Isospin Structure of the Multiperipheral Model and Charge Distributions*

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Several specific assumptions on the isospin structure of the dominant exchange mechanisms are examined in the framework of the multiperipheral model. The consequent predictions for the charged and neutral particle distributions are compared with the present data.

I. INTRODUCTION

The multiperipheral model (MPM) provides a valuable scheme for the classification and the understanding of the multiparticle production processes (MPPP). The main ideas of multiperipheralism have survived ten years of research, and the general features of the MPPP are surprisingly well reproduced by simple multiperipheral parametrizations.[1] However, the detailed study of particular production processes has not yet provided compelling evidence for (or against) the dominance of the multiperipheral dynamics; the statistics are not rich enough, and the number of free parameters is usually large.

It looks more promising, for the time being, to concentrate our attention on the general features of the MPPP, i.e., on quantities that are hopefully independent of the details of the dynamics and, therefore, of the particular reactions involved.

An obvious quantity of this kind is the average number of particles produced in high-energy inelastic collisions, $\langle n(s)\rangle$. A recent experiment[2] has confirmed the logarithmic behavior predicted by the MPM. More information is contained in the charge distribution function $P(n_+,n_0,s)$ that gives the probability of having $2n_+$ charged tracks and n_0 neutral particles (in a reaction with total charge 0) as a function of the energy. The knowledge of $P(n_+,n_0,s)$ obviously provides more detailed information than is contained in $\langle n(s)\rangle$. Still, this quantity is an extremely averaged one in the sense that all the final-state kinematical variables have been integrated over. There is actually good evidence that the charge distributions are, to a good approximation, universal—i.e., the same function $P(n_+,n_0,s)$ describes all reactions, provided an obvious shift is made to relate reactions with initial charges 2, 1, 0, and -1.[3]

The general feature of the charged particle distribution is in good agreement with a Poisson-like distributions,[4] providing a further hint for the validity of the MPM. In fact, this kind of distribution is predicted for the production of n identical bosons in all simpler MPM. For instance, Chew and Pignotti[5] obtained

$$P(n,s)=(g^2\ln s)^n s^{-g^2}/n!\,, \tag{1}$$

where g is the coupling of the boson to the multiperipheral chain. In the physically relevant situation in which most of the produced particles are pions, (1) cannot possibly hold because of charge and isospin conservation.

Several modifications have been proposed to take into account this constraint, like producing pairs of charged particles with a Poisson distribution,[3] or assuming a Poisson distribution for the probability of positive and negative particle production and multiplying the two to obtain the joint probability for production of a pair.[6] In the framework of the multiperipheral model, the function $P(n_+,n_0,s)$ is uniquely determined by the isospin structure of the exchanges. It is the purpose of the present paper to examine the predictions that follow from the assumption of the dominance of several well-defined and physically reasonable mechanisms. In Sec. II we introduce the models that we are going to consider, and we give physical justifications for their relevance. In Sec. III we study the models analytically, and in Sec. IV we examine their phenomenological consequences. Section V is devoted to some concluding remarks.

II. MODELS—GENERAL FEATURES

In this section we want to outline briefly the general (very simple) dynamical features common to the models that we want to consider, and to introduce the specific isospin structure of the various models, giving some justification for their selection and some hints as to the particular problems for which they can be relevant.

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[4] J. W. Elbert *et al.*, Nucl. Phys. **B19**, 85 (1970).
[5] G. F. Chew and A. Pignotti, Phys. Rev. **176**, 2112 (1968).
[6] D. Horn and R. Silver, Phys. Rev. D **2**, 2092 (1970); Ann. Phys. (N. Y.) (to be published).

As stressed in the Introduction, we feel that the charge distributions in which we are interested should not depend critically on the dynamics, and therefore we will keep all dynamical features to a maximum level of simplicity. Namely, we will assume the following.

(a) The integrated cross section for the multiperipheral production of n particles in a definite order is given by a simple Chew-Pignotti form,

$$\sigma(n,s)=(g^2\ln s)^n f(s)/n!\,,$$

where g^2 is an appropriately defined coupling constant and $f(s)$ behaves like s^{-1} in most multiperipheral models. However, this function is irrelevant to the problem of charge distributions.

(b) The matrix element for the production of n particles in a given order is sizable only in the phase-space region in which the longitudinal momenta of the particles produced multiperipherally are ordered in increasing magnitude (in the laboratory frame). This allows us to add the various permutations of the final particles incoherently, neglecting interference terms.

(c) We will for simplicity assume that all the produced particles are pions (i.e., isospin 1).

