

APPENDIX B: ORIGIN AND INTERPRETATION OF WAVE EQUATION (A1)

There is no complete closed solution of the relativistic two-body problem in quantum field theory even for small coupling constant α , let alone the large- α case that we are considering. The Klein-Gordon and Dirac equations provide nonperturbative closed solutions and correspond in perturbation theory to a summation of a whole class of diagrams; but they do not contain relativistic recoil corrections, which become essential when the masses of the constituents are comparable, as in positronium. The infinite-component wave equations remove completely the last difficulty yet allow closed solutions, because the Lorentz transformations are applied to the system as a whole and not to the relative coordinates.¹³ The method of constructing such wave equations is as follows. We start with the nonrelativistic

Hamiltonian, which in the case of dyonium is

$$H = \frac{1}{2m}\pi_r^2 + \frac{1}{2mr^2}(\mathbf{J}^2 - \nu^2) + \frac{\alpha}{r}.$$

We then transform H into an algebraic form to identify a complete linear space of states which in this case is a particular representation space of the dynamical group $SO(4,2)$ characterized by ν . The group $SO(4,2)$ contains Galilean as well as Lorentz boost operators; we replace the Galilean boost operators by the Lorentz boost operators to arrive at states with momentum P_μ . On this space of states with momentum P_μ , we construct a conserved current operator J_μ correctly generalizing the nonrelativistic mass spectrum (2). Because the external photon must be coupled to a conserved current, which is unique for a given mass spectrum, we can evaluate the form factors from the matrix elements of J_μ .

Troubles with Strict Conformal Symmetry*

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Formal low-energy theorems for matrix elements involving the stress tensor can be used to derive the differential equations that express scale and conformal invariance. We show that these equations, when applied to a single-particle matrix element of two fields, yield an integral representation that violates causality save for the trivial case where it reduces to the tree graphs of a free field. Although it is possible to construct conformally invariant, fully off-mass-shell amplitudes whose restriction to the mass shell yields a nontrivial result, these on-mass-shell amplitudes are not invariant under conformal transformations. This pathological behavior can be traced to an implicit assumption of analyticity when the stress tensor carries off small momenta that is, in general, false.

THERE has been considerable interest recently in the possibility that scale invariance and, perhaps, even the full conformal group, may have some role to play in high-energy physics.¹ Three general areas have been studied: the scaling behavior of field theory at short distances, theories with broken scale symmetry, and the structure of amplitudes that are strictly invariant under the conformal group. The study of strict conformal symmetry is motivated by the idea that, although many high-energy processes, such as elastic scattering, are by no means scale invariant, there may be other reactions of an inclusive nature, such as inelastic electroproduction, that do become conformally invariant at high energy. Thus, at high energy, these processes may be described by conformally invariant amplitudes with massless particles. We have shown²

that, when the amplitudes for massless particles are obtained as the limit of off-mass-shell amplitudes, fully conformally invariant structure functions can be constructed for the electroproduction process.³ On the other hand, the application of strict conformal symmetry to mass-shell amplitudes,⁴ for zero-mass external particles, gives rise to various difficulties. We shall discuss some of the troubles here.

We begin by considering⁵ off-mass-shell amplitudes that have a stress-tensor insertion carrying off momentum k . They obey divergence conditions (Ward-like identities) that yield low-energy theorems. A proper stress-tensor vertex is thereby determined including terms up to order k . The symmetry of the stress tensor, in conjunction with this determination in order 1, implies that the off-mass-shell amplitudes without the

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¹ A review of the recent work with references to the literature has been given by P. Carruthers, Phys. Repts. (to be published).

² D. G. Boulware, L. S. Brown, and R. D. Peccei, Phys. Rev. D **2**, 293 (1970). This paper also contains a brief introduction to the geometry of the conformal group.

³ We found, however, that invariance under the full conformal group gives no restrictions on these functions other than that of simple scale invariance.

