VII. DISCUSSION

We have demonstrated the existence of complex Regge poles in the solution of an ABFST model. Ball, Marchesini, and Zachariasen[4] have argued that the effects at moderate energies of a Regge cut can be approximated by a complex conjugate pair of Regge poles. One may ask whether the complex poles in our model are merely an approximation to the Regge cuts which we have neglected by cutting off the high-energy tail in the kernel. If this were the case, one would expect that the region in s of the kernel which is most important in the determination of the Regge-pole positions would be near the cutoff, 10 GeV^2. However, Shei[9] obtained essentially the same complex-pole positions as presented here using the trace approximation to a model with a sharp-ρ-pole kernel. From this result, we conclude that the most important region of the kernel is near m_ρ^2 and that the complex poles we have found are not an approximation to a Regge cut.

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Low-Energy Approach to the Kaon Electromagnetic Mass Splitting

KAZUO YAMADA
Department of Physics, Tokyo Institute of Technology, Oh-okayama, Tokyo 152, Japan
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The on-shell electromagnetic mass splitting of kaons is studied, on the assumption that it is purely electromagnetic in origin, in a Lagrangian model incorporating vector-meson dominance of the hadronic electromagnetic current. A simple symmetry-breaking term in Lagrangian, which preserves the invariance under the photon gauge transformation, gives the vector-meson mass spectrum and the negative ϕ-ω mixing angle systematically. The result for δm_K to order α (fine-structure constant) is cutoff dependent. A relatively low cutoff value, around 1 BeV, is required to yield a physically acceptable result. The discrepancy between the theoretical and the experimental values, $\delta m_K{}^{\rm th}-\delta m_K{}^{\rm expt}=5.44$–$6.22$ MeV, is most likely attributable to the high-energy contribution, which was neglected in the present approach.

I. INTRODUCTION

THE success of the current-algebra calculation of the pion electromagnetic mass splitting[1] has led people to extend the same method to calculate the kaon electromagnetic mass splitting, without notable success. The main difficulty in the kaon case comes from the fact that one cannot easily find the mechanism of the octet enhancement in this approach. From a general standpoint, this situation can be understood as follows. The joint success of current algebra and the hypothesis of partially conserved axial-vector current (PCAC) in the past several years has indicated that the strong interactions are nearly symmetrical under the chiral group $SU(3)\times SU(3)$. The symmetry $SU(3)\times SU(3)$ is broken by the presence of nonzero masses of the pseudoscalar mesons, and also by the presence of the electromagnetic and weak interactions.

Recently Dashen[2] has reached the conclusion that for first-order perturbations around an $SU(3)\times SU(3)$-symmetrical limit, there can be no dynamical mechanism which enhances the octet part of the pseudoscalar-meson mass splittings. This idea applies, in particular, to the electromagnetic mass splittings of the pseudoscalar mesons belonging to the same isomultiplet. One expects that the calculation of the $\Delta I=1$ kaon electromagnetic mass splitting in the usual current-algebra method will not work well, because the calculation of the pseudoscalar-meson mass splitting in this method is based on a first-order perturbation around an $SU(3)\times SU(3)$ limit [i.e., all the pseudoscalar mesons belonging to an $SU(3)$ octet have zero mass]. This is actually the case. However, the success of the calculation of the pion electromagnetic mass splitting is encouraging, and we may expect that one can calculate the kaon electromagnetic mass splitting simply by adding the high-energy contribution to the value obtained by the current-algebra method. This procedure would be justified by the work of Harari.[3] On the basis of the Regge-pole theory, he showed that, for the calculation of the $\Delta I=1$ electromagnetic mass splitting, it is essential to include the contribution from the high-energy part of the forward virtual Compton scattering process. The recent calculation due to Buccella *et al.*[4] indeed well reproduces the experimental value. As the low-energy contribution, these authors used the value obtained by Socolow.[5] It is our intent here to repeat the calculation

[1] T. Das, G. Guralnik, V. Mathur, F. Low, and J. Young, Phys. Rev. Letters **18**, 759 (1967); I. S. Gerstein, B. W. Lee, H. T. Nieh, and H. J. Schnitzer, *ibid.* **19**, 1064 (1967).
[2] R. F. Dashen, Phys. Rev. **183**, 1245 (1969).
[3] H. Harari, Phys. Rev. Letters **17**, 1303 (1966).
[4] F. Buccella, M. Cini, M. De Maria, and B. Tirozzi, Nuovo Cimento **64A**, 927 (1969).
[5] R. H. Socolow, Phys. Rev. **137**, B1221 (1965).

of the low-energy contribution to the kaon electromagnetic mass splitting δm_K by a somewhat different method from that used by Socolow, and also to confirm that the whole value of δm_K cannot arise within the framework of the low-energy approach[6] in any reasonable way. For this purpose we employ the phenomenological Lagrangian method, which reproduces the current-algebra result for the $q=0$ meson emission amplitude. The main point is the way of extrapolating the $q=0$ result to the large-momentum-transfer or hard-meson process. Assuming an explicit form for the Lagrangian, we derive the invariance condition under a photon gauge transformation. We also derive a phenomenological Lagrangian from the original one and then diagonalize it. This is done in Sec. II. The effective Lagrangian obtained in Sec. II is used in Sec. III to give the expression for δm_K in terms of the free parameters α_1, γ, and δ, which can be determined by the decay rates of the vector or the axial-vector mesons. Section IV contains the discussion.

II. LAGRANGIAN MODEL

A. Construction of Lagrangian

According to the work of Lee, Weinberg, and Zumino,[7] a massive Yang-Mills Lagrangian gives the algebra-of-fields commutators as the canonical commutation relations of the gauge fields. We require that the Lagrangian have the following properties: (i) The meson scattering amplitude calculated from the Lagrangian by taking only tree graphs agrees with the corresponding current-algebra result at vanishing meson four-momentum ($q=0$). (ii) When the electromagnetic fields are introduced into the Lagrangian, it is possible to calculate the photon process by relating it to the corresponding vector-meson process. Property (i) defines a phenomenological Lagrangian. Property (ii) is satisfied if the Lagrangian is so constructed that the resulting electromagnetic current operator is dominated by known neutral vector mesons (field-current identity). It is justified in Sec. II C that the Lagrangian given below, Eq. (1), has both properties (i) and (ii). Using these properties (i) and (ii), we can calculate the photon-meson scattering amplitude for a zero-momentum photon. For the calculation of the meson scattering amplitude, we take the following Lagrangian for the generalized massive Yang-Mills field plus matter fields as a model of $SU(3)\times SU(3)$ field algebra:

$$L=-\tfrac{1}{4}V_{\mu\nu,a}(Z_V)_{ab}V_{\mu\nu,b}-\tfrac{1}{4}A_{\mu\nu,a}(Z_A)_{ab}A_{\mu\nu,b} -\tfrac{1}{2}\phi_{\mu a}{}^V(M_V{}^2)_{ab}\phi_{\mu b}{}^V-\tfrac{1}{2}\phi_{\mu a}{}^A(M_A{}^2)_{ab}\phi_{\mu b}{}^A +L_m(\psi,D_\mu\psi,V_{\mu\nu,a},A_{\mu\nu,a})\,, \quad (1)$$

where Z_V and Z_A are symmetrical matrices and $M_V{}^2$ and $M_A{}^2$ are symmetrical mass matrices of the gauge fields. $\phi_{\mu a}{}^V$ and $\phi_{\mu a}{}^A$ ($a=0,\ldots,8$) are the vector and the axial-vector (gauge) fields, respectively, which constitute the basis of $(1+8)$ representations of $SU(3)$. $V_{\mu\nu}$ and $A_{\mu\nu}$ are given by

$$\begin{aligned} V_{\mu\nu,a}&=\partial_\mu\phi_{\nu a}{}^V-\partial_\nu\phi_{\mu a}{}^V +f\,f_{abc}(\phi_{\mu b}{}^V\phi_{\nu c}{}^V+\phi_{\mu b}{}^A\phi_{\nu c}{}^A)\,,\\ A_{\mu\nu,a}&=\partial_\mu\phi_{\nu a}{}^A-\partial_\nu\phi_{\mu a}{}^A +f\,f_{abc}(\phi_{\mu b}{}^A\phi_{\nu c}{}^V+\phi_{\mu b}{}^V\phi_{\nu c}{}^A)\,, \end{aligned} \quad (2)$$