The simplest model that we are going to consider (H model) assumes the multiple exchange of an $I=\frac{1}{2}$ object to be the dominant mechanism. Under this assumption it is straightforward to compute

$$P^H(n_+,n_0,s)=\frac{(g^2\ln s)^{2n_++n_0}}{(2n_+)!(n_0)!}\frac{(\frac{2}{3})^{2n_+}(\frac{1}{3})^{n_0}}{S^H(s)}\,, \qquad (2)$$

where we define the shadow function[7] $S^H(s)$ as

$$S^H(s)=\sum_{n_+,n_0}\frac{(\frac{1}{3}g^2\ln s)^{2n_++n_0}4^{n_+}}{(2n_+)!(n_0)!}\,. \qquad (3)$$

The factors $(\frac{1}{3})^{n_0}$ and $(\frac{2}{3})^{2n_+}$ are the squares of the Clebsch-Gordan coefficients for the $(1,0)\times(\frac{1}{2},\frac{1}{2})\to(\frac{1}{2},\frac{1}{2})$ and the $(1,1)\times(\frac{1}{2},-\frac{1}{2})\to(\frac{1}{2},\frac{1}{2})$ couplings, and a factor

$$\binom{2n_++n_0}{n_0}$$

is obtained on observing that the permutation of two oppositely charged particles leads to a configuration not allowed by $I=\frac{1}{2}$ exchange. The main interest of this model consists in the fact that the neutral particle production factorizes and therefore the model simulates an independent emission of the various charges. The charged particle distribution has the same structure[8] as one of those produced by Wang.[3]

[7] The shadow function $S(s)$ owes it name to the fact that it determines the asymptotic behavior of the imaginary part of the elastic amplitude through unitarity.

[8] We note, however, that in this model the average charge multiplicity $2\langle n^+\rangle$ can differ sizably from $\frac{2}{3}g^2\ln s$ at intermediate energies and, therefore, the distribution W^{II} of Ref. 3 and our (2) are not identical.

Even if this model has essentially only a formal interest, we can propose a physically relevant application in $p\bar{p}$ annihilation processes. From the data on the charge asymmetry[9] in $p\bar{p}\to\pi^+\pi^-$, we know that nucleon exchange dominates over Δ exchange at low energy, and the average subenergy of a pair of pions in $p\bar{p}$ annihilation should be generally very low,[10] as the total annihilation cross section behaves experimentally like s^{-1}. Also in πp and pp inelastic processes nucleon exchange has been successfully advocated[1] to parametrize the so-called central interactions. The pions emitted at the nucleon line in the model of Ref. 1 should follow the distribution (2).

The second model that we are going to consider (A model) was actually proposed by Chew and Pignotti.[5] The assumption that characterizes the A model is that the dominant multiperipheral mechanism is alternate exchange of $I=0$ and $I=1$ objects. Assuming for simplicity that the initial and final links have $I=0$ when the number of produced particles is even, the probability distribution is

$$P^A(n_+,n_0,s)=\frac{(g^2\ln s)^{2n_++n_0}}{(2n_++n_0)!}2^{n_+}\binom{n_++m}{m}\Big/S^A(s) \qquad (4)$$

with n_0 equal to $2m$ or $2m+1$, m integer.

The function $S^A(s)$ is defined in analogy with (3) by

$$S^A(s)=\sum_{n_+,n_0}\frac{(g^2\ln s)^{2n_++n_0}}{(2n_++n_0)!}2^{n_+}\binom{n_++m}{m}. \qquad (5)$$

Two physical justifications for this model can be proposed. In the framework of the AFS model,[11] this mechanism is relevant in the phase-space region which corresponds to large energies for the π-π cross sections, but we know that this phase-space region is actually quite small. On the other hand, the $I=0$ particle exchanged could be an ω or a P', and the $I=1$ could be a ρ or A_2 (or an elementary π). In the framework of the multi-Regge model the exchange of mesons (defined as Regge trajectories with intercept close to 0.5) is dominant, but it remains to be explained why $I=0$ and $I=1$ should be exactly alternate.[12] We can hopefully assume however that the predictions of models in which $I=1$ or $I=0$ exchanges can alternate in any (allowed) fashion, will be somehow intermediate between the A model and a third model (I model) in which $I=1$ exchanges dominate through the chain.

[9] L. Montanet, in *Proceedings of the Lund International Conference on Elementary Particles, 1969*, edited by G. von Dardel (Berlinska, Lund, Sweden, 1969), p. 201.

[10] In the general framework of the multiperipheral bootstrap it takes a large $p\bar{p}$ coupling (and a correspondently low $\pi\pi$ subenergy) to boost the low nucleon intercept to an effective $\alpha_{\text{out}}=0.5$. See P. Ting, Phys. Rev. **181**, 1942 (1969).

[11] D. Amati, S. Fubini, and A. Stanghellini, Nuovo Cimento **26**, 896 (1962). L. Bertocchi, S. Fubini, and M. Tonin, *ibid.* **25**, 626 (1962).

[12] This would be the case if the dominant exchanges were ω and ρ, or P' and A_2, but there is little evidence to justify either hypothesis.

In the I model, assuming for simplicity that the initial and final links have $I_z=0$, we obtain the distribution

$$P^I(n_+,n_0,s) = \frac{(g^2 \ln s)^{2n_++n_0}}{(2n_++n_0)!}(\tfrac{1}{2})^{n_++n_0}\binom{n_++n_0-1}{n_0}\Big/ S^I(s)\,, \quad (6)$$

and S^I is the shadow function for this model, defined in analogy with S^H and S^A.

In the last model (R model) the final pions are not directly produced by a multiperipheral mechanism but come from the decay of resonances for which a multiperipheral production mechanism is assumed. In this model there is a strong correlation between the neutral and charged particle production, and this seems in agreement with the present data.[4] To define this model more precisely, we assume that the dominant mechanism is the multiperipheral production of $I=0$ (σ,f_0) and $I=1$ (ρ) π-π resonances through multiple $I=1$ exchange. Therefore the R model corresponds to an extreme parametrization of the AFS model, in which the π-π cross section is assumed to be dominated by the s-channel production of $I=0$ and $I=1$ resonances. In view of the fact that the average π-π subenergy is of the order of 0.5 GeV^2, the model is not unreasonable. As pointed out in Ref. 13, the actual shape of the assumed resonance is not important, as all dynamical variables are integrated over. What really matters is a definite s-channel isospin character of the π-π cross section.

