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Photon-Photon Collisions, a New Area of Experimental Investigation in High-Energy Physics*

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The purpose of this paper is to show that the conditions of photon-photon collisions at high energy can be almost perfectly reproduced by using the "quasi-real" photon spectra originating from electron-positron colliding beams. We show that (a) the problem of background elimination can be properly solved by detecting both the outgoing electron and positron at very small angles with respect to their incident directions; (b) in spite of this very stringent restriction on phase space, and of possible additional restrictions due to experimental conditions, reasonably high counting rates will be achieved with the new electron-positron storage rings (of beam energy 2–3 GeV and of luminosity $\sim 10^{32}$ cm^{-2} sec^{-1}) now planned or under construction; and (c) these counting rates increase with rising beam energy. We discuss a number of applications: particle-pair creation (e^-e^+, $\mu^-\mu^+$, $\pi^-\pi^+$, K^-K^+, $p\bar{p}$, ...) or single-particle creation (π^0, η, η'). Angular distributions of particles produced are also shown.

I. INTRODUCTION

THE feasibility of photon-photon collision experiments was studied a few years ago by Csonka.[1] With regard to high-energy $\gamma\gamma$ collisions, he suggested the use of bremsstrahlung photons from electron accelerators or storage rings. Technical difficulties involved in this idea are, however, considerable.

The suggestion presented here is to use, instead of free photons, the "quasi-real" photon spectra originating from electron-positron colliding beams. Major technical difficulties are thus avoided, the experimental problem being reduced to that of proper detection of the outgoing particles.

This idea is not entirely new, although it has never been presented in this way before. Its main philosophy was already contained in two old papers, written by Low[2] and by Calogero and Zemach.[3] Low specialized his study to the case of π^0 production, through two virtual photons, in electron-electron and electron-positron collisions. The paper of Calogero and Zemach covered a wider scope; particle pair creation, in particular, the process $e+e \to e+e+\pi^-+\pi^+$, was considered by those authors.

Strangely enough, these fundamental papers were to a large extent forgotten by high-energy physicists for a number of years. It is true that in the first generation of e^-e^- and e^-e^+ storage rings (at Stanford, Orsay, and Novosibirsk) the luminosity was too weak for these processes to be studied seriously. Then, when the second generation was planned—at Frascati, Novosibirsk, Cambridge (U.S.A.), Stanford, Hamburg, and Orsay—the attention of most high-energy physicists was apparently so intensely focused on the—indeed very fascinating—electron-positron annihilation processes that the experiments suggested by Low and by Calogero and Zemach remained practically unmentioned.

In 1969, the authors[4] started a preliminary theoretical investigation of photon-photon collisions to be produced with electron-positron colliding beams. They were able to show (a) that the problem of background elimination—which is fundamental in most of these processes—can be properly solved by detecting both the outgoing electron and positron at small angles with respect to their incident directions; (b) that the cross sections are high enough, for a number of these processes, to obtain reasonably high counting rates with electron-positron colliding beams of beam energy 2–3 GeV and luminosity $\sim 10^{32}$ cm^{-2} sec^{-1}; and (c) that these cross sections increase with rising beam energy, in sharp contradistinction to the behavior of the cross sections for annihilation processes.

More recently, Balakin, Budnev, and Ginzburg,[5] and Brodsky, Kinoshita, and Terazawa[5] also considered the

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[1] P. L. Csonka, CERN Report No. TH. 772, 1967 (unpublished); Phys. Letters **24B**, 625 (1967).

[2] F. Low, Phys. Rev. **120**, 582 (1960). An improvement of this author's calculation was given by J. C. le Guillou, thèse de troisième cycle, Paris, 1965 (unpublished).

[3] F. Calogero and C. Zemach, Phys. Rev. **120**, 1860 (1960). A study of the process $e+e \to e+e+\pi^-+\pi^+$, but restricted to the $\sigma(0^{++})$ resonance, was also made by P. C. de Celles and J. E. Goehl, Jr., Phys. Rev. **184**, 1617 (1969).

[4] N. Arteaga-Romero, A. Jaccarini, and P. Kessler, Compt. Rend. **269B**, 153 (1969); **269B**, 1129 (1969). See also N. Arteaga-Romero, A. Jaccarini, and P. Kessler, Laboratoire de Physique Atomique Internal Report No. PAM 70-02, 1970 (unpublished); A. Jaccarini, thèse de troisième cycle, Paris, 1970 (unpublished); A. Jaccarini, N. Arteaga-Romero, J. Parisi, and P. Kessler, Nuovo Cimento Letters **4**, 933 (1970).

[5] S. J. Brodsky, T. Kinoshita, and H. Terazawa, Phys. Rev. Letters **25**, 972 (1970); V. E. Balakin, V. M. Budnev, and I. F. Ginzburg, Zh. Eksperim. i Teor. Fiz. v Pis'ma Redaktsiyu **11**, 559 (1970) [JETP Letters **11**, 388 (1970)]; in *Proceedings of the Fifteenth International Conference on High-Energy Physics, Kiev, 1970* (Academy of Sciences, U.S.S.R., Moscow, 1971).

electron-positron reactions involving photon-photon materialization, and stressed the quantitative importance of these reactions with respect to electron-positron annihilation. Energy dependence of the cross sections, and angular distributions of the particles produced, were investigated by these authors. However, they did not consider the problem of proper background suppression, and they used mainly rough approximations in their calculations.

In this paper, we are going to study (1) background elimination, and orders of magnitude of total cross sections, and (2) energy dependence of the cross sections, and angular distributions of the particles produced.

A number of applications are discussed. In the Appendix, the Williams-Weizsäker–type formulas used in our calculations are justified.

II. BACKGROUND ELIMINATION, AND ORDERS OF MAGNITUDE OF TOTAL CROSS SECTIONS

We are interested in reactions of the type

$$e^-+e^+\to e^-+e^++\text{anything};$$

however, we shall to a large extent confine our study to reactions

$$e^-+e^+\to e^-+e^++A^-+A^+,$$

where A is any charged particle (e,μ,π,K,p). To lowest order in electrodynamics, these reactions are represented by the six diagrams of Fig. 1. What we want

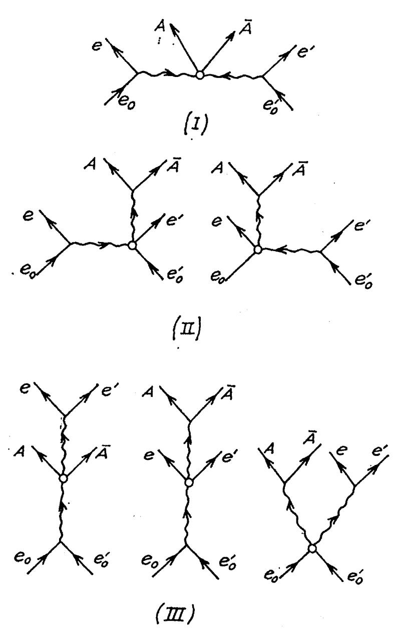

FIG. 1. Feynman diagrams for $e^-+e^+\to e^-+e^++A^-+A^+$. e_0 (e) is the incoming (outgoing) electron; e_0' (e') is the incoming (outgoing) positron. (For convenience, the latter is represented like a particle, not an antiparticle.) The circles represent sets of Compton-type subdiagrams.

(in order to reproduce, as perfectly as possible, the conditions of photon-photon collisions) is to define an experimental situation where practically only diagram I of Fig. 1 contributed, thus eliminating the background due to the five other diagrams.

