π^{-}-p polarization is approximately the negative of the π^{+}-p polarization[7]). Since there does not seem to be mirror symmetry in $K^{\pm}$-p polarization,[7] we would not expect the BZ predictions to be as well satisfied for K reactions as for π reactions. This observation is not in conflict with the data,[3] but greater accuracy is required in the K experiments to test it.

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Symmetry Breaking and Spin-Zero Mass Spectrum*

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We discuss the mass spectrum in a very general chiral $SU(3)\times SU(3)$ model of pseudoscalar and scalar mesons. The case of an exactly symmetric Lagrangian and spontaneous breakdown is covered, as well as the more realistic case when there is also some intrinsic symmetry breaking in the Lagrangian. Our treatment is facilitated by the derivation of a mass formula which holds for all situations. In the realistic case the structure of the theory forces the mass of the ninth pseudoscalar meson to come out right and leaves us with the freedom to choose the masses of all scalar mesons (except the κ) as high as we like. The structure of the intrinsic symmetry breaking term is predicted. We also include some discussion of the baryons in this model and estimate the corrections to the Cabibbo theory of semileptonic hyperon decay.

I. INTRODUCTION

At the present time it seems likely that the structure of strong-interaction physics is related to a broken $SU(3)\times SU(3)$ chiral symmetry group.[1] The discussion of symmetry breaking inevitably involves one with a fairly complicated nonlinear system, so it is advisable to study a relatively simple model in detail. This is done in the present paper for the $SU(3)$ "σ" model of mesons, from the "spontaneous-breakdown" point of view.[2]

Although there have been a number of interesting previous papers, we believe that the subject is far from exhausted. Generally, the previous work has been concerned either with special forms of interactions,[3] with discussions of nonlinear realizations of the chiral group,[4] or with generalizations[5] to gauge field theories. Here we develop a simple method for calculating the mass spectrum for all cases of interest. This lets us easily obtain the preceding results as well as interesting new ones, both for the situation when the Lagrangian is exactly chiral symmetric and when there is some symmetry breaking added. The former case represents a complete spontaneous breakdown of symmetry and gives some zero-mass bosons,[6] a thorough discussion of the resulting spectrum is presented. The latter case gives a more realistic mass spectrum, and we find that the mass of the ninth pseudoscalar meson is well predicted.

Taking the symmetric part of the interaction to be the most general nonderivative one possible, we use fields transforming as linear realizations of the chiral group. It will be noted that one advantage which has been claimed for nonlinear realizations[4]—that of suppressing information about hard-to-observe particles—actually occurs also for the mass spectrum when linear realizations are used. We do not include gauge fields, in order to avoid more complication of an already complicated system; this addition may be desirable. (It should be noted, though, that certain predictions of chiral theories, such as the meson-nucleon scattering lengths, seem to be independent[7] of the presence of gauge fields in the theory.)

In Sec. II the formalism for computing the meson mass spectrum is discussed. Isotopic spin invariance is not assumed at first, and in the following paper the

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same formulas will be used to discuss the case when electromagnetic perturbations are present.

The most interesting case is when a simple symmetry-breaking term is included (Sec. 5). The system is sufficiently restrictive to give us a prediction for the ninth pseudoscalar meson mass in terms of a parameter W, whose *range* is fairly well known from the study of weak interactions. It is found that the model gives a good value for this mass when W is anywhere in a physically reasonable region, and an absurd value when W is anywhere else. Thus the σ model seems to conspire with nature to make this mass come out right. In common with other authors,[8–11] we find that the symmetry-breaking part of the interaction is much closer to an $SU(2)\times SU(2)$ invariant than to an $SU(3)$ invariant. Our discussion is general in the sense that symmetry breaking is allowed both in the "vacuum" (spontaneous breakdown) and in the interaction itself.

We have investigated the situation both for the $U(3)\times U(3)$ and $SU(3)\times SU(3)$ groups. If the symmetry breaking is of simple form, the $SU(3)\times SU(3)$ case seems to be preferred (see the Appendix).

In Sec. VI a brief discussion of symmetry breaking in the derivative coupling part of the effective meson-baryon Lagrangian is given. This enables us to get an idea of the corrections to be expected in the Cabibbo theory,[12] of semileptonic decays. The corrections may easily be as large as 30% for the strangeness-changing modes.

II. GENERAL FORMALISM

The basic objects for our discussion are 18 spin-0 fields,

$$M_a{}^{\bar{b}}, \quad M_{\bar{a}}{}^b$$

where all indices take on the values 1, 2, and 3. These fields transform according to the (3,3*) and (3*,3) representations of the chiral $SU(3)+SU(3)$ or $U(3)\times U(3)$ groups. Explicitly, under a unitary transformation in the "left-handed" three-dimensional space of the form

$$x_a \to A_a{}^b x_b, \quad A_a{}^b (A_c{}^b)^* = \delta_a{}^c, \tag{1}$$

the fields transform as

$$M_a{}^{\bar{b}} \to A_a{}^c M_c{}^{\bar{b}}, \quad M_{\bar{a}}{}^b \to (A_b{}^c)^* M_{\bar{a}}{}^c. \tag{2}$$

Under the "right-handed" transformation

$$y_{\bar{a}} \to B_{\bar{a}}{}^{\bar{b}} y_{\bar{b}}, \quad B_{\bar{a}}{}^{\bar{b}} (B_{\bar{c}}{}^{\bar{b}})^* = \delta_{\bar{a}}{}^{\bar{c}}, \tag{3}$$

the fields transform as

$$M_a{}^{\bar{b}} \to (B_{\bar{b}}{}^{\bar{c}})^* M_a{}^{\bar{c}}, \quad M_{\bar{a}}{}^b \to B_{\bar{a}}{}^{\bar{c}} M_{\bar{c}}{}^b. \tag{4}$$

The Hermiticity property is defined as

$$(M_a{}^{\bar{b}})^\dagger = M_{\bar{b}}{}^a. \tag{5}$$

The operation of parity reversal results in the interchange

$$M_a{}^{\bar{b}}(\mathbf{x},t) \leftrightarrow M_{\bar{a}}{}^b(-\mathbf{x},t). \tag{6}$$

Thus the fields may be decomposed as

$$M_a{}^{\bar{b}} = S_a{}^b + i\phi_a{}^b, \tag{7a}$$

$$M_{\bar{a}}{}^b = S_a{}^b - i\phi_a{}^b, \tag{7b}$$

where $S_a{}^b$ is a scalar nonet and $\phi_a{}^b$ is a pseudoscalar nonet. These have the Hermiticity properties

$$(\phi_a{}^b)^\dagger = \phi_b{}^a, \quad (S_a{}^b)^\dagger = S_b{}^a. \tag{8}$$

The charge-conjugation operation gives also

$$\phi_a{}^b \to \phi_b{}^a, \quad S_a{}^b \to S_b{}^a. \tag{9}$$

Sometimes it is convenient to use matrix notation and not explicitly distinguish upper and lower or barred and unbarred indices. In this case we make the replacements

$$M_a{}^{\bar{b}} \to (M)_{ab}, \quad M_{\bar{a}}{}^b \to (M^\dagger)_{ab}. \tag{10}$$

We shall investigate the theory corresponding to the following Lagrangian density:

$$\mathcal{L} = -\tfrac{1}{2}\,\mathrm{Tr}(\partial_\mu M \partial_\mu M^\dagger) - V_0 - V_{\mathrm{SB}}, \tag{11}$$

where the matrix notation of (10) was used. V_0 will be taken as the most general charge-conjugation- and parity-conserving chiral invariant that can be made out of M and $M^\dagger$. On the other hand, V_{SB} will be considered to be a symmetry-breaking term of simple form.

