Search for $K_S \to \pi^+\pi^-\pi^0$

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A search for K_S decaying into the final state $\pi^+\pi^-\pi^0$ has been made by studying the time distribution of 3π decays of neutral K mesons produced in the reaction $K^+d \to K^0pp$. On the basis of 99 3π events, we find for the complex parameter $x+iy$, the ratio of the amplitude $K_S \to 3\pi$ to the amplitude $K_L \to 3\pi$, $x=0.47\pm0.20$, $y=-0.10_{-0.32}^{+0.37}$. Our result is consistent with no K_S decays into $\pi^+\pi^-\pi^0$ and no CP violation. Assuming $x=y=0$, we obtain $\Gamma_L(3\pi)=(2.71\pm0.28)\times10^6$ sec^{-1}.

I. INTRODUCTION

STUDIES of the decay $K_L \to \pi^+\pi^-\pi^0$ have been quite extensive. The $\Delta I=\frac{1}{2}$ rule has been tested by comparing properties of neutral-K decays (partial decay rates and slopes) with those of the charged-K decay.[1] There is some evidence that a $\Delta I=\frac{3}{2}$ amplitude of a few percent of the $\Delta I=\frac{1}{2}$ amplitude exists, but there is no evidence for a $\Delta I=\frac{5}{2}$ contribution. Violations of CP invariance in the decay $K_L{}^0 \to \pi^+\pi^-\pi^0$ have been searched for by looking for an asymmetry between the charged-pion distributions. No asymmetries have been found, and the limits are on the order of a few parts per thousand.[2] Furthermore, the spectrum of π^0's is linear with respect to the energy and there is no evidence for a quadratic term. Since the total isospin of the three pions in the final state can be $I=0$, 1, 2, or 3, these results imply that the pions in $K_L{}^0 \to \pi^+\pi^-\pi^0$ decay are predominantly in the CP-conserving, symmetric $I=1$ state; i.e., angular momentum states higher than the S states are suppressed by barriers.

CP invariance forbids $K_S \to \pi^+\pi^-\pi^0$ decay into S states. We assume that the $I=0$ state is considerably suppressed by angular momentum barriers, perhaps by a factor of 10^{-3} relative to the known K_L amplitude.[3] The $I=2$ state is suppressed by a combination of angular momentum barriers and the $\Delta I=\frac{1}{2}$ rule by a factor of about 10^{-2}. Since all searches to date have looked for a K_S amplitude comparable to the known K_L amplitude, we shall ignore the $I=0$ and $I=2$ states in K_S decay, leaving only the CP-violating $I=1$ and $I=3$ states. The technique used to find these decay amplitudes is to examine the time distribution of neutral K decays into $\pi^+\pi^-\pi^0$, looking for an interference between the known K_L amplitude and the possible K_S amplitude. With the amplitude ratio defined as

$$\frac{a_S(K_S \to 3\pi)}{a_L(K_L \to 3\pi)}=x+iy\,,$$

the approximation

$$|K^0\rangle=(1/\sqrt{2})\left(|K_S{}^0\rangle+|K_L{}^0\rangle\right),$$

and starting with an initial K^0 state, the intensity of 3π decays as a function of time is given by

$$N(x,y,t)=\tfrac{1}{2}N_0\Gamma_L(3\pi)\left[(x^2+y^2)e^{-\lambda_S t}+e^{-\lambda_L t}+2(x\cos\delta t-y\sin\delta t)e^{-\Lambda t}\right], \quad (1)$$

where N_0 is the number of K^0's produced, $\Gamma_L(3\pi)=|a_L|^2$ is the decay rate for $K_L{}^0 \to 3\pi$, and $\Lambda=\frac{1}{2}(\lambda_S+\lambda_L)$. We have used the following values[4]:

$$\delta=M(K_L)-M(K_S)=0.46\tau_S{}^{-1}\,,$$
$$\lambda_S=1/\tau_S=1.16\times10^{10}\text{ sec}^{-1}\,,$$
$$\lambda_L=1/\tau_L=1.86\times10^{7}\text{ sec}^{-1}\,.$$

Since the experimental data cover a range of decay configurations, averages of x^2 and y^2 are measured. The assumption, made in the analysis, that $\langle x^2\rangle=\langle x\rangle^2$ and $\langle y^2\rangle=\langle y\rangle^2$ is reasonable if the decay proceeds predominantly via S waves.

