitment as to the origin of A_1, A_2, and A_3. Here we are requiring them all to come from weak and electromagnetic effects. We can solve for t, Z_1, and Z_2 in terms of the A_i's and θ, so the scheme is at least consistent, even though no prediction can be made. Cabibbo and Maiani, on the other hand, assume weak and electromagnetic effects to cancel and set the left-hand sides of (27) equal to zero, obtaining the relation

$$\tan^2\theta = \frac{A_2}{A_3} \equiv \frac{g_0 - g_e}{g_0 - g_e + g}. \tag{28}$$

Curiously, with our results ($g/g_0 \simeq 38.8$, $g_e/g_0 \simeq -0.35$ at $W=2$) we still get the numerically reasonable value

$$\tan\theta \simeq 0.18,$$

even though the effects of symmetry breaking are included. From our present point of view we would have to regard (28) as a (possibly roughly true) relation[13] which has not been derived. Attempts can be made to modify the Cabibbo-Maiani scheme by introducing different sets of intermediate bosons, treating electromagnetic contributions differently, etc., but considering the speculative nature of all of this at the present stage, we do not expect to do any more than stimulate some thinking on the matter.

APPENDIX

Here we discuss the nonderivative part of the chiral invariant meson-baryon interaction. The derivative part was discussed in Sec. VI of I; we will continue to use the same notation.

In addition to giving scattering vertices, the interaction terms in chiral theories of this type give the baryon mass terms, since trilinear objects of the form $(\bar{N}NS)$ contribute to the mass term the quantity $\bar{N}N\langle S\rangle_0$. This procedure seems generally correct, since it leads to Golberger-Treiman type relations.

Many different chival-invariant meson-baryon interactions can be constructed. If two M's are used, we have

$$\begin{aligned} -\mathcal{L}^{(2)} &= (b_1/\alpha^2)\, \mathrm{Tr}(\bar{L}MRM^\dagger + \bar{R}M^\dagger LM) \\ &= b_1 \bar{N}_a{}^c N_c{}^a (\alpha_a\alpha_c/\alpha^2) + (\text{interaction terms}), \end{aligned} \tag{A1}$$

where b_1 is a real constant.

[13] For different speculations on the Cabibbo angle, see K. Tanaka and P. Tarjanne, Phys. Rev. Letters **23**, 1137 (1969).

If three M's are used, we have

$$\begin{aligned} -\mathcal{L}^{(3)} &= [Wb_2/2(\alpha_1\alpha_2\alpha_3)]\, \mathrm{Tr}(\bar{L}MRT^\dagger + \bar{R}M^\dagger LT) \\ &\quad + [Wb_3/2(\alpha_1\alpha_2\alpha_3)]\, \mathrm{Tr}(\bar{L}TRM^\dagger + \bar{R}T^\dagger LM) \\ &= W\bar{N}_a{}^c N_c{}^a (b_2\alpha_c/\alpha_a + b_3\alpha_a/\alpha_c) \\ &\qquad + (\text{interaction terms}), \end{aligned} \tag{A2}$$

where T is the "dual" tensor defined in Eq. (A3) of I.

If four M's are used, we have

$$\begin{aligned} -\mathcal{L}^{(4)} &= \frac{b_4}{\alpha^4}\{\mathrm{Tr}(\bar{L}MM^\dagger)\,\mathrm{Tr}(M^\dagger MR) \\ &\quad + \mathrm{Tr}(\bar{R}M^\dagger M)\,\mathrm{Tr}(MM^\dagger L)\} \\ &\quad + (b_5/\alpha^4)\,\mathrm{Tr}(\bar{L}MRM^\dagger MM^\dagger + \bar{R}M^\dagger LMM^\dagger M) \\ &\quad + (b_6/\alpha^4)\,\mathrm{Tr}(\bar{L}MM^\dagger MRM^\dagger + \bar{R}M^\dagger MM^\dagger LM) \\ &= b_4\bar{N}_a{}^a N_c{}^c (\alpha_a\alpha_c)^2/\alpha^4 \\ &\quad + \bar{N}_a{}^c N_c{}^a (b_5\alpha_a{}^3\alpha_c/\alpha^4 + b_6\alpha_a\alpha_c{}^3/\alpha^4) \\ &\qquad + (\text{interaction terms}). \end{aligned} \tag{A3}$$

It is clear that there are a considerable number of these terms. However, none of the above terms by themselves give either the Gell-Mann–Okubo relation or the Coleman-Glashow formula. Thus, to prevent ourselves from straying too far off the correct path, we will take the sum of all terms above and fit eight masses to the seven quantities $b_1, \ldots, b_6$, and $\epsilon_1 - \epsilon_2$. One sum rule is identically fulfilled:

$$\Sigma^+ + \Sigma^- = 2\Sigma^0 \tag{A4}$$

to first order in $\epsilon_1 - \epsilon_2$. This is reasonable (but not exact) experimentally. The four baryon masses, before including electromagnetism, are

$$\begin{aligned} N &= b_2 + b_3W^2 + b_1W + b_5W^3 + b_6W, \\ \Xi &= b_2W^2 + b_3 + b_1W + b_5W + b_6W^3, \\ \Sigma &= (b_2+b_3)W + b_1 + b_5 + b_6, \\ \Lambda &= (b_2+b_3)W + b_1\tfrac{1}{3}(2W^2+1) \\ &\quad + \tfrac{1}{3}[(b_4+b_5+b_6)(1+2W^4) + b_4(1-4W^2)]. \end{aligned} \tag{A5}$$

Three electromagnetic mass shifts are

$$\begin{aligned} p - n &= (\epsilon_1-\epsilon_2)(b_1W + b_2 - b_3W^2 + b_5W^3 + 3b_6W), \\ \Xi^- - \Xi^0 &= (\epsilon_1-\epsilon_2)(b_1W - b_2W^2 + b_3 + 3b_5W + b_6W^3), \\ \Sigma^+ - \Sigma^- &= (\epsilon_1-\epsilon_2)2[(b_2-b_3)W - (b_5-b_6)], \end{aligned} \tag{A6}$$

to first order in $\epsilon_1 - \epsilon_2$.

It is not necessary for our present purposes to give the expressions for $b_1, \ldots, b_6$ in terms of the masses. However, it is interesting to note that we have uniquely

$$\epsilon_1 - \epsilon_2 = (\Xi - N)^{-1}[(\Sigma^+ - \Sigma^-)W - (p-n) + (\Xi^- - \Xi^0)]. \tag{A7}$$

Using the Coleman-Glashow relation, we may put this in the simple form:

$$\epsilon_1 - \epsilon_2 = (W-1)(\Sigma^+ - \Sigma^-)/(\Xi - N). \tag{A8}$$