The solutions of the theory corresponding to (11) can exhibit symmetry breaking even when $V_{\mathrm{SB}}=0$. This situation is known as spontaneous breakdown and can be most conveniently explained, following Goldstone,[6,13] by taking (11) to be a *classical* Lagrangian. Then, as usual, we imagine all of space to be divided up into infinitesimal cubes, and the value of the field in each cube represents the amplitude of a (coupled) harmonic oscillator in that cube. The problem of finding the "free" Lagrangian is then the same as the problem of small-oscillation theory. Specifically, we must introduce "normal" coordinates corresponding to oscillations around the "equilibrium" point.[14] This equilibrium point is determined by imposing first the extremum condition

$$\langle \partial V/\partial M_a{}^{\bar{b}}\rangle_0 = \langle \partial V/\partial M_{\bar{a}}{}^b\rangle_0 = 0 \tag{12}$$

and second by requiring the extremum to be a minimum rather than a maximum. In (12), $V=V_0+V_{\mathrm{SB}}$, and

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[14] This is designated "ground state" or "vacuum" in the corresponding quantized theory.

the notation $\langle\ \rangle_0$ means that the enclosed object is taken at the equilibrium point. If parity is conserved, we have

$$\langle\phi_a{}^b\rangle_0=0\,, \tag{13a}$$

$$\langle S_a{}^b\rangle_0=\langle M_a{}^{\bar b}\rangle_0=\langle M_{\bar a}{}^b\rangle_0\,, \tag{13b}$$

and the normal coordinates are $\phi_a{}^b$ and

$$\tilde S_a{}^b=S_a{}^b-\langle S_a{}^b\rangle_0\,.$$

As in small-oscillation theory, we must expand (11) in terms of the normal coordinates. Depending on the location of the equilibrium point, the theory will only retain a portion of the original symmetry. There will also be zero-mass (Goldstone) particles if $V_{\mathrm{SB}}=0$. Making the expansion of (11) in normal coordinates gives

$$\begin{aligned}\mathcal{L}=&-\tfrac{1}{2}\operatorname{Tr}(\partial_\mu\phi\partial_\mu\phi)-\tfrac{1}{2}\operatorname{Tr}(\partial_\mu\tilde S\partial_\mu\tilde S)\\&-\tfrac{1}{2}\sum_{a,b,c,d}\left[\left\langle\frac{\partial^2V}{\partial\phi_a{}^b\partial\phi_c{}^d}\right\rangle_0\phi_a{}^b\phi_c{}^d\right.\\&\left.+\left\langle\frac{\partial^2V}{\partial S_a{}^b\partial S_c{}^d}\right\rangle_0\tilde S_a{}^b\tilde S_c{}^d\right]+(\text{interaction terms}).\end{aligned} \tag{14}$$

The coefficients of the bilinear nonderivative terms in (14) are, after suitable diagonalization, the squared masses of the appropriate particles. The condition that the equilibrium point be a stable one (minimum of V) means that these squared masses should be positive. In our subsequent discussion we shall consider (14) rather than (11) as the theory to be quantized.

To proceed further, it is useful to study briefly the invariants which can be formed from M and $M^\dagger$. These two can be combined to make a singlet, $M_a{}^{\bar b}M_{\bar b}{}^a=\operatorname{Tr}(MM^\dagger)$, and an octet in (for example) the left-handed space,

$$\mathcal{O}_a{}^b=M_a{}^{\bar c}M_{\bar c}{}^b-\tfrac{1}{3}M_d{}^{\bar c}M_{\bar c}{}^d\delta_a{}^b\,.$$

As is well known,[15] from the octet two independent $U(3)$ invariants can be constructed, i.e., $\mathcal{O}_a{}^b\mathcal{O}_b{}^a$ and $\mathcal{O}_a{}^b\mathcal{O}_b{}^c\mathcal{O}_c{}^a$. Thus, altogether we have three independent $U(3)\times U(3)$ invariants. In matrix notation these may be chosen as

$$\begin{aligned}I_1&=\operatorname{Tr}(MM^\dagger)\,,\\I_2&=\operatorname{Tr}(MM^\dagger MM^\dagger)\,,\\I_3&=\operatorname{Tr}(MM^\dagger MM^\dagger MM^\dagger)\,.\end{aligned} \tag{15}$$

If the symmetry of V_0 is reduced from $U(3)\times U(3)$ to $SU(3)\times SU(3)$, there is one more independent invariant. In $SU(3)\times SU(3)$ the transformation matrices of (1) and (3) are restricted by the additional condition

$$\det A=\det B=+1\,. \tag{16}$$

This has the consequence that the three-dimensional Levi-Civita symbol is an invariant tensor, so that we may form

$$\begin{aligned}I_4=&\,\epsilon_{abc}\epsilon^{\bar e\bar f\bar g}M_{\bar e}{}^aM_{\bar f}{}^bM_{\bar g}{}^c+\epsilon^{abc}\epsilon_{\bar e\bar f\bar g}M_a{}^{\bar e}M_b{}^{\bar f}M_c{}^{\bar g}\\\equiv&\,6(\det M^\dagger+\det M)\,.\end{aligned} \tag{17}$$

Under the transformation of Eq. (1), for example,

$$I_4\to6(\det A^*\det M^\dagger+\det A\det M)\,.$$

Thus I_4 is invariant when (16) holds. If $\det A\neq1$, it is easy to see that neither I_4 nor any analytic function of I_4 is invariant under (1). Since I_1, I_2, and I_3 are all invariant, I_4 or any analytic function of I_4 is independent of I_1, I_2, and I_3. We thus see also that I_3 (say) cannot be written as a function of I_1, I_2, and I_4.

It is also possible in the $SU(3)\times SU(3)$ case to form the invariant

$$I_5=6i(\det M^\dagger-\det M)\,.$$

I_5 changes sign under parity reversal, so V_0 must be a function of $(I_5)^2$. However, $(I_5)^2$ is not an independent invariant but is related to the others by

$$(I_5)^2=24(I_1)^3-72I_1I_2+48I_3-(I_4)^2\,. \tag{18}$$

Equation (18) is derived by setting $Q=MM^\dagger$ in the following formula,[15] which holds when Q is a diagonalizable 3×3 matrix:

$$Q^3-(\operatorname{Tr}Q)Q^2-\tfrac{1}{2}[\operatorname{Tr}Q^2-(\operatorname{Tr}Q)^2]Q-\det Q=0\,.$$

Thus, a convenient choice of invariants for discussing $SU(3)\times SU(3)$ is I_1, I_2, I_3, and I_4. By neglecting I_4, we will automatically be dealing with $U(3)\times U(3)$.