It is quite intuitive to think that such a situation can be realized by detecting both the outgoing electron and the outgoing positron at extremely small angles with respect to the corresponding incident directions. Since in diagram (I) both photons exchanged are spacelike and have predominantly very small q^2 values (because of the q^{-4} factors in the propagators), both the electron and the positron should be very sharply peaked at 0° scattering angle in this diagram's contribution. It may thus be expected that the order of magnitude of the contribution of I will not be drastically reduced by introducing very small experimental cutoff angles for both the electron and the positron. In diagrams II, of Fig. 1, one spacelike and one timelike photon are exchanged; here we may expect that only one of both outgoing $e^\pm$ particles will be sharply peaked at 0° scattering angle, whereas the other one will be (very roughly speaking) more or less isotropically distributed. Thus, the experimental cutoff considered, operating on on both $e^\pm$ particles, should have a much more drastic effect on diagrams II than on diagram I. This suppression effect should be still much stronger on diagrams III of Fig. 1, where both photons exchanged are timelike and therefore both outgoing $e^\pm$ particles should be (again roughly speaking) more or less isotropically distributed.

We were able to show numerically this effect of background suppression, at least with respect to diagrams II. In Figs. 2 and 3, we show, for an assumed beam energy $E_0=2$ GeV and a cutoff angle $\theta_{\max}=1°$ and 4 mrad, respectively (assumed, in each case, to be the same for the electron and the positron), the contribution of diagrams I and II of Fig. 1 separately for the following cases: (a) $A=e$; (b) $A=\mu$; (c) $A=\pi$; (d) $A=K$. The curves shown represent the differential cross sections $d\sigma/dM$, where M is the invariant mass of the pair A^-A^+ created. For the case $A=e$, only pairs with $M>100$ MeV were considered. In the cases $A=\pi$ and $A=K$, the contributions of diagrams II are shown both without and with resonant enhancements, such an enhancement being due to the ρ in the pion case and to the ϕ in the kaon case.

We do not reproduce here the corresponding curves calculated for $\theta_{\max}=3°$, which were already shown (except for the case $A=e$) in our preliminary papers.[4]

It can be seen from our curves that (except for a very small region near the threshold in the kaon case), the contribution of diagrams II is always and everywhere totally negligible with respect to that of diagram I. (Notice that, because of C invariance, there is no interference between diagram I and diagrams II.) The suppression of background, due to the angular cutoff, is thus extremely efficient, still more at $\theta_{\max}=4$

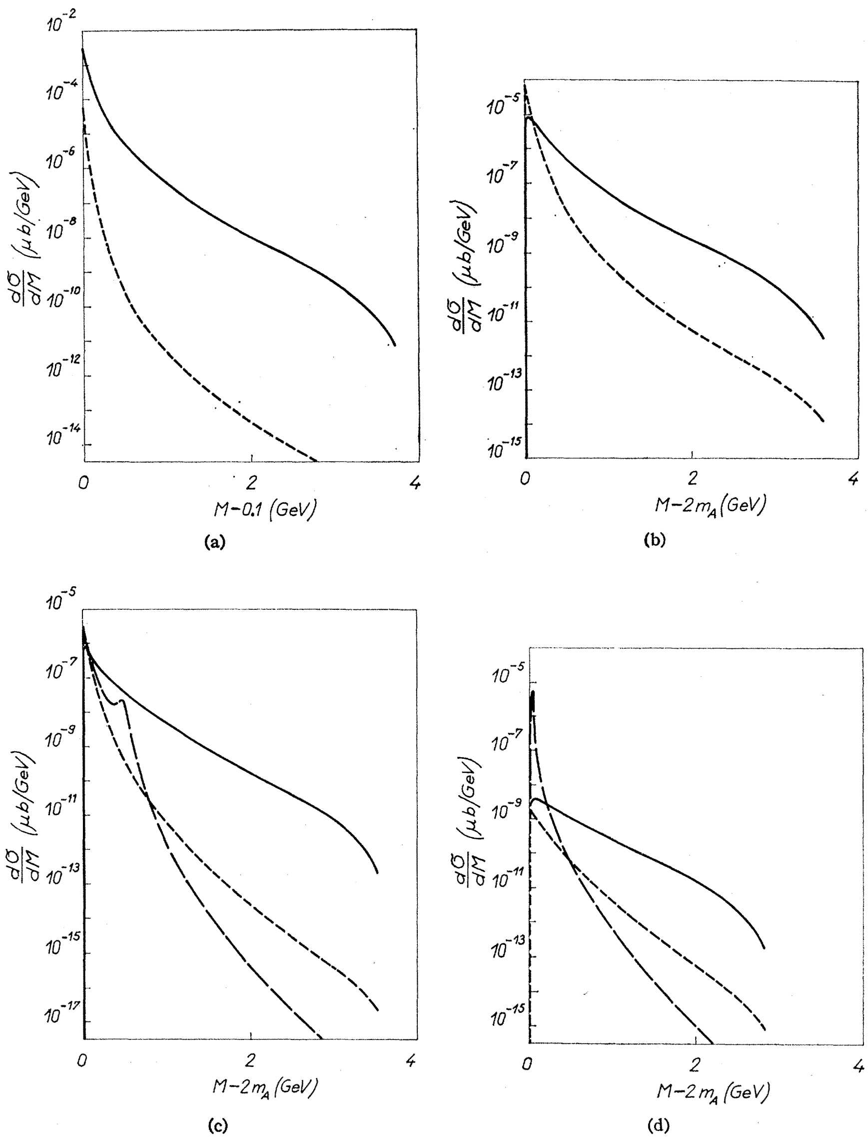

FIG. 2. Differential cross section $d\sigma/dM$ as a function of $M-2m_A$ (M=invariant mass of the pair A^-A^+; m_A=mass of A) at beam energy $E_0=2$ GeV, cutoff angle $\theta_{\max}=1°$ for both $e^\pm$. (a) Electron pair production (with lower cutoff $M_{\min}=0.1$ GeV). Solid line is the contribution of diagram I of Fig. 1, *divided by 10^4*; dashed line is the contribution of diagrams II of Fig. 1. (b) Muon pair production. Solid line is the contribution of diagram I of Fig. 1, *divided by 10^4*; dashed line is the contribution of diagrams II of Fig. 1. (c) Pion pair production. Solid line is the contribution of diagram I of Fig. 1, *divided by 10^4*; dashed line is the contribution of diagrams II of Fig. 1, according to QED; long-dashed line is the contribution of diagrams II of Fig. 1 with resonant (ρ) enhancement. (d) Kaon pair production. Solid line is the contribution of diagram I of Fig. 1, *divided by 10^4*; dashed line is the contribution of diagrams II of Fig. 1, according to QED; long-dashed line is the contribution of diagrams II of Fig. 1 with resonant (ϕ) enhancement.

mrad than at 1° or, *a fortiori*, at 3° (as was to be expected).

Once we have shown that diagrams II give a negligible contribution with respect to I, we may assume—because of the qualitative argument given above—that the contribution of diagrams III can be neglected *a fortiori*.

Considering now diagram I alone, we obtain, after integrating over M, the values for the total cross sections ($e^-+e^+\to e^-+e^++A^-+A^+$) at $E_0=2$ GeV. These are given in Table I. From these values, we may draw the following conclusions.