with real constant f and $SU(3)$ structure constants f_{abc}. L_m is a matter term, and ψ represent the matter fields. $D_\mu\psi$ is a covariant derivative of ψ. We may also define the field-current identity

$$J_{\mu a}{}^V=[(M_V{}^2)_{ab}/f]\phi_{\mu b}{}^V\,. \quad (3)$$

By using the canonical commutation rule for the gauge field, and the equation of motion for $\phi_{\mu a}{}^V$, one obtains the following equal-time commutators in an unrenormalized form[8]:

$$[J_{ka}{}^V(x),J_{lb}{}^V(x')]_{x_0=x_0'}=0\,, \quad (4)$$

$$[J_{4a}{}^V(x),J_{lb}{}^V(x')]_{x_0=x_0'}=[(M_V{}^2)_{ab}/f^2]\partial_l\delta(\mathbf{x}-\mathbf{x}') -f_{acd}(M_V{}^2)_{bc}(M_V{}^2)^{-1}{}_{de} \times\delta(\mathbf{x}-\mathbf{x}')J_{le}{}^V(x)\,, \quad (5)$$

$$[J_{4a}{}^V(x),J_{4b}{}^V(x')]_{x_0=x_0'}=-f_{abc}\delta(\mathbf{x}-\mathbf{x}')J_{4c}{}^V(x)\,. \quad (6)$$

(Equal-time commutators involving axial-vector currents are derived in the same way.)

The standard form of the field algebra is recovered only for the diagonal mass matrices. Therefore we assume

$$(M_V{}^2)_{ac}=(M_A{}^2)_{ac}=m^2\delta_{ac}+b\delta_{a0}\delta_{c0}\,. \quad (7)$$

The explicit forms of Z_V and Z_A are chosen in Sec. II D so as to yield the Gell-Mann–Okubo formula for the vector and axial-vector meson masses squared.

B. Gauge Invariance

We introduce the electromagnetic interaction into the Lagrangian (1) according to the prescription of Lee and Zumino.[9] We rewrite L as

$$L=L_0-\tfrac{1}{2}\phi_{\mu a}{}^V(M_V{}^2)_{ab}\phi_{\mu b}{}^V-\tfrac{1}{2}\phi_{\mu a}{}^A(M_A{}^2)_{ab}\phi_{\mu b}{}^A\,. \quad (8)$$

L_0 is identical with L except for the gauge-field mass terms. By replacing all $\phi_{\mu a}{}^V$ (including $\partial_\nu\phi_{\mu a}{}^V$) in L_0 by $\hat\phi_{\mu a}{}^V$, we obtain L_0'.

$$L_0'=L_0(\phi_{\mu a}{}^V\rightarrow\hat\phi_{\mu a}{}^V)\,,$$

where

$$\hat\phi_{\mu a}{}^V=\phi_{\mu a}{}^V-(e/f)\xi_a A_\mu \quad (9)$$

and

$$\begin{aligned}\xi_a&=1 &&(a=3)\\ &=3^{-1/2} &&(a=8)\\ &=0 &&\text{otherwise}.\end{aligned} \quad (10)$$

Here, e and f refer to the unrenormalized coupling

[6] We use the terminology of Okubo. See S. Okubo, Phys. Rev. Letters **18**, 256 (1967).

[7] T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).

[8] K. Kang, Phys. Rev. **177**, 2439 (1969).

[9] T. D. Lee and B. Zumino, Phys. Rev. **163**, 1667 (1967).

constants. The Lagrangian including the electromagnetic effect is as follows:

$$L'=-\tfrac{1}{4}F_{\mu\nu}{}^2+L_0'-\tfrac{1}{2}\phi_{\mu a}{}^V(M_V{}^2)_{ab}\phi_{\mu b}{}^V -\tfrac{1}{2}\phi_{\mu a}{}^A(M_A{}^2)_{ab}\phi_{\mu b}{}^A\,, \quad (11)$$

where $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ is the electromagnetic field tensor. The equation of motion for A_μ becomes

$$\partial_\mu F_{\mu\nu}=(e/f)[(M_V{}^2)_{3b}+3^{-1/2}(M_V{}^2)_{8b}]\phi_{\nu b}{}^V. \quad (12)$$

The gauge-invariance condition under the photon gauge transformation implies that the Lagrangian is invariant under the replacement $A_\mu\to A_\mu-\partial_\mu\Lambda(x)$. Thus L_0' must be invariant under the replacement

$$\hat{\phi}_{\mu a}{}^V\to\hat{\phi}_{\mu a}{}^V+(e/f)\xi_a\partial_\mu\Lambda. \quad (13)$$

This is equivalent to the invariance condition of L_0 under the replacement

$$\phi_{\mu a}{}^V\to\phi_{\mu a}{}^V+(e/f)\xi_a\partial_\mu\Lambda\,, \quad (14)$$

which depends on the symmetry property of L_0 under the strong interactions. We first note that any infinitesimal $SU(3)\times SU(3)$ transformation of $\phi_{\mu a}{}^V$ ($\phi_{\mu a}{}^A$, $V_{\mu\nu,a}$, or $A_{\mu\nu,a}$) is obtained by the combined application of the following two transformations, (15) and (16):

$$\begin{aligned}V_{\mu\nu,a}&\to V_{\mu\nu,a}+f\,f_{abc}\Lambda_b{}^V V_{\mu\nu,c}\,,\\ A_{\mu\nu,a}&\to A_{\mu\nu,a}+f\,f_{abc}\Lambda_b{}^V A_{\mu\nu,c}\,,\\ \phi_{\mu a}{}^V&\to\phi_{\mu a}{}^V-\partial_\mu\Lambda_a{}^V+f\,f_{abc}\Lambda_b{}^V\phi_{\mu c}{}^V\,,\\ \phi_{\mu a}{}^A&\to\phi_{\mu a}{}^A+f\,f_{abc}\Lambda_b{}^V\phi_{\mu c}{}^A\,,\end{aligned} \quad (15)$$

$$\begin{aligned}V_{\mu\nu,a}&\to V_{\mu\nu,a}+f\,f_{abc}\Lambda_b{}^A A_{\mu\nu,c}\,,\\ A_{\mu\nu,a}&\to A_{\mu\nu,a}+f\,f_{abc}\Lambda_b{}^A V_{\mu\nu,c}\,,\\ \phi_{\mu a}{}^V&\to\phi_{\mu a}{}^V+f\,f_{abc}\Lambda_b{}^A\phi_{\mu c}{}^A\,,\\ \phi_{\mu a}{}^A&\to\phi_{\mu a}{}^A-\partial_\mu\Lambda_a{}^A+f\,f_{abc}\Lambda_b{}^A\phi_{\mu c}{}^V\,,\end{aligned} \quad (16)$$

where $\Lambda_i{}^V$ ($i=1,\ldots,8$) and $\Lambda_i{}^A$ ($i=1,\ldots,8$) are coordinate-dependent infinitesimal parameters. The condition (14) determines $\Lambda_i{}^V$ ($i=1,\ldots,8$) and $\Lambda_i{}^A$ ($3\neq i\neq 8$) uniquely. We get

$$\Lambda_i{}^V=-(e/f)\xi_i\Lambda,\quad \Lambda_i{}^A(3\neq i\neq 8)=0. \quad (17)$$