Now let us rewrite (12) with V_0 considered to be a function of I_1, I_2, I_3, and I_4:

$$\begin{aligned}\sum_{i=1}^4V_i\left\langle\frac{\partial I_i}{\partial M_a{}^{\bar b}}\right\rangle_0+\left\langle\frac{\partial V_{\mathrm{SB}}}{\partial M_a{}^{\bar b}}\right\rangle_0&=0\,,\\\sum_{i=1}^4V_i\left\langle\frac{\partial I_i}{\partial M_{\bar a}{}^{b}}\right\rangle_0+\left\langle\frac{\partial V_{\mathrm{SB}}}{\partial M_{\bar a}{}^{b}}\right\rangle_0&=0\,,\end{aligned} \tag{19}$$

where we have defined

$$V_i\equiv\langle\partial V_0/\partial I_i\rangle_0\,. \tag{20}$$

With the parity-conserving solution of (13), the coefficients in (19) are explicitly

$$\begin{aligned}\left\langle\frac{\partial I_1}{\partial M_a{}^{\bar b}}\right\rangle_0&=\left\langle\frac{\partial I_1}{\partial M_{\bar a}{}^{b}}\right\rangle_0=\langle S_b{}^a\rangle_0\,,\\\left\langle\frac{\partial I_2}{\partial M_a{}^{\bar b}}\right\rangle_0&=\left\langle\frac{\partial I_2}{\partial M_{\bar a}{}^{b}}\right\rangle_0=2\langle(S^3)_b{}^a\rangle_0\,,\\\left\langle\frac{\partial I_3}{\partial M_a{}^{\bar b}}\right\rangle_0&=\left\langle\frac{\partial I_3}{\partial M_{\bar a}{}^{b}}\right\rangle_0=3\langle(S^5)_b{}^a\rangle_0\,,\\\left\langle\frac{\partial I_4}{\partial M_a{}^{\bar b}}\right\rangle_0&=\left\langle\frac{\partial I_4}{\partial M_{\bar a}{}^{b}}\right\rangle_0=3\epsilon^{aef}\epsilon_{bdh}\langle S_e{}^d\rangle_0\langle S_f{}^h\rangle_0\,.\end{aligned}$$

[15] See, e.g., S. Coleman, in *High Energy Physics and Elementary Particles* (International Atomic Energy Agency, Vienna, 1965).

Considered as matrices in a 3×3 space labeled by indices a and b, all the above objects are Hermitian and commute with each and so may be simultaneously diagonalized.[4] Thus we take our "equilibrium" point to satisfy

$$\langle M_a{}^{\bar{b}}\rangle_0=\langle M_{\bar{a}}{}^b\rangle_0=\langle S_a{}^b\rangle_0=\alpha_a\delta_a{}^b, \tag{21}$$

where the α_a are three real constants.

If $\langle\partial V_{\mathrm{SB}}/\partial M_a{}^{\bar{b}}\rangle_0$ and $\langle\partial V_{\mathrm{SB}}/\partial M_{\bar{a}}{}^b\rangle_0$ can also be simultaneously diagonalized with these others, then the choice (21) is sufficient as a solution of (19). We shall assume this to be the case.

Using (21) gives immediately

$$\begin{aligned}
\langle\partial I_1/\partial M_a{}^{\bar{b}}\rangle_0&=\delta_b{}^a\alpha_a,\\
\langle\partial I_2/\partial M_a{}^{\bar{b}}\rangle_0&=2\delta_b{}^a\alpha_a{}^3,\\
\langle\partial I_3/\partial M_a{}^{\bar{b}}\rangle_0&=3\delta_b{}^a\alpha_a{}^5,\\
\langle\partial I_4/\partial M_a{}^{\bar{b}}\rangle_0&=6\delta_b{}^af_a,
\end{aligned}$$

where $f_a=(\alpha_1\alpha_2\alpha_3)/\alpha_a$ if $\alpha_a\neq0$. If $\alpha_a=0$ we must first cancel α_a in the denominator against one of α_1, α_2, and α_3 in the numerator. Substituting these into (19) gives rise to only three nontrivial equations:

$$\alpha_a[V_1+2V_2\alpha_a{}^2+3V_3\alpha_a{}^4]+6f_aV_4=-\langle\partial V_{\mathrm{SB}}/\partial M_a{}^{\bar{a}}\rangle_0 \quad (a=1,2,3). \tag{22}$$

[We have assumed $\langle\partial V_{\mathrm{SB}}/\partial M_a{}^{\bar{a}}\rangle_0=\langle\partial V_{\mathrm{SB}}/\partial M_{\bar{a}}{}^a\rangle_0$, so that the second set of equations which might be formed from (19) just duplicates the first set.]

Next we compute the coefficients in (14) which give the pseudoscalar masses, as follows:

$$\left\langle\frac{\partial^2V_0}{\partial\phi_a{}^b\partial\phi_c{}^d}\right\rangle_0=\left\langle\left(\frac{\partial^2V_0}{\partial M_a{}^{\bar{b}}\partial M_{\bar{c}}{}^d}+\frac{\partial^2V_0}{\partial M_{\bar{a}}{}^b\partial M_c{}^{\bar{d}}}-\frac{\partial^2V_0}{\partial M_a{}^{\bar{b}}\partial M_c{}^{\bar{d}}}-\frac{\partial^2V_0}{\partial M_{\bar{a}}{}^b\partial M_{\bar{c}}{}^d}\right)\right\rangle_0.$$

Substituting in the relations

$$\frac{\partial^2V_0}{\partial M_a{}^{\bar{b}}\partial M_c{}^{\bar{d}}}=\sum_{i,j}\frac{\partial^2V_0}{\partial I_i\partial I_j}\frac{\partial I_i}{\partial M_a{}^{\bar{b}}}\frac{\partial I_j}{\partial M_c{}^{\bar{d}}}+\sum_i\frac{\partial^2I_i}{\partial M_a{}^{\bar{b}}\partial M_c{}^{\bar{d}}}\frac{\partial V_0}{\partial I_i},\quad\text{etc.}$$

gives

$$\begin{aligned}
\left\langle\frac{\partial^2V_0}{\partial\phi_a{}^b\partial\phi_c{}^d}\right\rangle_0
&=2\sum_{i,j}\left\langle\frac{\partial^2V_0}{\partial I_i\partial I_j}\right\rangle_0\left\langle\left(\frac{\partial I_i}{\partial M_a{}^{\bar{b}}}\frac{\partial I_i}{\partial M_{\bar{c}}{}^d}-\frac{\partial I_j}{\partial M_a{}^{\bar{b}}}\frac{\partial I_j}{\partial M_c{}^{\bar{d}}}\right)\right\rangle_0\\
&\quad+2\sum_iV_i\left\langle\left(\frac{\partial^2I_i}{\partial M_a{}^{\bar{b}}\partial M_{\bar{c}}{}^d}-\frac{\partial^2I_i}{\partial M_a{}^{\bar{b}}\partial M_c{}^{\bar{d}}}\right)\right\rangle_0\\
&=2\sum_iV_i\left\langle\left(\frac{\partial^2I_i}{\partial M_a{}^{\bar{b}}\partial M_{\bar{c}}{}^d}-\frac{\partial^2I_i}{\partial M_a{}^{\bar{b}}\partial M_c{}^{\bar{d}}}\right)\right\rangle_0.
\end{aligned}\tag{23a}$$

Note that the coefficients of $\langle\partial^2V_0/\partial I_i\partial I_j\rangle_0$ vanish. This does not occur in the case of the scalar masses, which are computed similarly to be

$$\begin{aligned}
\left\langle\frac{\partial^2V_0}{\partial S_a{}^b\partial S_c{}^d}\right\rangle_0&=4\sum_{i,j}\left\langle\frac{\partial^2V_0}{\partial I_i\partial I_j}\right\rangle_0\left\langle\frac{\partial I_i}{\partial M_a{}^{\bar{b}}}\right\rangle_0\left\langle\frac{\partial I_j}{\partial M_c{}^{\bar{d}}}\right\rangle_0\\
&\quad+2\sum_iV_i\left\langle\left(\frac{\partial^2I_i}{\partial M_a{}^{\bar{b}}\partial M_{\bar{c}}{}^d}+\frac{\partial^2I_i}{\partial M_a{}^{\bar{b}}\partial M_c{}^{\bar{d}}}\right)\right\rangle_0.
\end{aligned}\tag{23b}$$

