are of second order in chimeral $SU(3)$-breaking symmetry, and Eq. (26) is independent of corrections to pion PCAC. Taking $\Delta_K \approx -0.2$ [23] we see that the Eq. (22) is essentially unchanged for all practical purposes. We end wth a few comments.

(i) We observe that the main contributions to $\xi(0)$ depend only on the observables λ_+ and $F_K/F_\pi f_+(0)$. Taking $\lambda_+ \approx 0.08$ and $F_K/F_\pi f_+(0) \approx 1.28$ [19] we obtain $\xi(0) \approx -0.6$, as is favored by the recent trend in experimental data.[4]

(ii) In the present approach to K_{l3} decay, no smoothness assumption as a function of p^2 or k^2 has been invoked, in contrast to the earlier investigations.[24]

(iii) The scalar form factor does *not* exhibit a zero between $q^2 = (m_K - m_\pi)^2$ and $q^2 = (m_K + m_\pi)^2$, and hence does not satisfy the criterion of Ref. 5 to yield a large negative $\xi(0)$. One is, nonetheless, led here to predict $\xi(0) \approx -0.6$.

(iv) The soft-pion theorem of Eq. (2) is satisfied up to 10%, as may be verified from Eqs. (25) and (26).

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[23] See C. Chan and F. Meiere, Phys. Rev. **175**, 2222 (1968), and references therein.

[24] I. Gerstein and H. Schnitzer, Phys. Rev. **175**, 1876 (1968).

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Nonrelativistic Separable-Potential Quark Model of the Hadrons

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A nonrelativistic quark model of the hadrons based on nonlocal separable potentials is presented. The model contains a quark-antiquark force and an effective three-body force among quarks, replacing the two-body forces. For single-parameter potentials in which the strength of the interaction drops sharply as the quark relative momenta increase, reasonable values of the lifetimes and radii of the mesons are found. A baryon wave function useful for dynamical calculations is obtained. The validity of the nonrelativistic approximation for this model is confirmed.

I. INTRODUCTION

SINCE the work of Gell-Mann[1] and Zweig,[2] a growing number of physicists have found it helpful to discuss hadronic matter in terms of entities called quarks. These quarks may be elementary excitations (quasiparticles) of some underlying hadronic field, or they may be massive particles. Various reports of experimental effects which might be ascribed to quarks have appeared.[3–6] Although the question of the existence of quarks is still open, the utility of the quark concept is sufficient to justify the attempt to create dynamical models of the hadrons based on their assumed existence.

Nonrelativistic quark models were among the first to be investigated because of their simplicity and the wide array of theoretical tools which can be applied to them. Although justifications for the validity of these models have been presented,[7] the expectation value of the quark momenta is of the order of the quark mass if the usual Yukawa-type potential is used, invalidating the nonrelativistic approximation. However, there is no *a priori* reason for using a local potential in a quark model.

Consequently, we were led to consider nonrelativistic nonlocal separable-potential quark models of the hadrons. The use of separable-potential models to describe the scattering of subatomic particles has often been dismissed as unphysical because (a) they are nonlocal and (b) a single-term separable potential can

[1] M. Gell-Mann, Phys. Letters **8**, 274 (1964).
[2] G. Zweig, CERN Report No. 8479/TH472, 1964 (unpublished).
[3] M. Dardo, P. Penengo, and K. Sitte, Nuovo Cimento **58A**, 59 (1968).
[4] C. B. A. McCusker and I. Cairns, Phys. Rev. Letters **23**, 658 (1969).
[5] L. Kaufman and T. R. Mongan, Phys. Rev. D **1**, 988 (1970).
[6] W. T. Chu, Y. S. Kim, W. J. Beam, and N. Kwak, Phys. Rev. Letters **24**, 917 (1970).

