It is reliable in the case where small transfers are selected; otherwise it may become very rough, and even lead to gross overestimations.

(b) If we had chosen to make an exact calculation of $d\sigma/dM$ for diagram I of Fig. 1, we would have been compelled to perform a *fourfold* integration with the help of a computer. (The integration over $\cos\chi$ can be done analytically, at least in the Born-term model.) Even if the computer used were quite powerful, such a calculation would always involve some amount of error. Therefore, it is possible that such an "exact" calculation might have provided a result less accurate than our approximation procedure, where only one *single* integration was needed.

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Dashen-Weinstein Theorem in K_{l3} Decay: Soft-Pion Corrections

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The Mathur-Okubo sum rule in K_{l3} decay, together with Pagels's model for corrections due to partial conservation of axial-vector current, is invoked to estimate the $O(\epsilon^2)$ corrections to the Dashen-Weinstein theorem for $\xi(0)$. The corrections are found to be small, and we predict $\xi(0)\approx-0.6$ for $\lambda_+\approx0.08$.

THE problem of K_{l3} decay has attracted considerable attention in the literature.[1] The matrix element for this decay defines two form factors $f_+(q^2)$ and $f_-(q^2)$ through the relation

$$\langle\pi^0(k)|V_\mu^{4-i5}(0)|K^+(p)\rangle = (1/\sqrt{2})[f_+(q^2)(p+k)_\mu+f_-(q^2)(p-k)_\mu], \quad (1)$$

where $q=k-p$.

In this context, there are two important results which follow from Gell-Mann's current algebra and partial conservation of axial-vector current (PCAC) which enjoy the privileged status of "theorems": The first is the famous soft-pion theorem due to Callan and Treiman and to Mathur, Okubo, and Pandit,[2] which states that

$$f_+(p^2=m_K^2,\, k^2=m_\pi^2,\, q^2=m_K^2) + f_-(p^2=m_K^2,\, k^2=m_\pi^2,\, q^2=m_K^2) = F_K/F_\pi+O(\epsilon_\pi), \quad (2)$$

where ϵ_π is a parameter which measures the departure from the limit of exact $SU(2)\otimes SU(2)$ symmetry and massless pions, and $F_{\pi(K)}$ is the pion (kaon) decay amplitude. The other theorem of more recent origin is the Dashen-Weinstein[3] theorem for the form-factor ratio $\xi(0)\equiv f_-(0)/f_+(0)$:

$$\xi(0)+\lambda_+\frac{m_K^2-m_\pi^2}{m_\pi^2}=\frac{1}{2}\left(\frac{F_K}{F_\pi}-\frac{F_\pi}{F_K}\right)+O(\epsilon^2), \quad (3)$$

where

$$\lambda_+=m_\pi^2\left[\frac{d}{dt}\ln f_+(t)\right]_{t=0},$$

and ϵ is a measure of the breaking of $SU(3)\otimes SU(3)$ symmetry. This theorem is independent of any assumptions on the form of symmetry breaking.

It is well known that Eq. (2) is not directly amenable to experimental tests, since the point $q^2=m_K^2$ is not in the physical region of the decay: $m_l^2\leqslant q^2\leqslant(m_K-m_\pi)^2$.

The Dashen-Weinstein theorem [Eq. (3)] is somewhat more readily available for confrontation with experiment and is in good agreement with recent experimental data,[4] subject to the uncertainties in corrections of $O(\epsilon^2)$.

The possibility of a zero in the "scalar" form factor $f(q^2)\equiv f_+(q^2)(m_K^2-m_\pi^2)+f_-(q^2)q^2$ between $(m_K-m_\pi)^2$ and $(m_K+m_\pi)^2$ has been suggested to explain $\xi(0)\approx-1$,[5] as also has the proposal based on "weak" PCAC.[6] The latter proposal has been, however, criticized by Weinstein.[7]

In the present paper, we estimate corrections to Eq. (3) since it is rather essential to know whether or not the fair agreement with experimental data that one obtains from the Dashen-Weinstein formula [with $\lambda_+\approx0.08$ and $\xi(0)\simeq-0.6$] [4] is fortuitous.

We make the following assumptions:

(i) $SU(3)\otimes SU(3)$ chiral algebra of Gell-Mann;
(ii) PCAC hypotheses for pion and kaon fields;

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[1] See S. Weinberg, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968).