We obtain the charge distribution in two steps: We first find the probability distribution for the production of r $I=0$ resonances, $m_1\,\rho^+$ (and $m_1\,\rho^-$) and $m_2\,\rho_0$:

$$\tilde{P}(r,m_1,m_2,s) = \frac{(g_0^2 \ln s)^r}{r!} s^{-g_0^2} P^I(m_1,m_2,s)\,. \quad (7)$$

Here g_0^2 is the coupling constant for the production of the $I=0$ resonance, and g^2 (implicit in P^I) is the coupling for the production of an $I=1$ resonance; their relative value can be fixed by requiring that the π-π amplitude with $T=2$ in the crossed channel vanishes (as it does in the s channel). We obtain from this condition $g_0^2=\frac{1}{2}g^2$, but, as stressed in Ref. 13, the predictions of the model do not depend critically on this assumption. From (6) it is easy to obtain the pion distribution ($n_0=2m$ is necessarily even in this model):

$$P^R(n_+,n_0,s) = (\tfrac{1}{6}g^2 \ln s)^{n_++m} 2^{n_+} s^{-g^2/2} [S^I(s)]^{-1} \sum_{m_1=0}^{\min(n_+,m)} \sum_{m_2=0}^{n_+-m_1} \frac{(m_1+m_2-1)!\,3^{2m_1+m_2}2^{-m_2}}{(m_1-1)!(m_2)!(n_+-m_1-m_2)!(m-m_1)!(2m_1+m_2)!}\,. \quad (8)$$

III. ANALYTICAL STUDY OF MODELS

In this section we want to perform an analytical study of the distributions predicted by the various models in order to obtain some features (like the asymptotic behavior) of the phenomenologically relevant quantities.

The H model is very simple, and again it is convenient to start from it to establish the notations. The shadow function $S^H(s)$ can be explicitly computed[14] from (3),

$$S^H(s) = s^{g^2/3}[\cosh(\tfrac{2}{3}a)-1] \underset{s\to\infty}{\sim} \tfrac{1}{2}s^{g^2}, \quad (9)$$

where here and in the following, $a=g^2 \ln s$. The charged particle distribution is

$$C^H(n_+,s) = \sum_{n_0} P(n_+,n_0,s) = \frac{(\frac{2}{3}a)^{2n_+}}{(2n_+)![\cosh(\frac{2}{3}a)-1]} \quad (10)$$

and exhibits Poisson-like features. The average number of positive particles is

$$\langle n_+{}^H\rangle = \sum_{n_+,n_0} P(n_+,n_0,s)n_+ = \tfrac{1}{3}a\frac{\sinh(\frac{2}{3}a)}{\cosh(\frac{2}{3}a)-1} \underset{s\to\infty}{\sim} \tfrac{1}{3}a \quad (11)$$

and the average number of neutral particles is

$$\langle n_0{}^H\rangle = \sum_{n_+,n_0} P(n_+,n_0,s)n_0 \underset{s\to\infty}{\sim} \tfrac{1}{3}a\,. \quad (12)$$

The total average multiplicity behaves asymptotically as $g^2 \ln s$. The coefficient of the logarithmic increase (g^2) equals the power with which the shadow function increases, as usual in multiperipheralism.

Because the dependence on n_+ and n_0 of $P(n_+,n_0,s)$ factorizes, the average number of π^0 produced for a fixed number of charged particles n_+ does not depend on n_+:

$$A^H(n_+,s) = \sum_{n_0} n_0 P(n_+,n_0,s)\Big/\sum_{n_0} P(n_+,n_0,s) = \tfrac{1}{3}a\,. \quad (13)$$

In the A model the shadow function is given by

$$S^A(s) = \sum_{n,m} \frac{a^{2n+2m+1}}{(2n+2m+1)!} 2^n \binom{n+m}{n} \times\left(1+\frac{2n+2m+1}{a}\right), \quad (14)$$

where we separated the sum into two parts corresponding to an even and odd number of π^0, respectively. We note that the contribution from the terms with an even number of π^0 is the derivative with respect to a of the

[13] L. Caneschi and A. Schwimmer, Phys. Letters (to be published).

[14] We do not take into account events in which no charged particles are produced (i.e., the sum over n_+ starts from $n_+=1$).

contribution from the terms with odd n_{π^0}. Therefore, using the doubling formula for the Γ function, we can recast (14) in the form

$$S^A(s)=\pi^{1/2}\left(1+\frac{d}{da}\right)\sum_n\left(\frac{a^2}{2}\right)^{n+1/2}\times\frac{1}{n!}\sum_m\left(\frac{a}{2}\right)^{2m}\frac{1}{m!\Gamma(n+m+\frac{3}{2})}$$
$$=\pi^{1/2}\left(1+\frac{d}{da}\right)\left[\left(\frac{a}{2\sqrt{3}}\right)^{1/2}I_{1/2}(\sqrt{3}a)\right]$$
$$=\left(1+\frac{d}{da}\right)\left[\frac{1}{\sqrt{3}}\sinh(\sqrt{3}a)\right].\quad(15)$$