$\Lambda_3{}^A$ and $\Lambda_8{}^A$ are undetermined. For the general forms of Z_V and Z_A, kinetic parts of the gauge-field Lagrangian can preserve the Lorentz invariance only if $\Lambda_3{}^A=\Lambda_8{}^A=0$. Therefore we impose the requirement

$$\Lambda_3{}^A=\Lambda_8{}^A=0. \quad (18)$$

Thus the transformations (15) and (16) for $V_{\mu\nu,a}$ and $A_{\mu\nu,a}$ become

$$\begin{aligned}V_{\mu\nu,a}&\to V_{\mu\nu,a}+\epsilon(x)(f_{a3b}+3^{-1/2}f_{a8b})V_{\mu\nu,b}\,,\\ A_{\mu\nu,a}&\to A_{\mu\nu,a}+\epsilon(x)(f_{a3b}+3^{-1/2}f_{a8b})A_{\mu\nu,b}\,,\end{aligned} \quad (19)$$

with $\epsilon(x)=-(e/f)\Lambda(x)$. L_0 contains such terms as $V_{\mu\nu,a}(Z_V)_{ab}V_{\mu\nu,b}$, $A_{\mu\nu,a}(Z_A)_{ab}A_{\mu\nu,b}$, $D_{abc}V_{\mu\nu,a}A_{\mu\nu,b}P_c$, and $D_{abc}\epsilon_{\mu\nu\lambda\sigma}V_{\mu\nu,a}V_{\lambda\sigma,b}P_c$, where P_c ($c=1,\ldots,8$) represent fields of a pseudoscalar-meson octet. The gauge-invariance condition leads immediately to the following restrictions (i) and (ii) on $(Z_{V,A})_{ab}$ and D_{abc}, respectively[10]:

$$\text{(i)}\quad L_{ab}+L_{ba}=0\,,$$

where

$$L_{ab}=(f_{c3b}+3^{-1/2}f_{c8b})(Z_{V,A})_{ca}\,, \quad (20)$$

$$\text{(ii)}\quad D_{a'bc}(f_{a'3a}+3^{-1/2}f_{a'8a})+D_{ab'c}(f_{b'3b}+3^{-1/2}f_{b'8b}) +D_{abc'}(f_{c'3c}+3^{-1/2}f_{c'8c})=0. \quad (21)$$

C. Computational Rule for Photon Process

The equal-time commutators described in Sec. II A are written in an unrenormalized form. Following Lee and Zumino,[9] we assume the existence of the renormalized theory of the Lagrangian (1) even if the perturbation-series method fails. (Otherwise, the equal-time commutator between the renormalized current operators does not exist.) Then the equation of motion for the gauge field and the normalization condition of the current operator,

$$Q_a{}^{V,A}=-i\int J_{4a}{}^{V,A}(x)d^3x=-i\int[J_{4A}{}^{V,A}(x)]_R d^3x\,,$$

imply the renormalization independence of the current operator defined by Eq. (3).[11] These relations define the renormalized and the unrenormalized coupling constants. [$Q_a{}^{V,A}$ is an $SU(3)\times SU(3)$ generator and the subindex R implies the renormalized operator.] Thus Eqs. (4)–(6) and the related equal-time commutators involving the derivatives can be regarded as the relations between the physical current operators. From the work of Kang,[8] we know that the Weinberg second sum rule, derived from the equal-time commutators involving time derivatives, depends on the explicit forms of Z_V and Z_A. We take

$$(Z_V)^{-1}{}_{ab}=\delta_{ab}+\sqrt{3}\epsilon d_{ab8}. \quad (22)$$

ϵ is a symmetry-breaking parameter. This leads to the Gell-Mann–Okubo formula for the vector-meson mass squared through the single-particle saturation of the Weinberg second sum rule. One can easily see that the Z_V of (22) satisfies the gauge-invariance condition (20). In order to calculate the scattering amplitude for the low-momentum photon process, we first consider the L' given in Eq. (11). In this form, the photon couples to both the gauge-field and the hadronic electromagnetic current. The net effect is equivalent to adding the effective Lagraigian L_{int},

$$L_{\text{int}}=-ej_\mu{}^{\text{em}}A_\mu=-(em^2/f)(\phi_{\mu 3}{}^V+3^{-1/2}\phi_{\mu 8}{}^V)A_\mu\,, \quad (23)$$

to L given by Eq. (1) as the result of the equation of

[10] Similar conditions have also been considered by Schwinger. See J. Schwinger, Phys. Rev. **165**, 1714 (1968). Examples of the D_{abc} satisfying Eq. (21) are d_{abc}, f_{abc}, etc.

[11] N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. **157**, 1376 (1967).

motion. We can also construct a theory, which satisfies the field-current identity, by introducing the electromagnetic field into the gauge-field mass term by replacing $\phi_{\mu a}{}^V$ by $\phi_{\mu a}{}^V-(e/f)\xi_a A_\mu$. In the latter case, the photon couples only to the gauge-field and does not couple to the hadronic electromagnetic current. It is not a matter of convention, but of direct physical significance, that the photon couples to the hadronic electromagnetic current, or to the hadronic gauge field, or to something else.[7] We assume in the following that $\phi_{\mu 3,8}{}^V$ and A_μ in Eq. (11) represent the fields of the vector-meson and the photon, respectively. Then the lowest-order photon process can be calculated by using (23) as the effective interaction and noting the relation $eA_\mu=e_R(A_\mu)_R$ if the vector-meson emission amplitudes are known.[12] By using the field-algebra relations and the reduction technique, one can calculate the vector-meson emission amplitude in the zero-momentum-transfer ($q=0$) limit. It is known that the equivalent result is obtained by applying the Feynman-Dyson rule to the Lagrangian (1) and taking only the tree graph. The only effect of the higher-order graph on the soft-meson process ($q=0$) is to renormalize the trees.[13] Therefore we can regard the Lagrangian (1) with appropriate forms for $Z_{V,A}$ and $M_{V,A}{}^2$ as a phenomenological Lagrangian by replacing all the masses and coupling constants by corresponding renormalized ones. These techniques have already been used by Lee and Nieh, and by Wick and Zumino.[12]

D. Diagonalization of Phenomenological Lagrangian

A model constructed above can be used to reproduce the relations among masses and coupling constants which are derived from the Weinberg sum rules. It is accomplished by diagonalizations of the phenomenological Lagrangian. Taking the Lagrangian (1) with the conditions (7) and (20), we get a model Lagrangian of the field algebra which is consistent with gauge invariance. Further, replacing unrenormalized field operators, masses, and coupling constants by corresponding renormalized ones, we get a phenomenological Lagrangian which must by used in the tree approximation. Only in this form can diagonalization procedures have clear physical meaning. We first consider the vector gauge fields. The normalization condition of the kinetic part of L leads, with the aid of (22), to

$$\begin{aligned}\phi_{\mu,123}{}^V&=(1+\epsilon)^{1/2}\tilde{\phi}_{\mu,123}{}^V,\\ \phi_{\mu,4567}{}^V&=(1-\tfrac{1}{2}\epsilon)^{1/2}\tilde{\phi}_{\mu,4567}{}^V,\\ m_\rho{}^2=(1+\epsilon)m^2,&\quad m_{K^*}{}^2=(1-\tfrac{1}{2}\epsilon)m^2.\end{aligned}\tag{24}$$

Here the tilde over $\phi_{\mu a}{}^V$ means that it is a physical operator. We also define the ϕ-ω mixing angle by the relation[11]

$$\begin{aligned}\phi_{\mu 8}{}^V&=C_8(m_\phi{}^2\cos\theta_Y\,\tilde{\phi}_\mu-m_\omega{}^2\sin\theta_Y\,\tilde{\omega}_\mu),\\ \phi_{\mu 0}{}^V&=C_0(m_\phi{}^2\sin\theta_B\,\tilde{\phi}_\mu+m_\omega{}^2\cos\theta_B\,\tilde{\omega}_\mu).\end{aligned}\tag{25}$$

Then the diagonalization conditions of L become

$$\begin{aligned}\frac{m_\omega}{m_\phi}\tan\theta_Y=\frac{m_\phi}{m_\omega}&\tan\theta_B\equiv\tan\theta,\\ C_8=(mm_\phi)^{-1}[1+(m_\phi{}^2/m_\omega{}^2)&\tan^2\theta]^{1/2}\cos\theta,\end{aligned}\tag{26}$$

$$C_0=[(m^2+b)^{1/2}m_\omega]^{-1}\times[1+(m_\omega{}^2/m_\phi{}^2)\tan^2\theta]^{1/2}\cos\theta.\tag{27}$$

The mixing angle θ is determined by the equation

$$(1+b/m^2)y^2-2^{-1/2}(1+b/m^2\epsilon)y-1=0,\tag{28}$$

where $y=(1+b/m^2)^{-1/2}\tan\theta$. The ϕ- and ω-meson masses are given by

$$\begin{aligned}m_\phi{}^2&=m^2(1+\epsilon)(1-2\epsilon)\times[1+(1-\epsilon)y^2-2^{3/2}\epsilon y]^{-1}\cos^{-2}\theta,\\ m_\omega{}^2&=m^2(1+\epsilon)(1-2\epsilon)[(1+b/m^2)y^2+(1-\epsilon)(1+b/m^2)^{-1}+2^{3/2}\epsilon y]^{-1}\cos^{-2}\theta.\end{aligned}\tag{29}$$