(1) The order of magnitude of the total cross section is in no case substantially changed when going to smaller and smaller values of the cutoff angle $\theta_{\max}$ whereas, on the other hand, we have seen that the background-rejection effect then becomes better and better. We may thus state that, in principle, the smallest possible cutoff angles should be used experimentally.[6]

(2) For electron, muon, and pion pairs, the cross sections are high enough to ensure that reasonably high counting rates will be achieved with future electron-positron storage rings (such as, for instance, that under construction at DESY-Hamburg) characterized by

[6] We shall not discuss here the problem of the feasibility of such experiments where the electron and positron should be detected at very small angles, in coincidence with particles produced at large angle with respect to the beam axis. We would only like to stress that we discussed this problem in much detail with experimentalists, in particular, with Professor Waloschek (DESY) and Professor Haissinski (Orsay), and that, according to these discussions, it will probably be solved. In this paper we shall also leave aside the problem, brought to our attention by Professor Haissinski, of a possible experimental background, due to the fact that electrons (and positrons) having radiated bremsstrahlung photons and traveling forward with reduced energy may lead to fortuitous coincidences.

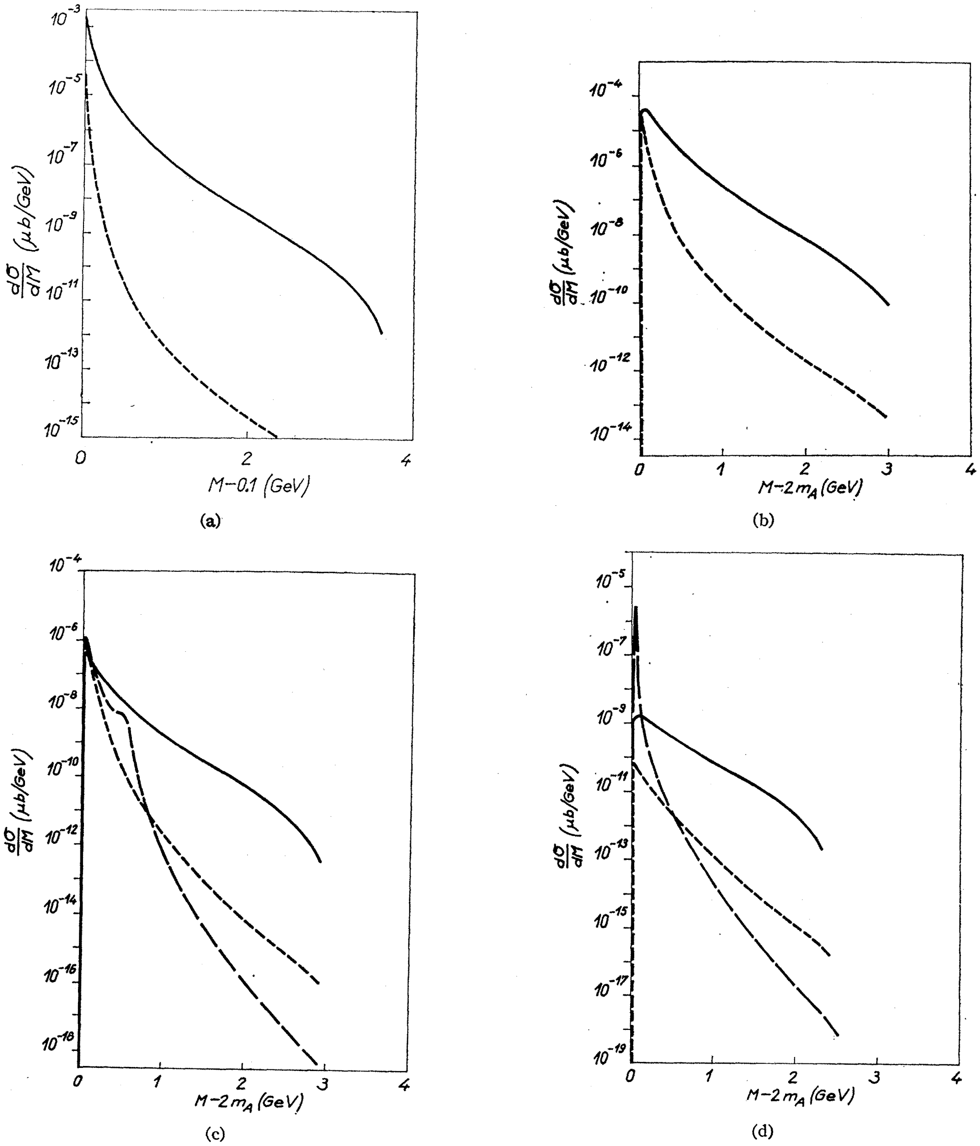

FIG. 3. Differential cross section $d\sigma/dM$ as a function of $M-2m_A$ at $E_0=2$ GeV, $\theta_{\max}=4$ mrad. (a) Electron pair production ($M_{\min}=0.1$ GeV); (b) muon pair production; (c) pion pair production; and (d) kaon pair production. All curves are characterized as in Fig. 2.

beam energies of a few GeV and luminosities of the order of 10^{32} cm^{-2} sec^{-1}. This statement should remain valid even when additional experimental cutoffs (on the energies of the scattered $e^{\pm}$ particles, and on the angles of the produced $A^{\pm}$ particles) are taken into account.

TABLE I. Total cross sections (in μb) for $e^-+e^+\rightarrow e^-+e^++A^-+A^+$ on the basis of diagram I of Fig. 1.

	$\theta_{\max}=3°$	$\theta_{\max}=1°$	$\theta_{\max}=4$ mrad
$A=e$ ($M>100$ MeV)	1.5	1.1	6.6×10^{-1}
$A=\mu$	2.4×10^{-2}	1.7×10^{-2}	9.0×10^{-3}
$A=\pi$	1.7×10^{-3}	1.2×10^{-3}	6.2×10^{-4}
$A=K$	2.5×10^{-5}	1.5×10^{-5}	6.2×10^{-6}

We now make some remarks about our calculations.

(i) In the case $A=e$, we neglected the exchange effect which appears because of the physical indistinguishibility between the scattered and the created electron (or positron). However, qualitative arguments (mainly the fact that the scattered electrons are predominantly high-energy, whereas the created electrons are predominantly low-energy particles) tell us that the exchange term should be small and thus should not modify substiantially the orders of magnitude obtained.

(ii) In our calculation of the contribution of the background diagrams II, we neglected the interference between those two diagrams. But even in the worst case (maximal constructive interefrence), this means only that our results for the contribution of II should be multiplied by a factor of 2. Obviously, such a factor does not change anything in our conclusions about background elimination.

(iii) To calculate the part of the process which corresponds to $\gamma+\gamma\rightarrow A^-+A^+$, the Born terms were systematically used by us. This procedure should be quite correct for $A=e$ and $A=\mu$, at least as long as we believe quantum electrodynamics (QED) to be valid for leptons and photons in the physical regions involved, but is doubtful for $A=\pi$ and $A=K$. Our main justification for the latter cases is that, in the present stage of strong-interaction theory, there is no better model available for such a calculation. On the other hand, we are well aware that calculations based upon the Born terms become rather bad at high energy, as soon as hadrons are involved; however, in our reactions, lower values of M always play a predominant role (see Figs. 2 and 3), and so we may hope that our calculation also has some legitimacy for the boson case.

(iv) So far, radiative corrections have been neglected throughout.

(v) Both $e^\pm$ particles being assumed to be scattered at very small angles, small-transfer approximations (of the Williams-Weizsäcker type) were applied systematically in our calculations. These approximations will be justified in the Appendix.