Therefore we obtain the simple expression

$$S^A(s)=\cosh(\sqrt{3}a)+(1/\sqrt{3})\sinh(\sqrt{3}a).$$

The function I in Eq. (15) is the Bessel function of imaginary argument; asymptotically the shadow function behaves like $[(\sqrt{3}+1)/2\sqrt{3}]S^{\sqrt{3}g^2}$. The distribution of charged pions, defined in analogy with (10), is given by

$$C^A(n_+,s)=\frac{1}{\sqrt{3}}\frac{a^n}{n_+!}\frac{I_{n_+-1/2}(a)+I_{n_++1/2}(a)}{I_{-1/2}(\sqrt{3}a)+(1/\sqrt{3})I_{1/2}(\sqrt{3}a)}.\quad(16)$$

We see that the "Poisson character" of the distribution is reproduced. The average number of charged (positive or negative) pions is

$$\langle n_+{}^A\rangle=\frac{a}{\sqrt{3}}\frac{\cosh(\sqrt{3}a)+\sqrt{3}(1-1/3a)\sinh(\sqrt{3}a)}{\sqrt{3}\cosh(\sqrt{3}a)+\sinh(\sqrt{3}a)},\quad(17)$$

which asymptotically behaves like $a/\sqrt{3}$. For the average number of neutral pions we get

$$\langle n_0{}^A\rangle=\frac{a}{\sqrt{3}}\frac{\cosh(\sqrt{3}a)+\sqrt{3}(1+2/3a)\sinh(\sqrt{3}a)}{\sqrt{3}\cosh(\sqrt{3}a)+\sinh(\sqrt{3}a)}\underset{a\to\infty}{\sim}\frac{a}{\sqrt{3}}.\quad(18)$$

Asymptotically, therefore, we have an equally increasing number of positive and neutral pions. For the correlation function between charged and neutral pions defined in analogy with (13) we get

$$A^A(n_+,s)=a\frac{(1+1/a)I_{n+1/2}(a)+I_{n+3/2}(a)}{I_{n-1/2}(a)+I_{n+1/2}(a)},\quad(19)$$

which for large a behaves like

$$A^A(n_+,s)\underset{a\sim\infty}{\longrightarrow}a-n_+,$$

monotonically decreasing as a function of n_+ in the region in which n_+ is much smaller than a. For large values of n_+ at fixed a, A levels off to the asymptotic value $\frac{1}{2}a$.

Let us now turn our attention to the I model. In this model the shadow function can be written as

$$S^I(s)=\sum_{n_+=1}^{\infty}\frac{(\frac{1}{2}a^2)^{n+}}{(2n_+)!}\sum_{n_0=0}^{\infty}\frac{(n_++n_0-1)!(2n_+)!(\frac{1}{2}a)^{n_0}}{(n_+-1)!(2n_++n_0)!n_0!}$$
$$=\sum_{n_+=1}^{\infty}\frac{(\frac{1}{2}a^2)^{n+}}{(2n_+)!}\Phi(n_+,2n_++1,\tfrac{1}{2}a).\quad(20)$$

Using the integral representation of the confluent hypergeometric function,

$$\Phi(c,d,x)=\frac{\Gamma(d)}{\Gamma(c)\Gamma(d-c)}\int_0^1 e^{xu}u^{c-1}(1-u)^{d-c-1}du,$$

we obtain the convenient integral representation of $S^I(s)$:

$$S^I(s)=\sum_{n=1}^{\infty}\frac{(\frac{1}{2}a^2)^n}{(n-1)!n!}\int_0^1 e^{au/2}u^{n-1}(1-u)^n du$$
$$=\frac{a}{\sqrt{2}}\int_0^1 e^{au/2}\left(\frac{1-u}{u}\right)^{1/2}I_1([2u(1-u)]^{1/2}a)du,\quad(21)$$

where I_1 is the Bessel function of imaginary argument. To obtain the asymptotic behavior of $S^I(s)$ when s (and therefore a) goes to infinity, we replace I_1 by the leading term of its asymptotic expansion

$$S^I(s)\underset{a\to\infty}{\sim}\left(\frac{a}{4\pi\sqrt{2}}\right)^{1/2}\int_0^1\exp\{\tfrac{1}{2}au+a[2u(1-u)]^{1/2}\}\times(1-u)^{3/4}u^{-1/4}du.\quad(22)$$

The asymptotic behavior of (22) can be evaluated by the standard saddle-point method:

$$S^I(s)\underset{s\to\infty}{\sim}\tfrac{1}{6}s^{g^2}.\quad(23)$$

The charged particle distribution in this model can be evaluated in a similar way:

$$C^I(n_+,s)=\sum_{n_0}P^I(n_+,n_0,s)=\frac{(\frac{1}{2}a^2)^{n+}}{(2n_+)!}\frac{\Phi(n_+,2n_++1,\frac{1}{2}a)}{S^I(s)}.\quad(24)$$

Using the asymptotic expansion of the confluent hypergeometric function, we obtain

$$C^I(n_+,s)\underset{s\to\infty}{\sim}6s^{-g^2}\frac{n_+}{n_++1}\frac{a^{n+}}{n_+!}.\quad(25)$$