[2] C. Callan and S. Treiman, Phys. Rev. Letters **16**, 153 (1966); V. Mathur, S. Okubo, and L. Pandit, *ibid.* **16**, 371 (1966).

[3] R. Dashen and M. Weinstein, Phys. Rev. Letters **22**, 1337 (1969); Fayyazuddin and Riazuddin, Phys. Rev. D **1**, 361 (1970).

[4] P. Innocenti *et al.*, in *Proceedings of the Fifteenth International Conference on High-Energy Physics, Kiev, 1970* (Academy of Sciences, U.S.S.R., Moscow, 1971); E. Dalley *et al.*, *ibid.*; C.-Y. Chien *et al.*, Phys. Letters **33B**, 627 (1970).

[5] K. Kang, Phys. Rev. Letters **25**, 414 (1970).

[6] R. Brandt and G. Preparata, Nuovo Cimento Letters **4**, 80 (1970).

[7] M. Weinstein, Phys. Rev. D **3**, 481 (1971).

(iii) field-current identity for the strangeness-changing axial-vector current;

(iv) the absence of poles in $f_+(q^2)$ and $f_-(q^2)$ at $q^2=0$;

(v) the validity of the Mathur-Okubo sum rule[8]

$$(m_K^2+m_\pi^2)f_+(\Delta)+(m_K^2-m_\pi^2)f_-(\Delta) = \frac{m_K^2F_K}{F_\pi}+\frac{m_\pi^2F_\pi}{F_K}+O(\epsilon_\pi\epsilon_K\epsilon_8^2), \quad (4)$$

where ϵ_K and ϵ_8 denote the departure from chimeral[8] and ordinary $SU(3)$ symmetries, respectively, and Δ is a function of m_π^2 and m_K^2 satisfying the conditions $\Delta=m_K^2$, m_π^2 and $O(\epsilon_8^2)$ in $SU(2)\otimes SU(2)$, chimeral $SU(3)$ and usual $SU(3)$ limits, respectively. This sum rule incorporates soft-pion corrections provided the best value of Δ is chosen (in the sense of Ref. 8).

As emphasized by Dashen,[9] assumption (ii) is correct only to zeroth order in symmetry breaking. Furthermore, assumption (iii) is valid as an operator identity only when $m_{(K)}=0$.[10] Therefore, a consistent treatment of the problem must necessarily depend on the inclusion of symmetry-breaking corrections to both (ii) and (iii). We will consider these points later, but first explore the consequences of assumptions (i)–(v) ignoring these corrections.

We start with the following identity for the axial-vector current:

$$\int d^4y d^4x\, e^{-iqy}\partial_x{}^\lambda[e^{iq_1x}\langle\pi^0(k)| \times T(A_\lambda{}^{1-i2}(x)A_\mu{}^{6-i7}(y))|K^+(p)\rangle]=0. \quad (5)$$

We take the limit $q_{1\lambda}\to 0$ and drop the term in $q_{1\lambda}$ (assuming that it is smooth) and employ assumptions (i) and (ii) for the pion field to obtain

$$\langle\pi^0(k)|V_\mu{}^{4-i5}(0)|K^+(p)\rangle(2\pi)^4\delta^4(k-q-p) = F_\pi\int d^4y\, e^{-iq\cdot y}\langle\pi^0(k)\pi^+(0)|A_\mu{}^{6-i7}(y)|K^+(p)\rangle. \quad (6)$$

The use of the field-current identity[11] [assumption (iii)]

$$A_\mu{}^{6-i7}(x)=g_{K_A}a_\mu(x)-F_K\partial_\mu\phi_{\bar{K}^0}(x) \quad (7)$$

in Eq. (6) leads to

$$\langle\pi^0(k)|V_\mu{}^{4-i5}(0)|K^+(p)\rangle = -F_\pi\left[\frac{ig_{K_A}}{m_{K_A}{}^2-q^2}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{m_{K_A}{}^2}\right)T^\nu+\frac{F_Kq_\mu}{m_K{}^2-q^2}T\right]. \quad (8)$$

In Eq. (8) the amplitudes T^ν and T stand for

$$\begin{aligned}(2\pi)^4\delta^4(k-q-p)T^\nu&\equiv\langle\pi^0(k)K_A{}^\nu(-q)|\pi^-(0)K^+(p)\rangle,\\ (2\pi)^4\delta^4(k-q-p)T&\equiv\langle\pi^0(k)K^0(-q)|\pi^-(0)K^+(p)\rangle.\end{aligned} \quad (9)$$

