ticles exchanged in the crossed channel. Similar treatment of diffraction processes has not proved fruitful, and the general consensus is that the Pomeranchuk singularity is not just a glorified Regge pole. Attempts at relating the nucleon form factors to hadron masses are legion: vector-meson dominance, Veneziano amplitudes, etc. Why not give up? Indeed, the fundamental implication of scale independence is that size and mass are two unrelated aspects of hadrons.

The separate dualism of Pomeranchukon and background,[5] and Reggeon and resonances,[19] among other considerations, led Harari[20] to suggest that scattering amplitudes may be composed of two independent parts: a geometrical-background-Pomeranchuk part and a dynamical-resonance-Regge part. Our proposal is to take the notion literally in the sense that diffraction scattering is the same regardless of the slope of the ordinary Regge trajectories. The consequence is a helicity selection rule, which so far agrees with experiment. A testable prediction is that electroproduced ρ^0's should be transversely polarized at high energy. Although one can contrive other interpretations of the available data (such as optical models, quark spin conservation, etc.), the virtue of our approach is that it deals only with direct observables.

I thank Y. Nambu for helpful suggestions and discussions.

[19] R. Dolen, D. Horn, and C. Schmid, Phys. Rev. **166**, 1768 (1968); G. Veneziano, Nuovo Cimento **57A**, 190 (1968).
[20] H. Harari, in *Lectures in Theoretical Physics*, edited by W. E. Britten *et al.* (Gordon and Breach, New York, 1969), Vol. XI-A, p. 553.

PHYSICAL REVIEW D VOLUME 3, NUMBER 7 1 APRIL 1971

CP Violation and the Strong Self-Interacting Intermediate-Vector-Boson Model

R. E. Marshak,* Y. W. Yang,† and J. S. Rao‡
University of Rochester, Rochester, New York 14627
(Received 14 September 1970)

The strong cubic intermediate-vector-boson (IVB) model of weak interactions, based on a triplet of strong self-interacting W bosons and treating CP violation as a maximal effect at the semiweak level, is reexamined. It is shown that the basic $CP=-1$ semiweak interaction must possess an essentially unique structure in order to reproduce the successes of the CP-conserving universal (V,A) current-current theory. The consequences to order g^3 (g is the semiweak coupling constant) of the strong cubic IVB model are worked out in detail. The most incisive test of this model at low energies is the detection of gross CP violation in $\Delta Y=1$ induced neutral-lepton-current decays of hadrons, in particular, the decays $K^+\to\pi^+l\bar{l}$ and $K_L^0\to\mu\bar{\mu}$. Under certain conditions, the detection of gross CP violation may be possible in $\Delta Y=0$ W production by neutrinos on nucleons.

I. INTRODUCTION

Two distinct types of strongly interacting intermediate-vector-boson (IVB) models have been considered[1] in an attempt to explain the small value of the weak interaction cutoff[2] and the large mass (compared to hadrons) of the hypothesized IVB particle. In the first type of strong IVB model, the W boson is assumed to interact strongly with hadrons and the usual CP-conserving weak-interaction predictions are maintained by making use of the additive triality quantum number. However, the CP-violation effect must be introduced in an *ad hoc* fashion. The second type of strong IVB model, which postulates strong self-interaction among the W bosons, goes further than the first type of model and directly incorporates the phenomenon of CP violation in weak interactions. An attractive version of this second type of model is the so-called strong cubic IVB model treated in several previous papers.[3]

In the strong cubic IVB model, the W bosons form a triplet (W^0,W^-,W^+) and interact strongly with each other via a cubic term (hence the name "strong cubic" model). More explicitly, one writes down for the W Lagrangian[4]

$$\begin{aligned}\mathcal{L}_0=&-\tfrac{1}{2}[\partial_\nu\bar{W}_\mu^{(a)}(x)-\partial_\mu\bar{W}_\nu^{(a)}(x)]\\&\times[\partial_\nu W_\mu^{(a)}(x)-\partial_\mu W_\nu^{(a)}(x)]-m_0^2\bar{W}_\mu^{(a)}(x)W_\mu^{(a)}(x)\\&-if_0\epsilon_{abc}[W_\mu^{(a)}(x)W_\nu^{(b)}(x)\partial_\mu W_\nu^{(c)}(x)\\&\qquad-\bar{W}_\mu^{(a)}(x)\bar{W}_\nu^{(b)}(x)\partial_\mu\bar{W}_\nu^{(c)}(x)],\quad(1)\end{aligned}$$

where $a=1$, 2, 3 (corresponding to charge 0, -1, $+1$),

* Present address: City College of New York, N. Y. 10031.
† Present address: Cheng Kung University, Taiwan.
‡ Present address: Cambridge University, Cambridge, England.
[1] Cf. R. E. Marshak, in *Proceedings of the Topical Conference on Weak Interactions*, edited by J. S. Bell (CERN, Geneva, 1969).
[2] Cf. B. L. Ioffe and E. P. Shabalin, Yadern. Fiz. **6**, 828 (1967) [Soviet J. Nucl. Phys. **6**, 603 (1968)]; R. N. Mohapatra, J. S. Rao and R. E. Marshak, Phys. Rev. **171**, 1502 (1968).
[3] S. Okubo, Nuovo Cimento **54A**, 491 (1968); Ann. Phys. (N. Y.) **49**, 219 (1968); R. E. Marshak, R. N. Mohapatra, S. Okubo, and J. S. Rao, Nucl. Phys. **B11**, 253 (1969). The last paper will be referred to as I.
[4] We shall see below that it is advisable to introduce into the W Lagrangian a quartic interaction term as well.

W_μ represents the IVB vector field, and f_0 is a strong coupling constant. In addition, the W's interact semiweakly with hadrons and leptons through a semiweak $CP=-1$ interaction of the form

$$\begin{aligned} H_{\text{s.w.}}=ig[&W_\mu^{(1)}(\alpha J_{\mu 3}^{\ 2}+\beta J_{\mu 3}^{\ 3})-\bar{W}_\mu^{(1)}(\alpha J_{\mu 2}^{\ 3}+\beta J_{\mu 3}^{\ 3}) \\ &+W_\mu^{(2)}(\gamma J_{\mu 2}^{\ 1}+\delta l_\mu)-\bar{W}_\mu^{(2)}(\gamma J_{\mu 1}^{\ 2}+\delta \bar{l}_\mu) \\ &+W_\mu^{(3)}(\gamma' J_{\mu 1}^{\ 3}+\delta' \bar{l}_\mu)-\bar{W}_\mu^{(3)}(\gamma' J_{\mu 3}^{\ 1}+\delta' l_\mu)], \end{aligned} \tag{2}$$

where α, β, γ, δ, γ', and δ' are real coefficients. As long as[5] $\alpha\neq 0$, the semiweak interaction (2) is the most general interaction which forbids $\Delta Y=2$ transitions[6] to order g^2 and g^3. It should be noted that interaction (2) does not yield any $CP=-1$ processes of order g because of conservation of (multiplicative) triality (see I). In order g^2, one obtains the usual CP-conserving weak processes. Because of Eq. (1), weak processes occurring in order g^3 do not vanish and this serves as the explanation of the observed CP violation in K_L^0 decay.

The coefficients in Eq. (2) are not arbitrary but are required to reproduce the results of the universal (V,A) current-current weak interaction[7]

$$H_{\text{w}}^{\text{c.c.}}=(G/\sqrt{2})\bar{J}_\mu J_\mu, \tag{3}$$

where $J_\mu=(\cos\theta J_{\mu 2}^{\ 1}+\sin\theta J_{\mu 3}^{\ 1}+l_\mu)$ is the Cabibbo current and $G/\sqrt{2}=g^2/m_W^2$. Equation (3) leads to a contribution of order g^2 to purely leptonic processes, $g^2\cos^2\theta$ and $g^2\sin^2\theta$ to $\Delta Y=0$ and $\Delta Y=1$ semileptonic processes, respectively, and $g^2\cos\theta\sin\theta$ to $\Delta Y=1$ hadronic weak processes. These features should be recaptured by Eq. (2) and hence the coefficients in Eq. (2) must satisfy the conditions[8]

$$\delta^2+\delta'^2=k, \tag{4a}$$

$$\gamma\delta=k\cos\theta, \quad \gamma'\delta'=k\sin\theta, \tag{4b}$$

$$\alpha\beta=k\cos\theta\sin\theta, \tag{4c}$$

where k is an arbitrary constant (since g can always be redefined). The model studied in I (with $\delta=\delta'=1$, $\gamma=\cos\theta$, $\gamma'=\sin\theta$) does not simultaneously[9] satisfy conditions (4a) and (4b) and hence we must readjust the coefficients in Eq. (2) in order to maintain universality of coupling. Since we have more coefficients than conditions in Eqs. (4a)–(4c), we would expect some ambiguity in the determination of the coefficients. However, we shall soon see that heuristic arguments lead to an essentially unique model (provided $\alpha\neq 0$).