In connection with point (v), we would like to stress that the q^2 values for both virtual photons of diagram I of Fig. 1 are really extremely small under the conditions defined, since they range from a few keV^2 to a few MeV^2, with the lower values predominating. So we may consider these photons as quasi-real and treat them, to a very good approximation, like photons on their mass shell. This means in particular that their longitudinal components may be ignored in the reactions considered, and that the electromagnetic form factors involved at photon-hadron vertices can be taken equal to 1.

Thus, once we "factorize out" the electron-photon and positron-photon vertices in diagram I, we are entitled to look on reactions of the type

$$e^-+e^+\rightarrow e^-+e^++A^-+A^+$$

as practically equivalent, under the conditions defined, to

$$\gamma+\gamma\rightarrow A^-+A^+,$$

where both γ's are free photons.

III. ENERGY DEPENDENCE OF CROSS SECTIONS, AND ANGULAR DISTRIBUTIONS OF PARTICLES PRODUCED

A. Energy Dependence of Total Cross Sections

In Fig. 4, we show the behavior of the total cross sections $\sigma(e^-+e^+\rightarrow e^-+e^++A^-+A^+)$, where A is identified with e, μ, π, and K, respectively, as functions of the beam energy E_0 which was varied between 1 and 10 GeV. A cutoff angle $\theta_{\max}=4$ mrad for the scattered electron and positron was used in all cases.

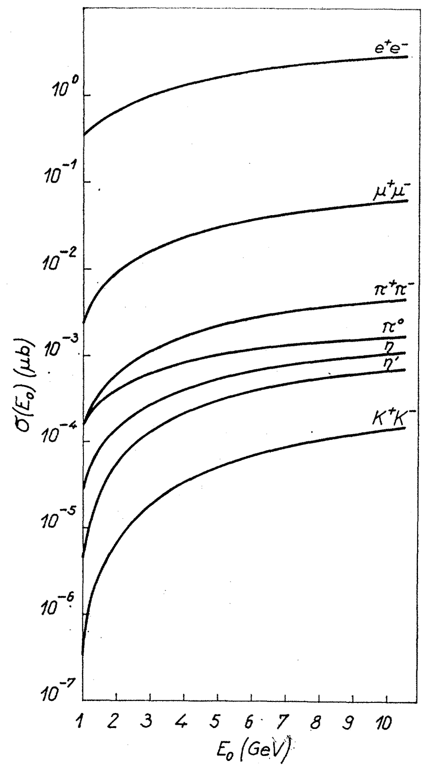

FIG. 4. Variation of the total cross section σ with the beam energy E_0, the $e^\pm$ cutoff angle being fixed at $\theta_{\max}=4$ mrad. The reactions considered are $e^-+e^+\rightarrow e^-+e^++A^-+A^+$, where $A^-A^+=e^-e^+$ (with $M_{\min}=0.1$ GeV), $\mu^-\mu^+$, $\pi^-\pi^+$, K^-K^+; $e^-+e^+\rightarrow e^-+e^++X$, where $X=\pi^0, \eta, \eta'$.

As can be seen, all cross sections increase with rising incident energy. This fact, which was already stressed before,[4,5] obviously makes a very sharp difference in the behavior of the cross sections of annihilation processes $(e^-+e^+\rightarrow A^-+A^+)$; it is all the more important because electron-positron storage rings with energies still much higher than those presently running or under construction may be planned in the future.

With respect to the creation of hadrons, the way our cross sections increase is in fact model dependent. But in any case, they *must* increase with rising E_0, just because at any value of the invariant mass M, the photon intensity becomes higher. This phenomenon is connected with the quite general effect—familiar to the experimentalists—that the multiplicities of secondaries are strongly increased when the energy of the primary beam becomes larger.

In Fig. 4, we also show the energy dependence of the cross sections for reactions of the type

$$e^-+e^+\rightarrow e^-+e^++X,$$

where X is any quasistable pseudoscalar particle, i.e., π^0, η, or η'. The corresponding diagrams (to lowest order in electrodynamics) are shown in Fig. 5. Here

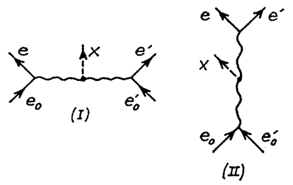

FIG. 5. Feynman diagrams for $e^-+e^+\to e^-+e^++X$. Characterization of the $e^\pm$ particles is the same as in Fig. 1.

again, taking a cutoff angle $\theta_{\max}=4$ mrad for both $e^\pm$, diagram II of Fig. 1 could be totally neglected, and both virtual photons could be treated as quasi-real. It is worthwhile to stress that for these reactions one would be able to give absolutely precise predictions, if the decay width $\Gamma(X\to 2\gamma)$ were exactly known. Here we took the values $\Gamma(\pi^0\to 2\gamma)\simeq 11$ eV [7]; $\Gamma(\eta\to 2\gamma)\simeq 1$ keV [7]; $\Gamma(\eta'\to 2\gamma)\simeq 5$ keV.

B. Angular Distributions of Particles Produced

Taking again $E_0=2$ GeV and $\theta_{\max}=4$ mrad, the angular distributions obtained for the cases $A=e$, μ, π, and K are shown in Fig. 6. Here, the abscissa ψ is the lab angle, with respect to the beam axis, of the momentum of the particle $A^\pm$ considered; the ordinate is defined as $\sigma^{-1}d\sigma/d\Omega$, which means that our values are normalized in such a way that one obtains 1 in all cases when integrating over Ω (the solid angle of the particle $A^\pm$).

As was to be expected, the curves are more peaked (forward and backward) where the mass of $A^\pm$ is smaller. Notice that the curve obtained for muons should have been slightly more peaked than that obtained for pions (owing to the slight difference in mass, and also to some difference in the dynamics); however, this difference was so small that it was impossible to distinguish both curves graphically with the logarithmic scale used.

Qualitatively speaking, it can be seen that, in spite of the nonisotropic character of these curves, no very drastic cutdown in the orders of magnitude of the cross sections for production of muon, pion, and kaon pairs will result from an angular cutoff such as may be required by the experimental conditions, i.e., a limitation of ψ to values ranging from some lower limit $\psi_{\min}$ (possibly of the order of 30° or so) to a higher limit $\pi-\psi_{\min}$. (Notice that the effects of such an angular restriction cannot be directly read off from our curves, since, in a quadruple coincidence experiment, not only the particle $A^\pm$ considered, but also its counterpart with the opposite charge, should be submitted to this restriction.)

For the electron pairs produced, the effect of such an angular cutoff will obviously be much more striking; but here the total cross section is so large (see Sec. II) that in any case we do not have to worry about the counting rates remaining after such a cutoff.

Considering the reactions $e^-+e^+\to e^-+e^++X$ ($X=\pi^0$, η, or η'), and assuming that in each case the particle X will be identified thanks to its decay into two photons (which provides about 100% of the total decay rate in the case of π^0, about 40% in the case of η,[8] and about 5% in the case of η [9]), we show in Fig. 7 the angular distribution calculated for either of these photons. Here the ordinate is $\sigma'^{-1}d\sigma'/d\Omega$, where σ' is defined as

$$\sigma'=\sigma(e^-+e^+\to e^-+e^++X)\Gamma(X\to 2\gamma)/\Gamma(X\to \text{total}).$$

The main problem, in an experiment where the two decay photons are to be observed, is that of contamination from double bremsstrahlung produced in the electron-positron collision. This problem is now under investigation.

