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Reaction $\pi^- p \to \eta n$ with Veneziano Terms

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The Veneziano amplitude for $\pi^- p \to \eta n$ scattering is constructed using suitable combinations of beta-function terms. If certain constraints for the pole structure are imposed, the amplitude shows agreement and consistency with the experimental data over the whole energy region. The shape of the high-energy $\pi^- p \to \eta n$ differential cross section is reproduced. The decay widths for the $\frac{5}{2}^+$ and $\frac{5}{2}^-$ pion-nucleon resonances are obtained, and Adler's self-consistency condition is also approximately satisfied.

I. INTRODUCTION

OVER the past year it has become clear that the attractive simplicity of the Lovelace-Veneziano form of the meson-meson scattering amplitude is difficult to retain in the meson-nucleon scattering processes. For example, the simplest possible representation, written down by Greenberg,[1] for pion-nucleon scattering leads to manifestly false results. He writes the representation of the invariant amplitudes A and B in terms of only one beta function each, and determines the residue parameters from a comparison with the Born diagram and the Adler self-consistency condition. However, when he extrapolates the resulting amplitudes to the ρ-meson pole, he gets the completely unacceptable coupling-constant relationship

$$g_{\rho NN} g_{\rho\pi\pi} = 2 g_{\pi NN}^2 .$$

Clearly, such an oversimplification has no basis; but one might still consider, as Bose and Gupta[2] have done, the physically plausible assumption of degenerate N_α, N_γ, and Δ_δ trajectories. Then the width of the D_{13} resonance comes out to be

$$\Gamma_{N_\gamma} = -\frac{E^* - m}{m^*} \frac{2}{3} q^{*2} \alpha' \left(\frac{g_{\pi NN}^2}{4\pi} - 2 \frac{g_{\rho\pi\pi} g_{\rho NN}}{4\pi} \right),$$

which is negative. The conclusion must again be drawn that the representation used is much too simple.

Fenster and Wali[3] have therefore written down representations for the invariant amplitudes $A^\pm$ and $B^\pm$ which contain eight to twelve terms each. They then determine the 29 residue parameters by imposing various constraints, such as (a) correct residues at the nucleon, $\Delta(1236)$, and $N_\gamma(1518)$ poles, (b) no $T=\frac{1}{2}$ state on the Δ trajectory, and $T=\frac{3}{2}$ states on the N trajectories, (c) correct forward charge-exchange cross sections, (d) correct S-wave scattering lengths, etc. With the residue parameters so determined, they are able to fit the t dependence of the forward charge-exchange differential cross section reasonably well.

In the present work, we have constructed a model for the process[4] $\pi N \to \eta N$ following the method of Fenster and Wali. This process, which is mediated by the exchange of only A_2 mesons in the t channel, has been considered by Logan and Sertorio[5] in the interference model (Regge poles+resonances), and by Miyamura[6] in the Veneziano model. Our work differs from that of Miyamura in at least two important respects. First, our representation for the invariant amplitudes contains many more terms than considered by Miyamura. Our enlarged representation, however, gives us the freedom to impose constraints in the manner of Fenster and Wali, ensuring thereby the presence of only physical particles on at least the parent trajectories. Secondly we have used, unlike Logan and Sertorio and Miyamura, an A_2 trajectory completely degenerate with the ρ-meson trajectory. This $\rho - A_2$ degeneracy is now fairly well established[7] and in any case it is a consequence of the application of Lovelace-Veneziano ansatz for meson-meson scattering.

The results of our calculation with this model for the differential cross section at various energies are very encouraging. In Sec. III the constants are evaluated, and then in Sec. IV we give a brief discussion of our results.

II. SCATTERING AMPLITUDE

As in pion-nucleon scattering, we write the scattering amplitude for our process in terms of the invariant amplitudes A and B as follows:

$$T = (m/4\pi W) \bar{u}(p_2)(-A + \gamma \cdot Q B) u(p_1),$$

where $Q = \frac{1}{2}(q_1 + q_2)$, and where p_1, p_2, q_1, and q_2 denote the four-momenta of the initial and the final nucleon, the initial pion, and the final η meson, respectively. These amplitudes satisfy the s-u crossing property

$$A(s,t,u) = A(u,t,s), \quad B(s,t,u) = -B(u,t,s),$$

where s, t, and u are the usual Mandelstam variables for the process. The asymptotic behavior of these ampli-