⁴ Such an application of strict conformal symmetry has been suggested by D. J. Gross and J. Wess, Phys. Rev. D **2**, 753 (1970).

⁵ Our method is a straightforward extension of that of Gross and Wess to off-mass-shell amplitudes.

stress tensor insertion are Lorentz invariant.⁶ The special condition that the stress tensor be traceless gives a further condition on the proper vertices which, when coupled with the low-energy theorem, implies that the amplitudes are invariant under the conformal group: The terms of order 1 require dilation invariance while the terms of order k require invariance under special conformal transformations.

The application of conformal invariance to the on-mass-shell single-particle matrix element of two field operators requires that it obey a single-parameter integral representation. This integral representation generally violates microscopic causality. When causality is enforced, the representation reduces to that of a trivial tree graph. We find, moreover, that the amplitude of two field operators has a structure that is entirely different from that of the corresponding amplitude for the source of the fields. Although the conformally invariant mass-shell amplitude is trivial, a wide class of conformally invariant off-mass-shell amplitudes can be constructed explicitly.⁷ We shall show by a simple example that the mass-shell limit of a conformally invariant off-mass-shell amplitude may not obey the differential equations that convey conformal symmetry. This pathological behavior results from a lack of analyticity near the mass shell $p^2=0$; the amplitude contains terms of the form $p^2 \ln p^2$ or $\gamma p \ln p^2$ that vanish on the mass shell but for which the operations of going on the mass shell and applying the conformal generators do not commute. This trouble is reflected in the failure of the stress-tensor low-energy theorem; it contains an implicit and invalid assumption that the amplitude is analytic near $p^2=0$.

In order to discuss the low-energy theorem, we must first make some definitions. The connected, n -point Green's function is related to its truncated part T by

$$\begin{aligned} \langle 0 | T^*(\chi_1(x_1) \cdots \chi_n(x_n)) | 0 \rangle_{\text{conn}} \\ = \int \frac{(d\hat{p})}{(2\pi)^{4n}} e^{i\sum_a p_a x_a} (2\pi)^4 \delta(\sum_b \hat{p}_b) S_1(\hat{p}_1) \cdots \\ \times S_n(\hat{p}_n) T(\hat{p}_1 \cdots \hat{p}_n). \quad (1) \end{aligned}$$

Here χ_a denotes either a spin-0 or a spin- $\frac{1}{2}$ field, and S_a is its full propagator. If we use the stress-tensor single-particle vertex $\Gamma^{\mu\nu}$,

$$\begin{aligned} \langle 0 | -T^*(\chi_1(x) T^{\mu\nu}(0) \chi_2(x')) | 0 \rangle \\ = \int \frac{d\hat{p} d\hat{p}'}{(2\pi)^8} e^{i(\hat{p}x - \hat{p}'x')} S(\hat{p}) \Gamma^{\mu\nu}(\hat{p}, \hat{p}') S(\hat{p}'), \quad (2) \end{aligned}$$

we can write the general stress-tensor matrix element

in terms of its proper part $T^{\mu\nu}(p_1 \cdots p_n)$,

$$\begin{aligned} \langle 0 | iT^*(\chi_1(x_1) \cdots \chi_n(x_n) T^{\mu\nu}(0)) | 0 \rangle_{\text{conn}} \\ = \int \frac{(d\hat{p})}{(2\pi)^{4n}} e^{i\sum_a p_a x_a} \left\{ \sum_b S_1(\hat{p}_1) \cdots \right. \\ \times [S_b(\hat{p}_b) \Gamma^{\mu\nu}(\hat{p}_b, \hat{p}_b + k) S_b(\hat{p}_b + k)] \cdots \\ \times S_n(\hat{p}_n) T(\hat{p}_1 \cdots \hat{p}_b + k \cdots \hat{p}_n) \\ \left. + S_1(\hat{p}_1) \cdots S_n(\hat{p}_n) T^{\mu\nu}(\hat{p}_1 \cdots \hat{p}_n) \right\}. \quad (3) \end{aligned}$$