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If we make the additional assumption that $\Delta I=\frac{5}{2}$ transitions do not occur, the $I=3$ final state may be neglected. Glashow and Weinberg[5] have shown that if the decay proceeds by the symmetric $I=1$ state and CPT invariance is valid, then

$$\frac{\mathrm{Re}(a_S a_L^*)}{|a_S|^2+|a_L|^2} \lesssim 0.01\,.$$

If $|a_S|$ is not much smaller than $|a_L|$, the amplitudes a_S and a_L must be relatively imaginary and $x\approx 0$. Thus, if symmetry, angular-momentum-barrier considerations, and isotopic-spin selection rules are invoked, the time distribution of $\pi^+\pi^-\pi^0$ decays depends, to a good approximation, only on the CP-violating parameter y.

Four experiments[6–9] on the time distribution of $K^0\to 3\pi$ decays have been reported; statistically the most significant of these is that of Behr *et al.*,[7] based on samples of 136 $K^0\to\pi^+\pi^-\pi^0$ and 54 $K^0\to 3\pi^0$ events. All four experiments are consistent with no CP violation in $K^0\to 3\pi$ decay. In this experiment we report on the analysis of 99 events resulting from $K^0\to\pi^+\pi^-\pi^0$ decay.

II. EXPERIMENTAL PROCEDURE

The events were obtained from the 30-in. BNL bubble chamber filled with deuterium, exposed to a separated 600-MeV/c K^+ beam at the AGS. In a previous publication,[10] hereafter referred to as I, we reported on the leptonic decays of neutral K mesons obtained from the same exposure. Since our method for finding three-pion decays was similar to that used for leptonic decays, we shall only emphasize the essential features.

The neutral K mesons were produced in the reaction

$$K^+ + d \to K^0 + p + p_s\,,$$

where p_s is a spectator proton. The neutral K can have a visible decay into $\pi^+\pi^-$, $\pi^+\pi^-\pi^0$, $\pi^+\pi^-\gamma$, $\pi\mu\nu$, or $\pi e\nu$. The pictures were scanned for all V's and possible production vertices. No criteria concerning the association of V's to production vertices were imposed in the scanning. The over-all scanning efficiency for the entire experiment was better than 95%.

Since the fraction of three-body decays expected was only about 1% of the two-body decays, extreme precautions were necessary to ensure complete removal of any background two-body decays. Once a V was found, the most likely production vertex was assumed to be its origin, and the whole event was measured. These events were processed through standard geometry and kinematic fitting programs and were then subjected to preliminary geometrical criteria, which were later made more restrictive.

Each V was fitted to the two-body decay mode $K_S\to\pi^+\pi^-$ under the hypotheses that it was not associated with any production origin (one-constraint fit) and that it was associated with the production vertex (three-constraint fit). An event for which the χ^2 probability was greater than 0.01% for the three-constraint fit was classified as a two-body decay and an over-all production and decay fit was attempted. About 50 000 $K_S\to\pi^+\pi^-$ events were obtained in this experiment. The χ^2 distributions for a sample of the three-constraint and one-constraint 2π fits, shown in I, are in agreement with the predictions. A V which failed the three-constraint fit was fitted to other possible production origins. A small number of V's made the one-constraint decay fit at a probability of better than 2×10^{-5} but failed all production fits. These events came from three sources. Some two-body decays came from charge-exchange interactions in the chamber window. Some came from K^0's produced in the chamber which scatter before decaying; these events have been analyzed separately. The remaining events are true three-body decays which make the one-constraint 2π fit. Most of the events lost are $\pi\mu\nu$ or $\pi e\nu$; loss of 3π events, estimated by Monte Carlo techniques to be $(0.7\pm 0.3)\%$, was necessary in order to guarantee the removal of the dominant 2π mode. Within the final fiducial volume, this loss is unbiased with respect to proper time and therefore does not affect the analysis of the time distributions, but necessitates a correction in the calculation of the total 3π decay rate.

An event which failed both the one- and three-constraint 2π fits was refitted to all possible production vertices under the assumption that the neutral K decayed by any one of the three-body decay modes $\pi e\nu$, $\pi\mu\nu$, $\pi^+\pi^-\pi^0$, and $\pi\pi\gamma$. The class of three-body events with visible spectators has four constraints for the over-all fit, while that with the invisible spectator has one constraint. The over-all χ^2 probability for a production of a K^0 followed by a three-body decay was required to be at least 2%. Each three-body candidate was remeasured twice, and was rejected if the χ^2 probability for the one-constraint 2π fit was greater than 2×10^{-5}, unless the fitted K^0 momentum had an unreasonable value. Neither of these cuts on χ^2 causes any time bias. For all three-body candidates the association with a production vertex was unique. The resulting uncertainty in the fitted K^0 momentum was approximately 5%.