[1] David F. Greenberg (unpublished).
[2] S. K. Bose and K. C. Gupta, Phys. Rev. **184**, 1572 (1969).
[3] S. Fenster and K. C. Wali, Phys. Rev. D **1**, 1409 (1970). Earlier references for the Veneziano-model calculations may be traced from here.
[4] Recently, M. L. Blackmon and K. C. Wali have utilized Miyamura's model to calculate both forward and backward differential cross sections for the reaction $\pi^- p \to \eta n$ [Phys. Rev. D **2**, 258 (1970)].
[5] P. K. Logan and L. Sertorio, Nuovo Cimento **52A**, 1022 (1967).
[6] O. Miyamura, Progr. Theoret. Phys. (Kyoto) **42**, 305 (1969).
[7] M. Jacob, Acta Phys. Austriaca, Suppl. No. 6 (1969).

tudes is known to be

$$A(s,t)\to s^{\alpha_M(t)},\quad B(s,t)\to s^{\alpha_M(t)-1},$$

for s large and t fixed and

$$A(s,t)\to s^{\alpha_B(u)-\frac{1}{2}},\quad B(s,t)\to s^{\alpha_B(u)-\frac{1}{2}},$$

for s large and u fixed. Here $\alpha_M(t)$ and $\alpha_B(u)$ denote the meson and baryon trajectories exchanged in the t and u channels, respectively.

With these properties, one can write down a Veneziano representation for the invariant amplitudes.

Our invariant amplitudes can be written as

$$\begin{aligned}(1/4\pi)A=&\mu_1C_N{}^+(1,\tfrac{3}{2})+\mu_2C_{N_\gamma}{}^+(1,\tfrac{3}{2})+\mu_3C_N{}^+(2,\tfrac{3}{2})\\&+\mu_4C_{N_\gamma}{}^+(2,\tfrac{3}{2})+\lambda_1C_{NN_\gamma}{}^+(\tfrac{3}{2},\tfrac{3}{2})+\lambda_2C_N(\tfrac{3}{2},\tfrac{3}{2})\\&+\lambda_3C_{N_\gamma}(\tfrac{3}{2},\tfrac{3}{2})+\lambda_4B_{N_\gamma}(\tfrac{1}{2},\tfrac{1}{2})+\lambda_5B_{N_\gamma N}{}^+(\tfrac{1}{2},\tfrac{3}{2}),\\(1/4\pi)B=&\beta_1B_N{}^-(1,\tfrac{1}{2})+\beta_2B_{N_\gamma}{}^-(1,\tfrac{1}{2})+\beta_3C_N{}^-(2,\tfrac{3}{2})\\&+\beta_4C_{N_\gamma}{}^-(2,\tfrac{3}{2})+\beta_5B_N{}^-(2,\tfrac{1}{2})+\beta_6B_{N_\gamma}{}^-(2,\tfrac{1}{2})\\&+\phi_1B_{NN_\lambda}{}^-(\tfrac{1}{2},\tfrac{1}{2})+\phi_2C_{NN_\lambda}{}^-(\tfrac{3}{2},\tfrac{3}{2})\\&+\phi_3B_N{}^-(\tfrac{1}{2},\tfrac{3}{2})+\phi_4B_{N_\gamma}{}^-(\tfrac{1}{2},\tfrac{3}{2}),\end{aligned}$$

where

$$B_x{}^\pm(\tfrac{1}{2}m,n)=\frac{\Gamma(\frac{1}{2}m-\alpha_x(s))\Gamma(n-\alpha(t))}{\Gamma(\frac{1}{2}m+n-\alpha_x(s)-\alpha(t))}\pm(s\to u),$$

$$B_{xy}{}^\pm(\tfrac{1}{2}m,\tfrac{1}{2}n)=\frac{\Gamma(\frac{1}{2}m-\alpha_x(s))\Gamma(\frac{1}{2}n-\alpha_y(u))}{\Gamma(\frac{1}{2}(m+n)-\alpha_x(s)-\alpha_y(u))}\pm(s\to u),$$

$$B_x(\tfrac{1}{2}m,\tfrac{1}{2}m)=\frac{\Gamma(\frac{1}{2}m-\alpha_x(s))\Gamma(\frac{1}{2}m-\alpha_x(u))}{\Gamma(m-\alpha_x(s)-\alpha_y(u))}.$$

Here x, y denote the fermion trajectories N, N_γ. The C functions $C(x,y)=\Gamma(x)\Gamma(y)/\Gamma(x+y-1)$ are defined in a similar fashion and $\beta_i(\mu i)$ denote the multiplicative constants for (s,t) terms in the B (A) amplitude and ϕ_i (λ_i) for (s,u) terms in the B (A) amplitude.