Here k is the momentum carried off by the stress tensor

$$k^\mu + \sum_a p_a^\mu = 0. \quad (4)$$

Since the stress tensor is conserved, the single-particle vertex obeys the divergence condition

$$\begin{aligned} k_\nu \Gamma^{\mu\nu}(p, p+k) = [(p+\lambda k)^\mu + \frac{1}{4} i \sigma^{\mu\nu} k_\nu] S^{-1}(p+k) \\ - S^{-1}(p) [(p+k-\lambda k)^\mu + \frac{1}{4} i \sigma^{\mu\nu} k_\nu], \quad (5) \end{aligned}$$

where $\lambda = \frac{1}{2}$ for spin $\frac{1}{2}$ while $\lambda = 0$ and the spin terms do not appear for spin 0. The general vertex obeys⁸

$$\begin{aligned} k_\nu T^{\mu\nu}(p_1 \cdots p_n) = - \sum_a (p_a^\mu + \lambda_a k^\mu + \frac{1}{4} i \sigma_a^{\mu\nu} k_\nu) \\ \times T(p_1 \cdots p_a + k \cdots p_n). \quad (6) \end{aligned}$$

We can expand the divergence condition (5) in powers of k and assemble the terms in the form

$$\begin{aligned} k_\nu \Gamma^{\mu\nu}(p, p+k) = \frac{1}{2} k_\nu \{ L^{\mu\nu} S^{-1}(p) + i [\frac{1}{2} \sigma^{\mu\nu}, S^{-1}(p)] \} \\ + k_\nu \{ D^{\mu\nu} + (2\lambda - 1) g^{\mu\nu} \} S^{-1}(p) \\ + k_\nu k_\rho \left\{ K^{\mu\nu\rho} + \lambda g^{\mu\nu} \frac{\partial}{\partial p_\rho} + \frac{1}{4} i \left(\sigma^{\mu\rho} \frac{\partial}{\partial p_\nu} + \sigma^{\nu\rho} \frac{\partial}{\partial p_\mu} \right) \right\} \\ \times S^{-1}(p) + O(k^3), \quad (7) \end{aligned}$$

where

$$\begin{aligned} L^{\mu\nu} &= p^\mu \frac{\partial}{\partial p_\nu} - p^\nu \frac{\partial}{\partial p_\mu}, \\ D^{\mu\nu} &= \frac{1}{2} \left(p^\mu \frac{\partial}{\partial p_\nu} + p^\nu \frac{\partial}{\partial p_\mu} \right), \\ K^{\mu\nu\rho} &= \frac{1}{2} \left(p^\mu \frac{\partial}{\partial p_\nu} \frac{\partial}{\partial p_\rho} + p^\nu \frac{\partial}{\partial p_\mu} \frac{\partial}{\partial p_\rho} - p^\rho \frac{\partial}{\partial p_\mu} \frac{\partial}{\partial p_\nu} \right). \end{aligned} \quad (8)$$

⁸ The divergence conditions can be derived by an external-source technique. This technique gives a divergence condition on the general matrix elements (2) and (3) which directly imply the proper vertex identities quoted in the text. Alternatively, the proper vertex identities can be inferred from the structure of Feynman graphs. They can also be obtained from the equal-time commutators that result from a derivative of a time-ordered product, but this method requires a careful definition of covariant T^* ordering, since stress-tensor matrix elements generally involve noncovariant contact terms. It has been used by C. G. Callan, S. Coleman, and R. Jackiw, Ann. Phys. (N.Y.) 59, 42 (1970).

⁶ If a manifestly translation-invariant notation is not used, then the terms of order k^{-1} in the low-energy theorem would require this invariance irrespective of the symmetry of the stress tensor.

⁷ This is done in Appendix A of Ref. 2.