Equations (4a) and (4b) can be satisfied by choosing $\delta=k^{1/2}\cos\theta$, $\delta'=k^{1/2}\sin\theta$, $\gamma=\gamma'=k^{1/2}$, i.e., by associating the "Cabibbo angle" with the lepton current. The condition (4c) can then be easily satisfied by a suitable choice of α and β (e.g., $\alpha=k^{1/2}\cos\theta$, $\beta=k^{1/2}\sin\theta$). This approach bears a similarity to the model treated by Segrè.[10] The objection to this model is that the "Cabibbo angle" appearing in Eq. (4c) is associated with the neutral hadron current and there is no reason why there should be any relation between this Cabibbo angle and the one associated with the charged lepton current. It is much more plausible to adopt the unified viewpoint that all Cabibbo angles are associated with hadron currents. This hypothesis produces a unique solution (assuming, of course, that the same lepton current is coupled to the $\Delta Y=0$ and $\Delta Y=1$ hadron currents so that $\delta=\delta'$), namely,[11]

$$\begin{aligned} &\delta=\delta'=\tfrac{1}{2}, \quad \gamma=\cos\theta, \quad \gamma'=\sin\theta, \\ &\alpha=(1/\sqrt{2})\sin\theta, \quad \beta=(1/\sqrt{2})\cos\theta. \end{aligned} \tag{5}$$

The $CP=-1$ semiweak interaction which is consistent with the universal (V,A) weak interaction thus becomes

$$\begin{aligned} H_{\text{s.w.}}=(ig/\sqrt{2})[&W_\mu^{(1)}(\sin\theta\, J_{\mu 3}^{\ 2}+\cos\theta\, J_{\mu 3}^{\ 3}) \\ &+W_\mu^{(2)}(\sqrt{2}\cos\theta\, J_{\mu 2}^{\ 1}+l_\mu/\sqrt{2}) \\ &+W_\mu^{(3)}(\sqrt{2}\sin\theta\, J_{\mu 1}^{\ 3}+\bar{l}_\mu/\sqrt{2})-\text{H.c.}]. \end{aligned} \tag{6}$$

It is easy to see that the semiweak interaction (6) automatically yields the $\Delta I=\frac{1}{2}$ rule for $\Delta Y=1$ hadron weak processes in order g^2 (since only the term $J_{\mu 3}^{\ 2}J_{\mu 3}^{\ 3}$ contributes to such processes), as well as the $\Delta I=\frac{1}{2}$ rule for $\Delta Y=1$ semileptonic processes in order g^2. For CP-violating hadronic weak processes in order g^3 (e.g., $K_L^0\to 2\pi$), there is, in general, a mixture of $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ matrix elements.

The interaction (6) can be recast in the more interesting form

$$\begin{aligned} H_{\text{s.w.}}=ig'\Bigg[&\frac{(W_\mu^{(2)}-\bar{W}_\mu^{(3)})}{\sqrt{2}}(\cos\theta\, J_{\mu 2}^{\ 1}+\sin\theta\, J_{\mu 3}^{\ 1}+l_\mu) \\ &+\frac{(W_\mu^{(2)}+\bar{W}_\mu^{(3)})}{\sqrt{2}}(\cos\theta\, J_{\mu 2}^{\ 1}-\sin\theta\, J_{\mu 3}^{\ 1}) \\ &+W_\mu^{(1)}(\sin\theta\, J_{\mu 3}^{\ 2}+\cos\theta\, J_{\mu 3}^{\ 3})-\text{H.c.}\Bigg], \end{aligned} \tag{7}$$

where $g'=g/\sqrt{2}$. The first term now contains the interaction of the usual Cabibbo current (consisting of charged hadron and lepton currents) with the nor-

[5] When $\alpha=0$, one may allow the $\Delta Y=-1$ hadron current $J_{\mu 3}^{\ 1}$ to interact with the $W_\mu^{(2)}$ boson field and the $\Delta Y=0$ hadron current $J_{\mu 1}^{\ 2}$ to interact with $W_\mu^{(3)}$; this "degenerate" case is considered below.

[6] $\Delta Y=2$ transition in order g^3 is forbidden by the small value of the $(K_L^0-K_S^0)$ mass difference (which is of order g^4).

[7] For a comprehensive discussion of the universal (V,A) current-current theory, see R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley, New York, 1969).

[8] Condition (4c) should actually be written in terms of the matrix elements, i.e., $\alpha\beta\langle f|J_{\mu 3}^{\ 2}J_{\mu 3}^{\ 3}|i\rangle=k\cos\theta\sin\theta\langle f|J_{\mu 1}^{\ 2}J_{\mu 3}^{\ 1}|i\rangle$, where i and f are the initial and final states, respectively. For the purposes of estimating the gross CP-violation effects, it suffices to work with the equality $\alpha\beta=k\cos\theta\sin\theta$.

[9] This was noted by L. Wolfenstein (private communication) and was pointed out by R. E. Marshak [in *Problems in Theoretical Physics*, essays in honor of N. N. Bogolubov (Soviet Academy Science, Moscow, 1969), p. 232] in connection with the strong *quadratic* IVB model.

[10] G. Segrè, Phys. Rev. **181**, 1996 (1969).

[11] The choice of α and β is dictated by experiment; e.g., interchanging the values of α and β would yield too small a branching ratio for $K_L^0\to 2\pi$.

malized combination $(W_\mu^{(2)}-\bar{W}_\mu^{(3)})/\sqrt{2}$. In the second term, the orthogonal combination of $W_\mu^{(2)}$ and $\bar{W}_\mu^{(3)}$ fields interacts with a purely charged hadron current while in the third term the neutral vector field $W_\mu^{(1)}$ interacts with a neutral hadron current which actually commutes with the "non-Cabibbo" charged hadron current in the second term.[12]

An even more convincing case for the form (7) of the semiweak interaction in the strong cubic IVB model has been pointed out by Bialynicki-Birula.[13] He notes that a quasi-Yang-Mills approach to the W Lagrangian suggests the following definition for the quasi-Yang-Mills fields $F_{\mu\nu}^{(a)}$:

$$F_{\mu\nu}^{(a)}(x)=\partial_\mu W_\nu^{(a)}(x)-\partial_\nu W_\mu^{(a)}(x) +if_0\epsilon_{abc}\bar{W}_\mu^{(b)}(x)\bar{W}_\nu^{(c)}(x). \quad (8)$$

It is easy to show that if the $W_\mu^{(a)}$ fields transform according to the triplet representation of SU_3, the same will be true of the $F_{\mu\nu}^{(a)}$ fields. The W Lagrangian (1) can then be rewritten in terms of the $F_{\mu\nu}^{(a)}$ fields as

$$\mathfrak{L}_0=-\tfrac{1}{2}\bar{F}_{\mu\nu}^{(a)}F_{\mu\nu}^{(a)}-m_0^2\bar{W}_\mu^{(a)}W_\mu^{(a)}, \quad (9)$$

where $\mathfrak{L}_0$ is manifestly invariant under SU_3. There is a subtle difference between the W boson Lagrangians (9) and (1), i.e., (9) contains an additional quartic interaction[14] among the W's; however, this quartic term leaves unchanged all the predictions of the semiweak interaction (7). If one now adds the electromagnetic field to $\mathfrak{L}_0$, one finds the interesting result that the combinations of $W^{(2)}$ and $\bar{W}^{(3)}$ used in Eq. (7) are precisely the ones which interact with the photon field. This places the semiweak interaction (7) on an attractive theoretical foundation.

While the coefficients in Eq. (7) differ from those used in I, the present model is basically the same as the one treated in that paper and the incisive test of the strong cubic IVB model remains the detection of gross CP-violation effects in weak processes. In view of the essential uniqueness of the semiweak interaction (7), it seems worthwhile to examine in greater detail several cases in which the interference between the strong cubic and electromagnetic contributions to the matrix element might result in gross CP violation. In an attempt to pinpoint the most promising experiments of this type, we examine the following weak processes in this paper:

$$K^+\to\pi^+l\bar{l}, \quad (10a)$$

$$K_L^0\to\pi^0l\bar{l}, \quad (10b)$$

$$\Sigma^+\to pe\bar{e}, \quad (10c)$$

$$\nu_\mu+N\to N+\mu+W. \quad (10d)$$

In connection with the decay (10b), we shall also comment briefly on the two-body decay mode $K_L^0\to\mu\bar{\mu}$ [forbidden in the SU_3 limit (see I)].

Before proceeding to an examination of the weak processes (10a)–(10d), we must mention the "degenerate" case $\alpha=0$, i.e., a strong cubic IVB model in which the $\Delta Y=-1$ neutral hadron current $J_{\mu3}{}^2$ is not allowed to interact with the $W_\mu^{(1)}$ field. In this case, the $W_\mu^{(2)}$ and $W_\mu^{(3)}$ fields may *both* interact with the Cabibbo current while still forbidding $\Delta Y=2$ transitions to order g^2 and g^3; this feat is achieved by dropping the second term as well as the $J_{\mu3}{}^2$ part ($\alpha=0$) of the third term in Eq. (7). Thus[15]

$$H_{\mathrm{s.w.}}=ig'\{[(W_\mu^{(2)}-\bar{W}_\mu^{(3)})/\sqrt{2}] \times(\cos\theta\, J_{\mu2}{}^1+\sin\theta\, J_{\mu3}{}^1+l_\mu) +\beta W_\mu^{(1)}J_{\mu3}{}^3-\mathrm{H.c.}\}, \quad (11)$$

where $\beta\sim1$ is fixed by the ratio of the $K_L^0\to2\pi$ to $K_S^0\to2\pi$ amplitudes. The strong cubic IVB model defined by (11) is, in order g^2, manifestly equivalent to the usual IVB model of a charged W boson interacting with the Cabibbo current except for $\Delta Y=0$ hadronic weak processes (e.g., the parity-violating part of the two-nucleon potential) which also receive contributions from the $J_{\mu3}{}^3J_{\mu3}{}^3$ term and thereby alter the isospin content of the $\Delta Y=0$ hadronic weak process. Equation (11) will also give rise to $CP=-1$ $\Delta Y=1$ hadronic weak processes in order g^3 (e.g., $K_L^0\to2\pi$) because of the presence of the $J_{\mu3}{}^3$ term; however, in contrast to Eq. (7), Eq. (11) leads to vanishing matrix elements in order g^3 for $\Delta Y=1$ semileptonic processes, e.g., for all the induced neutral-lepton-current decays (10a)–(10c) [because of the absence of the term $J_{\mu3}{}^2$ in the semiweak interaction (11)]. The model defined by (11) is therefore less interesting from the experimental point of view than that defined by (7). We shall comment further on the $\alpha=0$ model in later sections.