The off-shell amplitude T^ν may be expanded in terms of invariant amplitudes

$$T^\nu=k^\nu G(q^2)-q^\nu H(q^2). \quad (10)$$

Equations (1), (8), and (10) imply

$$f_+(q^2)=\frac{-i}{\sqrt{2}}\frac{F_\pi g_{K_A}}{m_{K_A}{}^2-q^2}G(q^2), \quad (11)$$

$$f_-(q^2)=\frac{F_\pi}{\sqrt{2}}\left\{\frac{2F_KT(q^2)}{m_K{}^2-q^2}+\frac{ig_{K_A}}{m_{K_A}{}^2-q^2}\left(1-\frac{2q\cdot k}{m_{K_A}{}^2}\right)G(q^2) - \frac{2ig_{K_A}}{m_{K_A}{}^2}H(q^2)\right\}. \quad (12)$$

We observe that the two form factors have been individually determined in terms of the off-shell $\pi K\to\pi K$ and $\pi K_A\to\pi K$ amplitudes. The Adler condition for T^ν ensures that $G(q^2)$ vanishes at $m_{K_A}{}^2$.[12] T and T^ν are related via the divergence condition:

$$T=i\frac{g_{K_A}}{F_Km_{K_A}{}^2}q_\mu T^\mu = i\frac{g_{K_A}}{F_Km_{K_A}{}^2}[(q\cdot k)G(q^2)-q^2H(q^2)]. \quad (13)$$

The derivation of Eq. (13) is straightforward[13] and involves the essential use of PCAC for the kaon field and of the field-current identity [Eq. (7)].

Given specific models for $T(q^2)$ and $G(q^2)$, with $H(q^2)$ being related via Eq. (13), one can compute $f_+(q^2)$ and $f_-(q^2)$. An obvious choice would be the celebrated Veneziano ansatz.[14] Such models have been exhibited in the literature.[15] In view of the arbitrariness in the selection of the Veneziano structures appropriate to these scatterings, and the more fundamental unresolved questions concerning the incorporation of currents[16] (i.e., variable q^2) into the Veneziano framework as well as the nature of the mechanism of $SU(3)\otimes SU(3)$ breaking compatible with the Veneziano form,[17] it seems clear that any such attempt would lead to model-dependent results. Therefore, we shall not pursue this aspect any further here. We shall be, however, specifically interested in extracting model-independent information on, and also the model-dependent corrections to, the observables in K_{l3} decay.

[8] V. Mathur and S. Okubo, Phys. Rev. D 2, 619 (1970); M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[9] R. Dashen, Phys. Rev. 183, 1245 (1969).

[10] R. Acharya, P. Narayanaswamy, and T. Santhanam, Phys. Rev. D 1, 3403 (1970).

[11] T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters 18, 1029 (1967); we use $p^2=+m^2$; $g_{K_A}\epsilon_\mu=\langle 0|A_\mu{}^{6-i7}(0)|K_A\rangle$.

[12] S. Adler, Phys. Rev. 137, B1022 (1965).

[13] H. Schnitzer, Phys. Rev. Letters 22, 1154 (1969); R. Arnowitt, M. Friedman, P. Nath, and Y. Srivastava, *ibid.* 22, 1158 (1969).

[14] G. Veneziano, Nuovo Cimento 57A, 190 (1968).

[15] Fayyazuddin and Riazuddin, Ann. Phys. (N. Y.) 55, 131 (1969).

[16] D. Freedman, Phys. Rev. D 1, 1133 (1970).

[17] J. Ellis and H. Osborn, Phys. Letters 31B, 580 (1970).