All of the terms that occur in Eq. (7) are symmetric in the indices μ and ν except those in the first pair of braces, which are antisymmetric. Since $T^{\mu\nu}$ is symmetric, so is $\Gamma^{\mu\nu}$ and these terms must vanish:

$$L^{\mu\nu}S^{-1}(p) + i[\frac{1}{2}\sigma^{\mu\nu}, S^{-1}(p)] = 0. \quad (9)$$

This is simply the statement of the Lorentz invariance of the propagator $S(p)$, and we find that this invariance follows directly from the divergence condition. It follows from the order-1 terms of the low-energy theorem. It is a simple matter to prove that a symmetric tensor is determined to $O(k)$ by its contraction with k_ν . Hence,

$$\begin{aligned} \Gamma^{\mu\nu}(p, p+k) &= [D^{\mu\nu} + (2\lambda - 1)g^{\mu\nu}]S^{-1}(p) \\ &+ k_\rho \left[K^{\mu\nu\rho} + \lambda g^{\mu\nu} \frac{\partial}{\partial p_\rho} + \frac{1}{4}i \left(\sigma^{\mu\rho} \frac{\partial}{\partial p_\nu} + \sigma^{\nu\rho} \frac{\partial}{\partial p_\mu} \right) \right] \\ &\times S^{-1}(p) + O(k^2). \quad (10) \end{aligned}$$

In an entirely similar manner, we find that the divergence condition (6) requires that the amplitude T be Lorentz invariant and provides the determination

$$\begin{aligned} T^{\mu\nu}(p_1 \cdots p_n) &= -\sum_a \left\{ [D_a^{\mu\nu} + \lambda_a g^{\mu\nu}] \right. \\ &+ k_\rho \left[K_a^{\mu\nu\rho} + g^{\mu\nu} \lambda_a \frac{\partial}{\partial p_{a\rho}} + \frac{1}{4}i \left(\sigma_a^{\mu\rho} \frac{\partial}{\partial p_{a\nu}} + \sigma_a^{\nu\rho} \frac{\partial}{\partial p_{a\mu}} \right) \right] \left. \right\} \\ &\times T(p_1 \cdots p_n) + O(k^2). \quad (11) \end{aligned}$$

These low-energy theorems are generally valid off the mass shell, but they may fail on the mass shell of a zero-mass particle since in this case the vertex may not be analytic for small k .

The special condition that the trace of the stress tensor vanish provides further identities for the vertices.⁹ For the single-particle vertex, one finds that

$$\Gamma^\mu_\mu(p, p+k) = (3\lambda - 1)[S^{-1}(p) + S^{-1}(p+k)]. \quad (12)$$

We expand this constraint in powers of k , and identify the terms of order 1 and order k with those previously determined by the low-energy theorem (10). The terms of order 1 give the statement of dilation invariance:

$$[D + 2(\lambda - 1)]S^{-1}(p) = 0, \quad (13)$$

where

$$D \equiv D^\mu_\mu = p \frac{\partial}{\partial p}. \quad (14)$$

The terms of order k give the statement of special

⁹ The trace identities can be derived by methods similar to those outlined in Ref. 8. However, in contrast to the divergence conditions which can always be maintained in the process of renormalization, the trace identities can be modified by renormalization. See K. G. Wilson, Phys. Rev. D 2, 1473 (1970); 2, 1478 (1970); and C. G. Callan, *ibid.* 2, 1541 (1970). We shall not consider such effects here.

conformal invariance:

$$\left[K^\rho + (\lambda + 1) \frac{\partial}{\partial p_\rho} - \frac{1}{2} i \sigma^{\rho\mu} \frac{\partial}{\partial p^\mu} \right] S^{-1}(p) = 0, \quad (15)$$

where

$$K^\rho \equiv K^\mu_{\mu\rho} = p \frac{\partial}{\partial p} \frac{\partial}{\partial p_\rho} - \frac{1}{2} p^\rho \frac{\partial^2}{\partial p^2}. \quad (16)$$

