points out that $\tilde{G}_5$ transforms physical mass states into unphysical (imaginary mass) states. These transformations (discussed in I) are generated by the covarient position operator X^μ. This problem may be avoided if one considers the XPM algebra as an algebra of observables rather than the group which it generates.

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Proton Form Factor, Magnetic Charges, and Dyonium*

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The relativistic quantum dynamics of two particles having both electric and magnetic charges (e,g) is discussed. It is shown that this model with its *strong long-range forces* is a physical realization underlying the dipole magnetic form factor $G_M(t)$ of the proton calculated in the relativistic $O(4,2)$ model. The mass spectrum gives a linear trajectory for low values of the principal quantum number N, contains N^2 degeneracy and parity doubling. The slope of the trajectory and the slope of the form factor can be related in principle to the masses of the constituents and the effective electric coupling $\alpha \equiv -(e_1e_2+g_1g_2)$.

PERHAPS the single most important piece of information we have at present about the structure of the proton is its magnetic form factor $G_M{}^p(t)$, which is known now[1] up to $t=25$ $(\text{GeV}/c)^2$, an extremely wide range of momentum transfer. Many other data involve small momentum transfers and are not sensitive enough to distinguish between various models. The rapid falloff of the form factor, going like $1/t^2$ for large t, can be understood in terms of the effects of multiparticle states in one form or another.[2] There is so far only one definite relativistic composite-particle model which yields the exact dipole form of $G_M(t)$ of proton and neutron, and also predicts $G_E(t)$ of these particles and their excited states.[3] The model gives further a mass spectrum, corresponding to linearly rising trajectories[4] (for not-too-large energy values) with a spectrum and degeneracy of states which has also appeared in Veneziano-type models. Physically, the model uses (in analogy to the H atom, but qualitatively distinct from it) forces inside the proton which are long-range, but strong. Mathematically, it makes use of the relativistic states provided by a specific fermion representation of the rest-frame dynamical group $O(4,2)$.[5] The representation used is quite different from that of the relativistic H atom.[6] Consequently, it is necessary to find a physical realization of this type of strong long-range forces.

The purpose of this paper is to show that the bound states of two spinless particles having both electric and magnetic charges contain the desired strong, long-range forces, and give precisely the fermion tower of states with the lowest total angular momentum $\frac{1}{2}$. This tower of states coincides with the representation of $O(4,2)$ used to calculate the form factors. We further derive a mass formula corresponding to linear Regge trajectories, and make some numerical order of magnitude estimates from the slopes of the trajectories and slopes of the form factors.

Since its introduction by Dirac,[7] the magnetic monopole problem has been extensively investigated.[8] More

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[1] D. H. Coward *et al.*, Phys. Rev. Letters **20**, 292 (1968) [final version: P. N. Kirk *et al.*, SLAC Report No. SLAC-PUB-659, 1969 (unpublished)].

[2] J. S. Ball and F. Zachariasen, Phys. Rev. **170**, 1541 (1968); D. Amati, L. Caneschi, and R. Jengo, Nuovo Cimento **98**, 783 (1968); A. Salam and J. Strathdee, Phys. Rev. D **1**, 3296 (1970).

[3] (a) A. O. Barut and K. Kleinert, Phys. Rev. **161**, 1464 (1967); (b) A. O. Barut, D. Corrigan, and K. Kleinert, Phys. Rev. Letters **20**, 167 (1968). For NN^* form factors, see D. Corrigan, B. Hamprecht, and H. Kleinert, Nucl. Phys. **B11**, 1 (1969).

[4] A. O. Barut, Acta Phys. Acad. Sci. Hung. **26**, 1 (1969).

[5] Various aspects of the model have been reviewed in my reports in *Lectures in Theoretical Physics*, edited by A. O. Barut and W. E. Brittin (Gordon and Breach, New York, 1968), Vol. X and in *Springer Tracts of Modern Physics*, edited by G. Hoehler (Springer, New York, 1969), Vol. 50.