Assumption (iv), which demands that $f_+(q^2)$ and $f_-(q^2)$ be free of poles at $q^2=0$, is certainly a reasonable physical assumption and has the consequences that

$$\lim_{q^2\to 0} q^2G(q^2)=\lim_{q^2\to 0} q^2H(q^2)=0\,. \qquad (14)$$

Equation (14) becomes relevant, for example, in the framework of a Veneziano model which is known to give rise to fixed poles in off-shell amplitudes such as $H(q^2)$, thereby contradicting Eq. (14).[18]

We observe that Eqs. (11)–(13) imply

$$f_-(q^2)=\frac{\sqrt{2}F_\pi F_K m_K{}^2}{q^2(m_K{}^2-q^2)}T(q^2)-\frac{(m_K{}^2-m_\pi{}^2)}{q^2}f_+(q^2)\,. \qquad (15)$$

We emphasize that the validity of Eq. (15) is in no way dependent on assumption (iii), since it is seen to follow directly from Eq. (6) on taking its divergence.

We now expand

$$f_\pm(q^2)=f_\pm(0)\left(1+\frac{\lambda_\pm}{m_\pi{}^2}q^2+\cdots\right) \qquad (16)$$

and employ the Weinberg expansion

$$T(q^2)=T(0)(1+aq^2+bq^4+\cdots)\,. \qquad (17)$$

Equations (15)–(17) imply

$$f_+(0)=\frac{\sqrt{2}F_\pi F_K}{m_K{}^2-m_\pi{}^2}T(0)\,, \qquad (18)$$

$$f_-(0)=\left(a+\frac{1}{m_K{}^2}\right)\sqrt{2}F_\pi F_K T(0)-\frac{m_K{}^2-m_\pi{}^2}{m_\pi{}^2}\lambda_+ f_+(0)\,. \qquad (19)$$

We evaluate a with the help of the Adler condition[12] (for the pion)

$$T(q^2=m_K{}^2,\, p^2=m_K{}^2,\, k^2=m_\pi{}^2)=0 \qquad (20)$$

and the Mathur-Okubo sum rule evaluated at the "best" value Δ_0, which is so chosen that there are no terms proportional to $\epsilon_\pi\epsilon_K\epsilon_8$ in Eq. (4). According to Ref. 8 the best value turns out to be

$$\Delta_0=\frac{(m_K{}^2-m_\pi{}^2)^2(4m_K{}^2+m_\pi{}^2)}{(2m_K{}^2+m_\pi{}^2)^2}\,. \qquad (21)$$

Thus we obtain

$$\xi(0)+\lambda_+\frac{m_K{}^2-m_\pi{}^2}{m_\pi{}^2}+1-\frac{F_K}{F_\pi f_+(0)}=\frac{m_\pi{}^2}{m_K{}^2-m_\pi{}^2}\left[\frac{1}{f_+(0)}\left(\frac{F_K}{F_\pi}+\frac{F_\pi}{F_K}\right)-2\right]+\lambda_+\frac{m_K{}^2(m_K{}^2-m_\pi{}^2)}{(2m_K{}^2+m_\pi{}^2)^2}\,. \qquad (22)$$

[18] R. Arnowitt, M. Friedman, and P. Nath, Phys. Rev. D **1**, 1813 (1970).

The right-hand side of Eq. (22) can be estimated numerically using the Glashow-Weinberg[19] relation

$$f_+(0)=\frac{F_K{}^2+F_\pi{}^2-F_\kappa{}^2}{2F_KF_\pi} \qquad (23)$$

and $\lambda_+\approx 0.08$, and is consistent with zero.

Since the validity of Eq. (23) does not depend on the specific manner of chiral breakdown,[20] it may be used in conjunction with the left-hand side of Eq. (22) to arrive at the Dashen-Weinstein theorem [Eq. (3)].

It is worth pointing out that Eq. (22) does *not* depend on the field-current identity, since it is a direct consequence of Eq. (15), *but it ignores PCAC corrections.* This could be amended by invoking the Dashen-Weinstein formalism.[21]

We shall adopt a more phenomenological path suggested by Pagels.[22] The essential idea is to modify assumption (ii) to read

$$\partial^\mu A_\mu{}^{\pi,K}(x)=F_{\pi,K}m_{\pi,K}{}^2\phi_{\pi,K}(x)-\Delta_{\pi,K}F_{\pi,K}J_{\pi,K}(x)\,, \qquad (24)$$

where $J_{\pi,K}(x)$ is the source of the (pion, kaon) field; $\Delta_\pi=1-\sqrt{2}Mg_A/gF_\pi$ is the deviation from the Goldberger-Treiman relation and hence measures the departure from exact $SU(2)\otimes SU(2)$ and massless pions. (Δ_K is defined in a similar fashion.) We observe that since $m_{\pi,K}{}^2=O(\epsilon_{\pi,K})$, the pole contribution is of order unity and the source term will be $O(\epsilon_{\pi,K})$ for small q^2, thereby including PCAC corrections to this order within this framework.