[7] G. Morpurgo, Physics **2**, 95 (1965).

support only one bound state in each partial wave. But, in constructing a quark theory of the hadrons, these may not be serious drawbacks. In the first place, we are no longer dealing with those forces for which a local-potential description has been shown to be necessary or desirable. Possibly, a nonlocal model may provide the most economical description of the forces between quarks. Second, the fact that a single-term separable potential can support only one bound state may be a definite advantage in the construction of quark models of the hadrons. It allows us to develop models which contain the bound states (hadrons), which are observed experimentally, and *no other* bound states. Furthermore, we find that separable potentials that provide reasonable values for hadron lifetimes and sizes also guarantee the validity of the nonrelativistic approach.

Separable-potential models are sufficiently simple that expressions for many physically important quantities can be obtained in closed form. We hope that this simplicity will allow deeper insight into the underlying physics of the quark models.

II. MODEL

To construct the proposed nonrelativistic separable-potential quark model of the hadrons we make the following assumptions. (a) The hadrons are bound states of the quarks of Gell-Mann and Zweig. (b) The quark-antiquark ($q\bar{q}$) force can be adequately described by an attractive single-term nonlocal separable S-wave ($l=0$) potential. Thus, the *only possible* $q\bar{q}$ bound states (mesons) in this theory are a pseudoscalar-meson nonet and a vector-meson nonet. (c) The force between quarks can be adequately described by an attractive single-term nonlocal separable *three-body* potential in the state of total orbital momentum $L=0$. The *only possible* three-quark bound states (baryons) in this theory are a baryon octet and decuplet. Furthermore, the difficulties introduced into three-body scattering theory by processes wherein a free particle interacts with a bound state of the other two are not present. Since we deal only with an effective three-body force, the so-called "disconnected diagrams" are not present in the Lippmann-Schwinger (LS) equation. Consequently, the Faddeev reduction of the three-body problem is not necessary and the three-body LS equation is not expected to have spurious solutions. We emphasize that we can avoid the complications of the Faddeev approach to the three-body problem and simply apply the three-body LS equation *if* we can replace the two-body forces by an effective three-body force. If we then assume that the three-body force is separable, we are left with a particularly simple soluble model of three-particle scattering which may be applicable to hadron physics.

Obviously, additional meson and baryon states can be obtained by introducing separable $q\bar{q}$ forces for $l\geq 1$ and separable qqq forces for $L\geq 1$. Furthermore, with only a slight increase in complication, n-term separable potentials can be introduced in each state of l or L. For example, a two-term separable $l=0$ $q\bar{q}$ force can be used to model a repulsive contribution to the over-all attractive $l=0$ $q\bar{q}$ force.

The applicability of our model is not limited to the quark theory of Gell-Mann and Zweig. The model can be used in any theory of the hadrons that involves the composition of hadrons from X particles in the combinations $X\bar{X}$ and XXX.

III. BASIC EQUATIONS

We treat the degenerate case in which all quark masses are assumed equal. Then, with an $SU(3)$-invariant force, all members of a given $SU(3)$ multiplet have the same mass. If the λ quark ($I=0$, $S=-1$) is assumed to be more massive than the p and n quarks ($I=\frac{1}{2}$, $S=0$), mass splittings will appear within the $SU(3)$ multiplets.[8] In addition, the symmetry breaking needed to reproduce the observed masses can be introduced by assigning different values for the potential parameters for different isospin states with a multiplet.

A. Quark-Antiquark Forces

We assume that the singlet and triplet quark-antiquark interaction can be adequately described by the two-body nonrelativistic partial-wave LS equation

$$T_l(p,p';k^2)=V_l(p,p')+\frac{2\mu}{\hbar^2}\int_0^\infty \frac{dq\,q^2V_l(p,q)T_l(q,p';k^2)}{k^2-q^2+i\epsilon}, \quad (1)$$

where the c.m. kinetic energy $E=\hbar^2k^2/2\mu$ and μ is the reduced mass of the quark-antiquark system.

Furthermore, we assume that the singlet and triplet quark-antiquark force in the lth partial wave can be described by the single-term attractive nonlocal separable potential:

$$V_l(p,p')=C_l^2g_l(p)g_l(p'). \quad (2)$$

Although we consider only $l=0$ in this paper, we shall state the equations for the more general case.