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The final geometrical criteria for the sample of three-body events were as follows: (a) The minimum projected K^0 length was 3 mm and the maximum projected length was 20 cm; (b) the dip angles of the K^0 and of the charged legs of the V were required to be less than 65°; (c) the opening angle between the charged legs of the V was required to be less than 170°. In addition, the K^0 momentum was required to be in the interval 100–650 MeV/c.

III. LIMITS ON BACKGROUND IN 3π SAMPLE

After rejecting all possible two-body decays we required that the event fit the $\pi^+\pi^-\pi^0$ decay mode at least at the 4% confidence level.

Monte Carlo calculations show that less than one event of the other three-body decay modes will fit the $\pi^+\pi^-\pi^0$ decay mode, and no time-dependent bias is introduced.

The accidental association of a stray track simulating a V with a production vertex and surviving our criteria is negligible.

The dominant $K_S \rightarrow \pi^+\pi^-$ decay mode can give rise to three types of contamination in the sample of events from which the $\pi^+\pi^-\pi^0$ decays are selected. The most serious type is caused by nuclear scattering of a decay pion near the vertex of the V. This nuclear scattering is completely dominated by the $\Delta(1238)$. Although half of the π^-n scatters give rise to events with no visible spectator, we estimate that less than one event remains in the sample. The other two sources of background come from Coulomb scattering and decay in flight of pions. For an in-flight decay of a π into a μ, $p_\pi \tan\theta$ must be less than 38 MeV/c, where p_π is the momentum of the π and θ is the space angle between the π and the μ. All events were subjected to a two-vertex fit to the K^0 production followed by a two-pion decay in which first one and then the other pion was considered unmeasured. When we used the fitted momentum of the unmeasured track for p_π and the angle between this fitted track and its measured direction for θ, all π decays in flight were rejected by the above criteria. The small Coulomb scatters were removed in a similar manner; all events for which $p_{\rm fit}\beta_{\rm fit}\theta < 2500$ MeV/c degrees were rejected, where $p_{\rm fit}$ and $\beta_{\rm fit}$ are the fitted momentum and velocity of the unmeasured track and θ is the space angle between the fitted and measured direction. From Monte Carlo calculations, we estimate that less than one Coulomb scatter will survive.

Since Dalitz pairs tend to be produced with low effective masses, the background from $K_S \rightarrow \pi^0\pi^0$ is completely negligible in the 3π sample. When a low effective mass is calculated from the charged tracks of a 3π decay, assumed to be electrons, the bubble density is always inconsistent with that of electrons.

IV. MAXIMUM-LIKELIHOOD ANALYSIS

An estimate of the parameters x and y can be obtained independently of the 3π decay rate $\Gamma_L(3\pi)$. The probability for the ith event is given by

$$L_i(x,y) = N(x,y,t_i) \Big/ \int_{t_i^{\rm min}}^{t_i^{\rm max}} N(x,y,t)dt\,, \qquad (2)$$

where $N(x,y,t)$ is given by Eq. (1), t_i is the proper time, and $t_i^{\rm min}$ and $t_i^{\rm max}$ are the limits of the detectable time interval for the ith event. The likelihood function is then given by

$$L(x,y) = \prod_{i=1}^{n} L_i(x,y)\,, \qquad (3)$$

where n is the number of events in the sample.

It is also possible to construct a likelihood function taking into account the 3π decay rate $\Gamma_L(3\pi)$ and the total number of events observed. The extended likelihood function L' is

$$L'(x,y) = [e^{-M(x,y)}M(x,y)^n/n!]L(x,y)\,, \qquad (4)$$

where $L(x,y)$ is given by Eq. (3) and n is the observed number of events. For $M(x,y)$, the expected number of 3π events, we take

$$M(x,y) = \epsilon_{3\pi}\int_0^\infty N(x,y,t)\epsilon(t)dt\,,$$

where $\epsilon(t)$ is the geometric detection efficiency determined from two-body decays and $\epsilon_{3\pi}$ is the efficiency for retaining 3π decays after geometric cuts are imposed.

The advantage of using $L(x,y)$ is that the final result is insensitive to any absolute detection efficiency calculation, but requires only that the events be obtained in a way unbiased with respect to the proper time. The advantage of using L' is that the total observed number of events adds a constraint to the result.

Since the number of events in this experiment is small (99 events), it is instructive to obtain estimates, with both likelihood functions, of the fluctuations expected in an analysis of this size sample. A large number of Monte Carlo experiments was generated, each consisting of 100 events. For each simulated event, the production-vertex coordinates and the K^0 kinematics were randomly chosen from a sample of fitted charge-exchange events. The decay time t_i was generated according to Eq. (1), assuming the decays were all K_L's (i.e., $x=y=0$). The coordinates of the simulated 3π decay vertex were calculated and, if this vertex lay within the fiducial volume, the event was accepted. Samples of generated events having a geometric detection efficiency identical to the actual data were thus obtained.