Equation (24) leads to the following consequences:

(i) Equations (11) and (12) will now receive contributions from the modification of both assumptions (ii) and (iii) of the order of $O(\epsilon_\pi)$ and $O(\epsilon_K)$, respectively; these corrections become relevant in constructing models for $f_\pm(q^2)$, and hence are of no interest to us here.

(ii) Equation (15) now reads

$$f_-(q^2)=\frac{\sqrt{2}F_\pi F_K}{q^2}(1-\Delta_\pi)\left[\frac{m_K{}^2}{m_K{}^2-q^2}-\Delta_K\right]T(q^2)-\frac{m_K{}^2-m_\pi{}^2}{q^2}f_+(q^2) \qquad (25)$$

and has the consequence that

$$\xi(0)_{\rm new}\approx\xi(0)_{\rm old}+\frac{m_K{}^2-m_\pi{}^2}{m_K{}^2}\Delta_K{}^2\,. \qquad (26)$$

Since $\Delta_K=O(\epsilon_K)$, the corrections due to kaon PCAC

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[22] H. Pagels, Phys. Rev. **182**, 1913 (1969).

are of second order in chimeral $SU(3)$-breaking symmetry, and Eq. (26) is independent of corrections to pion PCAC. Taking $\Delta_K \approx -0.2$[23] we see that the Eq. (22) is essentially unchanged for all practical purposes. We end wth a few comments.

(i) We observe that the main contributions to $\xi(0)$ depend only on the observables λ_+ and $F_K/F_\pi f_+(0)$. Taking $\lambda_+ \approx 0.08$ and $F_K/F_\pi f_+(0) \approx 1.28$[19] we obtain $\xi(0) \approx -0.6$, as is favored by the recent trend in experimental data.[4]

(ii) In the present approach to K_{l3} decay, no smoothness assumption as a function of p^2 or k^2 has been invoked, in contrast to the earlier investigations.[24]

[23] See C. Chan and F. Meiere, Phys. Rev. **175**, 2222 (1968), and references therein.

(iii) The scalar form factor does *not* exhibit a zero between $q^2 = (m_K - m_\pi)^2$ and $q^2 = (m_K + m_\pi)^2$, and hence does not satisfy the criterion of Ref. 5 to yield a large negative $\xi(0)$. One is, nonetheless, led here to predict $\xi(0) \approx -0.6$.

(iv) The soft-pion theorem of Eq. (2) is satisfied up to 10%, as may be verified from Eqs. (25) and (26).

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Nonrelativistic Separable-Potential Quark Model of the Hadrons

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A nonrelativistic quark model of the hadrons based on nonlocal separable potentials is presented. The model contains a quark-antiquark force and an effective three-body force among quarks, replacing the two-body forces. For single-parameter potentials in which the strength of the interaction drops sharply as the quark relative momenta increase, reasonable values of the lifetimes and radii of the mesons are found. A baryon wave function useful for dynamical calculations is obtained. The validity of the nonrelativistic approximation for this model is confirmed.

I. INTRODUCTION

SINCE the work of Gell-Mann[1] and Zweig,[2] a growing number of physicists have found it helpful to discuss hadronic matter in terms of entities called quarks. These quarks may be elementary excitations (quasiparticles) of some underlying hadronic field, or they may be massive particles. Various reports of experimental effects which might be ascribed to quarks have appeared.[3–6] Although the question of the existence of quarks is still open, the utility of the quark concept is sufficient to justify the attempt to create dynamical models of the hadrons based on their assumed existence.

Nonrelativistic quark models were among the first to be investigated because of their simplicity and the wide array of theoretical tools which can be applied to them. Although justifications for the validity of these models have been presented,[7] the expectation value of the quark momenta is of the order of the quark mass if the usual Yukawa-type potential is used, invalidating the nonrelativistic approximation. However, there is no *a priori* reason for using a local potential in a quark model.

Consequently, we were led to consider nonrelativistic nonlocal separable-potential quark models of the hadrons. The use of separable-potential models to describe the scattering of subatomic particles has often been dismissed as unphysical because (a) they are nonlocal and (b) a single-term separable potential can

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