When the potential (2) is inserted in the LS equation (1), we can solve for the quark-antiquark partial-wave T matrix in closed form, obtaining

$$T_l(p,p';k^2)=-C_l^2g_l(p)g_l(p')\left[1+\frac{2\mu C_l^2}{\hbar^2}\int_0^\infty\frac{dq\,q^2g_l^2(q)}{k^2-q^2+i\epsilon}\right]^{-1}.$$

A quark-antiquark bound state (meson) with binding

[8] J. J. J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969), pp. 34–40.

energy $E_B=-\hbar^2\alpha^2/2\mu$ will be manifested as a pole in the T matrix at $k^2=-\alpha^2$. Thus, for a given choice of the functional form $g_l(p)$ of the potential, the coupling strength C_l^2 needed to support a bound state with binding energy E_B can be determined from the condition

$$1=\frac{2\mu C_l^2}{\hbar^2}\int_0^\infty \frac{dq\, q^2 g_l^2(q)}{q^2+\alpha^2}. \tag{3}$$

The Schrödinger equation for the "radial" part of the quark-antiquark bound-state wave function in momentum space is

$$W_l(q)=\frac{2\mu}{\hbar^2}\frac{C_l^2}{q^2+\alpha^2}g_l(q)\int_0^\infty g_l(q')W_l(q')q'^2dq' \tag{4}$$

when the separable potential (2) is used. Thus, the Schrödinger equation is immediately solved and the integral appearing in Eq. (4) is simply a normalization constant A determined from the normalization condition

$$1=\int_0^\infty W_l^2(q)q^2dq$$

or

$$1=\left(\frac{2\mu}{\hbar^2}\right)^2 A^2C_l^4\int_0^\infty \frac{q^2g_l^2(q)}{(q^2+\alpha^2)^2}dq, \tag{5}$$

and thus

$$W_l(q)=\frac{2\mu}{\hbar^2}A\frac{C_l^2}{q^2+\alpha^2}g_l(q).$$

The radial part of the meson wave function in position space is

$$\frac{u_l(r)}{r}=\left(\frac{2}{\pi}\right)^{1/2}\int_0^\infty i^lW_l(q)j_l(qr)q^2dq, \tag{6}$$

where $j_l(qr)$ is the spherical Bessel function.

The normalization of $W_l(q)$ ensures the normalization

$$\int_0^\infty u_l^2(r)dr=1.$$

The expectation value of the square of the quark-antiquark relative momenta is

$$\hbar^2\langle p^2\rangle=\hbar^2\int_0^\infty W_l^2(p)p^4dp \tag{7}$$

and, for small quarks, the rms radius of the bound state is

$$r_{\rm rms}=\langle r^2\rangle^{1/2}=\left\{\int |\psi(\mathbf{r})|^2r^2d^3\mathbf{r}\right\}^{1/2}$$
$$=\left\{\int_0^\infty u_l^2(r)r^2dr\right\}^{1/2}. \tag{8}$$

B. Forces between Quarks

We assume that the force between quarks can be described by an effective three-body force when three quarks interact. We also assume that the interaction between three quarks in a state of total orbital angular momentum L can be described by the three-body nonrelativistic LS equation partial-wave analyzed in the manner of Omnés[9]:

$$T_L(w_1,w_2,w_3,w_1',w_2',w_3';Z)=V_L(w_1,w_2,w_3,w_1',w_2',w_3')+\int\frac{dw_1''dw_2''dw_3''V_L(w_1,w_2,w_3,w_1'',w_2'',w_3'')T_L(w_1'',w_2'',w_3'',w_1',w_2',w_3';Z)}{Z-(w_1''+w_2''+w_3'')+i\epsilon}, \tag{9}$$

where Z is the total c.m. energy:

$$Z=\frac{\hbar^2p_1^2}{2M_1}+\frac{\hbar^2p_2^2}{2M_2}+\frac{\hbar^2p_3^2}{2M_3}.$$

M_i is the quark mass and $w_i=\hbar^2p_i^2/2M_i$. Although Omnés carried out his angular momentum reduction for spinless particles, his analysis can easily be modified for particles with spin by using helicity states. In our consideration of three-quark states, we shall neglect the complications arising from quark spin and statistics.