Each 100-event Monte Carlo experiment was then analyzed independently of $\Gamma_L(3\pi)$ using $L(x,y)$ for the likelihood function. Approximately 75% of the experiments yielded a single maximum in the likelihood function with standard deviations in x and y of the

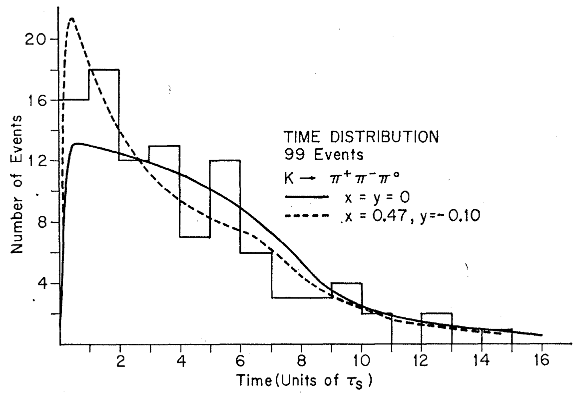

FIG. 1. Time distribution for 99 $K \to \pi^+\pi^-\pi^0$ decays. The solid curve is the expected distribution assuming all the decays come from K_L's, and the dashed curve is for our best estimates of x and y.

order of 0.2 and 0.6, respectively. The remaining 25% of the experiments exhibited two likelihood maxima of comparable magnitude. One maximum always occurred near the assumed $x=y=0$ values while the second maximum occurred at large and predominantly *negative* values of x and y. As the sample size was increased, the frequency of occurrence of the two maxima decreased. In those experiments where two maxima occur, the solution further from $x=y=0$ generally led to values of $\Gamma_L(3\pi)$ which were inconsistent with the measured rate. However, by using $L'(x,y)$, thus including $\Gamma_L(3\pi)$ in the likelihood function, the Monte Carlo study showed that unique solutions for x and y were always

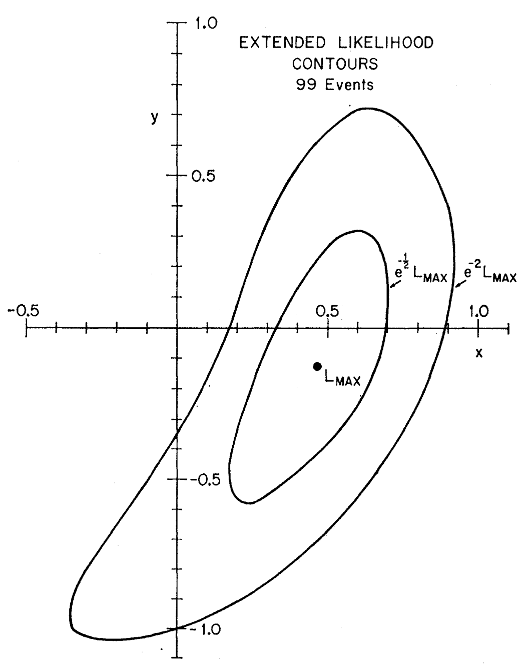

FIG. 2. Likelihood contour plot for 99 $K \to \pi^+\pi^-\pi^0$ decays. The solid curves are the $e^{-1/2}L_{\max}$ and $e^{-2}L_{\max}$ contours.

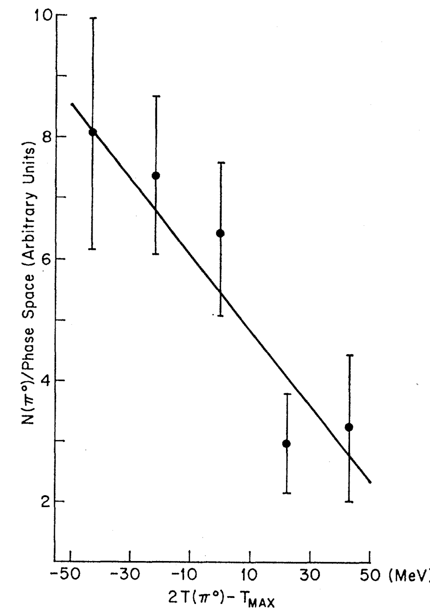

FIG. 3. Kinetic-energy spectrum of the π^0 in the K^0 rest frame.

obtained and the precision of the estimates of x and y was improved.