The final results, taking account of (21), are

$$\begin{aligned}
\left\langle\frac{\partial^2V}{\partial\phi_a{}^b\partial\phi_c{}^d}\right\rangle_0
&=2\delta_d{}^a\delta_b{}^c[V_1+2V_2(\alpha_a{}^2+\alpha_b{}^2-\alpha_a\alpha_b)\\
&\quad+3V_3(\alpha_a{}^4+\alpha_a{}^2\alpha_b{}^2+\alpha_b{}^4-\alpha_a{}^3\alpha_b-\alpha_b{}^3\alpha_a)]\\
&\quad-12V_4(\delta_b{}^a\delta_d{}^c-\delta_d{}^a\delta_b{}^c)\frac{\alpha_1\alpha_2\alpha_3}{\alpha_b\alpha_d}+\left\langle\frac{\partial^2V_{\mathrm{SB}}}{\partial\phi_a{}^b\partial\phi_c{}^d}\right\rangle_0,
\end{aligned}\tag{24}$$

$$\begin{aligned}
\left\langle\frac{\partial^2V}{\partial S_a{}^b\partial S_c{}^d}\right\rangle_0
&=2\delta_d{}^a\delta_b{}^c[V_1+2V_2(\alpha_a{}^2+\alpha_b{}^2+\alpha_a\alpha_b)\\
&\quad+3V_3(\alpha_a{}^4+\alpha_a{}^2\alpha_b{}^2+\alpha_b{}^4+\alpha_a{}^3\alpha_b+\alpha_b{}^3\alpha_a)]\\
&\quad+12V_4(\delta_b{}^a\delta_d{}^c-\delta_d{}^a\delta_b{}^c)\frac{\alpha_1\alpha_2\alpha_3}{\alpha_b\alpha_d}+\left\langle\frac{\partial^2V_{\mathrm{SB}}}{\partial S_a{}^b\partial S_c{}^d}\right\rangle_0\\
&\quad+4\delta_b{}^a\delta_d{}^cF_{ac},
\end{aligned}\tag{25}$$

where

$$\begin{aligned}
F_{ac}&=V_{11}\alpha_a\alpha_c+2V_{12}(\alpha_a\alpha_c{}^3+\alpha_c\alpha_a{}^3)+4V_{22}\alpha_a{}^3\alpha_c{}^3\\
&\quad+6V_{23}(\alpha_a{}^3\alpha_c{}^5+\alpha_a{}^5\alpha_c{}^3)+9V_{33}\alpha_a{}^5\alpha_c{}^5\\
&\quad+3V_{13}(\alpha_a\alpha_c{}^5+\alpha_a{}^5\alpha_c)+36\left(\frac{\alpha_1\alpha_2\alpha_3}{\alpha_a}\right)\left(\frac{\alpha_1\alpha_2\alpha_3}{\alpha_c}\right)V_{44}\\
&\quad+6\alpha_1\alpha_2\alpha_3\left[V_{14}\left(\frac{\alpha_a}{\alpha_c}+\frac{\alpha_c}{\alpha_a}\right)+2V_{24}\left(\frac{\alpha_a{}^3}{\alpha_c}+\frac{\alpha_c{}^3}{\alpha_a}\right)\right.\\
&\quad\left.+3V_{34}\left(\frac{\alpha_a{}^5}{\alpha_c}+\frac{\alpha_c{}^5}{\alpha_a}\right)\right]
\end{aligned}\tag{26}$$

and

$$V_{ji}=V_{ij}=\langle\partial^2V_0/\partial I_i\partial I_j\rangle_0. \tag{27}$$

[Note that $\alpha_1\alpha_2\alpha_3/\alpha_b\alpha_c$ occurring in (24)–(26) is a convenient shorthand notation. In the case when one of the α's in the denominator goes to zero, we define the expression so that particular α is first canceled off against an α in the numerator.]

Equations (24) and (25), which contain all the information about the mass spectrum of the system, are crucial for the work that follows. The V_i's and α_a's which appear in (24) and (25) are not unrestricted parameters, but must satisfy the extremum conditions (22).

So far we have not assumed that the system is necessarily isotopic-spin invariant. In this case we would have

$$\alpha_1=\alpha_2=\alpha \tag{28}$$

and (22) collapses into two equations. If these two equations are independent, it is convenient to construct their sum and difference:

$$(\alpha_a\mp\alpha_b)[V_1+2V_2(\alpha_a{}^2+\alpha_b{}^2\pm\alpha_a\alpha_b)+3V_3(\alpha_a{}^4+\alpha_b{}^4+\alpha_a{}^2\alpha_b{}^2\pm\alpha_a{}^3\alpha_b\pm\alpha_b{}^3\alpha_a)\mp6V_4\alpha_1\alpha_2\alpha_3/\alpha_a\alpha_b]=-\langle\partial V_{\mathrm{SB}}/\partial M_a{}^{\bar a}\rangle_0\pm\langle\partial V_{\mathrm{SB}}/\partial M_b{}^{\bar b}\rangle_0\quad(a\neq b). \tag{22'}$$

III. VECTOR AND AXIAL-VECTOR CURRENTS

We make the assumption that the vector and axial-vector currents computed from (11) according to the Noether prescription take part in the usual Cabibbo theory of semileptonic decays. This gives us, by appealing to experimental information, some idea of the important parameters α_a. The material in this section is, of course, well known.

The 18 pseudovector and vector currents whose integrated time components are the generators of $U(3)\times U(3)$ are, respectively,

$$(P_\mu)_a{}^b=(\alpha_a+\alpha_b)\partial_\mu\phi_a{}^b-(\phi_a{}^c\overleftrightarrow{\partial_\mu}\tilde S_c{}^b+\phi_c{}^b\overleftrightarrow{\partial_\mu}\tilde S_a{}^c) \tag{29}$$

$$(V_\mu)_a{}^b=i(\alpha_a-\alpha_b)\partial_\mu\tilde S_a{}^b+i(\phi_a{}^c\partial_\mu\phi_c{}^b-\partial_\mu\phi_a{}^c\phi_c{}^b)+i(\tilde S_a{}^c\partial_\mu\tilde S_c{}^b-\partial_\mu\tilde S_a{}^c\tilde S_c{}^b). \tag{30}$$

[For $SU(3)\times SU(3)$ we must deal instead with the 16 traceless objects: $(P_\mu)_a{}^b-\frac13\delta_a{}^b(P_\mu)_c{}^c$ and $(V_\mu)_a{}^b-\frac13\delta_a{}^b(V_\mu)_c{}^c$.]

The matrix elements which are "known" from the Cabibbo theory plus experiment are

$$\begin{aligned}(2q_0)^{1/2}\langle0|(P_\mu)_1{}^2|\pi^+(q)\rangle&\equiv iF_\pi q_\mu,\\(2q_0)^{1/2}\langle0|(P_\mu)_1{}^3|K^+(q)\rangle&\equiv iF_K q_\mu.\end{aligned} \tag{31}$$

The pion decay constant $|F_\pi|$ is numerically 1.01 in π^0 mass units and the ratio $|F_K/F_\pi|$ is conventionally taken to be 1.28, but there is some uncertainty associated with this determination.