II. GROSS CP VIOLATION IN $\Delta Y=1$ HADRON DECAYS WITH NEUTRAL LEPTON PAIRS

Throughout this section, we shall be working with the strong cubic IVB model defined by the semiweak interaction (7); as already noted, the semiweak interaction (11) will not contribute to the g^3 matrix elements for the $\Delta Y=1$ induced neutral-lepton-current hadron decays treated in this section. We consider the following three cases: (1) $K^+\to\pi^+l\bar{l}$; (2) $K_L^0\to\pi^0l\bar{l}$; (3) $\Sigma^+\to pe\bar{e}$.

Case 1: $K^+\to\pi^+l\bar{l}$. The matrix element of this charged kaon decay mode receives a purely semiweak

[12] The more precise statement is that the two currents commute for equal Lorentz subscripts, i.e., $[\cos\theta\, J_{\mu2}{}^1-\sin\theta\, J_{\mu3}{}^1, \sin\theta\, J_{\mu3}{}^2+\cos\theta\, J_{\mu3}{}^3]=(1-\delta_{\mu\nu})\times$const.

[13] I. Bialynicki-Birula (private communication).

[14] The quartic interaction prevents an infinite lower bound to the energy implied by a cubic interaction alone [I. Bialynicki-Birula (private communication)], and is a further argument in favor of the quasi-Yang-Mills W Lagrangian (9) over (1). Since the quartic interaction contributes to the W boson self-energy, it will change the estimate of the coupling constant f_0. However, we do not repeat the weak-interaction calculations based on (9) because the qualitative conclusions remain unchanged.

[15] The semiweak interaction (11) recalls the Mohapatra model [State University of New York at Stony Brook report, 1969 (unpublished)] developed for another purpose; R. N. Mohapatra derived the form (11) using a gauge field transformation, an octet of W bosons, and a weak cubic self-interaction among the W's.

$CP=-1$ contribution in order g^3 and a combined semiweak plus electromagnetic $CP=+1$ contribution in order g^2e^2. The interference between these two contributions with opposite CP quantum number may result in a gross CP-violation effect which we proceed to estimate. The diagram for $K^+\to\pi^+l\bar{l}$ in order g^3 is shown in Fig. 1. The matrix element deduced from Fig. 1 is

$$M=\frac{g'^3\sin\theta}{2}\int\frac{d^4q}{(2\pi)^4}\Delta_{\mu\alpha}(p_1-p_2)\Delta_{\nu\beta}(k_1-q)\Delta_{\delta\gamma}(k_2+q)$$
$$\times\langle\pi^+(p_2)|J_{\mu3}{}^2|K^+(p_1)\rangle\bar{l}(k_2)$$
$$\times\{\gamma_\delta(1+\gamma_5)[(-i\gamma\cdot q)/q^2]\gamma_\nu(1+\gamma_5)\}l(k_1)\Gamma_{\alpha\beta\gamma}. \quad (12a)$$

In Eq. (12a), $\Delta_{\mu\nu}(q)$ is the propagator of the W boson,

$$\Delta_{\mu\nu}(q)=\frac{\delta_{\mu\nu}+q_\mu q_\nu/m_W{}^2}{q^2+m_W{}^2}, \quad (12b)$$

whereas $\Gamma_{\alpha\beta\gamma}$ is the vertex for the cubic interaction of the W boson. From symmetry considerations, we may write

$$\Gamma_{\alpha\beta\gamma}=f[\delta_{\alpha\gamma}(p_1-p_2+k_2+q)_\beta-\delta_{\alpha\beta}(p_1-p_2+k_1-q)_\gamma$$
$$+\delta_{\beta\gamma}(k_1-k_2-2q)_\alpha], \quad (12c)$$

where f is a form factor depending on the various four-momenta which is replaced by f_0 in the numerical estimates.

The matrix element given in Eq. (12a) is divergent, and from other calculations,[2] it seems reasonable to retain only the most divergent term. If we further neglect terms of order $m_K{}^2/m_N{}^2$, we obtain

$$M=\frac{3g'G\sin\theta}{16\pi^2\sqrt{2}}\left(\frac{\Lambda^2f_0}{m_W{}^2}\right)\bar{u}(k_2)$$
$$\times[-if_+\gamma\cdot(p_1+p_2)(1+\gamma_5)$$
$$-if_-\gamma\cdot(p_1-p_2)\gamma_5]v(k_1), \quad (13a)$$

where $G/\sqrt{2}=g'^2/m_W{}^2$ (recall that g is replaced by g'), Λ is the cutoff, and f_+ and f_- are defined by

$$\langle\pi^+(p_2)|J_{\mu3}{}^2|K^+(p_1)\rangle$$
$$=f_+(p_1+p_2)_\mu+f_-(p_1-p_2)_\mu. \quad (13b)$$

To obtain the final numerical estimate, we must know f_0, Λ, and m_W; fortunately,[3] the combination $f_0\Lambda^2/m_W{}^2\simeq2\pi$ appears in the calculation of the self-mass of W so

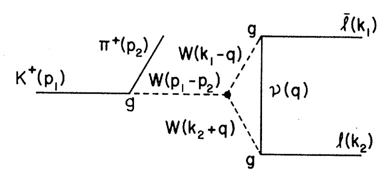

FIG. 1. g^3 contribution to $K^+\to\pi^+l\bar{l}$ decay.

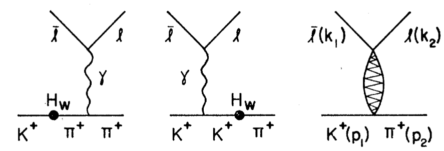

FIG. 2. g^2e^2 contributions to $K^+\to\pi^+l\bar{l}$ decay.

that Eq. (13a) depends only on g' (and therefore on m_W). Taking $m_W\sim5$ BeV as the "canonical value,"[1] we find for the g^3 ($CP=-1$) contribution to the $K^+\to\pi^+l\bar{l}$ decay rate (we distinguish between $K^+\to\pi^+e\bar{e}$ and $K^+\to\pi^+\mu\bar{\mu}$)

$$\frac{\Gamma(K^+\to\pi^+e\bar{e})}{\Gamma(K_{e3}{}^+)}\simeq3.1\times10^{-6} \quad (14a)$$

or[16]

$$\frac{\Gamma(K^+\to\pi^+e\bar{e})}{\Gamma(K^+\to\text{all})}\simeq1.5\times10^{-7}, \quad (14b)$$

and similarly

$$\frac{\Gamma(K^+\to\pi^+\mu\bar{\mu})}{\Gamma(K^+\to\text{all})}\simeq8.3\times10^{-8}. \quad (14c)$$

The g^2e^2 contribution to $K^+\to\pi^+l\bar{l}$ decay has been calculated by many authors.[17] The diagrams for this combined weak and electromagnetic contribution to $K^+\to\pi^+l\bar{l}$ decay are shown in Fig. 2.

The contributions of Figs. 2(a) and 2(b) cancel under the assumption that the pion and kaon electromagnetic form factors are equal; thus, the only diagram to consider is Fig. 2(c). The matrix element of Fig. 2(c) is

$$M=(G/\sqrt{2})\sin\theta\cos\theta\bar{u}(k_2)[\gamma\cdot(p_1+p_2)]v(k_1)A(q^2), \quad (15a)$$

where $q^2=(p_1-p_2)^2$ and $A(q^2)$ can be calculated by using Bég's[17] electromagnetic-induced neutral-current mechanism. The vector current (induced by one virtual photon) contributes to these processes; then $A(q^2)$ can be written as

$$A(q^2)=f_+(q^2)F_1{}^V(q^2), \quad (15b)$$

where $f_+(q^2)$ is the usual form factor of the $K\pi$ vertex and $F_1{}^V(q^2)$ is the induced form factor of the lepton current. $F_1{}^V(q^2)$ is evaluated by the use of dispersion relations and if only the 2π intermediate state is included, one finds

$$F_1{}^V(0)=e^2/4\gamma_{\rho\pi\pi}{}^2, \quad (15c)$$

where $\gamma_{\rho\pi\pi}{}^2$ is related to the width of the ρ meson

[16] The predicted g^3 branching ratio for $K^+\to\pi^+\nu\bar{\nu}$ is identical with that for $K^+\to\pi^+e\bar{e}$, namely, 1.5×10^{-7}. This level of accuracy should be achievable fairly soon [R. H. Hildebrand (private communication)]. The published upper limit is 1.2×10^{-6} [J. H. Klems, R. H. Hildebrand, and R. Stiening, Phys. Rev. Letters **24**, 1086 (1970)]. There will be, of course, no g^2e^2 contribution to $K^+\to\pi^+\nu\bar{\nu}$ decay.