If there are no qq bound states, and the two-body forces between quarks can be replaced by an effective three-body force, the three-body LS equation will not involve disconnected diagrams (which lead to δ functions in the kernel). Consequently, the three-body LS equation will not have spurious solutions and the Faddeev reduction of the three-body integral equation will not be necessary to obtain a valid solution to the three-body problem.

We assume that the three-body force among quarks in a state with given spin and total orbital angular momentum L can be described by a single-term attractive nonlocal separable three-body potential

$$V_L(w_1,w_2,w_3,w_1',w_2',w_3')=-C_L^2g_L(w_1,w_2,w_3)g_L(w_1',w_2',w_3'). \tag{10}$$

An algebraic solution to the three-body partial-wave LS equation (9) can be obtained for the separable

[9] R. L. Omnés, Phys. Rev. **134**, B1358 (1964).

three-body potential (10) in the form

$$T_L(w_1,w_2,w_3,w_1',w_2',w_3';Z)$$
$$=-C_L^2 g_L(w_1,w_2,w_3) g_L(w_1',w_2',w_3')$$
$$\times\left[1+C_L^2\int\frac{dw_1dw_2dw_3 g_L^2(w_1,w_2,w_3)}{Z-(w_1+w_2+w_3)+i\epsilon}\right]^{-1}.$$

A three-quark bound state (baryon) with total orbital angular momentum L will appear as a pole in the T matrix $T_L(w_1,w_2,w_3,w_1',w_2',w_3';Z)$ at an energy $E=-E_B$. Therefore, for a given choice of the potential form the coupling strength C_L^2 needed to support a bound state with binding energy E_B can be determined from

$$1=C_L^2\int\frac{dw_1dw_2dw_3 g_L^2(w_1,w_2,w_3)}{E_B+w_1+w_2+w_3}.$$

When the three-body separable potential (10) is used, the "radial" part of the three-quark bound-state wave function in momentum space is

$$\psi_L(w_1,w_2,w_3)=\frac{C_L^2 g_L(w_1,w_2,w_3)}{E_B+w_1+w_2+w_3}$$
$$\times\int g_L(w_1',w_2',w_3')\psi_L(w_1',w_2',w_3')dw_1'dw_2'dw_3'. \quad (11)$$

Again, the Schrödinger equation for the wave function is immediately solved and the integral in Eq. (11) is merely a normalization constant.

IV. MESONS

Upon considering weak and electromagnetic decays of the mesons in the quark model, Van Royen and Weisskopf[10] arrive at the result

$$|\psi(0)|^2\approx\tfrac{1}{2}m_\pi{}^2 m_{\text{meson}}. \quad (12)$$

Thus, if the meson is considered as a quark-antiquark bound state, the wave function at $\mathbf{r}=0$ depends only on the meson mass and not on the quark mass.

This surprising behavior can be guaranteed in our model by choosing a functional form $g_l(p)$ for the separable potential that falls off rapidly as momentum increases.

The binding energy of the quark-antiquark bound state (meson) is

$$\hbar^2\alpha^2/2\mu=(2M_Q-m)c^2,$$

where M_Q is the quark mass, m is the meson mass, and $2\mu=M_Q$. If the quark mass is above the lower limit of 6 GeV fixed by accelerator experiments, α^2 is quite large. We choose a quark mass of 10 GeV for convenience.

Now if the potential form $g_l(p)$ falls off rapidly with increasing p, we see from Eqs. (4) and (5) that $(q^2+\alpha^2)\approx\alpha^2$ for all values of the momentum important in the interaction, and the normalization (5) ensures that the meson wave function will be independent of the quark mass and the coupling strength. Furthermore, the rapid falloff with increasing p will guarantee the validity of the nonrelativistic approximation since $W_l^2(p)\approx g_l^2(p)$ in Eq. (7), resulting in a small expectation value of the momentum.