We see that the Poisson-like features are present also in this model, at least asymptotically. Formula (25) cannot be used to compute the average number of charged particles because the asymptotic expansion of ϕ is valid only for $a \gg n_+$. However, we can calculate $\langle n_+^I(s)\rangle$ using a representation similar to Eq. (22)[15]:

$$\langle n_+^I(s)\rangle = \sum_{n_+=1}^{\infty} n_+ C^I(n_+,s) = \frac{a^2}{2S^I(s)} \int_0^1 e^{au/2}(1-u) \times I_0([2u(1-u)]^{1/2}a)du \underset{s\to\infty}{\sim} \tfrac{1}{3}a \quad (26)$$

and, in an analogous way,

$$\langle n_0^I(s)\rangle = \sum_{n_0,n_+} n_0 P^I(n_+,n_0,s) = \frac{a}{2S^I(a)} \sum_{n=1}^{\infty} \frac{n(\frac{1}{2}a^2)^n}{(2n+1)!} \times \Phi(n+1,\, 2n+2,\, \tfrac{1}{2}a)$$

$$= \frac{a^2}{2\sqrt{2}S^I(a)} \int_0^1 e^{au/2}[u(1-u)]^{1/2} \times I_1([2u(1-u)]^{1/2}a)du \underset{s\to\infty}{\sim} \tfrac{1}{3}a. \quad (27)$$

The average number of π^0 at fixed n_+ is given in this model by

$$A^I(n_+,s) = \sum_{n_0} n_0 P^I(n_+,n_0,s) / \sum_{n_0} P^I(n_+,n_0,s)$$

$$= a\frac{n_+}{n_+ + \frac{1}{2}} \frac{\Phi(n_+ +1,\, 2n_+ +2,\, \frac{1}{2}a)}{\Phi(n_+,\, 2n_+ +1,\, \frac{1}{2}a)}$$

$$= a\frac{d}{da}[\ln\Phi(n_+,\, 2n_+ +1,\, \tfrac{1}{2}a)]. \quad (28)$$

At large a

$$A^I(n_+,s) \underset{a\to\infty}{\sim} \tfrac{1}{2}a - (n_+ +1) + \frac{n_+^2 - 1}{2a} + O(1/a^2) \quad (29)$$

and we expect a monotonically decreasing function of n_+ at fixed large s. However at small values of a, the third term in the expansion can give a small increase in n, this feature disappearing rapidly with increasing a. For n_+ large compared with a, we can use the asymptotic expansion at fixed a and $n \to \infty$, and we get

$$A^I(n_+,s) \underset{n_+\to\infty}{\sim} \tfrac{1}{4}a + O(1/n_+). \quad (30)$$

For the R model, analytical calculations are more cumbersome. However, using the results of the I model, a few basic quantities can still be calculated. The shadow function is given by

$$S^R(s) = \sum_{n_+,n_0} P^R(n_+,n_0,s)$$

$$= \sum_{n_1,n_2} \frac{(\frac{1}{6}a)^{n_1}}{n_2!} \frac{(\frac{1}{2}a)^{n_2}}{n_2!} \sum_{m_1,m_2} P^I(m_1,m_2,a)$$

$$= e^{a/2} S^I(s). \quad (31)$$

Using (22), we get for the asymptotic expansion of S^R

$$S^R(s) \underset{a\to\infty}{\sim} \tfrac{1}{6}e^{3a/2}. \quad (32)$$

The average number of charged pions is given by

$$\langle n_+^R(s)\rangle = \frac{1}{S^R(s)} \sum_{n_1,n_2,m_1,m_2} (m_1+m_2+n_2) \times \frac{(\frac{1}{6}a)^{n_1}}{n_1!} \frac{(\frac{1}{3}a)^{n_2}}{n_2!} P^I(m_1,m_2,\tfrac{1}{2}a). \quad (33)$$

The expression appearing in the numerator was calculated in (26). Therefore the asymptotic expression of $\langle n_+\rangle$ is

$$\langle n_+^R(s)\rangle \underset{a\to\infty}{\sim} \tfrac{2}{3}a + \tfrac{1}{3}a = a.$$

In an analogous way, we get for the neutral average number

$$\langle n_0^R(s)\rangle = \frac{1}{S^R(s)} \sum_{n_1,n_2,m_1,m_2} \frac{2n_1+m_1}{n_1!n_2!} \left(\frac{a}{6}\right)^{n_1} \left(\frac{a}{3}\right)^{n_2} \times P^I(m_1,m_2,s)$$

$$= \tfrac{1}{3}a + \sum_{m_1,m_2} m_1 P^I(m_1,m_2,s)/S^I(s) \underset{a\to\infty}{\sim} \tfrac{1}{3}a + \tfrac{2}{3}a = a.$$

The average total number of particles produced behaves therefore as $3g^2 \ln s$.

IV. PHENOMENOLOGICAL CONSEQUENCES

The function $P(n_+,n_0,s)$ derived in the previous sections contains in principle all the information about the charge distributions. Very few experiments, however, can determine the number of neutral particles present in the final state (none, to our knowledge, in the cosmic-ray energy region). Therefore, the charged particle distribution $C(n^+,s)$ defined in (10) is particularly relevant. The information contained in the function C can be exploited in different ways. We can fix an energy $\bar{s}$ and plot the dependence of the cross section on the number of prongs. The experimental information available on this dependence supports the Poisson-like structure common to all the models that we have considered. In Fig. 1 we compare the predictions of the four models with the data of Ref. 4. We see

[15] Another straightforward method is to define two formally different coupling constants g_+ and g_0 for the emission of positive and neutral particles, and to consider the relevant derivative of the shadow function.