[17] M. A. B. Bég, Phys. Rev. **132**, 426 (1963); cf. also S. Pakvasa and W. Simmons, Phys. Rev. **183**, 1215 (1969), and other references therein.

through the relation

$$\gamma_{\rho\pi\pi}^2/4\pi=3m_\rho^2\Gamma_\rho/(m_\rho^2-4m_\pi^2)^{3/2}. \quad (15d)$$

Insertion of Eqs. (15b)–(15d) into (15a) yields the g^2e^2 ($CP=+1$) contribution to the decay rates

$$\frac{\Gamma(K^+\to\pi^+e\bar{e})}{\Gamma(K_{e3}^+)}\simeq1.8\times10^{-5} \quad (16a)$$

or

$$\frac{\Gamma(K^+\to\pi^+e\bar{e})}{\Gamma(K^+\to\text{all})}\simeq8.5\times10^{-7}, \quad (16b)$$

and similarly

$$\frac{\Gamma(K^+\to\pi^+\mu\bar{\mu})}{\Gamma(K^+\to\text{all})}\simeq3.5\times10^{-7}. \quad (16c)$$

The g^2e^2 decay rates given in Eqs. (16b) and (16c) are comparable to (albeit somewhat larger than) the respective g^3 decay rates given in Eqs. (14b) and (14c) and we may therefore expect gross CP violation in both $K^+\to\pi^+e\bar{e}$ and $K^+\to\pi^+\mu\bar{\mu}$ decays. (It should be noted that experimental detection of CP violation may be less difficult with muons than with electrons despite the reduction in absolute rate.) We therefore look for interference terms in the total matrix element which are CP violating. The total matrix element for the decay is given by the sum of Eqs. (13a) and (15a), namely,

$$M=a\bar{u}(k_2)[-i\gamma\cdot(p_1+p_2)(C+\gamma_5) -i(f_-/f_+)\gamma\cdot(p_1-p_2)\gamma_5]v(k_1) \quad (17a)$$

with

$$C=1+iB, \quad (17b)$$

$$a=\frac{3g'G\sin\theta\, f_+}{8\pi\sqrt{2}},\qquad B=\frac{G\sin\theta\cos\theta\, f_+F_1{}^V(0)}{\sqrt{2}a}\simeq3.4. \quad (17c)$$

$B=0$ implies CP conservation whereas $B\sim1$ implies a large CP viloation. We note that the combined weak and electromagnetic contribution only appears as a correction to the scalar term in the matrix element (17a). [The comparable diagram for $K_L{}^0\to\pi^+l\bar{l}$ decay corrects only the pseudoscalar terms (see below).]

The CP-violating terms involve a correlation between the spins of the two leptons resulting from the $K^+\to\pi^+l\bar{l}$ decay, of the form $\hat{k}_1\cdot(\hat{\sigma}_1\times\hat{\sigma}_2)$, etc., where $\hat{\sigma}_1$ and $\hat{\sigma}_2$ are the lepton spins. The transition probability can be written in the more perspicuous form[18]

$$|M|^2\sim1+\alpha_T\hat{k}_1\cdot(\hat{\sigma}_1\times\hat{\sigma}_2)+\gamma_T(\hat{k}_1\cdot\hat{\sigma}_1)\hat{\sigma}_2\cdot(\hat{k}_1\times\hat{k}_2) +\rho_T(\hat{k}_1\cdot\hat{\sigma}_2)\hat{\sigma}_1\cdot(\hat{k}_1\times\hat{k}_2)+(\hat{k}_1\leftrightarrow\hat{k}_2,\,\hat{\sigma}_1\leftrightarrow\hat{\sigma}_2)\cdots. \quad (18)$$

[18] The situation is different for the neutral-kaon decay $K_L{}^0\to\pi^0l\bar{l}$, where one can expect different l and $\bar{l}$ energy spectra in the presence of CP violation (due to the two-photon rather than the one-photon vertex).

The nonvanishing of any of the six coefficients in Eq. (18) (α_T, γ_T, ρ_T, etc.) implies CP violation (assuming CPT invariance). In the kaon rest frame,

$$\alpha_T=-2B[(E_\pi+m_K+E_1)P\cdot k_2+E_2P\cdot k_1 -4m_l{}^2m_K(f_-/f_+)]|k_1|/I, \quad (19a)$$

$$\gamma_T=-2B\left[\frac{P\cdot(k_1-k_2)+2(m_K+E_\pi+E_1)m_K}{E_1}\right]\times\frac{|k_1|^2|k_2|}{I}, \quad (19b)$$

$$\rho_T=-4Bm_K|k_1|^2|k_2|/I, \quad (19c)$$

where

$$I=(2+B^2)(2P\cdot k_1P\cdot k_2-P^2k_1\cdot k_2)-B^2m_l{}^2P^2 -4m_l{}^2(f_-/f_+)^2(k_1\cdot k_2-m_l{}^2) -4m_l{}^2(f_-/f_+)P\cdot(k_1+k_2), \quad (19d)$$

with $P=p_1+p_2$ and E_1, E_2, k_1, k_2 the corresponding energies and momenta of the two leptons.

We have calculated the coefficients α_T, γ_T, and ρ_T as functions of E_1 and the results are given in Figs. 3(a)–3(c). In the region of leption kinetic energy where $d\Gamma/dE_l$ (the lepton energy spectrum of $K^+\to\pi^+l\bar{l}$) peaks, i.e., for $0.3T_{l\,\max}<T_l<0.8T_{l\,\max}$, α_T and γ_T are large (0.4–1.1 for electrons and 0.2–0.5 for muons) while ρ_T is probably too small to detect. The coefficient α_T is easier to measure experimentally, and for the more favorable case of muons the estimated value is ~0.5 at $T_l\simeq0.8T_{l\,\max}$.

It might be thought that a way to detect CP violation not requiring the difficult spin-correlation measurements would be to compare the energy spectra of $K^+\to\pi^+l\bar{l}$ and $K^-\to\pi^-l\bar{l}$. Unfortunately, these spectra remain identical in the presence of CP violation, as can easily be shown. The matrix element for $K^-\to\pi^-l\bar{l}$ is

$$M=-a\bar{u}(k_2)[-i\gamma\cdot(p_1+p_2)(C^*+\gamma_5) -i(f_-/f_+)\gamma\cdot(p_1-p_2)\gamma_5]v(k_1),$$

to be compared with the matrix element (17a) for $K^+\to\pi^+l\bar{l}$ decay. The difference in sign between the K^+ and K^- matrix elements and the change from $C=1+iB$ to $C^*=1-iB$ do not alter the energy spectrum[19] (which depends only on the magnitude of B).

Case 2: $K_L{}^0\to\pi^0l\bar{l}$. We next consider the neutral-kaon process $K_L{}^0\to\pi^0l\bar{l}$ which receives a g^3 ($CP=-1$) contribution essentially identical with that for the charged-kaon case. At first sight, it would seem advantageous to treat the lepton pair decay of the neutral kaon without π^0, namely, the decay $K_L{}^0\to\mu\bar{\mu}$ (the decay $K_L{}^0\to e\bar{e}$ has negligible probability because of the proportionality to $m_l{}^2$), but this decay vanishes in the SU_3 limit (see I). This is not true for $K_S{}^0\to\mu\bar{\mu}$ which,

[19] A. Pais and S. B. Treiman, Phys. Rev. 176, 1974 (1969).

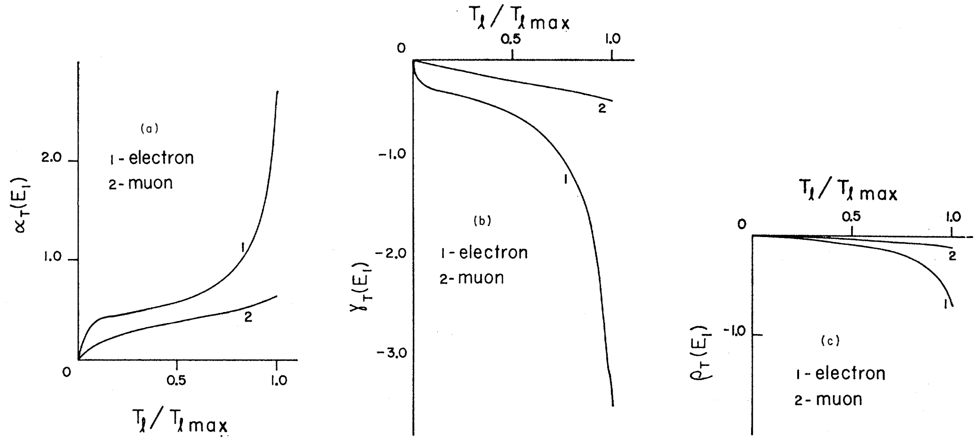

FIG. 3. (a) α_T coefficient for electrons and muons from $K^+ \to \pi^+ l\bar{l}$ decay. (b) γ_T coefficient for electrons and muons from $K^+ \to \pi^+ l\bar{l}$ decay. (c) ρ_T coefficient for electrons and muons from $K^+ \to \pi^+ l\bar{l}$ decay.

however, is much more difficult to measure. The decay $K_L{}^0 \to \mu\bar{\mu}$ should take place at the g^3 level in broken SU_3 symmetry. Since the manner of breaking SU_3 is unknown, it seems more useful to focus on the three-body decay $K_L{}^0 \to \pi^0 l\bar{l}$ which does not vanish in the limit of SU_3 symmetry (contrariwise, $K_S{}^0 \to \pi^0 l\bar{l}$ vanishes in this limit). After discussing $K_L{}^0 \to \pi^0 l\bar{l}$ decay, we shall nevertheless point out the experimental interest of $K_L{}^0 \to \mu\bar{\mu}$ decay.