From Eqs. (24), (28), and (29) we have

$$m_\phi{}^2\cos^2\theta+m_\omega{}^2\sin^2\theta=m^2(1-\epsilon)=\tfrac{1}{3}(4m_{K^*}{}^2-m_\rho{}^2).\tag{30}$$

For simplicity we put $b=0$ hereafter. Then the requirement $m_\phi>m_\omega$ fixes the sign of θ. We have from Eq. (28) that $\tan\theta=-1/\sqrt{2}$ or $\theta=-35.2°$. This value was first obtained by Akiba and Kang using the Weinberg sum rules.[14] A least-squares fit to the experimental value gives $m=859$ MeV, $\epsilon=-0.196$.[15] In summary, we have shown that the diagonalization procedures of the phenomenological Lagrangian lead to the same results as that of the Weinberg sum rules. (Coupling-constant relations can also be derived, but are not presented here.) Finally, we put relations (24) and (25) in the convenient form given by Brown, Munczek, and Singer.[16]

$$\begin{aligned}\phi_{\mu,123}{}^V&=(1/\sqrt{K_\rho})\tilde{\phi}_{\mu,123}{}^V,\quad \phi_{\mu,4567}{}^V=(1/\sqrt{K_*})\tilde{\phi}_{\mu,4567}{}^V,\\ \phi_{\mu 8}{}^V&=\frac{\cos\theta}{\sqrt{K_\phi}}\tilde{\phi}_\mu-\frac{\sin\theta}{\sqrt{K_\omega}}\tilde{\omega}_\mu,\quad \phi_{\mu 0}{}^V=\frac{\sin\theta}{\sqrt{K_\phi}}\tilde{\phi}_\mu+\frac{\cos\theta}{\sqrt{K_\omega}}\tilde{\omega}_\mu,\end{aligned}\tag{31}$$

where $K_i=m^2/m_i{}^2$ ($i=\rho$, ϕ, ω, K^*), $K_*\equiv K_{K^*}$, and $\tan\theta=-1/\sqrt{2}$.

E. Effective Lagrangian for Neutral-Vector-Meson-Kaon Scattering

In order to calculate the kaon electromagnetic mass splitting using the method described in Sec. II C, we add

[12] B. W. Lee and H. T. Nieh, Phys. Rev. **166**, 1507 (1968); G. C. Wick and B. Zumino, Phys. Letters **25B**, 479 (1967). As we use only the renormalized quantities, the subindices R are omitted hereafter.

[13] R. Dashen and M. Weinstein, Phys. Rev. **183**, 1261 (1969).

[14] T. Akiba and K. Kang, Phys. Rev. **172**, 1551 (1968).

[15] The values of masses used in this analysis are $m_\rho=765$ MeV, $m_{K^*}=890$ MeV, $m_\phi=1019$ MeV, and $m_\omega=783$ MeV.

[16] L. M. Brown, H. Munczek, and P. Singer, Phys. Rev. Letters **21**, 707 (1968).

the pseudoscalar meson terms to L_m of Eq. (1). We take the following L_m for this purpose:

$$L_m=-(1/2a^2)(D_\mu\xi_a)^2+i[\alpha_1 d_{abc}+\alpha_0(\delta_{ac}\delta_{b0}+\delta_{bc}\delta_{a0})]\times V_{\mu\nu,a}V_{\lambda\sigma,b}P_c\epsilon_{\mu\nu\lambda\sigma}+\beta_1 f_{abc}V_{\mu\nu,a}A_{\mu\nu,b}P_c+\beta_2 f_{abc}D_\mu P_a D_\nu P_b V_{\mu\nu,c}\,, \quad (32)$$

where a, α_i, and β_i are real constants. For the covariant derivatives of the pseudoscalar fields ξ_a ($\equiv\eta P_a$), we use the form given by Coleman *et al.*,[17] i.e.,

$$e^{i\gamma_5\xi}[\partial_\mu-if(\phi_\mu{}^V+\gamma_5\phi_\mu{}^A)]e^{-i\gamma_5\xi}=-iv_\mu-i(D_\mu\xi)\gamma_5\,. \quad (33)$$

In these expressions, $\phi_\mu{}^{V,A}$, V_μ, and $D_\mu\xi$ stand for 3×3 matrices defined by $\phi_\mu{}^V=\phi_{\mu a}{}^V\frac{1}{2}\lambda_a$, etc., f is identical with the one defined previously. Under charge conjugation, the 3×3 matrices $V_{\mu\nu}$ and $A_{\mu\nu}$ transform as $V_{\mu\nu}\to-V_{\mu\nu}{}^T$, $A_{\mu\nu}\to A_{\mu\nu}{}^T$.[18] The symmetrical (antisymmetrical) coupling of VVP (VAP) type in L_m was determined in this way. The sum of the gauge-field mass term and the kinetic term of the pseudoscalar field is diagonalized by regarding $\phi_{\mu a}{}^A$, apart from a factor, as the physical axial-vector field in the following way.[19]

Define $\hat{\phi}_{\mu a}{}^A$ by

$$\hat{\phi}_{\mu a}{}^A=\phi_{\mu a}{}^A+f(a^2m^2+f^2)^{-1}D_\mu'\xi_a\,, \quad (34)$$

with

$$D_\mu'\xi_a=D_\mu\xi_a-f\phi_{\mu a}{}^A\,, \quad (35)$$

$$\hat{m}^2=m^2+f^2/a^2\,, \quad (36)$$

$$\eta=(a^2+f^2/m^2)^{1/2}\,. \quad (37)$$

Then we see that

$$-(1/2a^2)(D_\mu\xi_a)^2-\tfrac{1}{2}m^2(\phi_{\mu a}{}^A)^2=-\tfrac{1}{2}(D_\mu'P_a)^2-\tfrac{1}{2}\hat{m}^2(\hat{\phi}_{\mu a}{}^A)^2\,. \quad (38)$$

The right-hand side no longer contains terms like $\partial_\mu P_a\hat{\phi}_{\mu a}{}^A$. We assume vector-meson dominance in the form

$$f/m\eta=1/\sqrt{2}\,, \quad (39)$$

which gives $\hat{m}^2=2m^2$. For definiteness, we also assume $Z_V=Z_A$ ($=Z$) in the following. This assumption leads, apart from a multiplicative factor, to the same spectrums of masses for the vector and axial-vector nonets, which are not in contradiction with experimental results.[20] Further, the condition $\Gamma(\phi\to\rho\pi)=0$ implies $\alpha_0=0$ in our model. Other constants in L_m are determined in Sec. III. Now we can readily write down the effective Lagrangians for the scattering of the neutral vector meson and the kaon. They are obtained by expanding the entire Lagrangian as follows:

$$L_{PVV}=\tfrac{1}{2}f(j_{\mu,K}-j_{\mu,K^0})\phi_{\mu3}{}^V+\tfrac{1}{2}\sqrt{3}f(j_{\mu,K}+j_{\mu,K^0})\phi_{\mu8}{}^V-\tfrac{1}{8}[(f/m^2)Z_{33}-\beta_2](j_{\nu,K}-j_{\nu,K^0})\partial_\mu F_{\mu\nu,3}-\tfrac{1}{8}\sqrt{3}[(f/m^2)Z_{33}-\beta_2](j_{\nu,K}+j_{\nu,K^0})\partial_\mu F_{\mu\nu,8}\,, \quad (40)$$