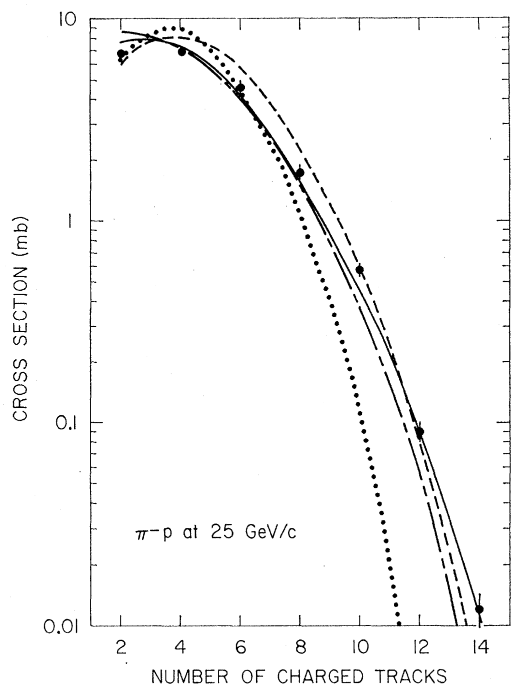

FIG. 1. Dependence of the cross section on the number of prongs at fixed energy. The data of Ref. 4 are compared with the predictions of the R model (solid line), the I model (dashed line), the A model (dot-dashed line), and the H model (dotted line).

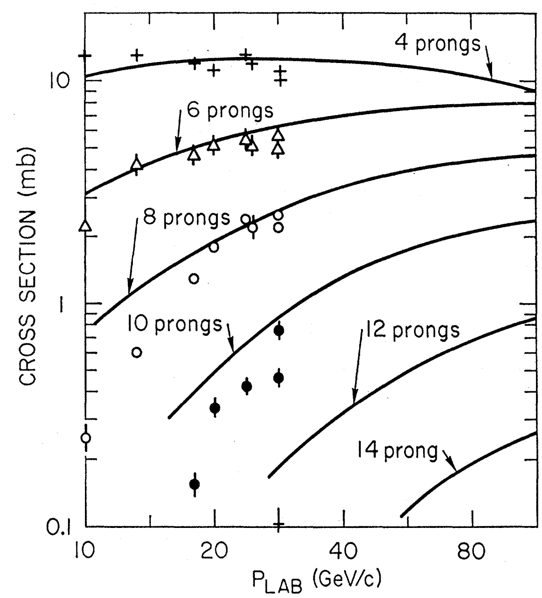

FIG. 2. Dependence of the cross sections with given number of prongs on the energy. The data from the compilation of Ref. 16 are compared with the predictions of the R model.

that the general shape is predicted by all models, even if the H model falls off too rapidly.

Another possibility is to study the s dependence of a given topology. Figure 2 shows the predictions of the R model compared with some data at accelerator energies. The prediction of the other models are qualitatively similar.[16] We note that every individual cross section decreases to zero in this kind of model as s raised to some negative power, and we see that the present data are compatible with this feature, even if no individual inelastic cross section shows a clear trend to decrease at the present energies. We note in particular that our model reproduces well the data on the basis of which a 2^{-n_+} rule for the (constant) asymptotic behavior of the cross sections with a definite number of prongs was proposed.[17] A third way of exploiting the information contained in $C(n_+,s)$ is to consider the probability of having a number $2n_+$ of charged tracks at an energy at which the average number of charged tracks is $2\langle n_+\rangle$. We obtain in this way a set of functions $P_{n_+}(\langle n_+\rangle)$ that is plotted in Fig. 3. The main interest of this kind of plot is in the fact that the set of functions $P_{n_+}(\langle n^+\rangle)$ looks experimentally universal, namely, independent of the particular type of reaction considered and this fact is, in our opinion, an indication that the details of the dynamics do not play an essential role in determining these distributions.

Let us now examine the neutral particle distributions. In all the models under consideration the average number of π^0 and of π^+ are the same asymptotically: Fig. 4 shows how this asymptotic limit is reached. We see that all models (except for the R model, which has by necessity the opposite behavior of the I model) predict a slight excess of π^+ over π^0 at low energies. In the A, I, and R models, the difference $\langle n_+\rangle-\langle n_0\rangle$ decreases very rapidly to a constant value, which survives asymp-

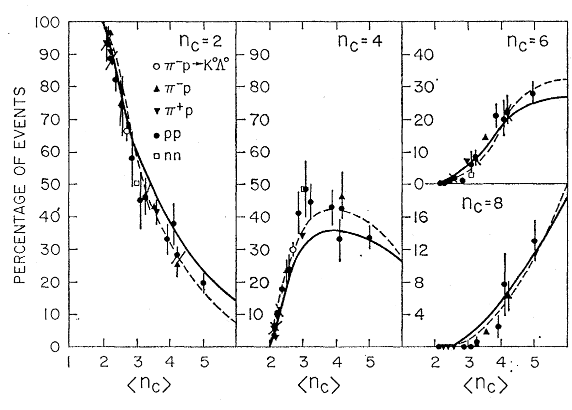

FIG. 3. The functions $P_{n_+}(\langle n_+\rangle)$ predicted by the H model (dashed line) and the R model (solid line), compared with the data from the compilations of Refs. 3 and 6. The predictions of the other two models (A, I) are intermediate between the H and R models.