We may write the g^3 contribution to $K_L{}^0 \to \pi^0 l\bar{l}$ decay in the form [cf. Eqs. (13a), (13b), and (17b)]

$$M = a\bar{u}(k_2)[-i\gamma\cdot(p_1+p_2)(1+\gamma_5) -i(f_-/f_+)\gamma\cdot(p_1-p_2)\gamma_5]v(k_1), \quad (20a)$$

where now f_- and f_+ are defined in terms of

$$\langle\pi^0(p_2)|J_{\mu 3}{}^2|K_L{}^0(p_1)\rangle = f_+(p_1+p_2)_\mu + f_-(p_1-p_2)_\mu. \quad (20b)$$

From Eq. (20a), we find for the g^3 branching ratios

$$\frac{\Gamma(K_L{}^0\to\pi^0 e\bar{e})}{\Gamma(K_L{}^0\to \text{all})} \simeq 1.1\times 10^{-6}, \quad (21a)$$

$$\frac{\Gamma(K_L{}^0\to\pi^0 \mu\bar{\mu})}{\Gamma(K_L{}^0\to \text{all})} \simeq 4.5\times 10^{-7}. \quad (21b)$$

We must next compute the combined weak and electromagnetic contribution to $K_L{}^0 \to \pi^0 l\bar{l}$ decay. In contrast to the charged-kaon case, this contribution is now of order g^2e^4 rather than g^2e^2; this follows from the vanishing of the one-photon vertex in the neutral-kaon case by virtue of charge-conjugation invariance. The two-photon contributions to $K_L{}^0 \to \pi^0 l\bar{l}$ decay are computed from the diagram shown in Fig. 4. This diagram may be considered to represent two types of contributions: (1) the induced neutral-current mechanism calculated by Bég,[17] which vanishes in the soft-pion limit, and (2) the soft-pion contribution which can be related to the matrix element for $K_S{}^0 \to l\bar{l}$ decay. In the Bég calculation, the matrix element for the g^2e^4 contribution to $K_L{}^0 \to \pi^0 l\bar{l}$ decay is approximated by writing it as a product of the matrix element of the effective hypercharge-changing vector neutral current (taken between the $K_L{}^0$ and π^0 states) and the matrix element of the induced hypercharge-conserving axial-vector neutral current (taken between vacuum and the lepton pair state), namely,

$$M = \langle\pi^0(p_2)|J_{\mu 3}{}^{2(V)}|K_L{}^0(p_1)\rangle\langle l(k_2)\bar{l}(k_1)|J_{\mu 3}{}^{3(A)}|0\rangle. \quad (22)$$

The first matrix element on the right-hand side of Eq. (22) is given by Eq. (20b) while the second is computed by postulating dominance by the one-pion intermediate state. Equation (22) becomes as a consequence

$$M = (G/\sqrt{2})\sin\theta\,\bar{u}(k_2)[F_1\gamma\cdot(p_1+p_2) - F_2\gamma\cdot(p_1-p_2)/2m_l]\gamma_5 v(k_1), \quad (23a)$$

where

$$F_1 = -f_+F_{\pi l\bar{l}}(-m_\pi{}^2)/2m_l \quad (23b)$$

and

$$F_2 = (m_K{}^2 - m_\pi{}^2)f_+F_3{}^A(q^2), \quad (23c)$$

with

$$F_3{}^A(q^2) = f_\pi F_{\pi l\bar{l}}(-m_\pi{}^2)/(q^2+m_\pi{}^2). \quad (23d)$$

In Eqs. (23b) and (23d), f_π is the pion decay amplitude and $F_{\pi l\bar{l}}(-m_\pi{}^2)$ is the pion-lepton pair vertex function evaluated on the pion mass shell. Numerical estimates will be given after we briefly consider the soft-pion contribution to the g^2e^4 matrix element.

The soft-pion calculation proceeds as follows: We have

$$\lim_{p_2\to 0}\langle\pi^0(p_2)\gamma(q_1)\gamma(q_2)|H_w|K_L{}^0(p_1)\rangle = (i/\sqrt{2}f_\pi)\langle 2\gamma|H_w|K_S{}^0\rangle. \quad (24)$$

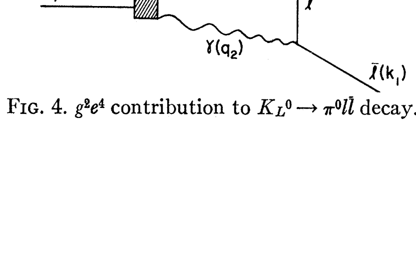

FIG. 4. g^2e^4 contribution to $K_L{}^0 \to \pi^0 l\bar{l}$ decay.

The $\langle 2\gamma|H_w|K_S^0\rangle$ matrix element may be written

$$\langle 2\gamma|H_w|K_S^0\rangle=(Ge^2/\sqrt{2})A(\epsilon_1\cdot\epsilon_2 q_1\cdot q_2-\epsilon_1\cdot q_2\epsilon_2\cdot q_1)\,, \quad (25)$$

where ϵ_1 and ϵ_2 are the polarizations of the two photons, and A is found from experiment. In the soft-pion limit, the matrix element for $K_L^0\to\pi^0\gamma\gamma$ assumes the same form:

$$\langle\gamma\gamma\pi^0|H_w|K_L^0\rangle=A'(\epsilon_1\cdot\epsilon_2 q_1\cdot q_2-\epsilon_1\cdot q_2\epsilon_2\cdot q_1)\,, \quad (26a)$$

but where, from Eq. (24), it follows that

$$A'=iGe^2A/2f_\pi\,. \quad (26b)$$

With the use of Eq. (26b), the leading term in the matrix element for $K_L^0\to\pi^0 l\bar{l}$ decay becomes

$$M=id\bar{u}(k_2)v(k_1)\,, \quad (27a)$$

where $d=-(3Ge^4Am_l/16\pi^2 f_\pi)\ln(\Lambda_1/m_K)$ and Λ_1 is a cutoff. From Eqs. (23a) and (27a), we compute the total g^2e^4 branching ratios for $K_L^0\to\pi^0 l\bar{l}$ decay:

$$\frac{\Gamma(K_L^0\to\pi^0 e\bar{e})}{\Gamma(K_L^0\to \text{all})}\simeq 3\times10^{-9}\,, \quad (27b)$$

$$\frac{\Gamma(K_L^0\to\pi^0 \mu\bar{\mu})}{\Gamma(K_L^0\to \text{all})}\simeq 2\times10^{-9}\,. \quad (27c)$$

We can now write down the total matrix element for $K_L^0\to\pi^0 l\bar{l}$ decay due to the g^3 contribution [Eq. (20a)] and both g^2e^4 contributions [Eqs. (23a) and (27a)]; we find

$$\begin{aligned}M=a\bar{u}(k_2)[&-i\gamma\cdot(p_1+p_2)(1+C'\gamma_5)\\ &+2m_l(f_-/f_+)(D+D'\gamma_5)]v(k_1)\,, \quad (28)\end{aligned}$$

where

$$C'=1+iB'\,, \quad (29a)$$

$$B'=\frac{G\sin\theta\cos\theta\, F_1}{2\sqrt{2}a}\simeq 0.03 \quad \text{(for electrons)}$$
$$\simeq 0.008 \quad \text{(for muons)}\,, \quad (29b)$$

$$D=\frac{idf_+}{2m_l f_- a}\simeq 0.4i \quad (\text{choosing } f_-\simeq -0.2)\,, \quad (30)$$

$$D'=1+iE'\,, \quad (31a)$$

$$E'=\frac{G\sin\theta\cos\theta\, f_+F_2}{4\sqrt{2}m_l a f_-}\simeq 1.0 \quad \text{(for electrons)}$$
$$\simeq 0.34 \quad \text{(for muons)}\,. \quad (31b)$$

It is seen from a comparison of Eqs. (28)–(31) with (17a) that the terms which are affected by the interference of the g^3 and g^2e^4 matrix elements for $K^0\to\pi^0 l\bar{l}$ decay are precisely the ones which are unchanged in the case of $K^+\to\pi^+ l\bar{l}$ decay. This has the consequence that at least one additional CP-violating term occurs in the transition probability for $K_L^0\to\pi^0 l\bar{l}$ decay, besides the six already present in the charged-kaon case. [cf. Eq. (18)—clearly, the coefficients α_T, γ_T, ρ_T, etc., are different functions[20] of B', D, and E'.] One of these additional terms is

$$|M|^2\sim\sigma_T(\hat{\sigma}_1-\hat{\sigma}_2)\cdot(\hat{k}_1\times\hat{k}_2)\,, \quad (32a)$$

where

$$\sigma_T=8m_K m_l(B'-E')|k_1||k_2|/I\,, \quad (32b)$$

with

$$\begin{aligned}I=(2+B'^2)(2P\cdot k_1 P\cdot k_2-P^2k_1\cdot k_2)&\\ -m_l^2P^2B'^2-4m_l^2[(|D|^2+1+E'^2)k_1\cdot k_2&\\ +(|D|^2-1-E'^2)m_l^2](f_-/f_+)^2&\\ -4m_l^2(1+B'E')(P\cdot k_1+P\cdot k_2)f_-/f_+\,.& \quad (32c)\end{aligned}$$

We have computed the most hopeful CP-violating coefficients for $K_L^0\to\pi^0 l\bar{l}$ decay, namely, α_T, γ_T, and σ_T. The first two are about 1% for muon kinetic energy, say, $T_\mu\sim 0.7T_{\mu\,\max}$, whereas σ_T is slightly larger. A search for σ_T is indicated but it is clear from these numbers that the prospect of detecting CP violation in $K^+\to\pi^+ l\bar{l}$ decay is more favorable.