IV. APPLICATIONS

A. $\gamma+\gamma\to\pi^-+\pi^+$

The $\gamma+\gamma\to\pi^-+\pi^+$ reaction is certainly the most important application for the near future. As has already been stressed,[3] conservation laws allow only dipion states with charge-conjugation number +1, parity +1, and even angular momentum to be produced here. It will be particularly interesting, in a first experiment, to look for the (not yet firmly established) 0^{++} resonance called σ or ϵ, with mass ~600–700 MeV and width ~150–400 MeV.[10] The experiment suggested has the double advantage that the ρ is completely absent (because of our background elimination) and that there is no spectator hadron.[11] Other states to look for are the 2^{++} resonances f, f', ...

It is a nice problem for theorists as well to apply modern theories of strong interactions (Regge poles, duality, Veneziano model, etc.), possibly in combination with the vector-dominance model, to these reactions. Presently, however, these theories have only little predictive power.

B. $\gamma+\gamma\to\mu^-+\mu^+$

One may consider the $\gamma+\gamma\to\mu^-+\mu^+$ reaction from two different points of view. One may believe in com-

[7] B. Richter, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), pp. 13–14.

[8] A. Barbaro-Galtieri *et al.*, Rev. Mod. Phys. **42**, 87 (1970); see p. 118.

[9] D. Bollini *et al.*, Nuovo Cimento **58A**, 289 (1968).

[10] Ref. 8, p. 128.

[11] There is some relation to the suggestion made by Renard [F. M. Renard, in Proceedings of the Moriond Meeting on Electromagnetic Interactions, 1969, p. 36 (unpublished); Nuovo Cimento **62A**, 475 (1969). See also M. J. Creutz and B. Einhorn, Phys. Rev. Letters **24**, 341 (1970); Phys. Rev. D **1**, 2537 (1970); A. Q. Sarker, Phys. Rev. Letters **25**, 1527 (1970).] to study the reaction $e^-+e^+\to\pi^-+\pi^++\gamma$ in the neighborhood of a vector-boson resonance, preferably the ϕ. The same structure ($\gamma\gamma\pi\pi$) would be involved, but in a different physical region. The experiment suggested by Renard has the advantage, with respect to ours, of requiring less energy and less luminosity; on the other hand, it involves difficult background problems, and obviously the photon in the final state is not a desirable spectator.

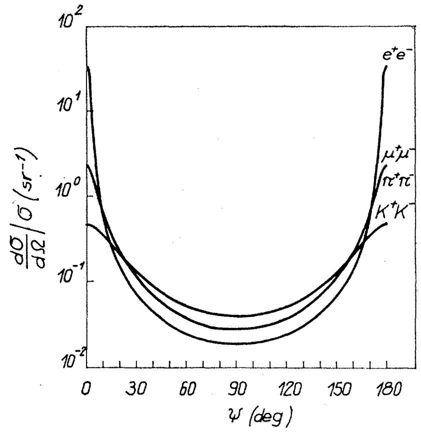

FIG. 6. Angular distribution in the lab frame for a particle $A^\pm$ emitted in the process $e^-+e^+\to e^-+e^++A^-+A^+$, where $A^-A^+=e^-e^+$ (with $M_{\min}=0.1$ GeV), $\mu^-\mu^+$, $\pi^-\pi^+$, K^-K^+. Beam energy: $E_0=2$ GeV; cutoff angle: $\theta_{\max}=4$ mrad. (Curves for $\mu^-\mu^+$ and $\pi^-\pi^+$ coincide.)

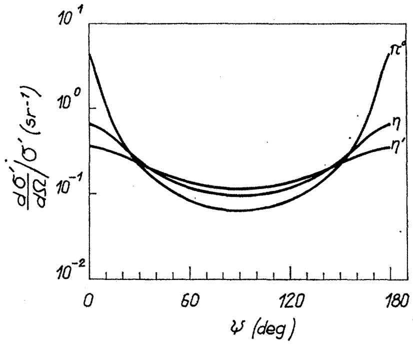

FIG. 7. Angular distribution in the lab frame for a photon emitted in the process $e^-+e^+\to e^-+e^++X$ (where $X=\pi^0$, η, η') followed by the decay $X\to 2\gamma$. Beam energy: $E_0=2$ GeV; cutoff angle: $\theta_{\max}=4$ mrad.

plete "normality" of the muon with respect to QED, and the theoretical predictions given for this process are assumed to be absolutely accurate. This experiment can then serve as a control experiment—i.e., an experiment for the checking the detection device (solid angles, acceptance factors, etc.), as well as radiative corrections and other corrections used for normalizing the cross sections—in order to ensure that the data obtained in the above-mentioned experiment (Sec. IV A) should be established on a firm ground.

Alternatively, one may believe that the muon is not entirely "normal"—as some very recent experimental results seem to show[12,13]—and then this reaction would provide a very clean experiment to look for an anomaly. In fact, any deviation, to be found experimentally, with respect to the Born-term calculation, would be due to such a muon anomaly (for instance, a dimuon exchanged in the s channel, or a μ^* exchanged in the t or u channel). Let us also stress that, from this point of view, the experiment considered would be much more significant than the annihilation process $e^-+e^+\to\mu^-+\mu^+$, because in the latter neither a dimuon (other than a 1^- state) nor a μ^* can appear.

C. $\gamma+\gamma\to e^-+e^+$

Since most people regard QED as valid for electrons and photons at least, the $\gamma+\gamma\to e^-+e^+$ reaction should essentially provide a control experiment.

[12] R. L. Thompson *et al.*, Phys. Rev. **177**, 2022 (1969); J. C. Montret, in Proceedings of the Moriond Meeting on Electromagnetic Interactions, 1970, p. II. 74 (unpublished). Both papers refer to a possible muon anomaly showing up in inelastic muon scattering at high energy (10–12 GeV) and mainly small momentum transfer.

[13] Talk given by Dr. Ramm at the Durham Conference on Elementary Particles, Durham, 1970 (unpublished). This talk referred to the possible occurrence of a "heavy muon" μ^* with mass ~430 MeV.

D. $\gamma+\gamma\to K^-+K^+$

As in the case of pion pairs, states with $C=+1$, $P=+1$, and even angular momentum will be studied in the $\gamma+\gamma\to K^-+K^+$ reaction. Unfortunately, the cross sections are low, so that one will probably have to wait for storage rings with energies and/or luminosities higher than those of the next generation, to perform such kaon pair-production experiments.

E. $\gamma+\gamma\to p+\bar{p}$

In the case of the $\gamma+\gamma\to p+\bar{p}$ reaction, again, and *a fortiori*, one will have to wait for a future generation of more powerful electron-positron storage rings. It is encouraging to notice that, in a recent experiment,[14] two events were apparently found for the inverse reaction ($p+\bar{p}\to\gamma+\gamma$) at an invariant mass squared of 5.1 GeV^2; this result should correspond to a rather high cross section (about 30 times higher than the value obtained in a Born-term calculation with a Dirac proton).

F. $\gamma+\gamma\to X$ ($X=\pi^0$, η, or η')

The values obtained through the Primakoff effect for $\Gamma(\pi^0\to 2\gamma)$ and $\Gamma(\eta\to 2\gamma)$[7] may be improved by performing the $\gamma+\gamma\to X$ ($X=\pi^0$, η, or η') experiments, and the value of $\Gamma(\eta'\to 2\gamma)$ determined.

A variant of the scheme suggested here was proposed and extensively studied by one of the authors (J.P.).[15] Namely, instead of detecting both $e^\pm$ in their forward directions, one should detect only one of them at very small scattering angle, and the other at large angle. Thus, only one of the photons exchanged would be quasi-real, whereas the other one would be highly virtual. In this way, one should be able to study experimentally the electromagnetic form factors of the $X\gamma\gamma$ vertices in the spacelike region.