The asymptotic behavior for the leading trajectories, for t fixed and $s\to\infty$, is

$$\begin{aligned}(1/4\pi)A(s,t)&\to\Gamma(1-\alpha(t)_y)\xi^+s^{\alpha(t)}(\mu_1+\mu_2),\\(1/4\pi)B(s,t)&\to\Gamma(1-\alpha(t))\xi^+s^{\alpha(t)-1}\\&\quad\times\{-\beta_1-\beta_2-[1-\alpha(t)](\beta_3+\beta_4)\},\end{aligned}$$

where

$$\xi^+=1+e^{i\pi\alpha(t)}.$$

The formula for the differential cross section in terms of the Pauli amplitudes f_1 and f_2 is well known. We have

$$f_{1,2}=\frac{[(E_1\pm m)(E_2\pm m)]^{1/2}}{8\pi W}[\pm A+(W\mp m)B],$$

where m is the nucleon mass, $W=\sqrt{s}$, and E_1 and E_2 are, respectively, the initial and final energies of the nucleons in the c.m. system:

$$E_{1,2}=[W^2+m^2-m_\pi{}^2(m_\eta{}^2)]/2W.$$

Here m_π and m_η are the pion and η-meson masses. Then, for small t,

$$\frac{d\sigma}{d\Omega}=|f_1+f_2|^2+\frac{t}{4q_s{}^2}(|f_1+f_2|^2-|f_1-f_2|^2).$$

q_s is the c.m. momentum, and in the limit of large s for forward scattering we approximate

$$f_1+f_2\quad\text{by}\quad\frac{m}{4\pi W}\left(A+\frac{s}{2m}B\right),$$

$$f_1-f_2\quad\text{by}\quad(1/8\pi)A,$$

and $4q_s{}^2$ by s.

III. EVALUATION OF CONSTANTS AND RESULTS

The states $N(938)$, $N_R(1470)$, $N_\gamma(1518)$, and $N_{11}(1530)$ occur in the energy region of interest. $N(938)$ and $N_\gamma(1518)$ are taken to be lying on two different trajectories. Taking the universal slope ~1 GeV^{-2}, intercepts are

$$\alpha_{N_\alpha}(0)=-0.38,\quad\alpha_{N_\gamma}(0)=-0.83.$$

We take the t-channel intercept

$$\alpha_\rho(0)=\alpha_{A_2}(0)=0.41.$$

In order to determine the high-energy (forward-charge-exchange) differential cross section, the amplitudes A and B are to be determined. For this purpose we impose the conditions (1) correct residue for the nucleon pole, (2) correct behavior of the amplitude at the $N_\gamma(1518)$ pole, and (3) spin conditions on the leading trajectories.

(a) Absence of spin-$\frac{1}{2}$ states on the N_γ trajectory:

$$\lambda_4+\lambda_5=0,\quad\beta_2+\beta_6-\phi_1+\phi_4=0.$$

(b) Absence of spin-$\frac{3}{2}$ states on the N_α trajectory:

$$\begin{gathered}-\mu_1-\mu_3+\lambda_1+\lambda_2=0,\\-0.12\mu_1+0.82\mu_3+1.35\lambda_1+0.9\lambda_2+1.0\lambda_5=0,\\\beta_1-\beta_3+\beta_5-\phi_1+\phi_2-\phi_3=0,\\0.18\beta_1+0.82\beta_3-0.82\beta_5-0.35\phi_1+1.35\phi_2-1.9\phi_3=0.\end{gathered}$$