The quantities D and K^ρ provide a differential-operator realization of the generators of the conformal group. The conformal equations (13) and (15) uniquely determine the propagators, save for an over-all constant, to be those of a massless free field,

$$S_0^{-1}(p) = p^2, \quad (17)$$

$$S_{1/2}^{-1}(p) = \gamma p. \quad (18)$$

In general, the special condition that the trace of the stress tensor vanishes gives the identity

$$T^\mu_\mu(p_1 \cdots p_n) = \sum_a (1 - 3\lambda_a) T(p_1 \cdots p_a + k \cdots p_n), \quad (19)$$

and the equations that convey the conformal symmetry of the truncated amplitude,

$$\sum_a [D_a + \lambda_a + 1] T(p_1 \cdots p_n) = 0, \quad (20)$$

$$\begin{aligned} \sum_a \left[K_a^\rho + (\lambda_a + 1) \frac{\partial}{\partial p_{a\rho}} - \frac{1}{2} i \sigma_a^{\rho\mu} \frac{\partial}{\partial p_{a\mu}} \right] \\ \times T(p_1 \cdots p_n) = 0. \quad (21) \end{aligned}$$

We should emphasize that the validity of these equations rests on the tacit assumption that an expansion in powers of k is permissible. This, as we shall find, is in general not true for amplitudes on a zero-mass shell.

As a first example of the pathologies that arise from on-mass-shell conformal invariance, we study two equivalent forms of a scalar amplitude, the source correlation function

$$\begin{aligned} W(x, p', p) &= (-\partial^2)(-\partial'^2) \langle p' | T(\phi(x)\phi(x')) | p \rangle_{\text{conn}} \Big|_{x'=0} \\ &= \int \frac{dq'}{(2\pi)^4} e^{iq'x} T(p', -p, q', -q) \Big|_{q=p-p'-q'} \quad (22) \end{aligned}$$

and the corresponding field correlation function

$$\begin{aligned} \Phi(x, p', p) &= \langle p' | T(\phi(x)\phi(0)) | p \rangle_{\text{conn}} \\ &= \int \frac{dq'}{(2\pi)^4} e^{iq'x} \frac{1}{q^2} \frac{1}{q'^2} T(p', -p, q', -q) \Big|_{q=p-p'-q'}. \quad (23) \end{aligned}$$

Here p' and p are on the zero-mass shell, $p'^2 = 0 = p^2$. It is a straightforward matter to transcribe the con-

formal equations for the truncated amplitude (20) and (21) into equations for W and Φ . We need not write down the equation of dilation invariance, for it simply requires that once a term of the appropriate dimension has been factored out of the amplitude, the remainder is a function of dimensionless parameters. Hence we may write

$$W(x, p', p) = (x^2)^{-2} F_1(\alpha, \beta, \gamma) + D\delta(x) \quad (24)$$

and

$$\Phi(x, p', p) = F_2(\alpha, \beta, \gamma), \quad (25)$$

where

$$\alpha = (p' + p)x, \quad \beta = (p' - p)x, \quad \gamma = (p' - p)^2 x^2. \quad (26)$$

The two dimensionless functions now obey the same equation of special conformal invariance:

$$\left[K'^\rho - K^\rho + \frac{\partial}{\partial p'_\rho} - \frac{\partial}{\partial p_\rho} + i(x^\rho x^\sigma \partial_\sigma - \frac{1}{2} x^2 \partial^\rho + x^\rho) \right] \times F_{1,2} = 0. \quad (27)$$

[The $\delta(x)$ that appears in Eq. (24) automatically obeys this equation.]