[14] D. L. Hartill *et al.*, Phys. Rev. **184**, 1415 (1969).

[15] J. Parisi, thèse de troisième cycle, Paris, 1970 (unpublished).

G. $\gamma+\gamma \rightarrow$ Any Hadrons

Doubtlessly, there is a very wide field of applications in the $\gamma+\gamma \rightarrow$ any hadron experiment. Let us only mention the possibility of studying all nonstrange mesonic resonances with $C=+1$, by direct formation, without any spectator hadron.

H. $\gamma+\gamma \rightarrow \gamma+\gamma$

From the theorists' point of view, the $\gamma+\gamma \rightarrow \gamma+\gamma$ reaction is a field of study in itself, because of its extreme complexity. A wide variety of hadrons may be exchanged in all three (s, t, and u) channels, and lepton pairs are exchanged as well.[16] Experimentally, however, one may wonder whether this effect will not be dominated, even at large emission angles of the outgoing photons, by double bremsstrahlung from the electron-positron collision. We are now investigating this question.

V. CONCLUSION

Since in the mind of many physicists electron-positron storage rings are still mainly associated with annihilation, it is perhaps not uninteresting to compare the respective merits and difficulties of both types of experiments, i.e., photon-photon materialization and electron-positron annihilation.

It appears that, in comparison with the annihilation processes, photon-photon collisions show two main disadvantages and two main advantages.

The disadvantages are as follows.

1. Greater experimental difficulties, since two additional particles (i.e., both $e^{\pm}$ scattered at small angle) are to be detected, together with the particles created.

2. Lower collision energy for the same beam energy. As can be seen from Figs. 2 and 3, the photon-photon materialization occurs predominantly in the region of small M, so that only a fraction of the total available energy is used for this process, whereas the full energy is used in electron-positron annihilation.

The advantages are as follows.

1. Higher cross sections. Very roughly speaking, we can see that the cross section for $e^-+e^+ \rightarrow e^-+e^+ +A^-+A^+$ becomes already comparable to that of $e^-+e^+ \rightarrow A^-+A^+$ (with the same A) at beam energies which are in general of the order of 1 GeV.[17] At $E_0=10$ GeV, photon-photon materialization should give rise to counting rates at least 2 or 3 orders of magnitude higher than the corresponding ones for electron-positron annihilation.

2. Greater physical interset, at least in our opinion (physical interest can rarely be defined in a completely objective way). In e^-e^+ annihilation, as long as it goes via one-photon exchange—and so far we do not know anything about the possible occurrence of annihilation via two photons—, only 1^{--} states can be produced. In $\gamma\gamma$ materialization, the possibilities offered for creation of hadronic states are incomparably wider; in particular, all angular momentum and parity states can be produced (except in a few cases, such as the production of pairs of pseudoscalar particles).

However, so far, we must not look upon these two kinds of phenomena from a point of view of competition or rivalry. From a practical standpoint, the important fact is that both converge to provide overwhelming reasons for building more powerful (higher energy, higher luminosity) electron-positron storage rings in the near future. Photon-photon collisions will then provide a new area of experimental investigation in high-energy physics.

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APPENDIX: DETAILS OF CALCULATIONS

To establish the double Williams-Weizsäcker–type approximation formula used in our calculation of diagram I of Fig. 1, we started from the generalized (invariant) helicity method developed by one of us (P.K.).[18] The original purpose of this method was to achieve considerable simplifications in the (exact) calculation of large classes of Feynman diagrams occurring in electrodynamics and weak interactions.

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[16] J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley, Cambridge, Mass., 1955), p. 287.

[17] In connection with this point, the question has been raised whether the recent Frascati measurements on annihilation were not contaminated to some extent by the type of processes studied here. We have not investigated this particular question. Let us only make the following comment: An accurate identification and measurement of the energies of the particles created appears as a sufficient—but also to a large extent necessary—condition to avoid any confusion between these two entirely different production mechanisms.

[18] P. Kessler, Laboratoire de Physique Atomique Internal Report No. PAM 68-05 (unpublished); Nucl. Phys. **B15**, 253 (1970).

It appears now that this method is also a very efficient tool for deriving valuable approximation methods such as that used here.

Let us state from the beginning that the main principle of this approximation was to neglect terms of order 1 or higher in t/M^2 and t'/M^2, where t and t' are the absolute values of the squares of the four-momenta of the left-hand and right-hand photon, respectively. This procedure is justified because the maximum value of t and t' is $E_0^2\,\theta_{\max}^2$; for $E_0=2$ GeV and $\theta_{\max}=4$ mrad, this quantity is 64 MeV^2; for $E_0=2$ GeV and $\theta_{\max}=1°$, it is about 1200 MeV^2. Remember that the minimum value of M^2 considered in our study is 10^4 MeV^2.

One may write the differential cross section in the general form

$$d\sigma=(4\pi\alpha)^4 dC\, D\,, \qquad \text{(A1)}$$

where dC is the kinematic factor (essentially the phase space of the outgoing particles, divided by the flux of incoming particles), and D is the dynamic factor.

The (quite trivial) calculation of dC gives (treating all $e^\pm$ particles as extreme relativistic)

$$dC=\frac{1}{(2\pi)^6}\frac{m^4}{8E_0^2}EE'dEdE'd(\cos\theta)\times d(\cos\theta')d\Phi\frac{k}{M}d(\cos\chi)\,, \qquad \text{(A2)}$$

where m is the electron mass; E_0 is the beam energy; E and E' are, in the lab frame, the energies of the outgoing electron and positron, respectively; θ and θ' are, also in the lab frame, the respective scattering angles of the e^- and e^+; Φ is the corresponding azimuthal angle; M is the total energy in the photon-photon c.m. frame; χ is, also in that frame, the emission angle of one $A^\pm$ particle with respect to the axis which carries the photon momenta; and finally, $k=\frac{1}{2}(M^2-4m_A^2)^{1/2}$, m_A being the mass of A.

Using $t=t_{\min}+2E_0E(1-\cos\theta)$, with $t_{\min}=m^2(E_0-E)^2/(E_0E)$, and the analogous expression for t', and defining $\tau=t/(4m^2)$, $\tau'=t'/(4m^2)$, $\omega=E_0-E$, $\omega'=E_0-E'$, we get

$$dC=\frac{1}{(2\pi)^6}\frac{m^8}{2E_0^4}d\omega d\omega' d\tau d\tau' d\Phi\frac{k}{M}d(\cos\chi)\,. \qquad \text{(A3)}$$

For D, the generalized helicity method[18] gives us in a straightforward way

$$D=\tfrac{1}{4}t^{-2}t'^{-2}I_{\lambda,\lambda+\nu}I_{\mu,\mu-\nu}'j_{\lambda,\mu}j_{\lambda+\nu,\mu-\nu}^{*}e^{i\nu\varphi}\,, \qquad \text{(A4)}$$

where $I_{\lambda,\lambda+\nu}$ is the polarization matrix of the photon emitted at the electron-photon vertex, this matrix being defined in the $\gamma\gamma$ c.m. frame (with the z axis along the photon momentum, and the $\gamma\gamma\to A^-A^+$ reaction plane defining the zx plane). The subscripts λ and $\lambda+\nu$ characterize the photon's spin component on the z axis, respectively, in the direct and in the conjugated amplitude. $I_{\mu,\mu-\nu}'$ is defined in an analogous way for the photon emitted at the positron-photon vertex. $j_{\lambda,\mu}$ and $j_{\lambda+\nu,\mu-\nu}^*$ are the (direct and conjugated) helicity amplitudes for the process $\gamma+\gamma\to A^-+A^+$. Finally, φ is, also in the $\gamma\gamma$ c.m. frame, the angle between the half-plane formed by the z axis and the outgoing electron's momentum, on the one hand, and that formed by the z axis and the outgoing positron's momentum, on the other hand.