V. RESULTS

The distribution in proper time of the $\pi^+\pi^-\pi^0$ events is shown in Fig. 1; the solid curve is the distribution expected for $x=y=0$, and is essentially the geometric detection efficiency. The χ^2 probability for $x=y=0$ is 30%.

Using the likelihood function $L'(x,y)$, which *includes the decay rate* $\Gamma_L(3\pi)$, we obtain as our best estimates[11]

$$x=0.47\pm0.20, \quad y=-0.10_{-0.32}^{+0.37}, \qquad (5)$$

where the errors include the statistical uncertainty in $\Gamma_L(3\pi)$. The dashed curve in Fig. 1 is the distribution expected for our best estimates, Eq. (5).

For the decay rate we have taken $\Gamma_L(3\pi)=(2.44\pm0.10)\times10^6$ sec^{-1}, the result obtained by Budagov *et al.*[12] in an analysis using the most precise experimental data. A systematic increase (decrease) in the value of $\Gamma_L(3\pi)$ of 0.1×10^6 sec^{-1} would leave x unchanged and would increase (decrease) y by about 0.15. Figure 2 shows the likelihood solution with the 1- and 2-

[11] An analysis of our 99-event sample without incorporating Γ_L into the likelihood function yields the following two solutions: (I) $x=0.36\pm0.30$, $y=-0.75\pm0.54$; (II) $x=-2.38_{-0.60}^{+2.30}$, $y=-1.60\pm0.37$. Solution I gives $\Gamma_L=(2.00\pm0.20)\times10^6$ sec^{-1}, which is in reasonable agreement with the measured value. Solution II leads to $\Gamma_L=(1.33\pm0.14)\times10^6$ sec^{-1}, which is inconsistent with the measured value.

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standard-deviation contours. From our Monte Carlo study, we conclude the probability of obtaining a value of $(x^2+y^2)^{1/2} \geq 0.5$ is 25% in an event sample of this size. We therefore have no evidence that either x or y is different from zero. This is consistent with the results of the previous experiments.[6–9]

If we now assume the final state of the decay is the symmetric $I=1$ state, then the decay only depends on the parameter y. Under that assumption we obtain

$$y=-0.66\pm0.27,$$

where the error includes the statistical uncertainty in $\Gamma_L(3\pi)$. A systematic shift in the value of $\Gamma_L(3\pi)$ of 0.1×10^6 sec^{-1} would change y by about 0.1.

Since our result is consistent with $x=y=0$, we can calculate $\Gamma_L(3\pi)$ assuming the observed events are due entirely to $K_L^0 \to \pi^+\pi^-\pi^0$. We find $\Gamma_L(3\pi)=(2.71 \pm0.28)\times10^6$ sec^{-1}, which is in excellent agreement with the value obtained by Budagov *et al.*[12] The π^0 kinetic-energy spectrum in the K^0 rest frame is shown in Fig. 3. Variation in the detection efficiency as a function of π^0 energy is negligible. The straight line drawn is from a fit using the matrix element squared:

$$|M|^2 \sim 1+2\alpha(M_K/M_\pi^2)(2T_{\pi^0}-T_{\max}).$$

We find $\alpha=-0.26\pm0.07$, which agrees with the known slope of the K_L decay, $\alpha=-0.20\pm0.014$.[1]

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Double-Regge-Pole Description of the Reaction $K^-n \to K^-\pi^-p$ at 5.5 GeV/c*

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New data on the reaction $K^-d \to K^-\pi^-pp_s$ at 5.5 GeV/c are presented. The experimental distributions are analyzed in terms of a double-peripheral Regge-pole model. The events were assigned to two dominant diagrams using the Van Hove angle. These diagrams, involving Pomeranchuk and meson exchanges, provide a good description of the reaction without additional form factors of the momentum transfer.

I. INTRODUCTION

In recent years the extension of the Regge-pole model from two-body and quasi-two-body reactions to reactions involving many particles has been studied by several groups.[1–7] In particular, the double-Regge-pole model (DRM) has been successful in describing many three-body final states. Because the Regge model was originally introduced as an asymptotic expansion, some of these fits were restricted to the relatively small fraction of events where all two-particle subenergies were larger than some prescribed limits. Berger[3] extended the model down to the threshold of one of the two-body subenergies involved and obtained satisfactory descriptions of several reactions.

Cuts in the subenergies can also provide a mechanism for selecting events appropriate to the different peripheral diagrams. This is not a unique method of selection, and Alexander *et al.*[4] used the four-momentum transfers at the beam and target vertices. Recently, Van Hove[8]

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