$$L_{PVA}=-\tfrac{1}{2}i[(2^{-1/2}f/m)Z_{44}-2\beta_1]\{[\phi_K(\hat{\phi}_{\nu,K}{}^A)^\dagger-\phi_K{}^\dagger\hat{\phi}_{\nu,K}{}^A]-[\phi_{K^0}(\hat{\phi}_{\nu,K^0}{}^A)^\dagger-\phi_{K^0}{}^\dagger\hat{\phi}_{\nu,K^0}{}^A]\}\partial_\mu F_{\mu\nu,3}$$
$$+\tfrac{1}{2}i[(2^{-1/2}f/m)(Z_{33}-Z_{44})+2^{-1/2}m\beta_2+2\beta_1]\{[\partial_\mu\phi_K(\hat{\phi}_{\nu,K}{}^A)^\dagger-\partial_\mu\phi_K{}^\dagger\hat{\phi}_{\nu,K}{}^A]-[\partial_\mu\phi_{K^0}(\hat{\phi}_{\nu,K^0}{}^A)^\dagger-\partial_\mu\phi_{K^0}{}^\dagger\phi_{\nu,K^0}{}^A]\}F_{\mu\nu,3}-\tfrac{1}{2}\sqrt{3}i[(2^{-1/2}f/m)Z_{44}-2\beta_1]\{[\phi_K(\hat{\phi}_{\nu,K}{}^A)^\dagger-\phi_K{}^\dagger\hat{\phi}_{\nu,K}{}^A]$$
$$+[\phi_{K^0}(\hat{\phi}_{\nu,K^0}{}^A)^\dagger-\phi_{K^0}{}^\dagger\hat{\phi}_{\nu,K^0}{}^A]\}\partial_\mu F_{\mu\nu,8}+\tfrac{1}{2}\sqrt{3}i[(2^{-1/2}f/m)(Z_{88}-Z_{44})+2^{-1/2}m\beta_2+2\beta_1]$$
$$\times\{[\partial_\mu\phi_K(\hat{\phi}_{\nu,K}{}^A)^\dagger-\partial_\mu\phi_K{}^\dagger\hat{\phi}_{\nu,K}{}^A]+[\partial_\mu\phi_{K^0}(\hat{\phi}_{\nu,K^0}{}^A)^\dagger-\partial_\mu\phi_{K^0}{}^\dagger\hat{\phi}_{\nu,K^0}{}^A]\}F_{\mu\nu,8}\,, \quad (41)$$

$$L_{PPVV}=-\tfrac{1}{4}f^2[(\phi_{\mu3}{}^V)^2(\phi_K\phi_K{}^\dagger+\phi_{K^0}\phi_{K^0}{}^\dagger)+2\sqrt{3}\phi_{\mu3}{}^V\phi_{\mu8}{}^V(\phi_K\phi_K{}^\dagger-\phi_{K^0}\phi_{K^0}{}^\dagger)+3(\phi_{\mu8}{}^V)^2(\phi_K\phi_K{}^\dagger+\phi_{K^0}\phi_{K^0}{}^\dagger)]$$
$$+\tfrac{1}{8}[(f^2/m^2)Z_{33}-f\beta_2](\phi_K\phi_K{}^\dagger+\phi_{K^0}\phi_{K^0}{}^\dagger)\partial_\mu F_{\mu\nu,3}\phi_{\nu3}{}^V+\tfrac{1}{8}\sqrt{3}[(f^2/m^2)Z_{33}-f\beta_2](\phi_K\phi_K{}^\dagger-\phi_{K^0}\phi_{K^0}{}^\dagger)\partial_\mu F_{\mu\nu,3}\phi_{\nu8}{}^V$$
$$+\tfrac{1}{8}\sqrt{3}[(f^2/m^2)Z_{88}-f\beta_2](\phi_K\phi_K{}^\dagger-\phi_{K^0}\phi_{K^0}{}^\dagger)\partial_\mu F_{\mu\nu,}\,\phi_{\nu3}{}^V+\tfrac{3}{8}[(f^2/m^2)Z_{88}-f\beta_2](\phi_K\phi_K{}^\dagger+\phi_{K^0}\phi_{K^0}{}^\dagger)\partial_\mu F_{\mu\nu,8}\phi_{\nu8}{}^V$$
$$+\tfrac{1}{16}[(f^2/m^2)(Z_{33}-Z_{44})-f\beta_2+(4\sqrt{2}/m)f\beta_1](\phi_K\phi_K{}^\dagger+\phi_{K^0}\phi_{K^0}{}^\dagger)(F_{\mu\nu,3})^2$$
$$+\tfrac{1}{16}\sqrt{3}[(f^2/m^2)(Z_{33}-2Z_{44}+Z_{88})-2f\beta_2+(8\sqrt{2}/m)f\beta_1](\phi_K\phi_K{}^\dagger-\phi_{K^0}\phi_{K^0}{}^\dagger)F_{\mu\nu,3}F_{\mu\nu,8}$$
$$+\tfrac{3}{16}[(f^2/m^2)(Z_{88}-Z_{44})-f\beta_2+(4\sqrt{2}/m)f\beta_1](\phi_K\phi_K{}^\dagger+\phi_{K^0}\phi_{K^0}{}^\dagger)(F_{\mu\nu,8})^2\,, \quad (42)$$

$$L_{PVV}=2\alpha_1 i\epsilon_{\mu\nu\lambda\sigma}\{[\phi_K\partial_\mu(\phi_{\nu,K}{}^V)^\dagger+\phi_K{}^\dagger\partial_\mu\phi_{\nu,K}{}^V](F_{\lambda\sigma,3}-3^{-1/2}F_{\lambda\sigma,8})-[\phi_{K^0}\partial_\mu(\phi_{\nu,K^0}{}^V)^\dagger+\phi_{K^0}{}^\dagger\partial_\mu\phi_{\nu,K^0}{}^V](F_{\lambda\sigma,3}+3^{-1/2}F_{\lambda\sigma,8})\}\,, \quad (43)$$

where

$$j_{\mu,K(K^0)}=i[\partial_\mu\phi_{K(K^0)}\phi_{K(K^0)}{}^\dagger-\phi_{K(K^0)}{}^\dagger\partial_\mu\phi_{K(K^0)}]\,,$$
$$F_{\mu\nu,i}=\partial_\mu\phi_{\nu i}{}^V-\partial_\nu\phi_{\mu i}{}^V\quad(i=3,8)\,.$$

In the above expressions, (40)–(43), $\phi_K(\phi_{K^0})$ stands for the charged (neutral) kaon field operator. $\hat{\phi}_{\mu,K}{}^A$ ($\hat{\phi}_{\mu,K^0}{}^A$) stands for the field operator of the charged (neutral) K_A meson, except for a normalization factor.

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[17] S. Coleman, J. Wess, and B. Zumino, Phys. Rev. **177**, 2239 (1969); C. G. Callan, S. Coleman, J. Wess, and B. Zumino, *ibid.* **177**, 2247 (1969). The choice of the form of covariant derivatives is not essential, except for the requirement that the time component of the current operator should satisfy Eq. (6) as the result of the equation of motion. One can easily see that the nonlinear $D_\mu\xi$ gives, by the canonical commutation rule, Eqs. (4) and (5) in exact forms, and Eq. (6) at least to order ξ^2. This is sufficient for our purpose.

[18] T implies "transposed."

[19] This $\hat{\phi}_{\mu a}{}^A$ is an axial-vector partner of the $\phi_{\mu a}{}^V$. The physical axial-vector meson field is obtained by further diagonalization as in Eq. (31) by replacing K_i by $K_i'=\hat{m}^2/m_i{}^2$.

[20] Particle Data Group, Rev. Mod. Phys. **42**, 87 (1970).

III. EVALUATION OF MASS SPLITTING

A. Expression for δm_K

The expression for $\delta m_K^2 = m_{K^+}^2 - m_{K^0}^2$ is given by[21]

$$\delta m_K^2 = [i/(2\pi)^4][\delta(p-p')]^{-1} \times(\langle K^+(p)|S_2|K^+(p')\rangle - \langle K^0(p)|S_2|K^0(p')\rangle)|_{p=p'} = \int \frac{M}{i(2\pi)^4} d^4q\,,$$

where S_2 is the S operator for Compton scattering in order e^2 with disconnected graphs removed. We divide the contribution to M of the effective Lagrangians (40)–(43) into five parts M_i. Here M_i are the contributions from the graphs of the following types. M_1 is the pole graph of second order in $j_\mu \phi_\mu^V$, plus the contact graph involving no derivative. M_2 is the pole graph of second order in $\partial_\mu F_{\mu\nu,3(8)} j_\nu$, plus the contact graph involving derivatives. M_{12} is the pole graph due to cross terms of $j_\mu\phi_\mu^V$ and $\partial_\mu F_{\mu\nu,3(8)} j_\nu$. M_3 is the pole graph with an axial-vector meson in the intermediate state. M_4 is the pole graph with a vector meson in the intermediate state. Equation (23) can be rewritten in the form

$$L_{\rm int} = -\frac{em^2}{f}\left(\frac{1}{\sqrt{K_\rho}}\tilde{\phi}_{\nu 3}{}^V + \frac{\cos\theta}{(3K_\phi)^{1/2}}\tilde{\phi}_\nu - \frac{\sin\theta}{(3K_\omega)^{1/2}}\tilde{\omega}_\nu\right)A_\nu\,. \qquad (23')$$