If $\phi_a{}^b$ in (29) is taken to represent the physical pseudoscalar particles, we have immediately

$$F_\pi=\alpha_1+\alpha_2,\quad F_K=\alpha_1+\alpha_3. \tag{32}$$

It is convenient, in the isotopic spin symmetry limit, to introduce[4] a parameter W, which characterizes the breaking of $SU(3)$ symmetry:

$$W=\alpha_3/\alpha. \tag{33}$$

In the $SU(3)$ limit, $W=+1$. We have, using (32),

$$W=2F_K/F_\pi-1, \tag{34}$$

which results in $W\simeq1.56$ (if $F_K>F_\pi>0$).

In a more general framework, the pseudoscalar fields in (29) may not be the physical ones but are related to the physical ones by

$$\pi_+=\pi^+{}_p/Z_\pi,\quad K_+=K^+{}_p/Z_K,\ \text{etc.}, \tag{35}$$

where Z_π, Z_K, etc., are some constants and the subscript p stands for physical. In this case (34) gets modified to

$$W=2(F_K/F_\pi)(Z_\pi/Z_K)-1. \tag{34'}$$

Since (Z_π/Z_K) is model dependent, we shall not take W to be fixed at 1.56 but, as an initial guess, shall consider the range $W=1$ [exact $SU(3)$] to roughly $W=2$ to be a sensible one.

IV. SPONTANEOUS BREAKDOWN SITUATION

If $V_{\mathrm{SB}}=0$, the Lagrangian (11) is exactly $U(3)\times U(3)$ or $SU(3)\times SU(3)$ invariant and all the vector and axial-vector currents formally satisfy

$$\partial_\mu(V_\mu)_a{}^b=\partial_\mu(P_\mu)_a{}^b=0.$$

However, as explained in Sec. II, the physical states may not exhibit the full symmetry. Now all information on the masses of the one-particle states is contained in (24) and (25). Thus, in our formalism, it is very easy to see what the one-particle spectrum looks like, and to infer the corresponding symmetry. In all cases (as expected by the Goldstone theorem), there are zero-mass particles which are apparently not observed in nature.

The mass spectrum corresponding to different types of spontaneous breakdown, under the assumption of isotopic spin invariance, $\alpha_1=\alpha_2=\alpha$, is listed[16] in Table I. Our notation for the mesons is (π,K,η,η') for the pseudoscalar nonet, and $(\epsilon,\kappa,\sigma,\sigma')$ for the corresponding scalar nonet. The squared mass of π^+ meson, for example, is identified with $\langle\partial^2V/\partial\phi_1{}^2\partial\phi_2{}^1\rangle_0$. The results in Table I were read off from (24) and (25) with the aid of (22) and (22′). The objects in brackets on the left-hand sides of (22) and (22′) appear together in (24) and (25), making the job very simple. Since the right-hand sides of (22) and (22′) are zero in this case, and the brackets on the left-hand sides are multiplied by α_a and $\alpha_a\pm\alpha_b$, we can only make a definite statement about the value of the object in the brackets when the appropriate one of α_a and $\alpha_a\pm\alpha_b$ is nonzero. This *ordinary* case is called case 1, and gives a spectrum which is characteristic of isotopic spin invariance. The other cases correspond to α_a and $\alpha_a\pm\alpha_b$ being zero in various combinations. We only remark that case 2, $\alpha=\alpha_3$, gives an $SU(3)$ spectrum; case 4, $\alpha=0$, $\alpha_3\neq0$ gives an $SU(2)\times SU(2)$ spectrum[17]; and case 5, $\alpha=-\alpha_3$, gives a chimeral $SU(3)$ spectrum, in the language of Mathur and Okubo.[10] Case 6, of course, corresponds to no spontaneous breakdown of symmetry.

[16] The zero entries (Goldstone bosons) in Table I have been given by Bardeen and Lee (Ref. 4).

[17] Note that examples of chiral $SU(2)$ multiplets are (i) scalar isosinglet, (ii) pseudoscalar isotriplet plus scalar isosinglet, and (iii) scalar isotriplet plus pseudoscalar isosinglet.

TABLE I. One-particle spectrum for the various spontaneous breakdown situations given in terms of V_i of (20) and F_{ij} of (26). The F_{ij} should be evaluated separately for each case. The quantity $\xi=8\alpha^2(V_2+3\alpha^2V_3)$. Since there is symmetry breaking, η^2 and η'^2 and σ^2 and σ'^2 are *mixtures* of the usual objects; which one deserves a prime is just a matter of convention. Thus σ_1^2 and $\sigma_1'^2$ are given by $6(\alpha^2-\alpha_3^2)V_3+(6/\alpha_3)(\alpha^2+2\alpha_3^2)V_4+4F_{11}+2F_{33}\pm\{[4F_{11}-2F_{33}+6(\alpha^4-\alpha_3^4)V_3+(6/\alpha_3)(3\alpha^2-2\alpha_3^2)V_4]^2+32(F_{13}+3\alpha V_4)^2\}^{1/2}$ and σ_3^2 and $\sigma_3'^2$ are given by $2\alpha^2V_2+9\alpha^4V_3+4F_{11}+2F_{33}\pm[(4F_{11}-2F_{33}+6\alpha^2V_2+15\alpha^4V_3)^2+32(F_{13})^2]^{1/2}$.

Case \ Squared mass	π^2	K^2	η^2	η'^2	ϵ^2	κ^2	σ^2	σ'^2
1. $\alpha\neq0,\ \pm\alpha_3$; $(\alpha_3\neq0)$	0	0	0	$-12\alpha_3[2+(\alpha^2/\alpha_3^2)]V_4$	$12(\alpha^2-\alpha_3^2)$ $\times[\alpha^4V_3+(2/\alpha_3)V_4]$	0	σ_1^2	$\sigma_1'^2$
2. $\alpha=\alpha_3\neq0$	0	0	0	$-36\alpha V_4$	$\xi-24\alpha V_4$	$\xi-24\alpha V_4$	$\xi-24\alpha V_4$	$\xi+12\alpha V_4$ $+12F_{11}$
3. $\alpha\neq0;\alpha_3=0$	0	0	0	$2V_1$	ξ	0	σ_3^2	$\sigma_3'^2$
4. $\alpha=0;\alpha_3\neq0$	$2(V_1+6\alpha_3V_4)$	0	$2(V_1-6\alpha_3V_4)$	0	$2(V_1-6\alpha_3V_4)$	0	$2(V_1+6\alpha_3V_4)$	$8\alpha_3^2V_2+24\alpha_3^4V_3$ $+4F_{33}$
5. $\alpha=-\alpha_3\neq0$	0	$\xi+24\alpha V_4$	0	$36\alpha V_4$	$\xi+24\alpha V_4$	0	$\xi+24\alpha V_4$	$\xi-12\alpha V_4$ $+12F_{11}$
6. $\alpha=\alpha_3=0$	$2V_1$	$2V_1$	$2V_1$	$2V_1$	$2V_1$	$2V_1$	$2V_1$	$2V_1$

As written, Table I describes the situation when the Lagrangian is $SU(3)\times SU(3)$ symmetric. To get the results for the $U(3)\times U(3)$ case we should set $V_4=V_{4i}=0$.

V. REALISTIC SITUATION

As we saw in Sec. IV, the assumption $V_{\rm SB}=0$ and spontaneous breakdown leads to unobserved zero-mass particles. Hence, some explicit intrinsic breaking must be included to construct a realistic theory.