[6] The dipole form of the form factor of the nonrelativistic H atom is really due to nonrelativistic definition and nonrelativistic kinematics, and of course cannot be correct for large t. The relativistic form factor of the atom actually goes like $1/t$. See A. O. Barut and A. Baiquni, IC Report No. IC/69/116 (unpublished) and Phys. Letters **30A**, 352 (1969).

[7] P. A. M. Dirac, Proc. Roy. Soc. (London) **A133**, 60 (1931); Phys. Rev. **74**, 817 (1948).

[8] I. Tamm, Z. Physik **71**, 141 (1931); M. Fierz, Helv. Phys. Acta **17**, 27 (1944); P. Banderet, *ibid.* **19**, 503 (1946); M. N. Saha, Phys. Rev. **75**, 1968 (1949); H. A. Wilson, *ibid.* **75**, 309 (1949), N. F. Ramsey, *ibid.* **109**, 225 (1958) (L); A. Salam and J. Tiomno, Nucl. Phys. **9**, 585 (1959); N. Cabibbo and E. Ferrari, Nuovo Cimento **23**, 1147 (1962); R. Katz, Am. J. Phys. **33**, 306 (1963); A. Goldhaber, Phys. Rev. **140**, B1407 (1965); D. Zwanziger, *ibid.* **137**, B647 (1965); S. Weinberg, *ibid.* **138**, B988 (1965); J. Schwinger, *ibid.* **144**, 1087 (1966), and in *Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, 1966*, edited by A. Perlmutter *et al.* (Freeman, San Francisco, 1966); G. Wentzel, Progr. Theoret. Phys. (Kyoto) Suppl. **37-38**, 163 (1966); A. Salam, Phys. Letters **22**, 683 (1966); B. Zumino, in *Strong and Weak Interactions: Present Problems*, edited by A. Zichichi (Academic, New York, 1967); A. Peres, Phys. Rev. Letters **18**, 50 (1967); J. G. Taylor, *ibid.* **18**, 713 (1967); E. Amaldi, in *Old and New Problems in Elementary Particles*, edited by G. Puppi (Academic, New York, 1968)—review article; J. Schwinger, Phys. Rev. **173**, 1536 (1968); A. Peres, *ibid.* **167**, 1449 (1968); C. A. Hurst, Ann. Phys. (N. Y.) **50**, 51 (1968); D. Zwanziger, Phys. Rev. **176**, 1480 (1968); **176**, 1489 (1968).

recently, Schiff[9] proposed to identify the "quarks" with spin-$\frac{1}{2}$ magnetic monopoles, and Schwinger[10] proposed to identify the "quarks" with spin-$\frac{1}{2}$ particles having both the electric and magnetic charges; he called these doubly charged particles "*dyons.*" Following this terminology, I call the simplest structure consisting of the bound states of two spinless dyons a "*dyonium.*" With respect to the existence of dyons, I should like to emphasize the following: Because of the invariance of all electromagnetic phenomena—in particular, of the energy-momentum tensor $T_{\mu\nu}$ of the electromagnetic field—under a rotation in the two-dimensional space spanned by the electric charge and the magnetic charge, and, simultaneously, by $\mathbf{E}$ and $\mathbf{B}$, the question is not whether magnetic charges exist. Rather the question should be, what is the electric and magnetic charge of each new particle, once the convention is made that one particle (say, the electron) has only an electric charge, as we have tacitly assumed in the historical development. In other words, we must consider *a priori* all particles to be endowed with both an electric charge e and a magnetic charge g. The observable quantities arising from the interaction of two particles can depend only on the invariant combinations $\alpha=-(e_1e_2+g_1g_2)$ and $\nu=(e_1g_2-e_2g_1)$. Dirac's symmetric form of the Maxwell's equations is then a logical consequence of the fact that all equations must be invariant under the rotation in charge space. Thus, it is perfectly sound, in fact necessary, to start with doubly charged particles, and we do this.