If we consider, for example, a potential form $g_l(p)$ which falls off rapidly and depends only on a single "range parameter" K determining the momentum values of importance in the interaction, the meson wave function will depend only on this range parameter. We can obtain a value for the range parameter K from the Van Royen-Weisskopf relation (12) and then calculate the meson radius from the wave function to check the validity of the model. Also, the expectation value of the momentum will be of the order of K, and if

$$\hbar^2K^2/2\mu\ll M_Qc^2 \quad\text{or}\quad \hbar^2K^2/(Mc)^2\ll 1,$$

the nonrelativistic approximation is justified.

As a simple example of a potential which falls off sufficiently rapidly, we choose a Gaussian form

$$g_l(p)=p^l\exp(-p^2/K^2).$$

The coupling strength required to produce a meson bound state is obtained from Eq. (3) where

$$\frac{1}{q^2+\alpha^2}=\frac{1}{\alpha^2}\left(1-\frac{q^2}{\alpha^2}\right)$$

and the error in the approximation $(q^2+\alpha^2)\approx\alpha^2$ is expected to be of the order of K^2/α^2. Thus

$$C_L^2=|E_B|\bigg/\int_0^\infty dq\, q^2 g_l^2(q)$$

and, in the strong-binding limit where the meson mass is zero,

$$C_L^2=\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{2}{K}\right)^{2l+3}\frac{2M_Qc^2}{1\times3\times5\times\cdots\times(2l+1)}.$$

From Eqs. (4) and (5), the normalized "radial" part of the bound-state wave function in momentum space is

$$W_L(p)=\left(\frac{2}{\pi}\right)^{1/4}\frac{(2/K)^{(2l+3)/2}}{[1\times3\times5\times\cdots\times(2l+1)]^{1/2}}\frac{\alpha^2p^l}{\alpha^2+p^2}$$
$$\times\exp(-p^2/K^2).$$

From Eq. (6), using the approximation $(q^2+\alpha^2)\approx\alpha^2$,

[10] R. Van Royen and V. F. Weisskopf, II, Nuovo Cimento **50A**, 617 (1967).

the radial wave function in position space is

$$\frac{u_l(r)}{r}=\left(\frac{2}{\pi}\right)^{1/4}\frac{K^{(2l+3)/2}r^l}{[1\times3\times5\times\cdots(2l+1)]^{1/2}}\exp(-K^2r^2/4).$$

Now

$$\psi_{l=0}(\mathbf{r})=\frac{1}{(4\pi)^{1/2}}\frac{u_{l=0}(r)}{r},$$

so

$$|\psi_{l=0}(\mathbf{r}=0)|^2=(8\pi^3)^{-1/2}K^3$$

and using the Van Royen-Weisskopf relation (12) we find for the pseudoscalar- and vector-meson nonets

$$K_M=(2\pi^3)^{1/6}(m_\pi^2m_M)^{1/3}.$$

It is clear that this weak dependence of K on the meson mass ($K_{\text{meson}}\sim m_{\text{meson}}^{1/3}$), which can be interpreted as an $SU(3)$-breaking isospin dependence of the potential parameters, will ensure that the Van Royen-Weisskopf relation holds for all members of a meson nonet. Henceforth we discuss only the pion.

The error in the approximation $(q^2+\alpha^2)\approx\alpha^2$ is of the order of

$$(2\pi^3)^{1/3}m_\pi^2/\alpha^2$$

or, in the strong binding limit,

$$(2\pi^3)^{1/3}m_\pi^2\hbar^2/2(M_Qc)^2=0.036\%.$$

The expectation value of the momentum is $\hbar^2\langle p^2\rangle=\frac{1}{4}(2l+3)\hbar^2K^2$, so in the pion case ($l=0$)

$$\hbar^2\langle p^2\rangle/(M_Qc)^2=5.4\times10^{-4},$$

justifying the nonrelativistic approximation.