Let us work in the limit of exact isotopic spin symmetry, and choose the most general $V_{\rm SB}$ which is linear in M and $M^\dagger$, namely,

$$V_{\rm SB}=-g_0(M_c{}^{\bar c}+M_{\bar c}{}^c)-g(M_3{}^{\bar 3}+M_{\bar 3}{}^3). \tag{36}$$

At the present time we shall not attempt to justify such a choice, except to note that it is about the simplest possible one and has been used by many authors. In the limit $g=0$ the Lagrangian is $SU(3)$ [or $U(3)$] symmetric while in the limit $g_0=0$ it is $SU(2)\times SU(2)$ symmetric. Recent work[8–11] has tended to indicate the surprising fact that $|g|\gg|g_0|$, but we shall not be required to make any assumption on this matter.

Before going further, we should specify whether V_0 is to be considered $U(3)\times U(3)$ symmetric or $SU(3)\times SU(3)$ symmetric. In the Appendix it is shown that, with (36), the $U(3)\times U(3)$ case does not lead to a realistic mass spectrum.[18] Some other forms of $V_{\rm SB}$ were investigated and also found to be unsatisfactory. Hence we shall consider the $SU(3)\times SU(3)$ case here. In other words, V_0 will be assumed to depend on I_4 as well as I_1, I_2, and I_3.

From (36) and (21) we have

$$\langle\partial V_{\rm SB}/\partial M_1{}^{\bar 1}\rangle_0=\langle\partial V_{\rm SB}/\partial M_2{}^{\bar 2}\rangle_0=-g_0,$$
$$\langle\partial V_{\rm SB}/\partial M_3{}^{\bar 3}\rangle_0=-(g+g_0),$$
$$\langle\partial^2V_{\rm SB}/\partial M_a{}^{\bar b}\partial M_{\bar c}{}^d\rangle=0,\quad\text{etc.}$$

Using these in (22), (22′), and (24) gives the squared pion and kaon masses

$$\pi^2=2g_0/\alpha, \tag{37}$$

$$K^2=\frac{2g_0}{\alpha}\frac{2+g/g_0}{1+W}. \tag{38}$$

(We are adopting a notation where each particle symbol also stands for its mass.) From (22), (22′), and (25) we find for the corresponding scalar particles

$$\epsilon^2=-2g_0/\alpha+4[V_1+4V_2\alpha^2+9V_3\alpha^4], \tag{39}$$

$$\kappa^2=\frac{2g_0}{\alpha}\frac{g/g_0}{W-1}. \tag{40}$$

Next we consider the neutral nonstrange pseudoscalar mesons. From (24), the matrix whose (ab) element is $\langle\partial^2V/\partial\phi_a{}^a\partial\phi_b{}^b\rangle_0$ (no sum) turns out to be

$$\begin{bmatrix} 2g_0/\alpha-12V_4\alpha_3 & -12V_4\alpha_3 & -12V_4\alpha \\ -12V_4\alpha_3 & 2g_0/\alpha-12V_4\alpha_3 & -12V_4\alpha \\ -12V_4\alpha & -12V_4\alpha & (2/\alpha_3)(g_0+g-6V_4\alpha^2) \end{bmatrix}. \tag{41}$$

The three roots of the secular equation of (41) are π^2, η^2, and η'^2. This leads to the following two equations for η^2 and η'^2:

$$\eta^2+\eta'^2=\frac{2g_0}{\alpha}+\frac{2}{\alpha W}(g_0+g)-\frac{12V_4\alpha}{W}(2W^2+1),$$
$$(\eta\eta')^2=-24V_4\left[\frac{g_0}{W}+2(g+g_0)\right]+\frac{4g_0}{\alpha^2W}(g+g_0). \tag{42}$$

[18] This has been noted by Gasiorowicz and Geffen (Ref. 5).

We can also compute the masses of the scalar isosinglets σ and σ' by using (25), but these involve the unknown quantities F_{ac} of (26). The mass of the scalar isovector given in (39) also involves undetermined objects. Thus we are free to take these scalar particles to have any masses whatsoever. The particles whose masses *are* constrained are the pseudoscalar nonet and the κ. From a practical point of view this is a fortunate situation.

Useful predictions may be extracted from the set of equations (37), (38), (40), and (42). We shall take as input the masses of the neutral π, neutral K, and η mesons. We shall use a system of units where the neutral pion mass is unity. As output we shall calculate the η' mass, the κ mass, and the quantity g/g_0 as a function of the basic parameter W of Eq. (33). g/g_0 is simply

$$g/g_0=(K/\pi)^2(1+W)-2. \tag{43}$$

Then the squared κ mass is

$$\kappa^2=\frac{\pi^2(g/g_0)}{W-1}. \tag{44}$$

Finally, the squared mass of the ninth pseudoscalar in terms of the input masses and W is

$$\eta'^2=\frac{2W^2K^2(\eta^2-K^2)+2W(K^2-\pi^2)(\eta^2-2K^2)+\pi^2(\eta^2-\pi^2)-2(K^2-\pi^2)^2}{2W^2(\eta^2-K^2)-2W(K^2-\pi^2)+(\eta^2-\pi^2)}. \tag{45}$$

Equation (45) is plotted in Fig. 1. There is a very broad minimum around $W=2.3$, and we see that quite reasonable values for η'^2 emerge when W is anywhere in the range 1.5–3. Values of W less than 0.75 or greater than 3.58 are clearly ruled out. The remarkable fact is that the σ model seems to militate for an η' mass in the right region when the value of W is what we might expect from the theory of weak interactions (see Sec. III). Exact agreement for η'^2 ($\eta'^2_{\text{expt}}\approx 50.4$) is obtained at $W=1.71$ and 3.00.

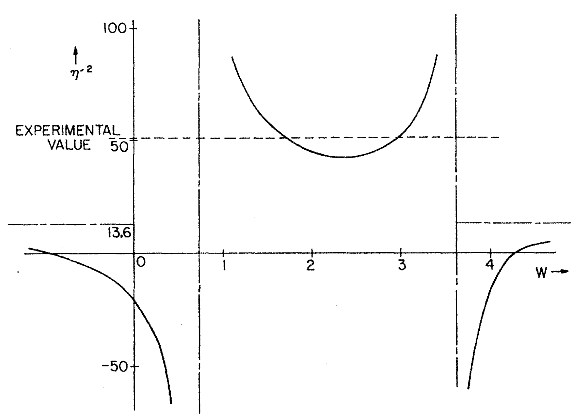

Fig. 1. η'^2 (in units of π_0^2) as a function of W.

g/g_0 is plotted in Fig. 2 and we notice that it is very large for the interesting range of W. Thus V_{SB} is much closer to an $SU(2)\times SU(2)$ rather than an $SU(3)$ invariant, as has been pointed out by Gell-Mann, Oakes, and Renner.[9] These authors worked in the limit where $W=1$ [$SU(3)$ invariance for the spontaneous breakdown situation]. We see from Fig. 2 that larger and more realistic values of W bring us even closer to the $SU(2)\times SU(2)$ limit. Actually, for the precise value $W=1$ the set of Eqs. (22) is no longer consistent unless $g=0$.

The squared mass of the κ is plotted in Fig. 3. Since this particle has not been decisively established, we can not say too much. Note that κ^2 tends to infinity as W approaches 1 from above. For $W=1.7$ the κ mass is 950 MeV.