By writing the derivatives that occur here in terms of derivatives with respect to parameters $\alpha, \beta,$ and $\gamma,$ one can identify three independent partial differential equations that are the coefficients of the three independent vectors $(p' + p)^\rho, (p' - p)^\rho,$ and x^ρ . Although these partial differential equations are of second order, they are linear in the variables $\alpha, \beta,$ and γ . Hence, the general solution can be obtained by taking a Fourier transform and solving the resulting first-order differential equations being careful to include various δ -function solutions. The result of the rather lengthy

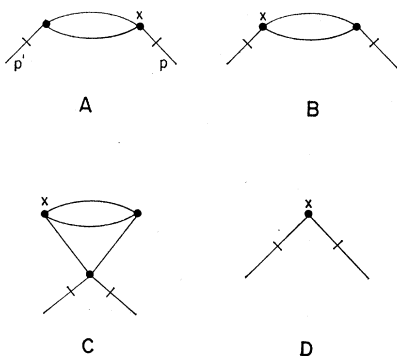


FIG. 1. Graphs labeled A, B, C, and D represent the corresponding conformal solutions for the source function W .

FIG. 2. Conformal graph for the field function Φ .

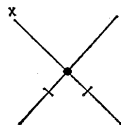
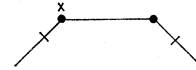


FIG. 3. Graph for the doubly truncated amplitude T_3 .



calculation that we have outlined is

$$F_{1,2}(\alpha, \beta, \gamma) = A_{1,2} e^{-\frac{1}{2}i(\beta - \alpha)} + B_{1,2} e^{-\frac{1}{2}i(\beta + \alpha)} + C_{1,2} e^{-\frac{1}{2}i\beta} \int_0^\infty \frac{dc}{c} \int db e^{i(b\beta + c\gamma)} e^{i(b^2 - 1)/4c} + E_{1,2} e^{-\frac{1}{2}i\beta} \int_{-\infty}^\infty dadbdc \times c^{-2} e^{i(a\alpha + b\beta + c\gamma)} e^{i(b^2 - 1 - a^2)/4c} f(a/c). \quad (28)$$

Consider the last term involving the triple integral. The integration over b can be done explicitly, giving an integrand that involves only $\alpha = (p' + p)x$ and $\gamma - \beta^2 = (p' - p)^2 x^2 - [(p' - p)x]^2$. In the brick-wall frame, $p = (E, 0, 0, E), p' = (E, 0, 0, -E)$, these variables become $\alpha = -2Ex^0$ and $\gamma - \beta^2 = 4E^2[x^2 - (x^3)^2]$, and we see that singularities will occur for spacelike intervals since x^2 occurs in the combination $x^2 - (x^3)^2$. Such spacelike singularities violate microscopic causality, and so this term cannot appear in a causal theory. We can evaluate the c integral in the $C_{1,2}$ term, obtaining an integrand that involves the Bessel function of imaginary argument, $K_0([-b(b+1)(p' - p)^2 x^2]^{1/2})$. This gives a causal solution if b is restricted to the interval $-1 < b < 0$, a restriction that is consistent with the conformal differential equations. Hence the general, causal, conformal solution is

$$F_{1,2} = A_{1,2} e^{ipx} + B_{1,2} e^{-ip'x} + C_{1,2} \int_{-1}^0 db e^{ib(p' - p)x} \times K_0([-b(b+1)(p' - p)^2 x^2]^{1/2}). \quad (29)$$

The source function W and field function Φ are simply related to this solution by Eqs. (24) and (25). (The terms involving A and B do not appear for Φ since they are disconnected.) The resulting functions are trivial; they correspond to those low-order graphs of a massless $\lambda\phi^4$ theory that are finite. The graphs for the source-function W are displayed in Fig. 1, with each graph labeled by its coefficient; the graph for the field function Φ is in Fig. 2. We must note that the application of the D'Alembertian operators $\partial^2 \partial'^2$ to the field function Φ does not produce all of the possible source-function W terms but only that involving the δ function. Thus, the implications of conformal symmetry depend upon whether one uses the full amplitudes or the truncated ones. The reason for this pathological behavior is clear. The graphs A, B, and C in Fig. 1 are not well defined because of the factor $(x^2)^{-2}$, and the