For small quarks (the Compton wavelength for a 10-GeV quark is $\hbar c/M_Qc^2=2.0\times10^{-2}$ F) the predicted value of the meson radius is found from

$$\langle r^2\rangle=(2l+3)/K^2.$$

For the pion this becomes

$$r_{\text{rms}}=\langle r^2\rangle^{1/2}=(3/\pi)^{1/2}2^{-1/6}/m_\pi=1.27\text{ F},$$

which is to be compared with experimental values for this somewhat poorly defined parameter ranging from 0.628 F to an upper bound of 4.5 F.[11]

V. BARYONS

In the baryon case, $|E_B|=(3M_Q-m_B)c^2$, where m_B is the baryon mass. If we choose three-body potential forms in Eq. (10) such that $g_L(w_1,w_2,w_3)\to0$ rapidly whenever any $w_i=\hbar^2p_i^2/2M_{Qi}$ becomes large, we may set $E_B+w_1+w_2+w_3\approx E_B$ in Eq. (11). This produces a three-quark bound-state wave function which does not depend on the quark mass. Such wave functions have been used by Mitra and Majumdar,[12] Kreps and De Swart,[13] and Licht and Pagnamenta[14] in earlier considerations of the quark model. Furthermore, the symmetry or antisymmetry properties of the wave function are determined by the properties of the function $g_l(w_1,w_2,w_3)$.

Consider potentials which depend only on the magnitude of the relative momenta between quarks

$$p_{ij}^2=(\mathbf{p}_i-\mathbf{p}_j)^2.$$

Since we work in the c.m. frame where

$$\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3=0$$

and all quark masses are equal, we find

$$p_{ij}^2=(2M/\hbar^2)(2w_i+2w_j-w_k),\quad k\neq i,\ j.$$

Then, for $L=0$, if we choose

$$g_{L=0}(w_1,w_2,w_3)=\exp(-p_{12}^2/12K^2)\times\exp(-p_{13}^2/12K^2)\exp(-p_{23}^2/12K^2),$$

we find

$$g_{L=0}(w_1,w_2,w_3)=\exp[-M(w_1+w_2+w_3)/2\hbar^2K^2].$$

Making the approximation $E_B+w_1+w_2+w_3=E_B$ in Eq. (11) [which is expected to cause an error of the order of $\frac{2}{3}(\hbar^2K^2/M_Q^2c^2)$ in the strong-binding limit], the three-body bound-state wave function of the baryon takes the form

$$\psi_{L=0}(w_1,w_2,w_3)=N\exp[-M(w_1+w_2+w_3)/2\hbar^2K^2]\tag{13}$$

or, in position space,

$$\psi_{L=0}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)=N'\exp[-(x_1^2+x_2^2+x_3^2)/K^2]\tag{14}$$

in the c.m. system, where N and N' are normalization constants. Thus we have a symmetric wave function which corresponds to the neglect of quark spin and statistics or the assumption of parastatistics for quarks.

Recently, Licht and Pagnamenta[14] have used just such a wave function to derive expressions for the proton electromagnetic form factor which are in good agreement with experiment.

The coupling strength required in Eq. (10) to support an $L=0$ three-body bound state with a binding energy E_B is

$$C_{L=0}^2=\left(\frac{M_Q}{2\hbar^2K^2}\right)^3E_B$$

or, in the strong-binding limit where the baryon mass is zero,

$$C_{L=0}^2=3M_Q^4c^2/8\hbar^6K^6.$$

For small quarks, if $\psi(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$ is a completely symmetric or antisymmetric wave function for total $L=0$

[11] R. A. Christensen, Phys. Rev. D 1, 1469 (1970).
[12] A. N. Mitra and R. Majumdar, Phys. Rev. **150**, 1194 (1966).
[13] R. E. Kreps and J. J. De Swart, Phys. Rev. **162**, 1729 (1967).
[14] A. L. Licht and A. Pagnamenta, Phys. Rev. D 2, 1150 (1970); **2**, 1156 (1970).

in the c.m. system where

$$\mathbf{x}_1+\mathbf{x}_2+\mathbf{x}_3=0,$$

then the baryon mass distribution is[13]

$$\rho(\mathbf{r})=\int d^3\mathbf{x}\,|\psi(\mathbf{r},\mathbf{x},-(\mathbf{r}+\mathbf{x}))|^2.$$

We determine the cutoff parameter K in Eq. (14) by setting the baryon rms radius

$$\langle r^2\rangle^{1/2}=\left[\frac{\int\rho(\mathbf{r})r^2d^3r}{\int\rho(\mathbf{r})d^3r}\right]^{1/2}$$

equal to the proton rms radius which we choose to be 0.80 F. The resulting value of K is 0.88 F^{-1}.