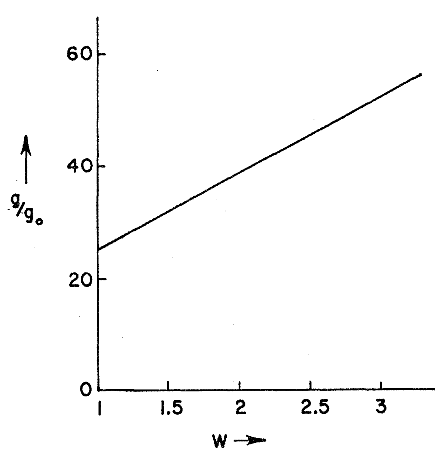

Fig. 2. g/g_0 as a function of W.

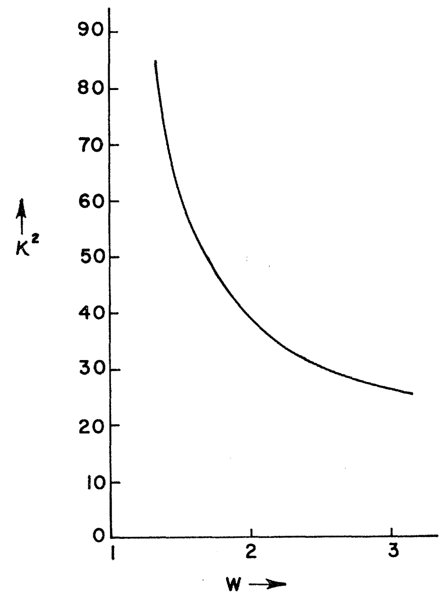

Fig. 3. κ^2 (in units of π_0^2) as a function of W.

Finally, we must form a conclusion about the reliability of our predictions. Since the tendency for the η' mass to come out correctly with reasonable W is so striking, we believe that this model possesses many of the features of the real physical situation. However, since the addition of other particles in the theory will probably have some effect, we should regard the numbers we get as a guide, rather than definitive.

VI. BARYONS

It is of some interest to construct an effective Lagrangain to describe the meson-baryon interactions. The most characteristic feature of the meson-baryon interactions in the σ model is that, because of the spontaneous breakdown condition (21), the baryon mass term arises automatically, leading to relations of the Goldberger-Treiman type.

For the baryon octet we shall adopt a notation such that, in the γ_5 diagonal representation,

$$N_a{}^b \equiv \begin{pmatrix} L_a{}^b \\ R_{\bar{a}}{}^{\bar{b}} \end{pmatrix}, \quad \bar{N}_a{}^b \equiv (\bar{R}_{\bar{a}}{}^{\bar{b}}, \quad \bar{L}_a{}^b). \tag{46}$$

The above two-component notation is sometimes convenient, since the barred and unbarred tensor indices are explicitly displayed. The kinetic part of the free-baryon Lagrangian is

$$-\mathrm{Tr}(\bar{L}\sigma_\mu \partial_\mu L + \bar{R}\tilde{\sigma}_\mu \partial_\mu R) \equiv -\mathrm{Tr}(\bar{N}\gamma_\mu \partial_\mu N). \tag{47}$$

We may consider both derivative and nonderivative meson-baryon interactions. The nonderivative ones contribute to the nucleon mass terms. It is necessary to include derivative terms if we want the axial-vector currents, computed from our Lagrangian according to Noether's theorem, to be identified as participants in the weak semileptonic interactions. Without derivative coupling, only the term (47) would contribute to the axial-vector current, and it would give an "F-type" current, which seems to be contradicted by experiment. We adopt the following as the simplest chiral-invariant derivative-type meson-nucleon interaction:

$$\begin{aligned} &-a\,\mathrm{Tr}[\bar{L}\sigma_\mu(M\overleftrightarrow{\partial}_\mu M^\dagger)L + \bar{R}\tilde{\sigma}_\mu(M^\dagger\overleftrightarrow{\partial}_\mu M)R] \\ &\quad -b\,\mathrm{Tr}[\bar{L}\sigma_\mu L(M\overleftrightarrow{\partial}_\mu M^\dagger) + \bar{R}\tilde{\sigma}_\mu R(M^\dagger\overleftrightarrow{\partial}_\mu M)] \\ &= -\tfrac{1}{2}a\,\mathrm{Tr}[\bar{N}(1-\gamma_5)\gamma_\mu(M\partial_\mu M^\dagger)N \\ &\quad +\bar{N}(1+\gamma_5)\gamma_\mu(M^\dagger\overleftrightarrow{\partial}_\mu M)N] \\ &\quad -\tfrac{1}{2}b\,\mathrm{Tr}[\bar{N}(1-\gamma_5)\gamma_\mu N(M\overleftrightarrow{\partial}_\mu M^\dagger) \\ &\qquad\qquad +\bar{N}(1+\gamma_5)\gamma_\mu N(M^\dagger\overleftrightarrow{\partial}_\mu M)], \end{aligned} \tag{48}$$

where a and b are two real constants.

Computing the vector and axial-vector currents from (47) and (48) in the usual way[19] and taking account of (21) gives

$$\begin{aligned} (V_\mu)_r{}^s &= i\bar{N}_t{}^s\gamma_\mu N_r{}^t[1-a(\alpha_r-\alpha_s)^2] \\ &\quad - i\bar{N}_r{}^t\gamma_\mu N_t{}^s[1+b(\alpha_r-\alpha_s)^2]+\cdots, \\ (P_\mu)_r{}^s &= i\bar{N}_t{}^s\gamma_\mu\gamma_5 N_r{}^t[1-\alpha(\alpha_r+\alpha_s)^2] \\ &\quad - i\bar{N}_r{}^t\gamma_\mu\gamma_5 N_t{}^s[1+b(\alpha_r+\alpha_s)^2]+\cdots. \end{aligned} \tag{49}$$

The three dots in (49) indicate that we have left out trilinear and quadrilinear terms. In terms of (49), the Cabibbo semileptonic Lagrangian is

$$\mathcal{L}^{\mathrm{SL}} = -(G/\sqrt{2})l_\mu\{[(P_\mu)_2{}^1+(V_\mu)_2{}^1]\cos\theta + [(P_\mu)_3{}^1+(V_\mu)_3{}^1]\sin\theta\} + \mathrm{H.c.}, \tag{50}$$

where l_μ is the lepton current, G is the Fermi constant, and θ is the Cabibbo angle. Now we are in a position to write the effective couplings, including the effects of symmetry breaking for decays of interest. These are given in Table II, as functions of the parameter W and the two quantities a and b. From measurements of neutron β decay, we may determine a since

$$g_A = 1 - 4a\alpha^2 \simeq 1.2, \tag{51}$$

where $\alpha \approx \frac{1}{2}$ (pion mass units) from (32). g_A is the usual "axial-vector renormalization" constant. The quantity b may be determined by measurement of any one other semileptonic mode (Table I shows that $\Sigma^\pm \to \Lambda e^\pm \nu$ are interesting candidates).

In the limit of no $SU(3)$ symmetry breaking, we have $W=1$. Then the vector current in (49) is pure F type and we regain the usual Cabibbo theory. In this case one finds $b \simeq -1.32$ in addition to $a \simeq -0.20$ from (51). Reference to Table II shows that the hadron part of the $\Lambda \to pe\nu$ matrix element (for example) is

$$-\gamma\sin\theta\,(1.22+0.85\gamma_5)$$

for $W=1$. To get some idea of the kind of corrections

TABLE II. Cabibbo matrix elements with symmetry breaking. γ and γ_5 are Dirac matrices.