[16] At large energy, however, the total cross section for a fixed multiplicity n behaves in the H, I, and A models like $(\ln s)^n e^{-\bar{n}(s)}$, whereas the R model predicts the slower decrease $(\ln s)^{n/2}e^{-1/2\bar{n}(s)}$.

[17] A. Wroblewski, Phys. Letters **32B**, 149 (1970). As expected, the predictions of our simple-minded models are systematically larger than the experimental results in the region of low s and large n, where phase-space effects play a dominant role and cannot be effectively represented by the $n!$ factor in the denominator of (1).

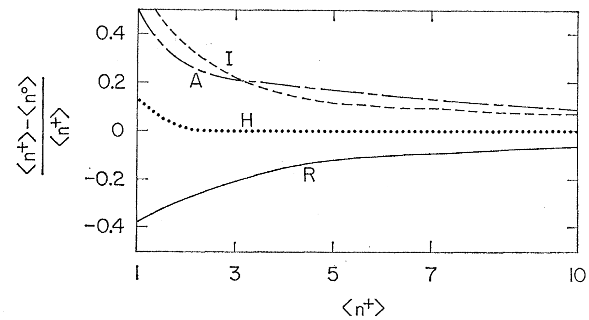

FIG. 4. Difference between the average number of charged and neutral particles produced in the various models.

totically. The few data available on $\langle n_0 \rangle$ are, however, not sufficient to test this prediction. A very interesting quantity for which some experimental data are available is the function $A(n_+,s)$ introduced in Sec. III that measures the correlation between charged and neutral particle emission. As argued in Ref. 13, the available data show a clear dependence of A on n_+, and therefore cast serious doubts on any model in which the neutral and charged particles are emitted independently (like the H model). Also the prediction of the I and A models (see Fig. 5) are not in agreement with the data of Ref. 4 that on the contrary support a mechanism of resonance production of the form of the R model.

A further interesting point related to the neutral particle distribution is suggested by the study of cosmic-ray events. In this kind of experiment only charged particles are detected, and also the momentum analysis is usually very difficult. Therefore, the only experimentally observable quantity is the scattering angle θ_{lab}. It has been observed that a rather high percentage of events presents large gaps in the ln $\tan\theta_{\text{lab}}$ distribution. Assuming limited values for all the transverse momenta, this fact corresponds to the existence of large gaps in the longitudinal momentum distribution of the charged particles. It is therefore possible to classify the charged particles produced in this kind of event into two (or more) clusters in such a way that the relative energy between any pair of charged particles belonging to different cluster is larger than, say, 3 GeV^2. These events are usually referred to as "two (or more) fireball" events, and their occurrence is a challenge to the multiperipheral scheme. Recently, DeTar and Snider[18] examined the problem and found suitable mechanisms within the MPM to account for the occurrence of this kind of reaction. One obvious explanation for the presence of a large gap between two charged particles in the framework of the MPM is that several neutral particles have been emitted between them along the multiperipheral chain and have gone undetected. DeTar and Snider estimated that the subsequent emission of two neutral particles is sufficient to produce a gap between the longitudinal momenta of the adjacent charged particles large enough to meet the phenomenological requirement for the classification of the event in the fireball category. It is therefore a relevant question to ask what is the probability in the various models for the subsequent emission of two or more neutral particles. We expect that the A, I, and R models in which the various charges are somehow correlated will give a higher probability than the H model, which simulates uncorrelated emission. In Fig. 6 we plot the function $O(n)$ that gives the probability of having an event with n particles in the final state without any subsequent pair of neutral particles. As expected, the A model (in which neutral particles always appear in pairs) gives the lowest probability, and the I model, in which charged particles are forced to appear in pairs, gives a smaller probability than the H model. The computation in the R model is less straightforward. We will assume that any $I=0$ resonance decaying into $2\pi^0$ produces a gap and, by phase-space considerations, we estimate that the relative energy of a pair of charged particles produced by the decay of two adjacent charged resonances exceeds 3 GeV^2 about 10% of the time. With this figure, the function $O(n)$ in the R model is practically equivalent to the one of the I model.

V. CONCLUSIONS

The charge distributions look like a promising ground for testing models of particle production. They are rather easy to observe experimentally; they look remarkably universal so that the data from different reactions can be combined to obtain better statistics, and this feature of universality hints to the independence of these distributions from the details of the dynamics.