The expectation value of the quark kinetic energy is, from Eq. (13),

$$\langle w_i\rangle=2\hbar^2K^2/M_Q.$$

Thus, if

$$2\hbar^2K^2/(M_Qc)^2\ll 1,$$

the nonrelativistic approximation is justified, and when $K=0.88\ \mathrm{F}^{-1}$,

$$2\hbar^2K^2/(M_Qc)^2=6\times10^{-4}.$$

The expected error introduced by the approximation $E_B+w_1+w_2+w_3\approx E_B$ is, in the strong-binding limit, $2\hbar^2K^2/3(M_Qc)^2=0.02\%$.

We have previously noted[5] that some apparently anomalous results for the primary cosmic-ray proton flux and the proton-carbon cross section at energies above about 225 GeV (obtained from the Proton satellites by Grigorov *et al.*[15–17]) might be ascribed, following a suggestion of Dooher,[18] to the breakup of the primary cosmic-ray protons into their constituent quarks.[19] Consequently, we used the wave functions derived from our quark model to estimate the cross section for diffraction dissociation of protons into quarks in proton-nucleus collisions at high energies. Our estimate for the proton-nucleus diffraction-dissociation cross section σ, which is based on Glauber's approach[20] and which applies in the high-energy limit, is $\sigma=2\pi R\Sigma$, where R is the nuclear radius,

$$\Sigma=\int_0^\infty J(b)[1-J(b)]db,$$

and

$$J(b)=\left[\int_0^\infty\rho(r)r^2dr\right]^{-1}\int_0^\infty(r^2-br)\rho(r)dr.$$

Using the DESY value for the strong-interaction nuclear radii,[21]

$$R(A)=(1.12\pm0.02)A^{1/3}\ \mathrm{F},$$

we estimate the cross section for diffraction dissociation of protons on carbon ($A=12$) as 24.6 mb. This estimate compares very favorably with the value of 26 mb for the diffraction-dissociation cross section for protons on carbon which we obtained from an analysis of Grigorov's data.[5] Similarly, using Glauber's formulation[20] and our quark-model meson wave function, the π-meson diffraction-dissociation cross section on carbon is 19.6 mb. It is worth noting that Refs. 3 and 4 indicate the possibility of quark fluxes requiring the production of quarks with cross sections in the mb range.

VI. SUMMARY

We have considered a nonrelativistic quark model of the hadrons in which the following assumptions are made.

(1) The quark-antiquark force can be described by an attractive S-wave nonlocal separable potential.

(2) The force among quarks can be described by an attractive *three-body* nonlocal separable potential in states with $L=0$.

Such a model contains only pseudoscalar- and vector-meson nonets and a baryon octet and decuplet.

We find that separable-potential forms which fall off rapidly as the relative momenta increases produce internal wave functions for the hadrons which do not depend on the quark mass and for which the nonrelativistic approximation is valid.

Furthermore, we find the following conclusions.

(1) A single-parameter Gaussian potential form can be obtained which provides adequate values for the meson radii and decay lifetimes and allows the Van Royen-Weisskopf relation to be satisfied by a weak dependence of the potential range on the meson mass, i.e., an $SU(3)$-breaking isospin dependence of the potential range.

(2) A three-body Gaussian potential form yields a proton wave function which has been used to derive expressions for the proton electromagnetic form factors which are in good agreement with experiment.

(3) The model predicts substantial cross sections for diffraction dissociation of hadrons into their constituent quarks in high-energy hadron-nucleus collisions.

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