The fact that the g^3 contribution to the $K_L^0\to\pi^0 l\bar{l}$ decay rate dominates the g^2e^4 contribution by a factor of 10^3 suggests taking another look at $K_L^0\to\mu\bar{\mu}$ decay. Since the latter decay can only take place in broken SU_3 symmetry (for the W bosons—see I), the g^3 contribution will be inhibited; this opens up the possibility of a larger CP-violation effect than in $K_L^0\to\pi^0\mu\bar{\mu}$ decay. If we allow for a 10% mass difference among the components of the W triplet (the same as for hadrons), we can expect CP violation in $K_L^0\to\mu\bar{\mu}$ decay to be of the order of 50% and its branching ratio to be decreased to about 5×10^{-9} [cf. Eq. (21b)]. Present measurements[21] place an upper limit of about 2×10^{-7} for the $K_L^0\to\mu\bar{\mu}$ branching ratio and another factor of 100 increase in intensity might provide us with a good test of CP violation in K_L^0 decay.

Case 3: $\Sigma^+\to pe\bar{e}$. The last decay we consider is $\Sigma^+\to pe\bar{e}$; this process, like the decay process $K^+\to\pi^+ l\bar{l}$, receives g^3 and g^2e^2 contributions. The g^3 contribution shown in Fig. 5 yields a matrix element whose most divergent part (neglecting terms of order m_N^2/m_W^2) can be written in the form

$$\begin{aligned}M=-ia\bar{u}(p_2)[(\gamma_\mu G_1-\sigma_{\mu\nu}q_\nu G_2)&\\ +(\gamma_\mu F_1+iq_\mu F_2)\gamma_5]u(p_1)&\\ \times\bar{u}(k_2)\gamma_\mu(1+\gamma_5)v(k_1)\,.& \quad (33a)\end{aligned}$$

The matrix element (33a) now possesses exactly the same structure as that for $\Sigma^-\to ne\bar{\nu}_e$ decay, namely,

$$\begin{aligned}M=i\sin\theta(G/\sqrt{2})\bar{u}(p_2)[(\gamma_\mu g_1-\sigma_{\mu\nu}q_\nu g_2)&\\ +(\gamma_\mu f_1+iq_\mu f_2)\gamma_5]u(p_1)&\\ \times\bar{u}(k_2)\gamma_\mu(1+\gamma_5)v(k_1)\,.& \quad (33b)\end{aligned}$$

The quantities F_i, G_i, f_i, and g_i in Eqs. (33a) and

[20] For details see Y. W. Yang, Ph.D. thesis, University of Rochester, 1970 (unpublished).

[21] H. Foeth *et al.*, Phys. Letters **30B**, 282 (1969).

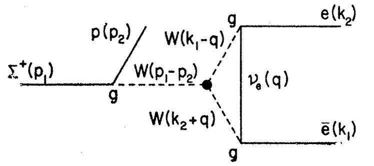

FIG. 5. g^3 contribution to $\Sigma^+ \to pe\bar{e}$ decay.

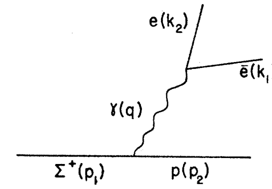

FIG. 6. g^2e^2 contribution to $\Sigma^+ \to pe\bar{e}$ decay.

(33b) are, of course, the form factors and are related through isospin invariance, i.e., $F_i/f_i = G_i/g_i = \sqrt{2}$. From Eqs. (33a) and (33b), it is easy to show that

$$\frac{\Gamma(\Sigma^+ \to pe\bar{e})}{\Gamma(\Sigma^- \to ne\bar{\nu}_e)} \simeq 3\times 10^{-6} \tag{34a}$$

or

$$\frac{\Gamma(\Sigma^+ \to pe\bar{e})}{\Gamma(\Sigma^+ \to \text{all})} \simeq 1.7\times 10^{-9}. \tag{34b}$$

The combined weak and electromagnetic contribution to $\Sigma^+ \to pe\bar{e}$ decay, of order g^2e^2, is given by Fig. 6. The matrix element can be calculated in standard fashion by expressing it in terms of the $\Sigma^+ \to p\gamma$ vertex, namely,

$$M = i\epsilon_\nu(q)\bar{u}(p_2)(C+D\gamma_5)\sigma_{\nu\mu}q_\mu u(p_1). \tag{35}$$

The matrix element for $\Sigma^+ \to pe\bar{e}$ is then

$$\begin{aligned} M &= e\bar{u}(p_2)(C+D\gamma_5)\sigma_{\nu\mu}q_\mu u(p_1)(1/q^2)\bar{u}(k_2)\gamma_\nu v(k_1) \\ &= e\bar{u}(p_2)[(m_\Sigma+m_N)C\gamma_\mu-(m_\Sigma-m_N)D\gamma_\mu\gamma_5 \\ &\qquad -i(p_1+p_2)_\mu(C+D\gamma_5)]u(p_1) \\ &\qquad\qquad \times(1/q^2)\bar{u}(k_2)\gamma_\mu v(k_1). \end{aligned} \tag{36}$$

Using Eq. (36) and the values of C and D given by Mohan[22] yields a g^2e^2 branching ratio

$$\frac{\Gamma(\Sigma^+ \to pe\bar{e})}{\Gamma(\Sigma^+ \to \text{all})} \simeq 2.8\times 10^{-6}. \tag{37}$$

We see that the g^2e^2 contribution to the $\Sigma^+ \to pe\bar{e}$ decay rate is about 10^3 times the g^3 contribution [cf. Eq. (34b)] and hence we can anticipate a CP-violation effect of the order of several percent. In fact, the total matrix element [i.e., the combination of Eqs. (33a) and (36)] is

$$\begin{aligned} M = -ia\{&\bar{u}(p_2)[(G_1+(m_\Sigma+m_N)G_2 \\ &+i(m_\Sigma+m_N)Ce/aq^2)\gamma_\mu+(F_1-i(m_\Sigma-m_N)De/aq^2) \\ &\times\gamma_\mu\gamma_5-i(p_1+p_2)_\mu(G_2+ieC/aq^2)+iq_\mu\gamma_5F_2 \\ &+(p_1+p_2)_\mu(De/aq^2)\gamma_5]u(p_1)\bar{u}(k_2)\gamma_\mu v(k_1) \\ &+\bar{u}(p_2)[\gamma_\mu G_1-\sigma_{\mu\nu}q_\nu G_2+\gamma_\mu\gamma_5F_1 \\ &+iq_\mu\gamma_5F_2]u(p_1)\bar{u}(k_2)\gamma_\mu\gamma_5v(k_1)\}. \end{aligned} \tag{38}$$

From Eq. (38) we can derive an expression for $|M|^2$ which exhibits the CP-violating terms, namely,

$$|M|^2 \sim 1+A_T\hat{\sigma}_\Sigma\cdot(\hat{k}_1\times\hat{k}_2)+\cdots, \tag{39}$$

where the most interesting coefficient A_T is a function of the parameters in Eq. (38). The measurement of the electron-positron correlation from polarized Σ^+ provides a method for detecting gross CP violation (since we accept the CPT theorem, T noninvariance is equivalent to CP violation) which is not available for kaon decay. The coefficient A_T is plotted as a function of the electron kinetic energy in Fig. 7. As expected, the maximum effect is about 5%.

III. CP VIOLATION IN W BOSON PRODUCTION

Thus far we have only considered induced neutral-lepton current $\Delta Y=1$ decay processes as a test of the strong cubic IVB model defined by Eq. (7); as remarked in Sec. I, the degenerate model defined in Eq. (11) cannot give rise to such $\Delta Y=1$ decays to order g^3. We now examine briefly the $\Delta Y=0$ W-boson production process in high-energy neutrino-nucleon collisions as a possible test of gross CP violation implicit in both formulations (7) and (11) of the strong cubic IVB model. The W-production reaction is

$$\nu_\mu+N \to \mu+N+W \tag{40}$$

and receives a contribution of first-order semiweak and second-order electromagnetic interactions except for the fact that this ge^2 matrix element carries the signature $CP=-1$ in the strong cubic IVB model. The contributing diagrams in order ge^2 are shown in Figs. 8(a) and 8(b). Because of the strong cubic self-interacting term, the W-production reaction can also receive a $CP=+1$ g^2 contribution from the pure semiweak interaction as shown in Fig. 9. By virtue of the opposite CP parity of the two contributions given by Figs. 8 and 9, the possibility of gross CP violation exists.

Let us try to estimate the relative magnitude of the g^2 and ge^2 contributions. The diagram in Fig. 9 yields

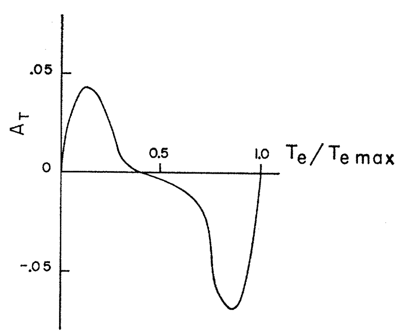

FIG. 7. A_T coefficient for $\Sigma^+ \to pe\bar{e}$ decay.

[22] L. R. Ram Mohan, Phys. Rev. **179**, 1561 (1969); R. H. Graham and S. Pakvasa, *ibid.* **140**, B1144 (1965).

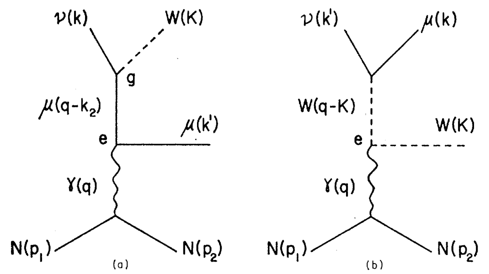

FIG. 8. (a) Electromagnetic contribution to W production (μ intermediate state). (b) Electromagnetic contribution to W production (W intermediate state).