Consider now the system of two spinless dyons. It is important to note that, because the conserved total angular momentum J satisfying the angular momentum commutation relations $J=\mathbf{r}\times\boldsymbol{\pi}-(\nu/c)\mathbf{r}/r$, $\{\boldsymbol{\pi}=\mathbf{p}-\nu[\mathbf{r}\times\hat{n}\mathbf{r}\cdot\hat{n}]/r[r^2-(\mathbf{r}\cdot\hat{n})^2]\}$ has an extra term proportional to $\nu=e_1g_2-e_2g_1$, two spinless dyons can produce, in a first-quantized theory, a half-integer spin state,[11] because of the quantization of $\mathbf{J}$ along $\mathbf{r}$:

$$\nu/\hbar c\equiv(e_1g_2-g_1e_2)/\hbar c=-\mu\,,\quad \mu=0,\pm\tfrac{1}{2},\pm1,\ldots\,. \tag{1}$$

For each quantized value of $\nu/\hbar c$, the states of the system correspond to a degenerate unitary irreducible representation of the group $O(4,2)$ with the lowest spin $|\nu|/\hbar c$, labeled by the quantum numbers $|Njm\rangle$. The parity connects the two representations ν and $-\nu$. (ν is a Casimir operator.) The nonrelativistic dynamical problem has been investigated extensively[8,10]; the Hamiltonian can be solved exactly, most easily from the $O(2,1)$ dynamical group of the radial wave equation. We obtain the mass formula ($\nu/\hbar c=\mu$, m = reduced mass):

$$M_N=M_{\min}+\frac{1}{2}\left(\frac{e^2}{\hbar c}\right)^2 mc^2\frac{N^2-[\frac{1}{2}+(|\mu|+\frac{1}{4})^{1/2}]^2}{N^2[\frac{1}{2}+|\mu|+(\frac{1}{4}+|\mu|)^{\frac{1}{2}}]}\,, \tag{2}$$

where

$$N=\tfrac{1}{2}+[(j+\tfrac{1}{2})^2-\mu^2]^{1/2}+n'\,,\quad n'=0,1,2,\ldots\,. \tag{3}$$

This formula contains a fine-structure splitting, and for $|\nu|\neq0$ the $O(4)$ symmetry is broken. However, it is by now well understood that the use of the dynamical group $O(4,2)$ is not at all restricted to the $O(4)$ symmetric situations, and in fact we are interested in the more general broken $O(4)$ symmetry.[12]

The formalism of the dynamical group allows us to treat the system relativistically, which has not been done until now.

In the relativistic theory, we start from a basis of a representation of $O(4,2)$ representing the rest-frame states, then define the states with momentum P_μ by applying pure Lorentz transformations $e^{i\xi\cdot\mathbf{M}}$ to the rest-frame states, and, finally, introduce a *conserved current operator* $J_\mu(k)$ as a function of group generators and momenta. The current-conservation condition, or, equivalently, a "free" infinite-component wave equation on the representation space, determines then the mass spectrum, and via *the "minimal" coupling with* $J_\mu(k)$, one evaluates form factors. With this approach it is possible, for example, to take into account automatically the recoil corrections in the H atom to all orders,[6,13] except Lamb-shift-type terms,[14] which are small for the H atom. These terms, however, cannot be neglected if α is large; in fact they may dominate. For this reason the coefficients in $J_\mu(k)$ were treated as parameters in the $O(4,2)$ model discussed in Ref. 3(b). Thus, there is still a gap in relating the parameters of the relativistic current J_μ to the masses of the constituents of dyonium.

It is, however, instructive to see what happens to the relativistic Balmer formula when α is large. Thus, for the special values given in (A2) and (A3) (i.e., neglecting Lamb-shift-type terms), we obtain the mass formula

$$M^2=(m_1^2+m_2^2)+2m_1m_2N/(N^2+\alpha^2)^{1/2}\,, \tag{4}$$

where N is essentially as given by Eq. (3), and m_1 and m_2 are the masses of the two dyons.

The difference with the H atom is that $\alpha=-(e_1e_2+g_1g_2)$, the effective electric coupling, is now large. For example, for a dyonium with total electric charge $e=e_1+e_2$, total magnetic charge $g=g_1+g_2=0$, and $\nu/\hbar c$

[9] L. I. Schiff, Phys. Rev. Letters **17**, 714 (1967); Phys. Rev. **160**, 1257 (1967).