(c) Absence of spin-$\frac{5}{2}$ states on the N_γ trajectory:

$$\begin{gathered}-\mu_2-\mu_4-\lambda_1-\lambda_3+0.5\lambda_4+0.5\lambda_5=0,\\-1.36\mu_2+0.64\mu_4-0.8\lambda_1-1.7\lambda_3-0.15\lambda_4+0.4\lambda_5=0,\\-0.21\mu_2+0.15\mu_4+0.09\lambda_1-0.47\lambda_3-0.12\lambda_4-0.05\lambda_5=0,\\\beta_2-2\beta_4+\beta_6-\phi_1+2\phi_2+\phi_4=0,\\0.21\beta_2-0.3\beta_4-0.15\beta_6-0.11\phi_1-0.18\phi_2+1.17\phi_4=0,\\1.36\beta_2+1.28\beta_4-0.64\beta_6+1.2\phi_1-1.6\phi_2+3.7\phi_4=0.\end{gathered}$$

Five more equations may be set up by a comparison with the Feynman diagrams at the nucleon and N_γ poles. Writing the $N^*N\pi$ and $N^*N\eta$ vertices as

$$(g_{1,2}{}^*/m_\pi)U_\mu(P')iq_\mu\gamma_5u(P),$$

where $U_\mu(P')$ and $u(P)$ are the Dirac spinors for the spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ particles, and the spin-$\frac{3}{2}$ propagator in the form

$$\frac{-i}{P^2-M^2}\left[g^{\mu\nu}-\frac{2}{3}\frac{P^\mu P^\nu}{M^2}-\frac{1}{3}\gamma^\mu\gamma^\nu+\frac{1}{3}\frac{\gamma\cdot P}{M^2}(P^\mu\gamma^\nu-\gamma^\mu P^\nu)\right]\times(\gamma\cdot P+M),$$

we easily find the contributions of the N_γ resonance to the f_1 and f_2 amplitudes in the form

$$f_1=\frac{1}{4\pi}\left(\frac{g_1^*g_2^*}{m_\pi^2}\right)\frac{1}{3}\frac{qq'[(E_1^*-m)(E_2^*-m)]^{1/2}}{M^2-s},$$

$$f_2=\frac{1}{4\pi}\left(\frac{g_1^*g_2^*}{m_\pi^2}\right)\frac{qq'[(E_1^*-m)(E_2^*-m)]^{1/2}}{M^2-s}\cos\theta_s.$$

From the partial-wave amplitude f_{2-} at resonance, we then relate the coupling constants to the widths and obtain

$$\frac{1}{12\pi}\frac{g_1^*g_2^*}{m_\pi^2}\frac{(qq')^{3/2}}{N_-}=m^*(\Gamma_1\Gamma_2)^{1/2},$$

where

$$N_\pm=[(E_1^*\pm m)(E_2^*\pm m)]^{-1/2}$$

and Γ_1 and Γ_2 are the two partial widths for the processes $N^*\to N+\pi$ and $N^*\to N+\eta$.[8] In this way we obtain the following equations.

Nucleon pole[9]*:*

$$\beta_1+\beta_5+\phi_1+\phi_3=5.25.$$

N_γ pole:

$$-0.18\mu_2+0.82\mu_4+0.9\lambda_1+1.35\lambda_3-0.35\lambda_4+0.9\lambda_5=-0.04,$$

$$-\mu_2-\mu_4+\lambda_1+\lambda_3-\lambda_4-\lambda_5=-10.67,$$

$$-0.18\beta_2+0.82\beta_4-0.82\beta_6-0.1\phi_1-0.9\phi_2-2.4\phi_4=-0.02,$$

$$\beta_2-\beta_4+\beta_6+\phi_1-\phi_2-\phi_4=18.10.$$

We mention that the method of Fenster and Wali[3] does not involve an explicit evaluation of the Feynman diagrams but leads to the same results. Their method consists in expanding the invariant amplitudes at resonance in terms of the partial-wave amplitudes:

$$\frac{1}{4\pi}A\to\frac{1}{2W_Jqq'}\{-[W_-(W_J+m)+3W_+(W_J-m)\times\cos\theta_s]f_{2-}+\cdots\},$$

$$\frac{1}{4\pi}B\to\frac{1}{2W_Jqq'}\{[-W_-+3W_+\cos\theta_s]f_{2-}+\cdots\},$$

and in expressing the partial-wave amplitudes in terms of widths:

$$f_{l\pm}=\frac{e^{i\delta_{l\pm}}\sin\delta_{l\pm}}{(qq')^{1/2}}\to\frac{W_J\Gamma_{l\pm}}{(qq')^{1/2}(s_J-s)},$$

where

$$W_\pm=\{[(W_J\pm m)^2-m_\pi^2][(W_J\pm m)^2-m_\eta^2]\}^{1/2}$$

and W_J is the mass of the resonance considered. If one now writes the amplitudes in the form