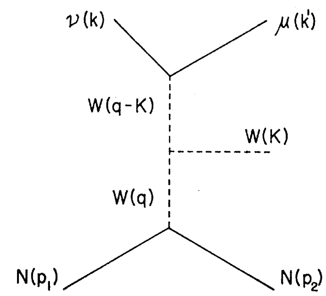

FIG. 9. Strong cubic IVB contribution to W production.

the matrix element

$$M=g'^2\cos\theta\,[\bar{u}(p_2)J_{\mu3}{}^3u(p_1)]\epsilon^\beta(K)\bar{u}(k')\gamma_\nu(1+\gamma_5)v(k)\times\Delta_{\mu\alpha}(q)\Delta_{\nu\gamma}(q-K)\Gamma_{\alpha\beta\gamma},\quad(41)$$

where $\epsilon^\beta(K)$ is the polarization vector of the W boson, $K\cdot\epsilon=0$, $\Delta_{\mu\alpha}$ and $\Delta_{\nu\gamma}$ are the propagators of W, and $\Gamma_{\alpha\beta\gamma}$ is the strong W vertex function defined by Eq. (12c). In writing down the g^2 matrix element (39), we have used the "nondegenerate" version of the strong cubic IVB model; the "degenerate" version gives essentially the same result since $\beta\sim1\sim\cos\theta$ [cf. Eq. (11)]. The hadron matrix element can be expressed in the form

$$\begin{aligned}\bar{u}(p_2)&J_{\mu3}{}^3u(p_1)\\&=\bar{u}(p_2)[A\gamma_\mu+iCP_\mu+B\gamma_\mu\gamma_5+iEq_\mu\gamma_5]u(p_1),\quad(42)\\&\quad P=p_1+p_2,\quad q=p_1-p_2\end{aligned}$$

where A, B, C, and E are related to the np form factors. Averaging over initial spins and summing over final spins, the square of the g^2 matrix element becomes

$$|M|^2=g^4\cos^2\theta\,M_{\mu\mu'}L_{\nu\nu'}A_{\mu\beta\nu}A_{\mu'\beta'\nu'}\times(\delta_{\beta\beta'}+K_\beta K_{\beta'}/m_W{}^2),\quad(43a)$$

where

$$A_{\mu\beta\nu}=T_{\alpha\beta\gamma}\Delta_{\mu\alpha}(q)\Delta_{\nu\gamma}(q-K),\quad(43b)$$

$$L_{\nu\nu}'=-(k_\nu k_{\nu'}'+k_{\nu'}k_\nu'-\delta_{\nu\nu'}kk')+k_\sigma k_\rho'\cdot\epsilon_{\nu'\sigma\nu\rho},\quad(43c)$$

$$\begin{aligned}M_{\mu\mu'}=&m_1m_2[(|A|^2-|B|^2)\delta_{\mu\mu'}\\&-|C|^2P_\mu P_{\mu'}+|E|^2q_\mu q_{\mu'}]+m_1[A^*CP_{2\mu}P_{\mu'}\\&+AC^*P_\mu P_{2\mu'}-E^*Bq_{\mu'}P_{2\mu}-EB^*q_\mu P_{2\mu'}]\\&+m_2[A^*CP_{1\mu}P_{\mu'}+AC^*P_\mu P_{1\mu'}\\&-E^*Bq_{\mu'}P_{1\mu}-EB^*q_\mu P_{1\mu'}]\\&-[(|A|^2+|B|^2)(P_{2\mu}P_{1\mu'}+P_{1\mu'}P_{2\mu}-P_1\cdot P_2\delta_{\mu\mu'})\\&-|C|^2P_1\cdot P_2P_\mu P_{\mu'}-|E|^2q_\mu q_{\mu'}P_1\cdot P_2\\&-A^*BP_{2\rho}P_{1\sigma}\epsilon_{\mu'\rho\mu\sigma}-B^*A\epsilon_{\mu'\rho\mu\sigma}P_{2\rho}P_{1\sigma}].\quad(43d)\end{aligned}$$

From Eq. (41) the total cross section may be computed in terms of the strong W coupling constant f_0 for a specified value of m_W; the results are shown in Table I for $m_W\sim5$ BeV as a function of incident neutrino energy (in the laboratory system). We have included values for $m_W=1.4$ BeV and $m_W=1$ BeV so that we can compare the g^2 with the ge^2 cross section calculated on the basis of the standard IVB model by Lee, Markstein, and Yang.[23] This comparison makes it clear that the ge^2 are substantially larger than the g^2 cross sections for this low-mass range. We must decide whether this large factor persists in the vicinity of $m_W\sim5$ BeV.

Let us recall that the minimal coupling between W and the electromagnetic field is given by

$$\begin{aligned}H_{\text{int}}{}^{W\gamma}=&ieA_\mu(x)\{\bar{W}_\nu(x)[(\partial_\mu-ieA_\mu(x))W_\nu(x)\\&-(\partial_\nu-ieA_\nu(x))W_\mu(x)]-W_\nu(x)[(\partial_\mu+ieA_\mu(x))\bar{W}_\nu(x)\\&-(\partial_\nu+ieA_\nu(x))\bar{W}_\mu(x)]\}.\quad(44)\end{aligned}$$

Using Eqs. (44) and (7), we obtain the matrix element for the diagrams in Fig. 8:

$$\begin{aligned}M=ige^2\epsilon^\beta(K)\Bigg\{&\bar{u}(k')\frac{\gamma\cdot q\gamma_\alpha+2k_\alpha'}{(k'-q)^2+m_\mu{}^2}\delta_{\lambda\beta}\gamma_\lambda(1+\gamma_5)v(k)\\&-\bar{u}(k')\gamma_\lambda(1+\gamma_5)v(k)\frac{1}{(K-q)^2+m_W{}^2}\\&\times\bigg[2K_\alpha\delta_{\beta\lambda}-q_\lambda\delta_{\alpha\beta}+q_\beta\delta_{\alpha\lambda}\\&-\frac{(K-q)_\lambda}{m_W{}^2}(K_\alpha q_\beta-q\cdot K\delta_{\alpha\beta})\bigg]\Bigg\}\\&\times(1/q^2)\langle p_2|j_\alpha{}^{\text{e.m.}}(0)|p_1\rangle.\quad(45)\end{aligned}$$

The calculations of LMY[23] are based on Eq. (45), but unfortunately do not extend beyond $m_W=1.4$ BeV. Since a calculation of the ge^2 contribution for $m_W\sim5$ BeV using (45) is very complicated, we have resorted to a dodge. We first compute the cross section (denoted by $\tilde{\sigma}$) corresponding to the ge^2 diagram of Fig. 8(b) (which is the analog of the g^2 diagram of Fig. 9) for the same values of m_W and the same ratio E_ν/E_{th} (E_{th} is the threshold neutrino energy for W production). The results are shown in Table II.

It is seen from Table II that[24] $\tilde{\sigma}\gg\sigma$ but that as long

[23] T. D. Lee. P. Markstein, and C. N. Yang, Phys. Rev. Letters 1, 429 (1961); A. C. T. Wu, C. P. Yang, R. Fuchel, and S. Heller, *ibid.* 12, 57 (1964).

[24] The large difference between $\tilde{\sigma}$ [due to the diagram in Fig. 8(b)] and σ_{LMY} [due to the diagrams in Fig. 8(b) *plus* 8(a)] is explained by the large cancellation between the two diagrams for small q^2 due to gauge invariance.

TABLE I. g^2 contribution to the W-production cross section.

m_W (BeV)	E_ν (BeV)	σ/f_0^2 (cm^2)
5	20	1.1×10^{-45}
	30	6.6×10^{-43}
	40	7.9×10^{-42}
	50	3.4×10^{-41}
	75	2.6×10^{-40}
	100	8.6×10^{-40}
1.4	4	1.5×10^{-42}
	5	5.65×10^{-42}
	6	1.01×10^{-41}
1	4	1.7×10^{-42}

as one chooses the same E_ν/E_{th} ratio, the ratio of cross sections $\tilde{\sigma}/\sigma_{\text{LMY}}$ stays approximately constant ($\sim$40). We therefore calculate $\tilde{\sigma}$ for $m_W \sim 5$ BeV and $E_\nu/E_{\text{th}} = 2.1$ and find $\tilde{\sigma} = 2.9 \times 10^{-36}$ cm^2; from this value of $\tilde{\sigma}$, we estimate that the $CP = -1$ ge^2 total cross section is about 7×10^{-38} cm^2 for $m_W \sim 5$ BeV and $E_\nu \sim 40$ BeV. The corresponding $CP = +1$ g^2 total cross section is about $8 \times 10^{-42} f_0^2$ cm^2, which is less than the ge^2 total cross section by the very substantial factor $9 \times 10^3 f_0^2$. This large ratio in the total cross sections can be reduced by taking advantage of the different q^2 behavior of the photon and W-boson propagators; thus the ratio of ge^2 to g^2 differential cross sections can be reduced by as much as a factor of 10 by observing muons from reaction (40) with kinetic energies in excess, say, of 50% of the maximum kinetic energy. Moreover, the factor f_0^{-2} in the ratio can easily introduce a further reduction of a factor 10, leading, under favorable circumstances to a CP-violation effect of the order of 10%. If the W-boson production process (40) actually takes place, CP violation can be detected by means of a measurement of the transverse polarization of the emergent muon.