[10] J. Schwinger, Science **165**, 757 (1969).

[11] We are aware that according to Schwinger's field-theoretical arguments (Ref. 10), only integer values are allowed in Eq. (1), But this result does not seem to be conclusive (see G. Wentzell, Ref. 8). From the algebraic point of view that we are using, half-integer values must be introduced.

[12] For the exact solutions of Klein-Gordon and Dirac-type dyonium, see A. O. Barut and G. Bornzin, J. Math. Phys. **12**, 841 (1971).

[13] A. O. Barut and A. Baiquni, Phys. Rev. **184**, 1342 (1969).

[14] Because α is now large, the Lamb-shift-type effects are actually larger than the fine-structure-type effects. [This is borne out by the empirical fact that $\frac{1}{2}^-$ N^* lies higher than $\frac{3}{2}^-$ level. See the assignment in A. O. Barut, Phys. Letters **26B**, 308 (1968).]

$=\frac{1}{2}$, $\alpha/\hbar c \cong \frac{1}{4}(\hbar c/e^2) \cong 137/4$. (This is a lower limit; α would be even larger if e is the difference of two large numbers.) Thus, for low-lying states we have from (4) the linear trajectory in the quantum number N:

$$M_N{}^2=(m_1{}^2+m_2{}^2)+\lambda N\,, \tag{5}$$

where the slope is determined in terms of the masses of the constituent and α:

$$\lambda=2m_1m_2/(\alpha/\hbar c)\,. \tag{6}$$

The magnetic form factor of the dynonium obtained from the matrix elements of the conserved current operator $J_\mu(k)$ is given by

$$G_M(t)=\text{const}(1-\cosh^2\theta t/4M^2)^{-2}\,, \tag{7}$$

where M is the ground-state mass (proton) and the parameter $\cosh^2\theta$ is also given in terms of the masses of the constituents[3,6] in the special case (A3): $\cosh^2\theta = M_p{}^2/q^2$, where q^2 is the magnitude of the center-of-mass three-momentum at the bound state. The slope of the form factor is then given by[15]

$$\left.\frac{dG}{dt}\right|_{t=0}=2\frac{\cosh^2\theta}{4M_p{}^2}=\frac{-2m_1{}^2}{[M^2-(m_1-m_2)^2][M^2-(m_1+m_2)^2]}$$
$$=(2m_2{}^2)^{-1}\alpha^2/(\alpha^2-N^2)\,. \tag{8}$$

We now make some estimates using the empirical slope of the Regge trajectories and the slope of the form factors assuming, for the sake of an order-of-magnitude estimate, the simplified (although incomplete) Eqs. (5) and (8).[16] From the slope of the trajectories (with respect to N), $\lambda\approx 1$ GeV^2, we find $m_1m_2\approx\frac{1}{2}\alpha/\hbar c\approx 16$ GeV^2, using the value of $\alpha/\hbar c\approx 34$ discussed above. On the other hand, from the empirical form-factor slope of approximately $1/2m_2{}^2\approx 2.8$ (GeV)$^{-2}$, we see that one of the masses of the constituents must be small, about $\frac{1}{2}$ GeV, and the other therefore large, like an atom. These numbers must be taken as order-of-magnitude estimates only; the more accurate values will depend on the more detailed mass formulas[14] and other empirical consequences of the model.

A model of this type opens up a number of new ways of looking into the problem of hadron structure and hadron interactions. For example, we can associate meson towers with the representations $\nu=0$, and obtain essentially the same trajectory as in Eq. (5), which explains the approximate equality of the slopes of mesons and baryon trajectories. Furthermore, the absolute values of the magnetic moments of the dyonium [there are several of them possible with dyons $(\pm e, \pm g)$] can now be computed, and will be due to both e_i and g_i (although $g=g_1+g_2=0$). We further expect a great symmetry in the states of a dyonium atom, but the scattering of two dyoniums (i.e., dyonium molecule) would have a broken symmetry. Ordinary hadron systems (pp or πp) correspond in this model to dyonium molecules,[17] a remarkable electromagnetic origin for strong interactions. To conclude, we have tried to show in this paper that the electromagnetic form factor is one of the important experimental manifestations of the relativistic dynamics of the dyonium model.[18]