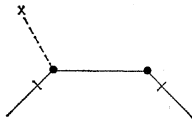


FIG. 4. Graph for the singly truncated amplitude M_3 .

integrations involved in attaching the external legs do not exist. It is not possible to renormalize these graphs and still maintain conformal invariance. We would like to emphasize that the pathology is a general one which appears for all field weights; in general, the solutions for W and Φ are different but still inconsistent.

A different example of conformal difficulties is provided by the theory of coupled, massless, spin-0 and spin- $\frac{1}{2}$ fields. In particular, we consider the two lowest-order amplitudes with two and one truncation corresponding to the graphs of Figs. 3 and 4. On the mass shell, the doubly truncated amplitude becomes

$$T_3(\gamma p' = 0) = \gamma x (x^2)^{-2} e^{-ip'x}, \quad (30)$$

while the singly truncated amplitude is given by

$$M_3(\gamma p' = 0) = \frac{1}{\pi^2} \int dy \frac{1}{(x-y)^2} \frac{\gamma y}{(y^2)^2} e^{-ip'y} \\ = \frac{\gamma x}{x^2} \frac{1 - e^{-ip'x}}{ip'x}. \quad (31)$$

The amplitudes are obviously dilation invariant. However, although the doubly truncated amplitude T_3 is invariant under special conformal transformations, the singly truncated amplitude M_3 is not. In order to investigate this, we consider the off-mass-shell extension

$$M_3(\gamma p' \neq 0) = \int_0^1 d\alpha e^{-i\alpha p'x} \{ (\gamma x/x^2) [\alpha(1-\alpha)p'^2 x^2]^{1/2} \\ \times K_1([\alpha(1-\alpha)p'^2 x^2]^{1/2}) - i\gamma p'(1-\alpha) \\ \times K_0([\alpha(1-\alpha)p'^2 x^2]^{1/2}) \}. \quad (32)$$

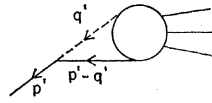


FIG. 5. Type of graph that gives non-analytic terms at the mass shell.

It is a straightforward matter to show that this off-mass-shell amplitude is invariant under special conformal transformations although its restriction to the mass shell is not. This pathological behavior arises because the amplitude is not analytic at $p'^2=0$. In fact, near the mass shell we have

$$M_3(\gamma p' \sim 0) = M_3(\gamma p' = 0) \\ + \frac{1}{2} \int_0^1 d\alpha e^{-i\alpha p'x} (1-\alpha) \\ \times [\frac{1}{2}\alpha\gamma x p'^2 + i\gamma p'] \ln(p'^2 x^2). \quad (33)$$

Although the terms involving $p'^2 \ln p'^2$ and $\gamma p' \ln p'^2$ vanish on the mass shell, they are not annihilated by the special conformal differential operator, but rather give a finite contribution when the mass-shell limit is taken after this operator has been applied. This contribution cancels the terms that make the mass-shell amplitude not invariant under the transformation.

We have found that the conformal invariance of the full theory is not present in the on-mass-shell amplitudes. It might be objected that this difficulty appears only in amplitudes that are partly in configuration space, as is the case with our example, and that, from the low-energy theorem for the stress tensor, one would find the fully on-mass-shell amplitudes to be conformally invariant. This is not true. The $p'^2 \ln p'^2$ or $\gamma p' \ln p'^2$ behavior generally results from the momentum integration in a graph of the form shown in Fig. 5 unless some form factors vanish on the mass shell. The logarithmic terms destroy the stress-tensor low-energy theorem at $p'^2=0$. They arise from the infrared integration region. Thus, only the introduction of finite particle masses can prevent their appearance, but this spoils the conformal symmetry.