Carrying out the tensorial factorization, taking parity conservation and angular momentum conservation into account, one gets the (still exact) expression

$$\begin{aligned}D=\tfrac{1}{4}t^{-2}t'^{-2}[&2I_{++}I_{++}'(|j_{++}|^2+|j_{+-}|^2)+2I_{++}I_{00}'|j_{+0}|^2\\&+2I_{00}I_{++}'|j_{0+}|^2+I_{00}I_{00}'|j_{00}|^2\\&+4(\cos\varphi)I_{+0}I_{+0}'\operatorname{Re}(j_{+0}j_{0+}^*-j_{+-}j_{00}^*)\\&+2(\cos2\varphi)I_{+-}I_{+-}'|j_{+-}|^2]\,,\end{aligned} \qquad \text{(A5)}$$

where the subscripts are the explicit values of the spin components λ, ... (we use $\pm$ for ±1).

Now, a large number of terms written on the right-hand side will vanish in our approximation.

(a) Terms in $\cos\varphi$ and $\cos2\varphi$ will vanish, because we shall integrate over Φ, and we have $\varphi\simeq\Phi$. This approximate equality between φ and Φ can be shown geometrically in the following way: Both photons are practically aligned along the electron-positron beam axis, which becomes also the photon-photon collision axis, i.e., the z axis. Then, the Lorentz transformation from the lab frame to the $\gamma\gamma$ c.m. frame is just a translation along the same axis, and it is obvious that such a translation does not change the azimuthal angle of both outgoing $e^\pm$ particles. This geometric argument was confirmed by an algebraic calculation, where we neglected only terms in m^2 and in t/M^2, t'/M^2.

(b) Terms containing $|j_{+0}|^2$, $|j_{0+}|^2$, and $|j_{00}|^2$ (i.e., partially or totally longitudinal terms) can be neglected, because they are proportional respectively to t'/M^2, t/M^2, and tt'/M^4.

We are thus left with

$$D\simeq\tfrac{1}{2}t^{-2}t'^{-2}I_{++}I_{++}'(|j_{++}|^2+|j_{+-}|^2) \qquad \text{(A6)}$$

(where implicitly the integration over Φ has been taken into account).

The expression for I_{++} is easily calculated to be (neglecting only terms in m^2, t/M^2, t'/M^2)

$$I_{++}\simeq\tau(x+1)-1\,,\quad \text{with}\quad x=M^{-4}(8E_0\omega'-M^2)^2.$$

Expressing M^2 in the lab frame, one gets (neglecting only terms in m^2)

$$M^2=4\omega\omega'-2EE'(1+\cos\Theta)\,,$$

where Θ is the lab angle between both outgoing $e^\pm$ particles. In this last equation, the second term on the right-hand side can be neglected since it contains only terms of the order of t, t', and $(tt')^{1/2}$. Thus

$$M^2\simeq4\omega\omega'\,,\quad\text{whence}\quad x\simeq\omega^{-2}(2E_0-\omega)^2=(1+\tau_{\min})/\tau_{\min}\,,$$

and finally $I_{++}\simeq[(1+2\tau_{\min})/\tau_{\min}]\tau-1$. Similarly, $I_{++}'\simeq[(1+2\tau'_{\min})/\tau'_{\min}]\tau'-1$.

Substituting these expressions in (A6), using now (A1) and (A3), and integrating over Φ, τ, and τ', we obtain

$$d\sigma\simeq(\alpha^4/128\pi E_0^4)FF'd\omega d\omega'(k/M)\times(|j_{++}|^2+|j_{+-}|^2)d(\cos\chi)\,, \quad \text{(A7)}$$

where we define

$$F=\frac{1+2\tau_{\min}}{\tau_{\min}}\ln\frac{\tau_{\max}}{\tau_{\min}}-\frac{1}{\tau_{\min}}+\frac{1}{\tau_{\max}}\,,$$

$$F'=\frac{1+2\tau'_{\min}}{\tau'_{\min}}\ln\frac{\tau'_{\max}}{\tau'_{\min}}-\frac{1}{\tau'_{\min}}+\frac{1}{\tau'_{\max}}\,,$$

with

$$\tau_{\min}=\omega^2/4E_0(E_0-\omega)\,,$$

$$\tau_{\max}=\tau_{\min}+E_0(E_0-\omega)(1-\cos\theta_{\max})/2m^2\,,$$

and analogous definitions for $\tau'_{\min}$ and $\tau'_{\max}$.

Now we compare our result with the cross section for $\gamma+\gamma\rightarrow A^-+A^+$ in the case of two free photons colliding with opposite momenta, their respective energies being ω and $\omega'[=M^2/(4\omega)]$:

$$\sigma_{\gamma\gamma}(\omega,\omega')=\frac{\pi\alpha^2}{2}\frac{k}{M^3}\int(|j_{++}|^2+|j_{+-}|^2)d(\cos\chi)\,. \quad \text{(A8)}$$

Here, the helicity amplitudes j_{++} and j_{+-} are the same as those considered before for the quasi-real photons (once we neglect terms in t/M^2, t'/M^2). Thus, combining (A7) and (A8), we obtain the Williams-Weizsäcker–type formula

$$d\sigma\simeq(\alpha^2M^2/64\pi^2E_0^4)FF'd\omega d\omega'\sigma_{\gamma\gamma}(\omega,\omega')\,. \quad \text{(A9)}$$

Using the relation $2\omega d\omega'=M\,dM$ (at constant ω), and noticing that $\sigma_{\gamma\gamma}$ depends only on M, we obtain the final formula used in our calculation of diagram I of Fig. 1:

$$\frac{d\sigma}{dM}\simeq\sigma_{\gamma\gamma}(M)\frac{\alpha^2M^3}{128\pi^2E_0^4}\int\frac{d\omega}{\omega}FF'\,. \quad \text{(A10)}$$

If we want to extend this result to the case [diagram I or Fig. 5] where, instead of a pair A^-A^+, one single particle X (with mass m_X) is produced, we use

$$\sigma_{\gamma\gamma}(M)=(4\pi^2/m_X^2)\Gamma(X\rightarrow2\gamma)\delta(M-m_X)\,,$$

where $\Gamma(X\rightarrow2\gamma)$ is the partial width for 2-photon decay of X. Substituting this expression in (10), we get for the total cross section of $e^-+e^+\rightarrow e^-+e^++X$

$$\sigma\simeq\Gamma(X\rightarrow2\gamma)\frac{\alpha^2m_X}{32E_0^4}\int\frac{d\omega}{\omega}FF'\,. \quad \text{(A11)}$$

Let us go over now to diagrams II of Fig. 1. Using the same small-transfer approximation as above, and an addition an exact factorization formula (due to Bordes and Jouvet[19]) for timelike virtual particles, we obtain for one of these diagrams (the left-hand one, for instance)

$$\frac{d\sigma}{dM}\simeq\Gamma(M)\frac{\alpha^3}{4\pi}\frac{m^2}{M^2E_0^4}\int\frac{d\omega}{\omega}Ff_\omega\tau'_{\max}\,, \quad \text{(A12)}$$

where f_ω is given by

$$f_\omega=\frac{4\omega E_0-M^2}{4\omega E_0}+\frac{4\omega E_0}{4\omega E_0-M^2}$$

and $\Gamma(M)$ is the "partial width" for decay of the heavy photon into A^-A^+. If A is a lepton, this width is given by

$$\Gamma(M)=(\alpha/3M^2)(M^2+2m_A^2)(M^2-4m_A^2)^{1/2}\,.$$

If A is a boson, it is given (not accounting for a resonant enhancement) by

$$\Gamma(M)=(\alpha/12M^2)(M^2-4m_A^2)^{3/2}\,.$$

To obtain the total contribution of both diagrams II of Figs. 1, we multiplied the result (A12) by 2 (thereby neglecting the interference term).