Encouraged by the success of the multiperipheral model in predicting the energy dependence of the total multiplicity and in hinting a general Poisson-like

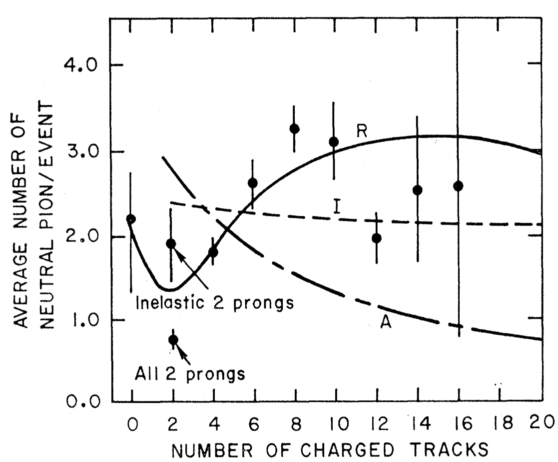

FIG. 5. The function $A(n_+,s)$ predicted by the A, I, and R models compared with the data of Ref. 4 on the $\pi^- p$ interaction at $p_{\text{lab}} = 25$ GeV/c. The H model predicts a constant behavior.

[18] C. E. DeTar and D. R. Snider, Phys. Rev. Letters 25, 410 (1970).

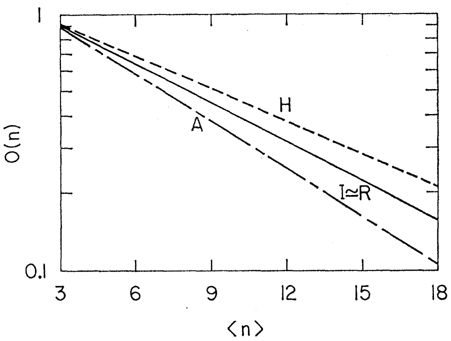

FIG. 6. The function $O(n)$ which gives the probability of having a high multiplicity event without the subsequent emission of π^0.

structure of the charged particle distribution in good agreement with the data, we have exploited the consequences of several detailed assumptions on the isospin structure of the dominant exchanges. The main results of this analysis have been the following.

(a) The average multiplicity of each charge grows logarithmically with the same coefficient in each model. The difference $\langle n^+\rangle - \langle n^0\rangle$ goes to 0 in the H model, to a positive constant in the A and I models, and to a negative constant in the R model, and can be sizeable in the region of intermediate energies.

(b) The charged particle distribution function $C(n_+,s)$ shows Poisson-like features for all the models in good agreement with the data. These distributions do not particularly favor an isospin structure over another.

(c) The function $A(n_+,s)$ that measures the correlation between neutral and charged particles emissions is, on the contrary, very sensitive to the models. The available data are rather preliminary, but they seem to rule out the sharp decrease with n_+ predicted by the A model, the nearly constant behavior predicted at $s=50$ GeV^2 by the I model, and the n_+ independence predicted by models in which the emission of charged and neutral particles is not correlated (like the H model). The resonance production (R) model, on the contrary, predicts a rapid rise of $A(n_+,s)$ for low values of n_+, the correlation being due to the decay of the charged resonances. At large values of n_+, phase-space effects eventually take over and $A(n,s)$ decreases to a constant limit. These qualitative features are in reasonable agreement with the data.

(d) The introduction of a definite isospin structure in the multiperipheral model is likely to increase the correlation between the emission of the various charges. In particular we found that in all the models considered (A,I,R) the probability of emitting two or more neutral particles in a row is considerably larger than in the H model that simulates independent emission. In view of this result, we feel that the mechanism of the subsequent emission of several neutral particles can be proposed as a major explanation of the occurence of "fireballs" in the framework of the multiperipheral model.

(e) If the total cross sections approach a constant limit asymptotically, it is enough to multiply our probability distribution by this constant to obtain the partial cross sections $\sigma(n_+,n_0,s)$ for the various reactions. It is clear that in this framework every individual cross section decreases to 0 asymptotically like $(\ln s)^{2n_+ + n_0}/S(s)$. It seems generally impossible to accommodate a finite limit for an infinite number of the partial cross section without forcing the average multiplicity to approach a constant limit itself (or to grow at most as ln ln$s$). In the framework of the multi-Regge bootstrap[5] diffractive effects (i.e., inelastic Pomeranchukon exchange) can be introduced only if the intercept of the Pomeranchuck trajectory is slightly lower than 1 (and therefore also the diffective contributions vanish asymptotically).

(f) One of the difficulties of the multiperipheral scheme is that in any model with direct emission of pions (including therefore the H, A, and I models considered here), the value of g^2 determined from the coefficient of the logarithmic increase of the multiplicity is unreasonably large for interpretation as a true coupling constant. On the contrary, this value is exactly what is needed to produce through unitarity a Pomeranchuk trajectory around 1 and a self-consistent meson trajectory around 0.5.[5,19] In a scheme of resonance production, like the R scheme, the multiperipheral production of n resonances corresponds to a final multiplicity $2n$. The coupling constant required to reproduce the observed multiplicity is therefore one-half of the one needed in the direct production models, and its value corresponds now to acceptable resonances widths.[13] The shadow function $S^R(s)$ however also increases with a power roughly half of what is needed and, consequently, the intercept of the Pomeranchuck trajectory generated by the shadow in this model is very low.[20] This shortcoming could possibly be overcome by introducing in the R model a diffractive mechanism corresponding, for instance, to a Pomeranchuk-dominated large-subenergy tail in the π-π cross section.

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[19] L. Caneschi and A. Pignotti, Phys. Rev. **180**, 1525 (1969); **184**, 1915 (1969); G. F. Chew and W. R. Frazer, *ibid.* **181**, 1914 (1969).

[20] D. M. Tow, Phys. Rev. D **2**, 154 (1970).