Then we apply the Feynman rule to get the expressions for $(\delta m_K^2)_i$, by taking the negative of the effective Lagrangians (40)–(43) and (23′) as the effective interaction Hamiltonians.[22] Here

$$\delta m_K^2 = \sum_i (\delta m_K^2)_i$$

and

$$(\delta m_K^2)_i = \int \frac{M_i}{i(2\pi)^4} d^4q\,. \qquad (44)$$

(i runs over 1, 2, 12, 3, and 4.) In the expressions for $(\delta m_K^2)_i$, we must always use the values obtained in Sec. II D for the vector-meson masses and the ϕ-ω mixing angle θ. Integrations over vitrual photon four-momentum can be carried out by using the standard Feynman technique. $(\delta m_K^2)_2$ and $(\delta m_K^2)_3$ are cutoff dependent. We obtain the following results. (The notation is explained at the end.)

$$(\delta m_K^2)_1 = \frac{3\alpha}{4\pi} m^2 \frac{1}{6y_0} \times\{(K_\rho K_\phi)^{-1} I_1[(m_\rho/m_K)^2,(m_\phi/m_K)^2]\cos^2\theta + (K_\rho K_\omega)^{-1} I_1[(m_\rho/m_K)^2,(m_\omega/m_K)^2]\sin^2\theta\}\,, \qquad (45)$$

where

$$I_1[x_1,x_2] = \frac{\Delta}{\Delta x}F_1(x)\Big|_{x=x_2}^{x=x_1} \equiv \frac{F_1(x_1)-F_1(x_2)}{x_1-x_2}$$

($\Delta/\Delta x$ should be replaced by $\partial/\partial x$ when $x_1=x_2$),

$$F_1(x) = x\ln x - (x-4)^2 W(x-2,1)\,,$$

$$W(a,b) \equiv \int_0^1 (u^2+au+b)^{-1} du\,.$$

$$(\delta m_K^2)_2 = (3\alpha/8\pi) m^2 b_3 b_8 \times\{(K_\rho K_\phi)^{-1} I_2[(m_\rho/m_K)^2,(m_\phi/m_K)^2]\cos^2\theta + (K_\rho K_\omega)^{-1} I_2[(m_\rho/m_K)^2,(m_\omega/m_K)^2]\sin^2\theta\}\,, \qquad (46)$$

where

$$I_2[x_1,x_2] = (b_0+\tfrac{1}{8}y_0)\ln(\Lambda/m_K)^2 + \tfrac{3}{16}y_0 + \tfrac{1}{4}y_0(\Delta/\Delta x)F_2(x)\Big|_{x=x_2}^{x=x_1}\,,$$

$$F_2(x) = (\tfrac{1}{12}x^2 - \tfrac{1}{2}x - 4b_0/y_0)x\ln x - \tfrac{1}{12}x^2(x-4)^2 W(x-2,1) - \tfrac{1}{6}x^2\,. \qquad (47)$$

$$(\delta m_K^2)_{12} = (3\alpha/8\pi) m^2 (b_3+b_8) \times\{(K_\rho K_\phi)^{-1} I_{12}[(m_\rho/m_K)^2,(m_\phi/m_K)^2]\cos^2\theta + (K_\rho K_\omega)^{-1} I_{12}[(m_\rho/m_K)^2,(m_\omega/m_K)^2]\sin^2\theta\}\,,$$

where

$$I_{12}[x_1,x_2] = \tfrac{1}{6} + (\Delta/\Delta x)F_{12}(x)\Big|_{x=x_2}^{x=x_1}\,,$$

$$F_{12}(x) = (-\tfrac{1}{12}x^2 + \tfrac{1}{2}x)\ln x + \tfrac{1}{12}x(x-4)^2 W(x-2,1)\,.$$

$$(\delta m_K^2)_3 = (3\alpha/8\pi) m^2 (K_*)^{-1}\{(K_\rho K_\phi)^{-1} I_3 \times[(m_\rho/m_K)^2,(m_\phi/m_K)^2]\cos^2\theta + (K_\rho K_\omega)^{-1} I_3[(m_\rho/m_K)^2,(m_\omega/m_K)^2]\sin^2\theta\}\,, \qquad (48)$$

where

$$I_3[x_1,x_2] = -(a_4^2 - \tfrac{1}{8}ya_3a_8)[\ln(\Lambda/m_K)^2 - \ln(2/y)] - \tfrac{1}{6}a_4^2 + (5/48)ya_3a_8 - \tfrac{1}{3}a_4(a_3+a_8) + (\Delta/\Delta x)F_3(x)\Big|_{x=x_2}^{x=x_1}\,,$$

$$F_3(x) = \frac{1}{6}\left(\frac{2}{y}-1\right)^3 [a_4^2 + a_3a_8 - a_4(a_3+a_8)] \times\frac{1}{x}\ln(1-\tfrac{1}{2}y) - \frac{1}{24}ya_3a_8x^2 + A(x)\ln(\tfrac{1}{2}xy) + B(x)W\left(x-1-\frac{2}{y},\frac{2}{y}\right).$$

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[21] M. Cini, E. Ferrari, and R. Gatto, Phys. Rev. Letters **2**, 7 (1959).

[22] T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962); J. Reiff and N. Veltman, Nucl. Phys. **B13**, 545 (1969).

$A(x)$ and $B(x)$ are given by

$$A(x)=(1/48)ya_3a_8x^3+[\tfrac{1}{12}a_4^2-(1/24+y/16)a_3a_8+\tfrac{1}{6}a_4(a_3+a_8)]x^2$$
$$+[(1/2y+\tfrac{3}{4})a_4^2-\tfrac{1}{16}y(2/y-1)^2a_3a_8-(1/2y+\tfrac{1}{4})a_4(a_3+a_8)]x$$
$$-\tfrac{1}{4}(2/y-1)^2a_4^2+(y/48)(2/y-1)^2(10/y+1)a_3a_8-\tfrac{1}{12}(2/y-1)^3[a_4^2+a_3a_8-a_4(a_3+a_8)](1/x),$$

$$B(x)=-(1/48)ya_3a_8x^4+\tfrac{1}{12}[-a_4^2+(1+y)a_3a_8-2a_4(a_3+a_8)]x^3$$
$$+[\tfrac{1}{3}(1-1/y)a_4^2+(1/6y-\tfrac{1}{3}-\tfrac{1}{8}y)a_3a_8+(5/12)(2y+1)a_4(a_3+a_8)]x^2$$
$$+[(2/y^2+4/3y-\tfrac{1}{2})a_4^2+(-4/3y^2+1/y+\tfrac{1}{3}+y/12)a_3a_8-(1/y^2+1/3y+\tfrac{1}{4})a_4(a_3+a_8)]x$$
$$-\tfrac{1}{3}(2/y-1)^2(1/y-1)a_4^2+(1/48)y(2/y-1)^2(28/y^2-4/y-1)a_3a_8$$
$$-\tfrac{1}{12}(2/y-1)^2(2/y+1)a_4(a_3+a_8)-\tfrac{1}{12}(2/y-1)^4[a_4^2+a_3a_8-a_4(a_3+a_8)](1/x).$$

$$(\delta m_K^2)_4=\frac{3\alpha}{4\pi}m^4\frac{64\alpha_1^2}{3f^2}(K_*)^{-1}\{(K_\rho K_\phi)^{-1}I_4[(m_\rho/m_K)^2,(m_\phi/m_K)^2]\cos^2\theta$$
$$+(K_\rho K_\omega)^{-1}I_4[(m_\rho/m_K)^2,(m_\omega/m_K)^2]\sin^2\theta\}, \quad (49)$$

where

$$I_4[x_1,x_2]=\tfrac{1}{6}+(\Delta/\Delta x)F_4(x)|_{x=x_2}{}^{x=x_1},$$
$$F_4(x)=-\tfrac{1}{6}(2/y-1)^3[\ln(1-y)](1/x)+[-\tfrac{1}{12}x^2$$
$$+\tfrac{1}{4}(1/y+1)x+\tfrac{1}{4}(1-1/y^2)+\tfrac{1}{12}(1/y-1)^3(1/x)]\ln(xy)$$
$$+(1/12x)[x^2-2x(1+1/y)+(1-1/y)^2]^2$$
$$\times W(x-1-1/y,\ 1/y).$$