A last detail to be given concerns the limits of integration over ω in our formulas (A10)–(A12). In order to keep our extreme-relativistic approximation for the electrons and positrons valid everywhere, we made a lower cutoff (obviously of no practical importance) on the energies of the outgoing $e^\pm$ particles, namely,

$$E_{\min}=E'_{\min}=\epsilon \quad \text{with} \quad \epsilon\simeq10\text{ MeV}\,.$$

Thus, our limits of integration were

$$\omega_{\min}=M^2/4(E_0-\epsilon)\,,\quad \omega_{\max}=E_0-\epsilon\,.$$

The extension of our calculations to angular distributions of the particles produced was trivial.

Let us conclude with two remarks.

(a) The Williams-Weizsäcker–type approximation formula obtained, Eq. (A10), is much more precise than the semiclassical one,[20] and also somewhat better than corresponding formulas given previously by various people, among them one of the authors,[21] which were derived from field theory, but not in a strictly invariant way. On the other hand, it is obvious that such an approximation must be used with some caution:

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[19] G. Bordes and B. Jouvet, Compt. Rend. **257**, 1007 (1963). See also P. Kessler and Ph. Leruste, Cahiers Phys. **18**, 189 (1964); **18**, 201 (1964).

[20] See W. Heitler, *The Quantum Theory of Radiation* (Clarendon, Oxford, 1954), 3rd ed., pp. 414–418.

[21] P. Kessler, Nuovo Cimento **17**, 809 (1960).

It is reliable in the case where small transfers are selected; otherwise it may become very rough, and even lead to gross overestimations.

(b) If we had chosen to make an exact calculation of $d\sigma/dM$ for diagram I of Fig. 1, we would have been compelled to perform a *fourfold* integration with the help of a computer. (The integration over $\cos\chi$ can be done analytically, at least in the Born-term model.) Even if the computer used were quite powerful, such a calculation would always involve some amount of error. Therefore, it is possible that such an "exact" calculation might have provided a result less accurate than our approximation procedure, where only one *single* integration was needed.

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Dashen-Weinstein Theorem in K_{l3} Decay: Soft-Pion Corrections

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The Mathur-Okubo sum rule in K_{l3} decay, together with Pagels's model for corrections due to partial conservation of axial-vector current, is invoked to estimate the $O(\epsilon^2)$ corrections to the Dashen-Weinstein theorem for $\xi(0)$. The corrections are found to be small, and we predict $\xi(0)\approx -0.6$ for $\lambda_+\approx 0.08$.

THE problem of K_{l3} decay has attracted considerable attention in the literature.[1] The matrix element for this decay defines two form factors $f_+(q^2)$ and $f_-(q^2)$ through the relation

$$\langle\pi^0(k)|V_\mu^{4-i5}(0)|K^+(p)\rangle = (1/\sqrt{2})[f_+(q^2)(p+k)_\mu+f_-(q^2)(p-k)_\mu], \quad (1)$$

where $q=k-p$.

In this context, there are two important results which follow from Gell-Mann's current algebra and partial conservation of axial-vector current (PCAC) which enjoy the privileged status of "theorems": The first is the famous soft-pion theorem due to Callan and Treiman and to Mathur, Okubo, and Pandit,[2] which states that

$$f_+(p^2=m_K^2,\, k^2=m_\pi^2,\, q^2=m_K^2) + f_-(p^2=m_K^2,\, k^2=m_\pi^2,\, q^2=m_K^2) = F_K/F_\pi+O(\epsilon_\pi), \quad (2)$$

where ϵ_π is a parameter which measures the departure from the limit of exact $SU(2)\otimes SU(2)$ symmetry and massless pions, and $F_{\pi(K)}$ is the pion (kaon) decay amplitude. The other theorem of more recent origin is the Dashen-Weinstein[3] theorem for the form-factor ratio $\xi(0)\equiv f_-(0)/f_+(0)$:

$$\xi(0)+\lambda_+\frac{m_K^2-m_\pi^2}{m_\pi^2}=\frac{1}{2}\left(\frac{F_K}{F_\pi}-\frac{F_\pi}{F_K}\right)+O(\epsilon^2), \quad (3)$$

where

$$\lambda_+=m_\pi^2\left[\frac{d}{dt}\ln f_+(t)\right]_{t=0},$$

and ϵ is a measure of the breaking of $SU(3)\otimes SU(3)$ symmetry. This theorem is independent of any assumptions on the form of symmetry breaking.

It is well known that Eq. (2) is not directly amenable to experimental tests, since the point $q^2=m_K^2$ is not in the physical region of the decay: $m_l^2\leqslant q^2\leqslant (m_K-m_\pi)^2$.

The Dashen-Weinstein theorem [Eq. (3)] is somewhat more readily available for confrontation with experiment and is in good agreement with recent experimental data,[4] subject to the uncertainties in corrections of $O(\epsilon^2)$.

The possibility of a zero in the "scalar" form factor $f(q^2)\equiv f_+(q^2)(m_K^2-m_\pi^2)+f_-(q^2)q^2$ between $(m_K-m_\pi)^2$ and $(m_K+m_\pi)^2$ has been suggested to explain $\xi(0)\approx -1$,[5] as also has the proposal based on "weak" PCAC.[6] The latter proposal has been, however, criticized by Weinstein.[7]

In the present paper, we estimate corrections to Eq. (3) since it is rather essential to know whether or not the fair agreement with experimental data that one obtains from the Dashen-Weinstein formula [with $\lambda_+\approx 0.08$ and $\xi(0)\simeq -0.6$] [4] is fortuitous.

We make the following assumptions:

(i) $SU(3)\otimes SU(3)$ chiral algebra of Gell-Mann;
(ii) PCAC hypotheses for pion and kaon fields;

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[1] See S. Weinberg, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968).

[2] C. Callan and S. Treiman, Phys. Rev. Letters **16**, 153 (1966); V. Mathur, S. Okubo, and L. Pandit, *ibid.* **16**, 371 (1966).

[3] R. Dashen and M. Weinstein, Phys. Rev. Letters **22**, 1337 (1969); Fayyazuddin and Riazuddin, Phys. Rev. D **1**, 361 (1970).

[4] P. Innocenti *et al.*, in *Proceedings of the Fifteenth International Conference on High-Energy Physics, Kiev, 1970* (Academy of Sciences, U.S.S.R., Moscow, 1971); E. Dalley *et al.*, *ibid.*; C.-Y. Chien *et al.*, Phys. Letters **33B**, 627 (1970).

[5] K. Kang, Phys. Rev. Letters **25**, 414 (1970).

[6] R. Brandt and G. Preparata, Nuovo Cimento Letters **4**, 80 (1970).

[7] M. Weinstein, Phys. Rev. D **3**, 481 (1971).