IV. DISCUSSION

Two strong cubic IVB models based on a pure $CP = -1$ semiweak interaction have been formulated [the model defined by Eq. (7) will be referred to as IVB model A and the one defined by Eq. (11) as IVB model B] which, in order g^2, duplicate the essential features of the universal CP-conserving (V,A) current-current (c.c.) model while at the same time explaining, in order g^3, the small CP violation observed in $K_L{}^0$ decay. Some consequences of these two IVB models with respect to gross CP violation have been derived in this paper but, before summarizing our results in this regard, we should note the deviations in order g^2 between the usual c.c.

TABLE II. Total ge^2 cross section for W production using Fig. 8(b).

m_W (BeV)	E_ν/E_{th}	E_ν (BeV)	σ_{LMY} (cm^2)	$\tilde{\sigma}$ (cm^2)	$\tilde{\sigma}/\sigma_{\text{LMY}}$
0.6	2.1	2.0	4.2×10^{-38}	1.61×10^{-38}	38
1	2.1	3.5	3.1×10^{-38}	1.23×10^{-38}	40
1.4	2.1	5.5	3.0×10^{-38}	1.35×10^{-36}	45

TABLE III. Isospin contributions of different models to parity-violating two-nucleon potential.

Hadronic weak process	c.c. model	IVB Model A	IVB Model B
$\Delta Y = 0$, $\Delta I = 0, 2$	1	3	3
$\Delta Y = 0$, $\Delta I = 1$	1/16	3/16	1/16

model and IVB models A and B. All three models yield identical predictions in order g^2 for the purley leptonic and semileptonic processes (always assuming that $q^2/m_W{}^2 \ll 1$). However, there are deviations among the three models in connection with the more equivocal hadronic weak processes.

Thus, IVB model A automatically predicts the $\Delta I = 1$ rule in $\Delta Y = 1$ hadronic weak processes at the g^2 level, whereas IVB model B requires a dynamical origin for this $\Delta I = \frac{1}{2}$ rule (as does the usual c.c. model). For the $\Delta Y = 0$ hadronic weak processes, in particular, for the parity-violating (but CP-conserving) two-nucleon potential, the three models make different predictions, at the g^2 level, which are given in Table III. In principle, it should be possible to measure[25] the absolute strengths of both the $\Delta I = 0, 2$ (dominated by ρ-meson exchange) and $\Delta I = 1$ (dominated by π-meson exchange) contributions to the parity-violating two-nucleon potential.

When we turn to the question of gross CP violation, the two IVB models are indistinguishable with regard to the magnitude of the effect predicted in $\Delta Y = 0$ W production in neutrino-nucleon collisions (which is an optimistic 10% effect). However, $\Delta Y = 1$ W production in neutrino-nucleon collisions (e.g., the reaction $\nu_\mu + N \to \mu + \Lambda + W$) and the $\Delta Y = 1$ induced neutral-lepton-current decays of hadrons treated in Sec. II are an entirely different matter. The absence of the $J_{\mu 3}{}^2$ hadron current in the semiweak interaction defining IVB model B forbids these two classes of processes in order g^2 and g^3, respectively. This is clearly not the case for IVB model A, which predicts nonvanishing matrix elements in the lowest permissible orders for all $\Delta Y = 1$ weak processes. We have not gone into detail concerning the $\Delta Y = 1$ W-production processes, but have examined in some detail the $\Delta Y = 1$ decay processes.

Our study of the $\Delta Y = 1$ decays has focused on the four decays $K^+ \to \pi^+ l\bar{l}$, $K_L{}^0 \to \mu\bar{\mu}$, $K_L{}^0 \to \pi^0 l\bar{l}$, and $\Sigma^+ \to pe\bar{e}$. We list in Table IV the ratio of $CP = -1$ to $CP = +1$ matrix elements (as a measure of CP violation) for the four decays (lumping together the electron and muon pair decay modes for the first and third decays). In each case, the $CP = -1$ contribution is of order g^3, whereas the $CP = +1$ contribution is either of order g^2e^2 or g^2e^4, as indicated in the second column of Table IV. In the last column we have also listed the branching ratio due to the dominant matrix element. If we recall that $g \sim e^4$ (assuming $m_W \sim 5$ BeV), then the numbers in

[25] Cf. F. C. Michel, Phys. Rev. **133**, B239 (1964); B. H. J. McKellar, Phys. Rev. Letters **20**, 1542 (1968); P. Olesen and J. S. Rao, Phys. Letters **29B**, 233 (1969).

TABLE IV. Summary of predictions of IVB Model A for $\Delta Y=1$ hadron decays emitting lepton pairs.

$\Delta Y=1$ decay process	Order of semiweak plus electromagnetic matrix element	Predicted ratio of $CP=-1$ to $CP=+1$ matrix elements	Predicted branching ratio due to dominant matrix element
$K^+\to\pi^+l\bar{l}$	g^2e^2	0.3	0.5 to 8.5×10^{-7}
$K_L{}^0\to\mu\bar{\mu}$	g^2e^4	~2	$\sim5\times10^{-9}$
$K_L{}^0\to\pi^0l\bar{l}$	g^2e^4	30	0.5 to 1.1×10^{-6}
$\Sigma^+\to pe\bar{e}$	g^2e^2	0.03	3×10^{-9}

the third column of Table IV require some explanation. The only decay in Table IV which is explained in a straightforward fashion is $\Sigma^+\to pe\bar{e}$ decay, where we expect the ratio of the g^3 to g^2e^2 matrix elements to be of order $g/e^2\sim\frac{1}{10}$ and this is confirmed. On the other hand, the estimated ratio of the two matrix elements for $K^+\to\pi^+l\bar{l}$ decay is larger by a factor of 10, an effect which follows from the relative suppression of the $K^+\pi^+\gamma$ vertex with respect to the $\Sigma^+p\gamma$ vertex (the former vertex vanishes for a real photon, whereas the latter does not). The ratio of the matrix elements for $K_L{}^0\to\pi^0ll$ is also "abnormal," since one expects $g^3/g^2e^4\sim1$. The enhancement of the g^3 with respect to the g^2e^4 matrix element finds its explanation in the fact that the $CP=-1$ and $+1$ matrix elements are associated with the ${}^1S_0$ and ${}^3P_0$ states, respectively. The explanation of the number for $K_L{}^0\to\mu\bar{\mu}$ decay has already been given in terms of a hypothesized amount ($\sim10\%$) of broken SU_3 symmetry.

Quite apart from explaining the numbers listed in Table IV, the most favorable candidates for the detection of gross CP violation appear to be the first two decays. The last two decays seem equally unfavorable, except that to measure electron-pair correlation from polarized Σ^+ decay may be easier than to measure muon-pair spin correlation from $K_L{}^0\to\pi^0l\bar{l}$ decay. As a practical test of the strong cubic IVB model A, it is suggested that, first, the existence of the induced neutral-lepton-current decay $K^+\to\pi^+\nu\bar{\nu}$ must be established at the 10^{-8}–10^{-7} level, i.e., at the g^3 rather than the g^4 level in the matrix element, in order to justify the laborious search for gross CP violation in $K^+\to\pi^+l\bar{l}$ or $K_L{}^0\to\mu\bar{\mu}$ decay. Failure to observe any of the $\Delta Y=1$ induced neutral-lepton-current decay of hadrons in order g^3 would eliminate IVB model A but not B. The latter could then be tested by searching for gross CP violation in direct W production by neutrinos on nucleons. The observation of gross CP violation either in accordance with the predictions of IVB model A or B, would serve as a striking confirmation of the idea that CP violation is not a small accident, but must be incorporated into the basic structure of weak-interaction theory at the semiweak level through the mechanism of a set of strong self-interacting intermediate vector bosons (the "fifth force" in nature).

ACKNOWLEDGMENTS

This work was supported in part by the U. S. Atomic Energy Commission. We acknowledge valuable discussions with Professor S. Okubo, Professor I. Bialynicki-Birula, and Professor N. Christ. We also thank T. C. Yang for checking several points in the paper.

PHYSICAL REVIEW D VOLUME 3, NUMBER 7 1 APRIL 1971

Field-Theoretic Model for Low-Energy $J=\frac{3}{2}^+$ K^+p Scattering

L. B. RÉDEI
Department of Theoretical Physics, University of Umeå, Umeå, Sweden
(Received 13 May 1970)

Assuming broken SU_3 Yakawa coupling between the octet baryons and pseudoscalar octet mesons, we discuss low-energy K^+p scattering. We obtain a broad ($\Gamma=1000$ MeV) resonance in the $J=\frac{3}{2}^+$ channel with mass 1750 MeV. The phase shift does not reach $\pi/2$ in the resonance region. The calculated cross section is in qualitative agreement with the experimental data near the resonance energy.

I. INTRODUCTION

In two recent papers[1,2] (subsequently referred to as I and II), we have treated low-energy elastic π^+p and $\bar{K}\Xi$ scattering, assuming Yukawa coupling between the octet baryons and pseudoscalar octet mesons and using the SU_3 values for the coupling constants[3] with the phenomenologically determined value[4] $\alpha=0.75$ for the F/D mixing parameter. We used the physical values for the masses. Virtual baryon-antibaryon pair-creation processes were neglected. An approximate expression for the off-energy-shell T matrix was obtained by summing to infinite order those terms in the perturbation expansion which did not contain more than two

[1] L. B. Rédei, Nucl. Phys. **B10**, 419 (1969).

[2] L. B. Rédei, Nucl. Phys. **B17**, 38 (1970).

[3] See, e.g., S. G. Gasiorowicz, *Elementary Particle Physics* (Wiley, New York, 1966).

[4] G. Ebel *et al.*, in *Proceedings of the Lund Conference on Elementary Particles*, edited by G. von Dardel (Berlingska Boktryckeriet, Lund, Sweden, 1969).