In the above expressions,

$$a_3=Z_{33}-\delta,\quad a_8=Z_{88}-\delta,\quad a_4=Z_{44}-\gamma,$$
$$b_3=Z_{33}+\delta,\quad b_8=Z_{88}+\delta,\quad b_4=Z_{44}-2\gamma,$$
$$b_0=[2b_4-\tfrac{1}{2}(b_3+b_8)]/b_3b_8, \quad (50)$$
$$\delta=-m^2\beta_2/f,\quad \gamma=-2\sqrt{2}m\beta_1/f,$$
$$y_0=(m_K/m)^2,\quad y=(m_K/m_{K^*})^2.$$

Using the values of Z_{aa} and θ in Sec. II D, and taking $m_K=496$ MeV, we have

$$(\delta m_K^2)_1=(3\alpha/4\pi)m^2(1.673), \quad (51)$$

$$(\delta m_K^2)_2=(3\alpha/4\pi)m^2(-0.357+0.397\delta-0.021\delta^2+1.771\gamma), \quad (52)$$

$$(\delta m_K^2)_{12}=(3\alpha/4\pi)m^2(0.136+0.127\delta), \quad (53)$$

$$(\delta m_K^2)_3=(3\alpha/4\pi)m^2(0.717+0.026\delta-0.001\delta^2-1.605\gamma+0.897\gamma^2-0.028\gamma\delta), \quad (54)$$

$$(\delta m_K^2)_4=(3\alpha/4\pi)m^4(64\alpha_1^2/3f^2)(0.092), \quad (55)$$

where only the convergent parts were collected. The total cutoff-dependent term is given by

$$(\delta m_K^2)_{\rm cutoff}=(3\alpha/4\pi)m^2\ln(\Lambda/m_{K^*})^2(-0.031-0.481\delta+0.040\delta^2-0.962\gamma-0.528\gamma^2). \quad (56)$$

Λ is a cutoff momentum for the virtual photon in self-energy graphs.

B. Determination of α_1, γ, δ, and δm_K

The above expressions for $(\delta m_K^2)_i$ still contain the unknown parameters α_1, γ, and δ. Using Eqs. (23′) and (40)–(43), we find that these parameters are determined by the decay rates of the vector and the axial-vector mesons. In our model, we get the following expressions:

$$\Gamma(\rho\to\pi\pi)=\frac{1}{12m^2}\left(\frac{3-(1+\epsilon)\delta}{4}\right)^2\left(\frac{f^2}{4\pi}\right)(m_\rho^2-4m_\pi^2)^{3/2}, \quad (57)$$

$$\Gamma(K^{*+}\to K\pi)=\frac{1}{16m^2}\left(\frac{3-(1-\tfrac{1}{2}\epsilon)\delta}{4}\right)^2\left(\frac{f^2}{4\pi}\right)\times\left(m_{K^*}^2+\frac{(m_K^2-m_\pi^2)^2}{m_{K^*}^2}-2(m_K^2+m_\pi^2)\right)^{3/2}, \quad (58)$$

$$\Gamma(A_1\to\rho\pi)=\frac{1}{24}\frac{f^2}{4\pi}\left(\frac{m_\rho^2}{m^2}\right)^3 m_\rho(\tfrac{1}{2}a^2-1)^{1/2}$$
$$\times\{(a^2+4)[Z_{33}-\delta+a(\delta-\gamma)]^2+2a(2-a^2)(\delta-\gamma)[Z_{33}-\delta+a(\delta-\gamma)]+(a^2-2)^2(\delta-\gamma)^2\}, \quad (59)$$

where

$$a=\tfrac{1}{2}[3-(m_\pi/m_\rho)^2].$$

We determine the value of $f^2/4\pi$ by relating it to the decay rate of the ρ meson into the lepton pair

$$\Gamma(\rho\to l^+l^-)=\tfrac{1}{3}\alpha^2\left(\frac{f^2}{4\pi}\right)^{-1}\frac{m^2}{m_\rho}\left[1+O\left(\frac{m_l^4}{m_\rho^4}\right)\right]. \quad (60)$$

From the experimental value[20] $\Gamma(\rho\to e^+e^-)/\Gamma(\rho\to \text{all})=(6.0\pm0.6)\times10^{-5}$, we get $f^2/4\pi=2.3\pm0.7$. For definiteness, we take $f^2/4\pi=2.3$ in the following. The value of α_1 is obtained from the rates of VVP-type decays:

$$\Gamma(\omega\to\pi^0\gamma)=\frac{(8\alpha_1 m)^2}{4\pi}\left(\frac{f^2}{4\pi}\right)^{-1}(0.138)\ \text{MeV}, \quad (61)$$

$$\Gamma(\pi^0\to 2\gamma)=\frac{(8\alpha_1 m)^2}{4\pi}\left(\frac{f^2}{4\pi}\right)^{-2}(2.46)\ \text{eV}. \quad (62)$$

TABLE I. Decay widths in MeV for $A_1 \to \rho\pi$, $\rho \to \pi\pi$, and $K^{*+} \to K\pi$ as functions of the parameters γ and δ. ($f^2/4\pi = 2.3$).

	$-\gamma$ \ $-\delta$	1.2	1.4	1.6	1.8	2.0	2.2	2.4
$\Gamma(A_1 \to \rho\pi)$	0.0	40	31	24	17	12	8	6
	0.2	77	65	53	43	34	27	20
	0.4	127	111	95	81	69	57	47
	0.6	189	169	150	132	116	101	87
	0.8	264	239	217	195	175	156	138
$\Gamma(\rho \to \pi\pi)$		94	102	110	119	127	137	146
$\Gamma(K^{*+} \to K\pi)$		45	50	55	61	66	71	77

The experimental value $\Gamma(\omega \to \pi^0\gamma) = 1.19 \pm 0.35$ MeV, combined with $f^2/4\pi = 2.3$, results in $\Gamma(\pi^0 \to 2\gamma) = 9.2 \pm 2.7$ eV, which is consistent with the observed width 7.2 ± 1.2 eV. Further, our γ and δ are related to the ratio of the longitudinal and the transverse coupling constant g_L and g_T, respectively, in the A_1 decay amplitude. In the notation of Gilman and Harari,[23]

$$\left|\frac{g_T}{g_L}\right|^2 = \left|\frac{Z_{33} - \delta + a(\delta - \gamma)}{\frac{1}{2}a(Z_{33} - \delta) + \delta - \gamma}\right|^2. \qquad (63)$$

With our choice of $f^2/4\pi$, the decay widths for $A_1 \to \rho\pi$, $\rho \to \pi\pi$, and $K^{*+} \to K\pi$ are given in Table I. The values of $|g_T/g_L|^2$ are given in Table II ($f^2/4\pi = 2.3$). If we assume that the A_1 decays predominantly via the $A_1 \to \rho\pi$ mode, we have[20] $\Gamma(A_1 \to \rho\pi) = 95 \pm 35$ MeV. This value gives an experimental upper limit for $\Gamma(A_1 \to \rho\pi)$. We also have $\Gamma(\rho \to \pi\pi) = 125 \pm 20$ MeV, $\Gamma(K^{*+} \to K\pi) = 50.1 \pm 0.8$ MeV,[20] and $|g_T/g_L|^2 = 0.64 \pm 0.25$.[24] From Tables I and II, we see that it is difficult to get over-all fits by choosing values of δ and γ if we take the existing experimental values too seriously. The experimental situation is still uncertain, in particular, for the A_1-decay data. Therefore the ranges $\delta = (-1.4)$–(-2.0), $\gamma = 0.0$–(-0.4) would be acceptable. The sum of $(\delta m_K)_i$, $(i \neq 4)$, can be written from Eqs. (51)–(54) as follows:

$$\sum_{i=1,2,12,3} (\delta m_K)_i = A + B \ln(\Lambda^2/m_{K^*}{}^2). \qquad (64)$$

The cutoff-independent term A is shown in Table III. The above ranges of δ and γ imply $A = 1.25$–1.84 MeV

TABLE II. Values of $|g_T/g_L|^2$ as functions of the parameters γ and δ.