APPENDIX A: WAVE EQUATION

The infinite-component wave equation underlying the mass formula (4) can be taken to be of the form

$$(J_\mu P^\mu+\beta S+\gamma)\tilde{u}(p)=0\,, \tag{A1}$$

where

$$J_\mu=\alpha_1\Gamma_\mu{}'+\alpha_2P_\mu+\alpha_3P_\mu S'+i\alpha_4L_{\mu\nu}q^\nu\,, \tag{A2}$$
$$\Gamma_\mu{}'=\Gamma_\mu+\Lambda_\mu,\quad S'=S+\Lambda$$

Here Γ_μ and S are $O(4,2)$ generators, and

$$q=[\alpha_1{}^2P_\mu P^\mu-(\alpha_3P_\mu P^\mu+\beta)^2]^{1/2}$$

is the magnitude of the center-of-mass momentum. The constant parameters are given in terms of the masses of the constituents m_1, m_2 and the coupling parameter α (in analogy to the relativistic H atom *without* Lamb-shift-type terms):

$$\alpha_1=1\,,\ \alpha_2=-\alpha/2m_1\,,\ \alpha_3=1/2m_1\,,\ \beta=(m_1{}^2-m_2{}^2)/2m_1\,,$$
$$\gamma=\alpha(m_1{}^2+m_2{}^2)/2m_1\,,\quad \alpha_4=0\,. \tag{A3}$$

As compared to the previous wave equations,[3,4,5,13] the new terms Λ_μ and Λ are responsible for the spin-orbit splitting and have the effect that the quantum number n is replaced by N.[12] The particular choice of the parameters α_i, β, γ given above reduces the set of parameters to m_1, m_2, and α. In Ref. 3(b), a direct experimental determination of these parameters is given.

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[15] It is worth noting that the singularity of the $O(4,2)$ form factor (7) coincides with the singularity of the triangular diagram at $t_1=4m_2{}^2-(m_2{}^2+m_1{}^2-M^2)^2/m_1{}^2$, where m_2 is the internal line coupled to the photon. This equality, first noticed in the case of the H-atom equation [A. O. Barut and H. Kleinert, Phys. Rev. **160**, 1149 (1967)] can also be used to derive the mass formula. However, this does not mean that the $O(4,2)$ vertex is equivalent to a triangular diagram; in particular, the normal threshold is obviously missing in our vertex. See also C. Fronsdal, Phys. Rev. **171**, 1811 (1968); P. Budini and G. Calucci, Nuovo Cimento (to be published).

[16] In the nonrelativistic kinematics, these two quantities, i.e., dM/dn and dG/dt, are sufficient to obtain α: $\alpha/\hbar c=\sqrt{2}(dM/dn)\times(dG/dt)^{1/2}$. A previous estimate (Ref. 5) based on this formula gave $\alpha\approx 3$ for the proton. However, in the relativistic kinematics, these two quantities are not sufficient to determine the three unknowns α, m_1, and m_2, unless we use the independent estimate of α given above.

[17] This interpretation of the broken symmetry of hadron and the difference between symmetry of the multiplets and symmetries of scattering was discussed extensively several years ago. See A. O. Barut, in *High Energy Physics and Elementary Particles* (IAEA, Vienna, 1965), pp. 679–694, and in *Non-Compact Groups in Particle Physics*, edited by Y. Chow (Benjamin, New York, 1966), pp. 1–22.

[18] A detailed semiclassical study of the dyonium can be found in the author's contribution in *Topics in Modern Physics—A Tribute to E. U. Condon* (Colorado Associated U. P., Boulder, 1971). For details and tables on form factors see A. O. Barut, in *Proceedings of the Second Coral Gables Conference on Fundamental Interactions*, edited by H. Odabasi and W. E. Brittin (Gordon & Breach, New York, 1970), pp. 199–220.