$$A=\frac{a+c\cos\theta_s}{s_J-s},\quad B=\frac{b+d\cos\theta_s}{s_J-s},$$

one easily gets

$$\frac{a}{4\pi}=-\frac{1}{2(qq')^{3/2}}W_-(W_J+m)\Gamma_\gamma=-0.044,$$

$$\frac{c}{4\pi}=-\frac{3}{2(qq')^{3/2}}W_+(W_J-m)\Gamma_\gamma=-1.571,$$

$$\frac{b}{4\pi}=-\frac{1}{2(qq')^{3/2}}W_-\Gamma_\gamma=-0.018,$$

$$\frac{d}{4\pi}=\frac{3}{2(qq')^{3/2}}W_+\Gamma_\gamma=2.665.$$

We checked that both methods give the same results. Finally, solving our equations, we obtain the following values for our parameters:

$\mu_1=1.72\lambda_2+3.49$,	$\beta_1=2.72\phi_3+13.69$,
$\mu_2=7.52$,	$\beta_2=0.89$,
$\mu_3=-0.72\lambda_2-5.37$,	$\beta_3=-2\phi_3-6.02$,
$\mu_4=-2.18$,	$\beta_4=-4.47$,
$\lambda_1=-1.85$,	$\beta_5=-3.72\phi_3-11.84$,
$\lambda_2\approx-5.69$,	$\beta_6=3.69$,
$\lambda_3=-3.46$,	$\phi_1=3.40$,
$\lambda_4=-7.57$,	$\phi_2=-4.47$,
$\lambda_5=7.57$,	$\phi_3=-4.26$
	$\phi_4=-1.18$.

Using these values of the residue parameters, we evaluated the differential cross sections at high energies and compared them with the available experimental data.[10] It is seen from Fig. 1 that the agreement with experiment is quite satisfactory.

[8] $(\Gamma_1\Gamma_2)^{1/2}=0.00603$ GeV [from Particle Data Group, Rev. Mod. Phys. **42**, 87 (1970)].

[9] We take here $g_{\eta NN}^2/g_{\pi NN}^2\approx0.12$, which is in accordance with the more recent value of Coleman *et al.*, Phys. Letters **30B**, 659 (1969), where $g_{\eta NN}^2/g_{\pi NN}^2=0.18\pm0.06$.

[10] O. Guisan *et al.* [Phys. Letters **18**, 200 (1965)] and J. S. Danburg *et al.* [Phys. Letters **30B**, 270 (1969)] have recently reported new data in the process $\pi^+n\to\eta p$. They have also shown that a simple Veneziano-type model without satellite terms reproduces the experimental data.

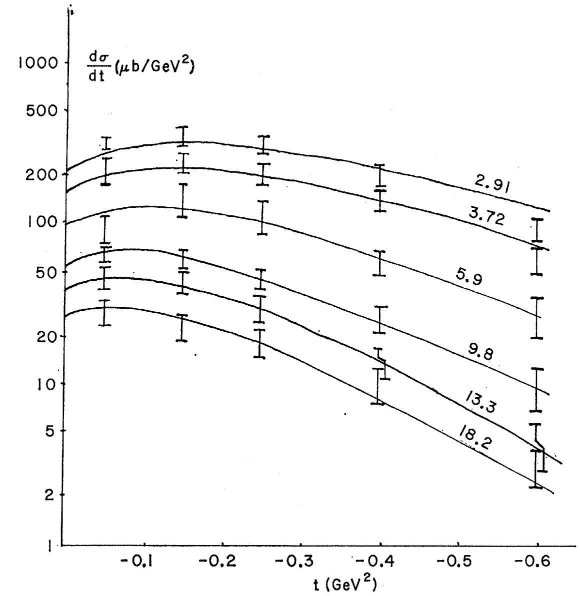

FIG. 1. Differential cross section for $\pi^- p \to \eta n$ in the small-t region. The solid curves are obtained from the Regge-pole limits of the formulas discussed in Sec. II. Experimental results are from Ref. 10. The labels on each curve specify the beam momentum in GeV/c.