$-\gamma$ \ $-\delta$	1.2	1.4	1.6	1.8	2.0	2.2	2.4
0.0	1.17	1.01	0.85	0.66	0.45	0.25	0.07
0.2	1.39	1.28	1.17	1.03	0.89	0.79	0.57
0.4	1.54	1.45	1.37	1.27	1.16	1.05	0.92
0.6	1.64	1.57	1.50	1.42	1.34	1.25	1.16
0.8	1.71	1.66	1.60	1.54	1.47	1.40	1.32

[23] F. Gilman and H. Harari, Phys. Rev. Letters **18**, 1150 (1967).

[24] J. Ballam *et al.*, Phys. Rev. Letters **21**, 934 (1968). The corrected experimental value is cited in P. Horwitz and P. Roy, Phys. Rev. **180**, 1430 (1969), or in S. G. Brown and G. B. West, *ibid.* **180**, 1613 (1969). A more recent value obtained by Ballam *et al.*, $|g_T/g_L| = (0.48 \pm 0.12) \times (m_A/m_\rho)$, is a little smaller than the value cited in the text. See J. Ballam *et al.*, Bull. Am. Phys. Soc. **14**, 573 (1969).

TABLE III. The cutoff-independent term A of δm_K in MeV defined by $\sum(\delta m_K)_i = A + B \ln(\Lambda^2/m_{K^*}{}^2)$, with $i = 1, 2, 12$, and 3.

$-\gamma$ \ $-\delta$	1.2	1.4	1.6	1.8	2.0	2.2	2.4
0.0	1.91	1.76	1.60	1.44	1.27	1.11	0.94
0.2	1.91	1.75	1.59	1.43	1.26	1.09	0.92
0.4	2.00	1.84	1.68	1.51	1.34	1.17	1.00
0.6	2.18	2.02	1.85	1.69	1.52	1.35	1.17
0.8	2.45	2.29	2.12	1.96	1.78	1.61	1.44

and $B = 0.93$–1.80 MeV. The value of B is not small compared with that of A. Therefore the whole expression for δm_K is critically dependent on the cuotff momentum Λ. For the moment, we neglect the cutoff-dependent term. This is equivalent to taking $\Lambda = m_{K^*} \approx 0.9$ BeV and the possible justifications will be given in Sec. IV. The contribution of the vector-meson intermediate state $(\delta m_K)_4$ is determined by $(\alpha_1/f^2)^2$. Using Eqs. (55) and (61), with the experimental value[20] for $\Gamma(\omega \to \pi^0\gamma)$, we obtain

$$(\delta m_K)_4 = 0.34_{-0.09}^{+0.10} \text{ MeV}. \qquad (65)$$

Thus our model for the kaon electromagnetic mass splitting predicts, with $\Lambda = m_{K^*}$ for the cutoff-dependent term, $\delta m_K = 1.50$–2.28 MeV for $\delta = (-1.4)$–(-2.0) and $\gamma = (0.0)$–(-0.4). Taking $\delta m_K{}^{\text{expt}} = -3.94$ MeV,[19] we get $\delta m_K{}^{\text{th}} - \delta m_K{}^{\text{expt}} = 5.44$–$6.22$ MeV. Note that we cannot make δm_K negative with any choice of Λ, if $\Lambda \geq m_{K^*}$, because the coefficient of the cutoff-dependent term is positive in the allowed ranges of δ and γ.

IV. DISCUSSION

The most troublesome problem in the present approach to the mass splitting is the appearance of the cutoff-dependent term. Previous authors[25] have treated this term by choosing a suitable cutoff so as to reproduce the experimental δm_K, or by imposing the condition that makes the coefficient of the cutoff-dependent term vanish. These procedures are not applicable to our case because the coefficient of the cutoff-dependent term is positive. Even if the coefficient were negative (e.g., $\delta = \gamma = 0$), it only means a different choice of the method of extrapolating the field-algebra result ($q = 0$) to the high-energy virtual process ($q^2 \to \infty$), and so we think it unattractive. A physically acceptable result should not be sensitive under minor changes of the extrapolation method, which depends on the parameters δ and γ. A different extrapolation method gives a different coefficient of the cutoff-dependent term. Thus the success of the calculation of the on-shell pion electromagnetic mass splitting due to Gerstein *et al.*[1] should be ascribed not to the smallness of the coefficient of the cutoff-dependent term, but to a relatively low-cutoff momentum, $\Lambda \approx$ (vector-meson mass). This is a possible justification for taking $\Lambda = m_{K^*}$ in our calculation of δm_K. Further, as was noted by Scott,[26] the empirical "double-

[25] See, for example, K. Tanaka, Nuovo Cimento **60A**, 589 (1969).

[26] D. M. Scott, Phys. Rev. **187**, 2153 (1969).

pole" form of the nucleon form factor would suggest an effective cutoff in the range of the vector-meson masses. This would be another reason for taking $\Lambda=m_{K^*}$. For the range $B=0.93$–1.80 MeV taken in Sec. III, $B\ln(\Lambda^2/m_{K^*}{}^2)$ takes values of (-0.29)–(-0.56) MeV for $\Lambda=m_\rho$ and 0.22–0.43 MeV for $\Lambda=m_\phi$. It is difficult to estimate realistic error bounds. The uncertainty lies partly in the determination of δ and γ, and partly in the dynamical assumptions including the value of Λ. From the above discussion we believe, perhaps too optimistically, that the result for δm_K given in Sec. III is correct within 1 MeV. Finally we mention the contribution to δm_K from the high-energy virtual processes. In place of the tadpole dominance, the pole dominance in the angular momentum plane has been used as a possible explanation of the octet enhancement.[2,3] The estimation due to Buccella *et al.* of the high-energy contribution from this viewpoint gives (-5.7)–(-7.1) MeV as the sum of the subtraction and the asymptotic contribution.[4] When combined with the low-energy contribution obtained in Sec. III, the above value well reproduces the experimental δm_K. Thus we conclude that the low-energy approach as employed in the present work offers a reliable method of calculation only for the low-energy contribution, and that the octet-enhancement mechanism cannot be made to appear in any reasonable way within the low-energy approach.

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Families of Indefinitely Rising Trajectories and Gell-Mann's Program

R. CASALBUONI
Istituto di Fisica dell'Università, Padova, Italy
AND
R. GATTO
Istituto di Fisica dell'Università, Padova, Italy
and
Istituto Nazionale di Fisica Nucleare, Sezione di Padova, Italy
AND
G. LONGHI
Istituto di Fisica dell'Università, Firenze, Italy
and
Istituto Nazionale di Fisica Nucleare, Sezione di Firenze, Italy
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Infinite-component wave equations giving rise to a linear mass spectrum and to families of parallel linear trajectories are considered. A general discussion is given of invariant equations for wave functions belonging to $SL(2,C)$ representations and the mass spectra that arise are examined. The simplest possibility corresponds to a higher-derivative equation that gives a linearly rising timelike spectrum that is free of continuum spacelike solutions. Discrete spacelike solutions are absent for the simplest choices of the $SL(2,C)$ representation. The currents and the commutators among the current components are calculated by setting up for the higher-derivative equation a Lagrangian formalism and a quantization procedure based on the action principle. An explicitly factorized model is considered, with respect to the internal symmetry group, and possible nonfactorized extensions are examined. A typical feature of the current commutators is the appearance of Schwinger terms which, besides satisfying known general requirements, also appear in commutators between time components of currents. An alternative interpretation of the physical system in terms of a bound-state equation is presented. The interpretation, in terms of a bound system in two space dimensions, leads to a extension to three space dimensions, again formulable as an infinite-component wave equation. The system describes a family of parallel linearly rising trajectories spaced by one unit of angular momentum. No continuum spacelike spectrum is present, and discrete spacelike solutions are absent for physical choices of the representation of the internal spin group.

I. INTRODUCTION

INFINITE-COMPONENT wave equations have been widely discussed in the literature.[1,2] They have appeared of interest also in connection with Gell-Mann's program of saturation of current commutation rela-

[1] The results described in the present paper were summarized in R. Casalbuoni, R. Gatto, and G. Longhi, Nuovo Cimento Letters **2**, 159 (1969); **2**, 166 (1969).

[2] For general references, see Y. Nambu, Phys. Rev. **160**, 1171 (1967); C. Fronsdal, *ibid.* **171**, 1811 (1968); L. O'Raifeartaigh, in *Proceedings of the Fifth Coral Gables Conference on Symmetry Principles at High Energy*, edited by A. Perlmutter *et al.* (Benjamin New York, 1968); R. C. Hwa, Nuovo Cimento **56A**, 107 (1968); **56A**, 127 (1968).