APPENDIX B: ORIGIN AND INTERPRETATION OF WAVE EQUATION (A1)

There is no complete closed solution of the relativistic two-body problem in quantum field theory even for small coupling constant α, let alone the large-α case that we are considering. The Klein-Gordon and Dirac equations provide nonperturbative closed solutions and correspond in perturbation theory to a summation of a whole class of diagrams; but they do not contain relativistic recoil corrections, which become essential when the masses of the constituents are comparable, as in positronium. The infinite-component wave equations remove completely the last difficulty yet allow closed solutions, because the Lorentz transformations are applied to the system as a whole and not to the relative coordinates.[13] The method of constructing such wave equations is as follows. We start with the nonrelativistic Hamiltonian, which in the case of dyonium is

$$H=\frac{1}{2m}\pi_r^2+\frac{1}{2mr^2}(\mathbf{J}^2-\nu^2)+\frac{\alpha}{r}.$$

We then transform H into an algebraic form to identify a complete linear space of states which in this case is a particular representation space of the dynamical group $SO(4,2)$ characterized by ν. The group $SO(4,2)$ contains Galilean as well as Lorentz boost operators; we replace the Galilean boost operators by the Lorentz boost operators to arrive at states with momentum P_μ. On this space of states with momentum P_μ, we construct a *conserved current operator* J_μ correctly generalizing the nonrelativistic mass spectrum (2). Because the external photon must be coupled to a conserved current, which is unique for a given mass spectrum, we can evaluate the form factors from the matrix elements of J_μ.

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Troubles with Strict Conformal Symmetry*

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Formal low-energy theorems for matrix elements involving the stress tensor can be used to derive the differential equations that express scale and conformal invariance. We show that these equations, when applied to a single-particle matrix element of two fields, yield an integral representation that violates causality save for the trivial case where it reduces to the tree graphs of a free field. Although it is possible to construct conformally invariant, fully off-mass-shell amplitudes whose restriction to the mass shell yields a nontrivial result, these on-mass-shell amplitudes are not invariant under conformal transformations. This pathological behavior can be traced to an implicit assumption of analyticity when the stress tensor carries off small momenta that is, in general, false.

THERE has been considerable interest recently in the possibility that scale invariance and, perhaps, even the full conformal group, may have some role to play in high-energy physics.[1] Three general areas have been studied: the scaling behavior of field theory at short distances, theories with broken scale symmetry, and the structure of amplitudes that are strictly invariant under the conformal group. The study of strict conformal symmetry is motivated by the idea that, although many high-energy processes, such as elastic scattering, are by no means scale invariant, there may be other reactions of an inclusive nature, such as inelastic electroproduction, that do become conformally invariant at high energy. Thus, at high energy, these processes may be described by conformally invariant amplitudes with massless particles. We have shown[2] that, when the amplitudes for massless particles are obtained as the limit of off-mass-shell amplitudes, fully conformally invariant structure functions can be constructed for the electroproduction process.[3] On the other hand, the application of strict conformal symmetry to mass-shell amplitudes,[4] for zero-mass external particles, gives rise to various difficulties. We shall discuss some of the troubles here.

We begin by considering[5] off-mass-shell amplitudes that have a stress-tensor insertion carrying off momentum k. They obey divergence conditions (Ward-like identities) that yield low-energy theorems. A proper stress-tensor vertex is thereby determined including terms up to order k. The symmetry of the stress tensor, in conjunction with this determination in order 1, implies that the off-mass-shell amplitudes without the

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[1] A review of the recent work with references to the literature has been given by P. Carruthers, Phys. Repts. (to be published).

[2] D. G. Boulware, L. S. Brown, and R. D. Peccei, Phys. Rev. D 2, 293 (1970). This paper also contains a brief introduction to the geometry of the conformal group.

[3] We found, however, that invariance under the full conformal group gives no restrictions on these functions other than that of simple scale invariance.

[4] Such an application of strict conformal symmetry has been suggested by D. J. Gross and J. Wess, Phys. Rev. D 2, 753 (1970).

[5] Our method is a straightforward extension of that of Gross and Wess to off-mass-shell amplitudes.