IV. SOME INELASTIC WIDTHS ON N_α TRAJECTORY

In this section, we briefly mention the calculated $N\eta$ inelastic widths of some resonances on the N_α trajectory. Picking up the highest power of $\cos\theta_s$ in the expansion of the invariant amplitudes at $\alpha_N(S)=J$, we find

$$\frac{A}{4\pi} \to \frac{1}{(J-\frac{3}{2})!}\frac{(2qq'\alpha'\cos\theta_s)^{J-1/2}}{J-\alpha_N(s)} \times[-(\mu_1+\mu_3)+(-1)^{J+1/2}(\lambda_1+\lambda_2)]$$

and

$$\frac{B}{4\pi} \to \frac{(2qq'\alpha'\cos\theta_s)^{J-1/2}}{J-\alpha_N(s)}\left[\frac{\beta_1+\beta_5}{(J-\frac{1}{2})!}-\frac{\beta_3}{(J-\frac{3}{2})!} +(-1)^{J-1/2}\left(\frac{\phi_1+\phi_3}{(J-\frac{1}{2})!}-\frac{\phi_2}{(J-\frac{3}{2})!}\right)\right].$$

Making a partial-wave expansion, we then obtain, for $l=J-\frac{1}{2}$, the following expression for the partial widths:

$$(\Gamma_1^+\Gamma_2^+)^{1/2} = \frac{\Gamma(J+\frac{1}{2})(\sqrt{\pi})(qq'\alpha')^J[(E_1+m)(E_2+m)]^{1/2}d^+(m^*)}{\Gamma(J+1)4m^{*2}(\alpha')^{3/2}},$$

where

$$d_J^+(m^*)=\frac{1}{(J-\frac{3}{2})!}[-(\mu_1+\mu_3)+(-1)^{J+1/2}(\lambda_1+\lambda_2)] +(m^*-m)\left[\frac{\beta_1+\beta_5}{(J-\frac{1}{2})!}-\frac{\beta_3}{(J-\frac{3}{2})!} +(-1)^{J-1/2}\left(\frac{\phi_1+\phi_3}{(J-\frac{1}{2})!}-\frac{\phi_2}{(J-\frac{3}{2})!}\right)\right].$$

The widths of the parity doublets are given by

$$(\Gamma_1^-\Gamma_2^-)^{1/2} = \frac{\Gamma(J+\frac{1}{2})(\sqrt{\pi})(qq'\alpha')^J[(E_1-m)(E_2-m)]^{1/2}d^-(m^*)}{\Gamma(J+1)4m^{*2}(\alpha')^{3/2}},$$

where

$$d_J^-(m^*)=-d_J^+(-m^*).$$

Thus the calculated and observed $N\eta$ widths of $\frac{5}{2}^+$ and $\frac{5}{2}^-$ resonances are as follows:

$J=\frac{5}{2}$	Calculated (GeV)	Experimental[11] (GeV)
$(\Gamma_1^+\Gamma_2^+)^{1/2}$	0.01	<0.13
$(\Gamma_1^-\Gamma_2^-)^{1/2}$	0.005	<0.017

V. DISCUSSION

The approach utilized in this paper for constructing Veneziano model in meson-baryon scattering is due to Fenster and Wali but it was already advocated by Oehme,[12] when he suggested using a superposition of Veneziano terms and adjusting "the coefficients in such a way that all resulting residues are non-negative."

However, if we wish to make the residues of all the daughters non-negative, we might need an infinite number of terms. Clearly one has to make a choice in this approach to cut down the number of terms, and we have shown in this simple example of a η production in π-N interaction that such a choice can be made in close analogy with the pion-nucleon charge-exchange scattering calculation.

We have also calculated the $N\eta$ decay widths of the $\frac{5}{2}^+$ and $\frac{5}{2}^-$ pion-nucleon resonances and found reasonable agreement with experimental data.

Finally we mention that we have not been able to obtain good agreement with the Adler condition. Writing the Adler condition in the form[13]

$$A(s=u=m^2,\, t=m_\eta^2)=g_{NN\pi}g_{NN\eta}/m,$$

we find that the left-hand side is approximately 8 whereas the right-hand side is about 6. This sort of disagreement with the Adler condition has also been noted by Miyamura[6] and by Fenster and Wali.[3]

[11] Particle Data Group, Rev. Mod. Phys. **42**, 87 (1970).
[12] R. Oehme, Nuovo Comento Letters **1**, 420 (1969).
[13] S. L. Adler, Phys. Rev. **137**, B1022 (1965).