Decay	Hadronic matrix element
$n \to pe\nu$	$\cos\theta\,[\gamma+(1-4a\alpha^2)\gamma\gamma_5]$
$\Sigma^- \to \Lambda e\nu$	$\cos\theta\,[-(4/\sqrt{6})\alpha^2(a+b)\gamma\gamma_5]$
$\Sigma^+ \to \Lambda e^+\nu$	$\cos\theta\,[-(4/\sqrt{6})\alpha^2(a+b)\gamma\gamma_5]$
$\Sigma^- \to \Sigma^0 e\nu$	$\sqrt{2}\cos\theta\,\{\gamma+[1+2\alpha^2(b-a)]\gamma\gamma_5\}$
$\Xi^- \to \Xi^0 e\nu$	$-\cos\theta\,[\gamma+(1+4\alpha^2 b)\gamma\gamma_5]$
$\Lambda \to pe\nu$	$(1/\sqrt{6})\sin\theta\,\{[-3+\alpha^2(1-W)^2(2a-b)]\gamma + [-3+\alpha^2(1+W)^2(2a-b)]\gamma\gamma_5\}$
$\Sigma^- \to ne\nu$	$-\sin\theta\,\{[1+\alpha^2 b(1-W)^2]\gamma + [1+\alpha^2 b(1+W)^2]\gamma\gamma_5\}$
$\Xi^- \to \Lambda e\nu$	$(1/\sqrt{6})\sin\theta\,\{[3+\alpha^2(1-W)^2(2b-a)]\gamma + [3+\alpha^2(1+W)^2(2b-a)]\gamma\gamma_5\}$
$\Xi^- \to \Sigma^0 e\nu$	$\frac{1}{2}\sqrt{2}\sin\theta\,\{[1-\alpha^2 a(1-W)^2]\gamma + [1-\alpha^2 a(1+W)^2]\gamma\gamma_5\}$
$\Xi^0 \to \Sigma^+ e\nu$	$\sin\theta\,\{[1-\alpha^2 a(1-W)^2]\gamma + [1-\alpha^2 a(1+W)^2]\gamma\gamma_5\}$

[19] We are following Ref. 7 and also J. Schechter, Y. Ueda, and G. Venturi, Phys. Rev. **177**, 2311 (1969).

that may be expected in a more realistic case, let us still use $b \simeq -1.32$ but take $W=1.7$. Then this becomes

$$-\gamma \sin\theta\,(1.18+0.54\gamma_5)\,.$$

The above treatment is of course crude; form factors have been neglected and (48) was assumed to be the only derivative-type interaction term. Nevertheless, the moral is clear; a reasonably large (30%) correction to the axial-vector part of the matrix element may be expected.

Still to be discussed are the nonderivative meson-baryon couplings; we postpone this to the following paper. Here we just remark that it is possible for all the different octet baryon masses to arise by the spontaneous breakdown mechanism from a chiral-invariant meson-baryon interaction, but that this requirement by itself is too weak to give us any additional information.

APPENDIX

Here we consider the $U(3)\times U(3)$ case. V_0 is no longer allowed to depend on I_4, so we must set $V_4=0$. Thus [when $V_{\rm SB}$ is given by (36)] (41) becomes the diagonal matrix

$$\begin{bmatrix} 2g_0/\alpha & 0 & 0 \\ 0 & 2g_0/\alpha & 0 \\ 0 & 0 & 2(g_0+g)/\alpha \end{bmatrix}. \tag{A1}$$

The diagonal entries in (A1) are the π, η, and η' masses. Evidently the π and one of the isoscalars are degenerate in clear contradiction to nature. The same unfortunate conclusion holds if $V_{\rm SB}$ is modified to include any combination of the following terms:

$$\begin{gathered}(M_3{}^{\bar c}M_{\bar c}{}^3+M_{\bar 3}{}^cM_c{}^{\bar 3})\,,\quad (M_c{}^{\bar c}+M_{\bar c}{}^c)I_1\,,\\ (M_3{}^{\bar 3}+M_{\bar 3}{}^3)I_1\,,\quad (M_c{}^{\bar a}M_{\bar a}{}^bM_b{}^{\bar c}+M_{\bar c}{}^aM_a{}^{\bar b}M_{\bar b}{}^c)\,,\\ (M_3{}^{\bar a}M_{\bar a}{}^bM_b{}^{\bar 3}+M_{\bar 3}{}^aM_a{}^{\bar b}M_{\bar b}{}^3)\,.\end{gathered} \tag{A2}$$

It appears more promising to try for $V_{\rm SB}$ a form that looks something like I_4. Define the "dual" objects

$$\begin{aligned} T_a{}^{\bar b} &= \epsilon_{amn}\epsilon^{\bar b\bar f\bar g}M_{\bar f}{}^mM_{\bar g}{}^n\,,\\ T_{\bar b}{}^a &= (T_a{}^{\bar b})^\dagger\,,\end{aligned} \tag{A3}$$

and try

$$V_{\rm SB}=-\tfrac{1}{2}g_0'(T_c{}^{\bar c}+T_{\bar c}{}^c)-\tfrac{1}{2}g'(T_3{}^{\bar 3}+T_{\bar 3}{}^3)\,. \tag{A4}$$

Using (A4), the squared pion and kaon masses come out to be

$$\begin{aligned} \pi^2 &= 2g_0'W\,,\\ K^2 &= [2/(1+W)](2g_0'+g')\,,\end{aligned} \tag{A5}$$

while the squared masses of π, η, and η' are the roots of the secular equation of

$$2\begin{bmatrix} g_0'(1+W)+g' & g_0'+g' & g_0' \\ g_0'+g' & g_0'(1+W)+g' & g_0' \\ g_0' & g_0' & 2g_0'/W \end{bmatrix}. \tag{A6}$$

The system (A5) and (A5) is more restrictive than the corresponding set (37), (38), and (41); it does not yield as convincing a solution. Specifically, the analog of (45) is here

$$W^4+2W^2\frac{\pi^2}{(\eta\eta')^2}(\pi^2-\eta^2-\eta'^2)+4\left(\frac{\pi^2}{\eta\eta'}\right)^2=0\,. \tag{A7}$$

This gives $W=\pm0.35$ or ±0.21, none of which is consistent with the *usual* theory of weak interactions.

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Electromagnetic Perturbation of the Pseudoscalar-Mass Spectrum*

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Using the formalism developed in the preceding paper, we investigate electromagnetic perturbations in a rather general chiral $SU(3)\times SU(3)$ model of mesons. The meson and octet baryon mass shifts can be successfully correlated, and it is found that the electromagnetic breaking term in the Lagrangian may be of the same order of magnitude as the chiral-symmetry-breaking term. We also discuss the speculation that all strong symmetry breaking may be of electromagnetic and weak origin.

I. INTRODUCTION

In the preceding paper[1] (hereafter designated I) we dealt with symmetry breaking in a very general chiral $SU(3)\times SU(3)$ model of spin-0 mesons. A mass formula was derived which was true when the Lagrangian contained any chiral-invariant nonderivative part and some additional specific symmetry-breaking part. It was found that the resulting mass spectrum seemed in agreement with nature. For example, there was a striking tendency for the mass of the ninth pseudoscalar meson to come out right when a certain parameter W

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[1] J. Schechter and Y. Ueda, preceding paper, Phys. Rev. D 3, 168 (1971).